

# Some properties of Punchthrough Pions

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PART 1

## 1 Introduction

The following work is inspired by a request to address the question of constraints on calorimeter thickness from muon system considerations. In previous muon notes [1] [2] we have used a sample of single pions to study the effects of punchthrough in the LHC environment. We feel it is enlightening to look at some of the properties independent of the environment in which the pions are created. Then, with these results in hand, we can understand the previous conclusions and try to extrapolate to studies which have not yet been completed.

The following properties have been studied here:

- the number of particles of various types above a given momentum threshold as a function of interaction length;
- the momenta of these particles as a function of interaction length;
- the multiplicities, mean and rms multiplicities of these particles as a function of interaction length.

In the first section we discuss the sample of events. In the next section we describe the results. In section 4 we discuss the results in terms of the ATLAS physics and finally we conclude.

## 2 Sample of Events

The same sample of events that described in [1] was used here. The particles indicated in table 2 were analyzed in a detector setup consisting of 8.5 interaction lengths of liquid argon calorimeter followed by layers of iron placed 0.5 interaction lengths ( $\lambda = 20.8$  cm) apart. The iron is instrumented with 1mm thick scintillator.

## 3 Results

The numbers of particles seen as a function of interaction length are shown in Figures 1 through 8. For all cases, the photon rates have been scaled down by a factor of 100 (assuming therefore that interactions in the chambers correspond to those after 1% of a radiation length). Also, the neutron rates have been scaled by a factor of 1000, consistent with the kind of efficiencies the detector may have in responding to neutrons. The scale is set such that it reads "detectable particles per pion". The results may be summarized as follows:

- For pions having momenta of 5 GeV/c, the rates of muon decay in flight are dominant for 9 or more interaction lengths. These decays yield the same particle rate as that of the electrons and hadrons at nine interaction lengths. The electrons and hadrons themselves have similar rates. The neutron and photon rates remain quite low.
- For pions having momenta of 10 GeV/c, the electron, then the hadron rates dominate the muon rates up to 10 interaction lengths. Above 10 interaction lengths, the (decay) muon rates dominate once again.
- The behaviour seen for pions having momenta of 10 GeV/c is repeated for the pions having momenta of 20 GeV/c, except the cross over point for muon rates to dominate occurs at 13 interaction lengths.
- For all remaining pion energies, the dominant contribution to rate comes from electrons, followed closely by the hadrons. The muons become less and less important as the pion energy goes up. The photon and neutron rates remain at the same relative separation to one another and to the hadron and electron rates. The muon rates can be seen to have two slopes, one due to the decay of pions in the shower and the second due to the decay in flight of the primary pion.

It is of course important to keep in mind that the neutron and photon rates may be moved according to the assumption on efficiency for detection. It also becomes interesting to ask how sensitive the particle rates are to the momentum cut. The answer to this is considered in two ways. First, figures 9 through 32 show the same graphs as above, but for cuts of 5, 10, 15, and 20 GeV/c on the momentum of the various punchthrough particles. With these cuts, one sees that mainly the muons and hadrons are left. Which of the two dominates depends on the momentum of the initial pion and the number of interaction lengths. For low momentum pions, the muon decays dominate. For higher momentum pions, the hadron punchthrough dominates, but perhaps only up to some value of lambda where the muon rate again dominates. The dominant component is shown schematically in Fig. 133. In this figure, the white region indicates the portion of  $p_\pi - \lambda$  space where hadron punchthrough dominates and the shaded region indicates combinations of pion momentum and calorimeter depth where the muon decay dominates. This is shown for cuts of 0, 5, 10, 15, and 20 GeV/c on the hadron momentum. Kinematically unallowed regions are indicated by the cross hatching.

Given that the hadrons and muons are the interesting high momentum particles, one may ask what fraction survive as a function of interaction length. The slopes of the hadron dependencies are plotted in Fig. 134 as a function of the pion momentum and shown for various momentum cutoffs for the hadrons. Indeed, the slope is not one! There is an dependence on the original pion energy and on the energy of the hadron. However, one sees that one gains an exponential decrease of order 0.5 to 0.8 of an interaction length as one increases the material. The muons on the other hand are quite insensitive to the amount of material and one gains very little. This has important implications when viewed in light of fig. 133. In those regions of  $p_\pi - \lambda$  space dominated by hadron punchthrough, there is a rapid gain of particle filtering as material is added. However, in those regions

dominated by muon decay, additional material does not reduce the particle flux by very much.

The second way in which the momentum dependence has been investigated is shown in figures 33 through 72. These figures contain the electron, hadron, muon, neutron, and photon momentum spectra from the various pion momenta studied and for 10, 11, 12, 13 interaction lengths. In order to get a flavor for the energy dependence function, the spectra were fit to two exponentials where possible:

$$(1) \quad N_{part} = A \exp(-Bp) + C \exp(-Dp)$$

The two values B and D are shown in the figures (as P1 and P2) in units of  $(\text{GeV}/c)^{-1}$ . Since these fits are not meant to be a phenomenological description, the results are only shown on the histograms themselves so one can see the meaning of the fits. In many cases they give a poor description. Clearly two exponentials are too naive. If one looks at large values of lambda for high momentum electrons or lower values of lambda for low momentum electrons, one sees that the scatter in the spectrum is not only statistical and hence fitting is difficult: there must be correlations. A better understanding of these kinds of correlations would be needed before sensibly motivated functional forms for the fits could be used.

For the hadrons the exponential decay slope in the momentum spectrum is of order 1  $(\text{GeV}/c)^{-1}$ . The fall off of electron rate with momentum is about 150 times faster than that of hadrons and it is notable that a 10 MeV/c change in momentum threshold typically means a decrease in electron rate by 70% to 90%. A 1 MeV/c change corresponds to a decrease in electron rate of between 10% and 25%. Neutron rates typically fall off a factor of 50 more rapidly with momentum than do charged hadrons. The photon rates fall off with two components, one that is 200 and the other 1200 times more rapid than hadrons.

It is also interesting to look at the multiplicity per event for the various particle classes as well as plots of the mean (and rms) multiplicity as a function of interaction length. The multiplicities are shown in figures 73 to 112 and the averages and RMS's are shown in figures 113 to 132. The electron multiplicities grow from about 1.2 for pions having momenta of 5 GeV/c to about 10 for pions having momenta of 1000 GeV/c. For hadrons, the average multiplicity shows a very slow increase with pion momentum and is typically one for lower momentum pions and increase to 6 for the highest momentum pions. Muons tend to come singly for all momenta and lambda. Neutron average multiplicities range from 2 to 10 as pion momentum increases, but for the high momentum pions, there is a factor of two dependence on interaction length over 5 interaction lengths. Photons show average multiplicities ranging from 3 at for low momentum pions to 16 for higher momentum pions.

Structure in these average multiplicities is explained by large tails in these multiplicity distributions: the rising averages are accompanied by increase in RMS as well. The electrons and the photons show the most structure. This can be understood as follows. For the electrons, one observes that the lambda dependence of the number of electrons per pion follows that of hadrons as long as there is no cut on the electron momentum (figures 1 through 8). It is likely therefore that the electrons are accompanying the hadrons and there can be large fluctuations. Similarly, the same is true of the photon relationship to neutrons.

## 4 Discussion for ATLAS

In the case of ATLAS, it is interesting then to ask the implications of these pion punchthrough particle properties. First, the extreme sensitivity of the electron rate to the momentum threshold is noteworthy. If we consider the electrons leaving the back of the calorimeter, then, for a magnetic field of 0.1 T, and assuming a 10 cm distance to the nearest chamber, the effective momentum threshold is  $0.3 \times 0.1 \times 0.1 \text{ GeV}/c = 3 \text{ MeV}/c$ . Therefore the electron rates in the first muon chamber would be about 30% of that calculated here. Since these electrons are so soft, it is likely therefore that they would be considered mainly a source of noise or accidental hits in the first layer of muon chambers. Since they seem to be correlated to the hadrons, one can imagine that a punchthrough hadron will produce not only its own hit but, for the first muon layer, there will be additional hits from the accompanying electron.

Next, one has to ask why decays of pions in flight have been said to dominate the punchthrough rates in previous studies. It is because the dominant cross section at LHC is the minimum bias cross section. The pions have a transverse momentum of 0.6 GeV/c and this means that even at  $\eta = 3$  the average pion momentum is around 6 GeV/c. From the single pion calculations it is easy to see that the decay of these pions dominates. Although one only reduces this rate by ranging out these muons, the source spectrum of pions is very steeply falling. Nonetheless, as shown in [2], the decrease of the rates of these muons is rather slow as a function of lambda.

Another point to be considered is the possibility of punchthrough hadrons causing real triggers. These hadrons have a rather hard momentum spectrum, falling to 30% of their original rate at 1 GeV/c. Furthermore they tend to be accompanied by very soft electrons that can create hits in the first muon layer.

## 5 Conclusions

There is a delicate interplay of the identity of the particles having the dominant rate when created by the punchthrough of a pion. The electrons have a very soft spectrum and hence rates will depend strongly on the momentum sensitivity of any apparatus behind the calorimeter. The muons must also be treated with care; while they are usually quite hard and the products of the decay of the pion before it interacts with the calorimeter, there is a softer component in hard pions that comes from decays inside the pion hadronic shower. The hadrons dominate in certain regions of  $p_\pi - \lambda$  space and have a fairly hard momentum spectrum; therefore, the hadrons may become an issue when one is considering the trigger rates.

No general statements can be made about dominant rates for all experiments. It is critical to consider the spectrum of incoming pions and fold that spectrum together with the punchthrough spectra considered here. However since the sources of pions in hadron colliders are basically minimum bias events, and since the momentum of the pions varies slowly with the center of mass energy, general conclusions can be drawn over a large range of hadron collider energies.

## References

- [1] A.Cheplakov et al., ASCOT/EAGLE Internal Note MUON-NO-006, 1992
- [2] A.Cheplakov et al., ASCOT/EAGLE Internal Note MUON-NO-013, 1992

$p_{\pi^+} (GeV/c)$	5	10	20	40	100	300	500	1000
$N_{tot}^{event}$	300K	80K	80K	40K	20K	8K	4.5K	1K

Table 1: Statistics of single pion showers generated at different pion momenta and  $\eta = 0$ .

Particle Group	Members	Minimum Momentum (Mev/c)
muon	$\mu^+, \mu^-$	10.0
hadron	$\pi^+, \pi^-$ $K^+, K^-$ p	10.0
electron	$e^+, e^-$	1.0
neutron	n	0.0
photon	$\gamma$	0.3

Table 2: Particles considered in the study of punchthrough of single pions.

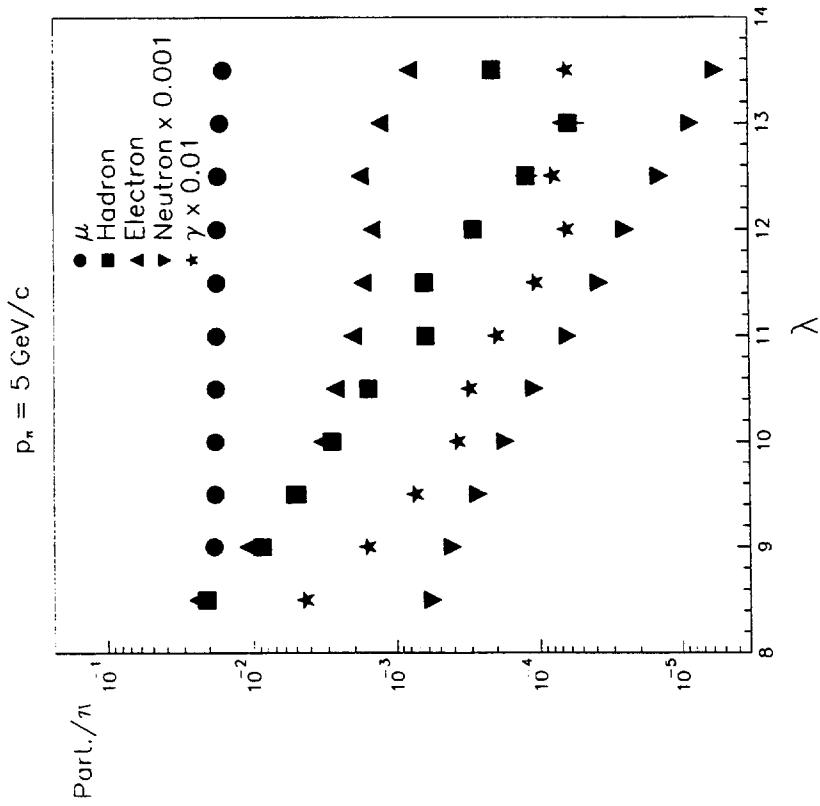


Figure 1: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 5 GeV/c.

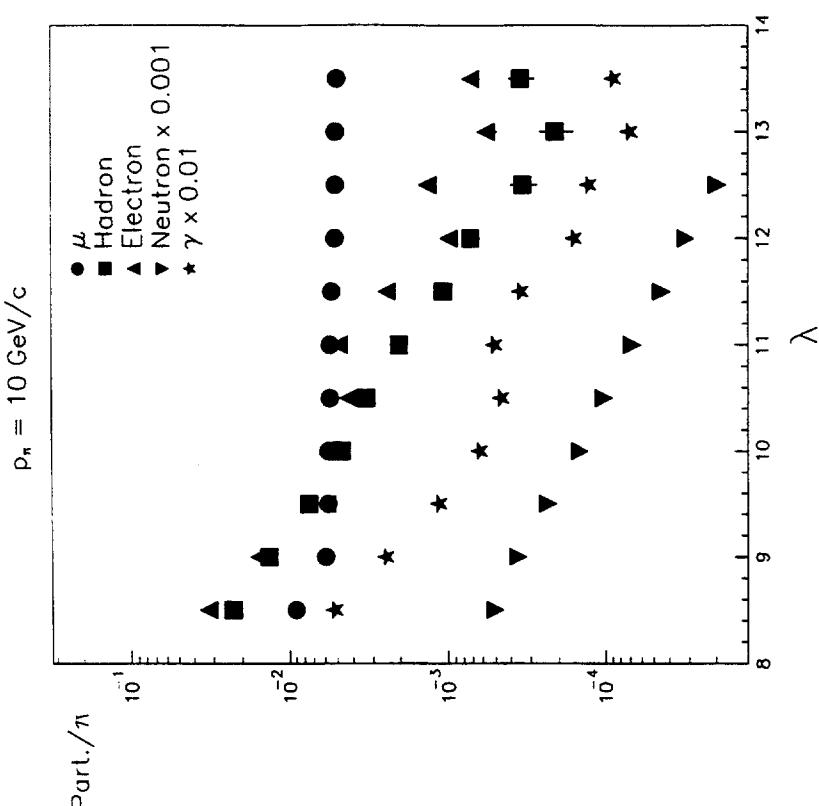


Figure 2: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 10 GeV/c.

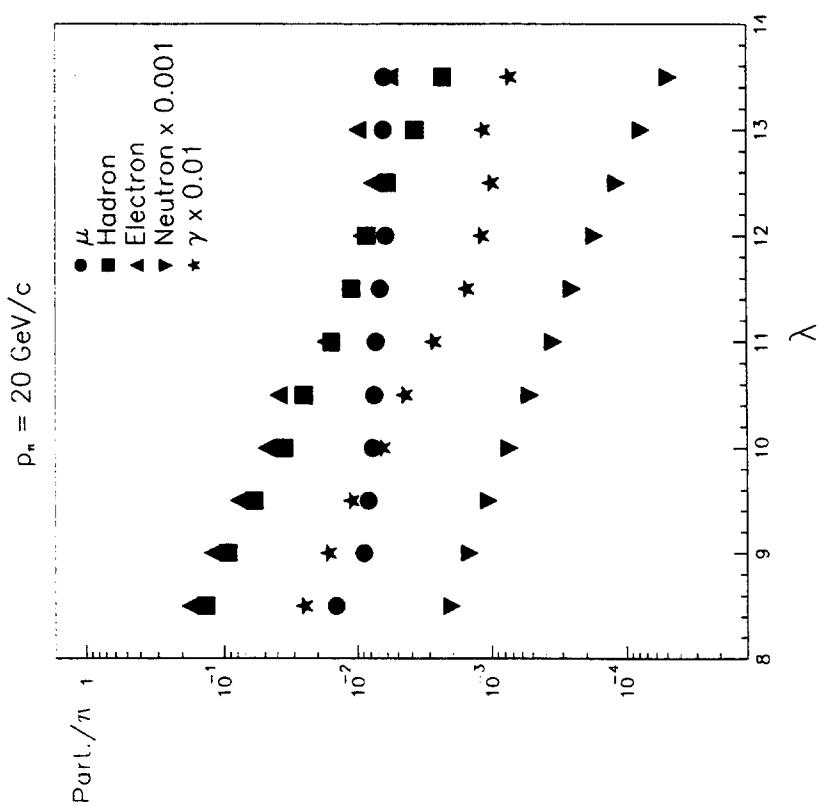


Figure 3: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of  $20 \text{ GeV}/c$ .

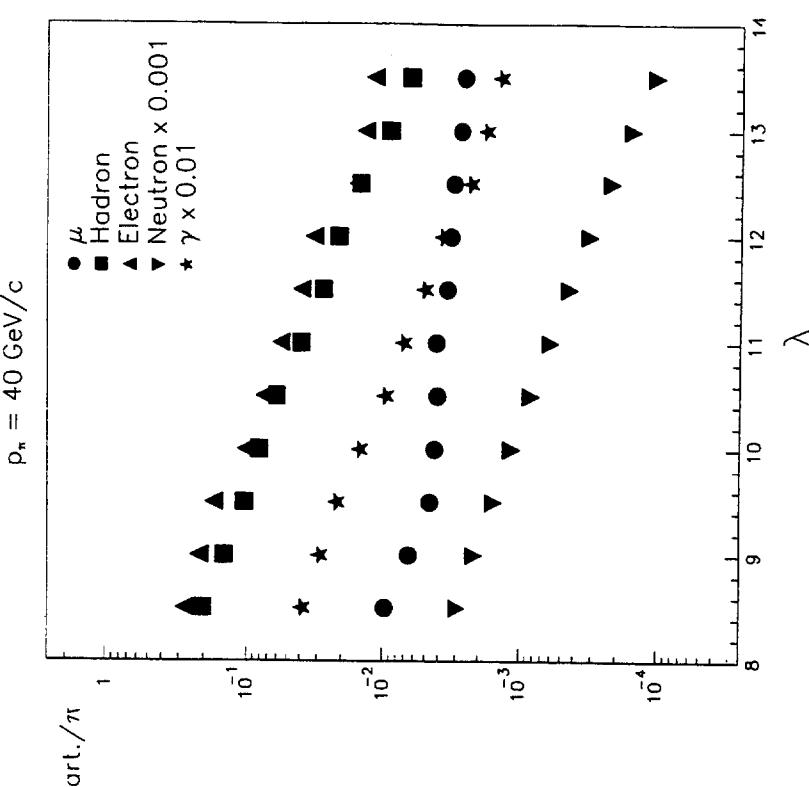


Figure 4: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of  $40 \text{ GeV}/c$ .

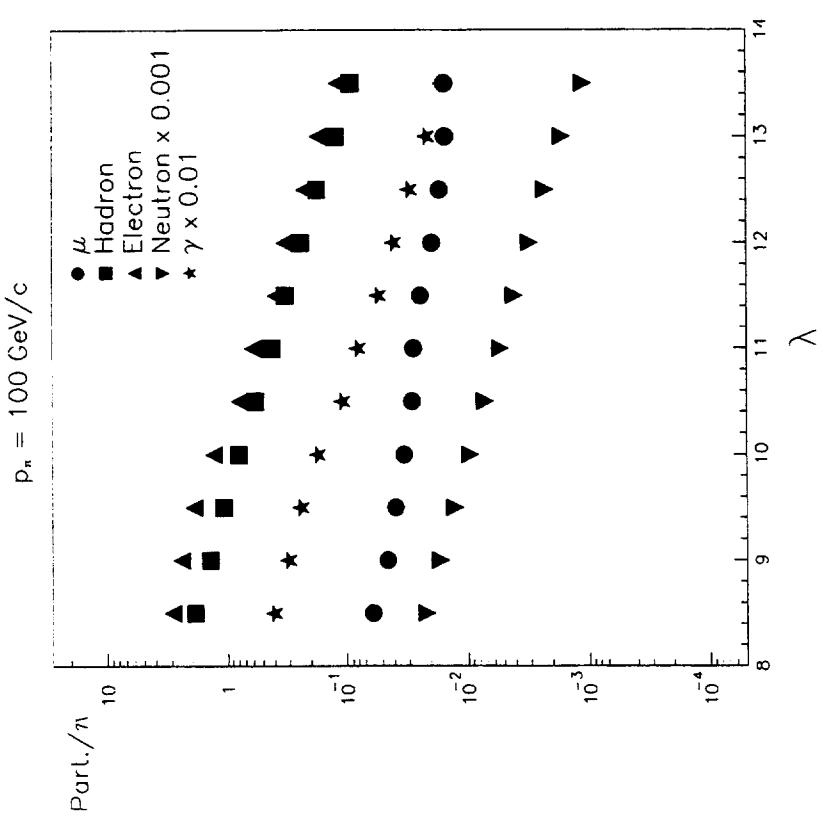


Figure 5: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 100 GeV/c.

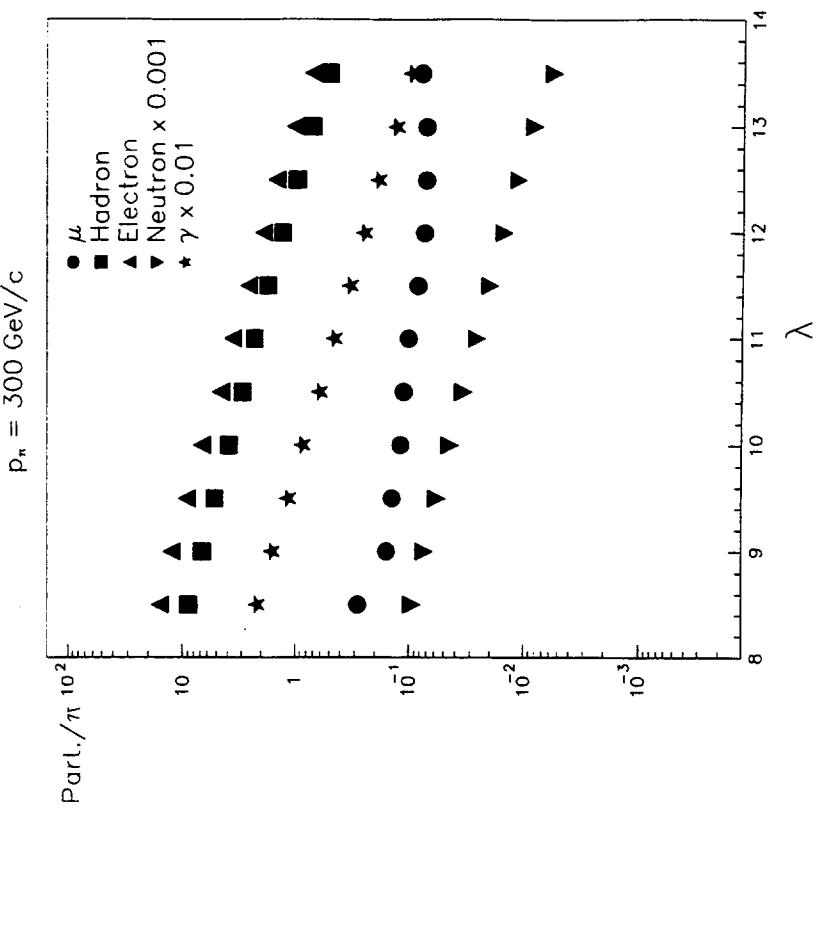


Figure 6: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 300 GeV/c.

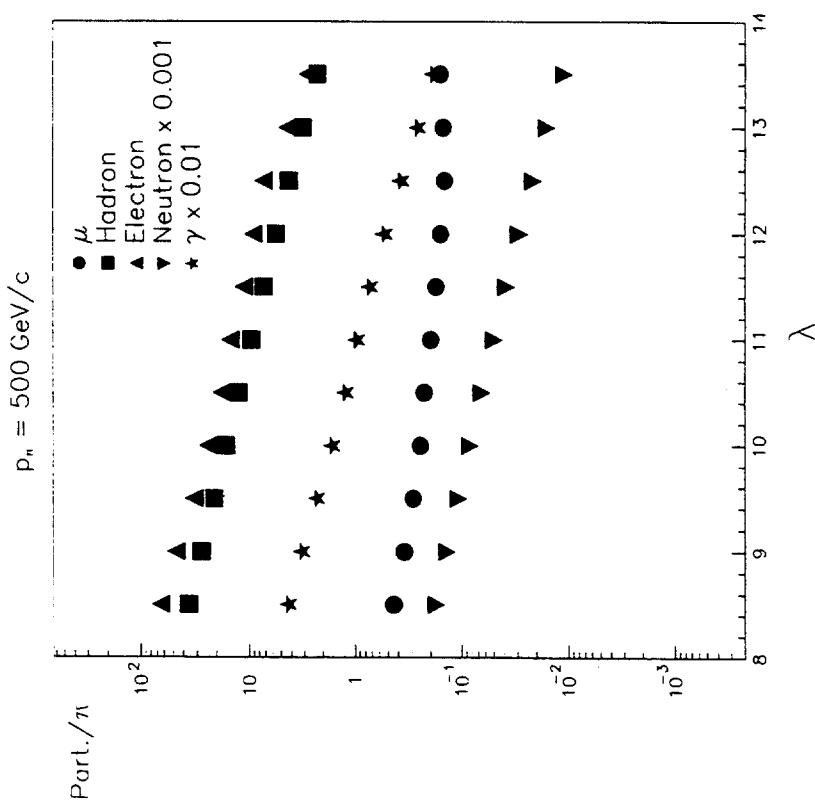


Figure 7: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of  $500 \text{ GeV}/c$ .

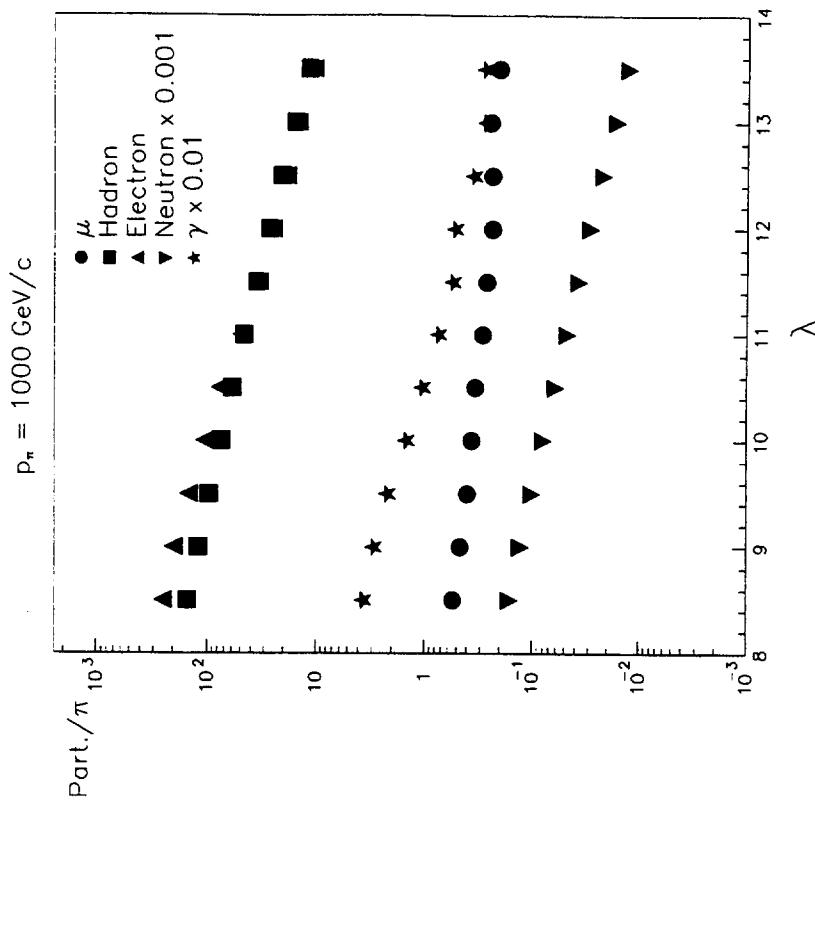


Figure 8: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of  $1000 \text{ GeV}/c$ .

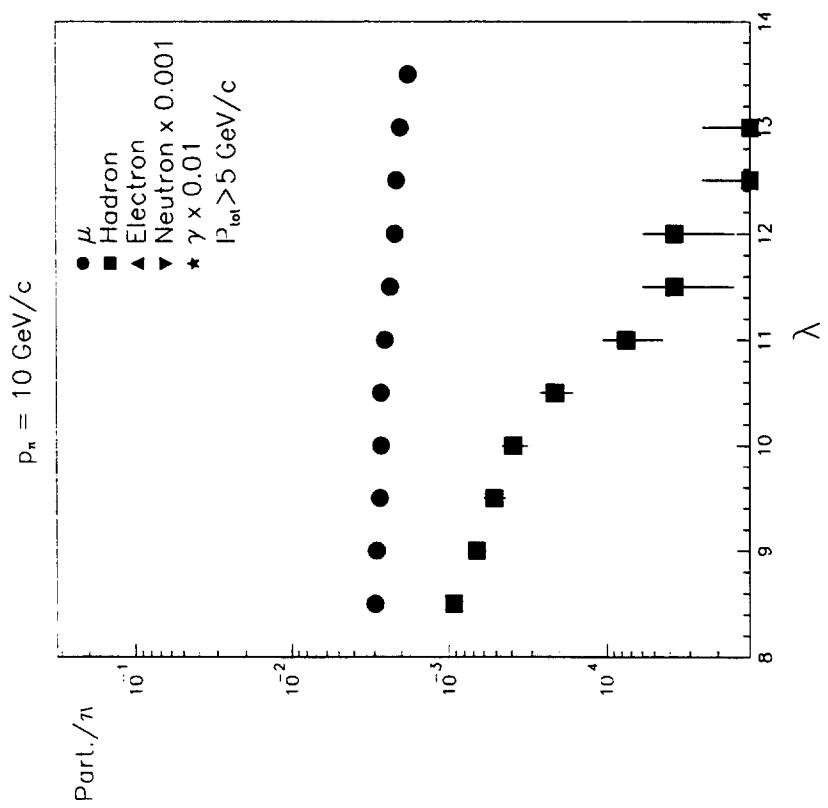


Figure 9: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 10 GeV/c. A momentum cut of 5 GeV/c has been placed on the particles.

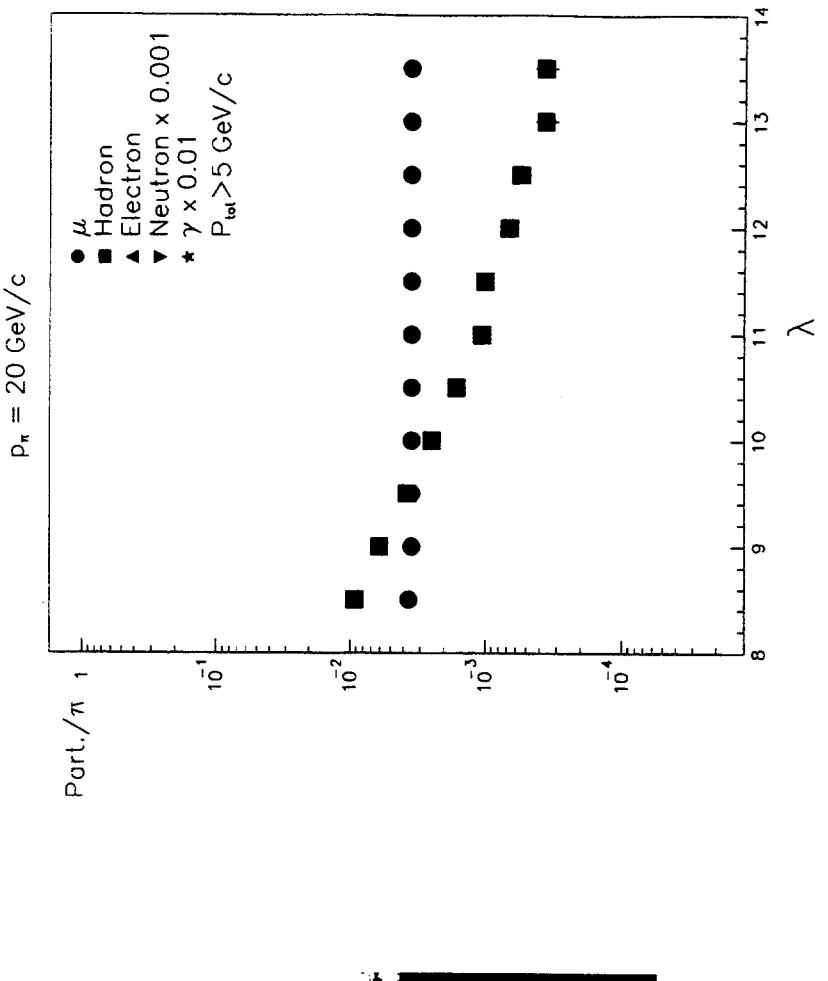


Figure 10: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 20 GeV/c. A momentum cut of 5 GeV/c has been placed on the particles.

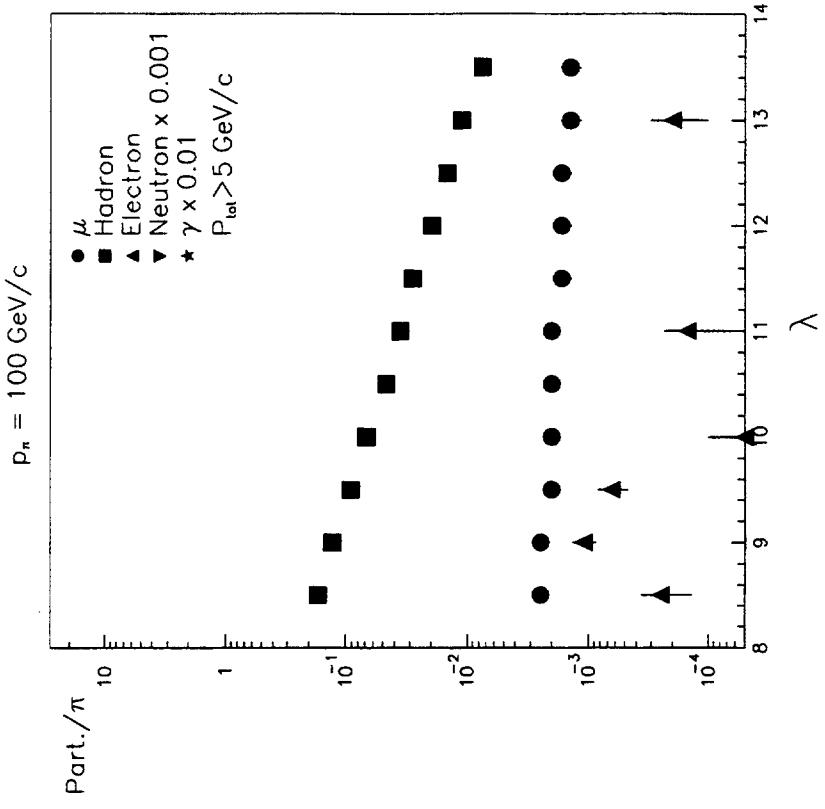
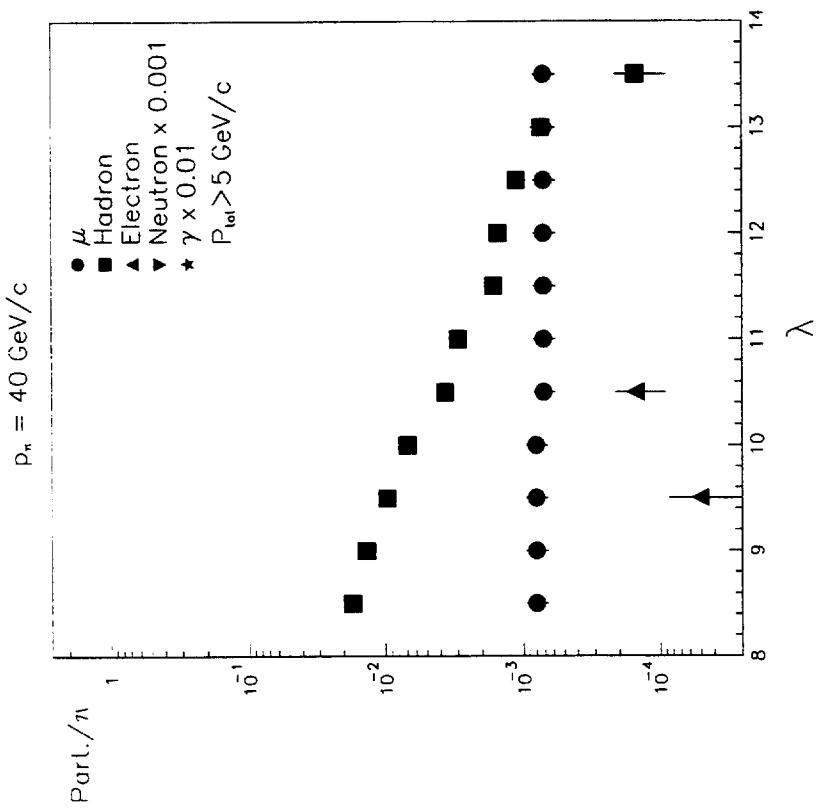


Figure 11: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 40  $\text{GeV}/c$ . A momentum cut of 5  $\text{GeV}/c$  has been placed on the particles.

Figure 12: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 100  $\text{GeV}/c$ . A momentum cut of 5  $\text{GeV}/c$  has been placed on the particles.

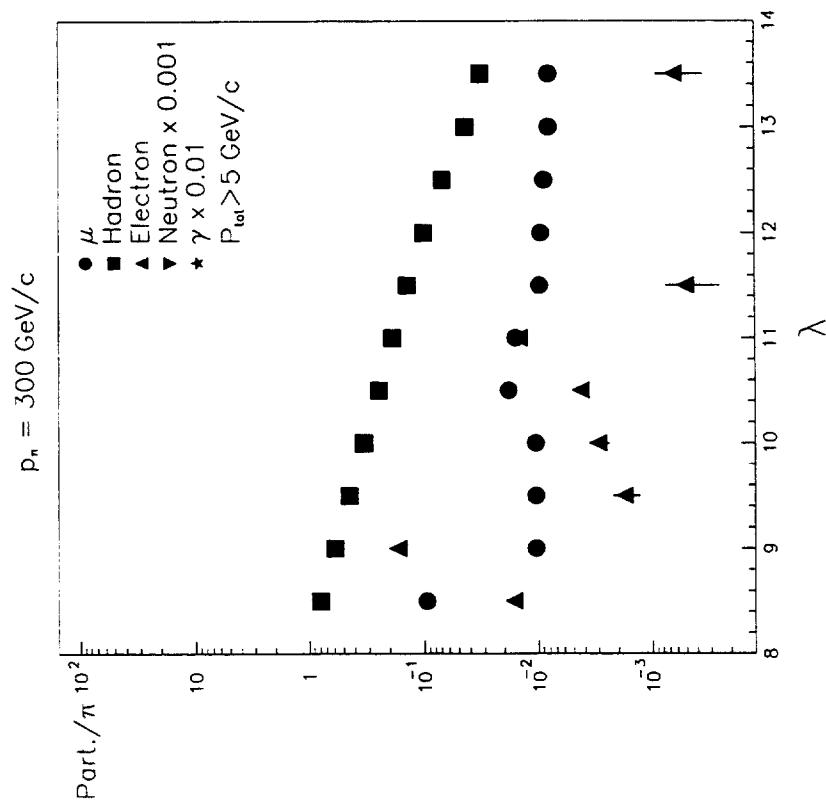


Figure 13: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of  $300 \text{ GeV}/c$ . A momentum cut of  $5 \text{ GeV}/c$  has been placed on the particles.

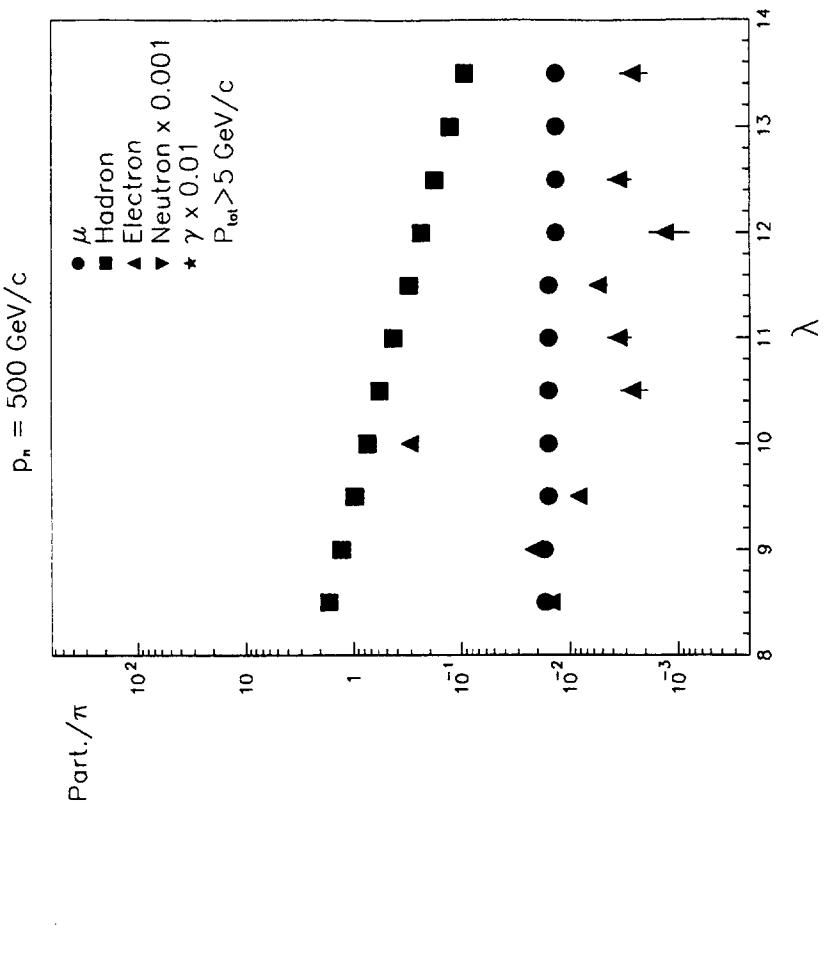


Figure 14: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of  $500 \text{ GeV}/c$ . A momentum cut of  $5 \text{ GeV}/c$  has been placed on the particles.

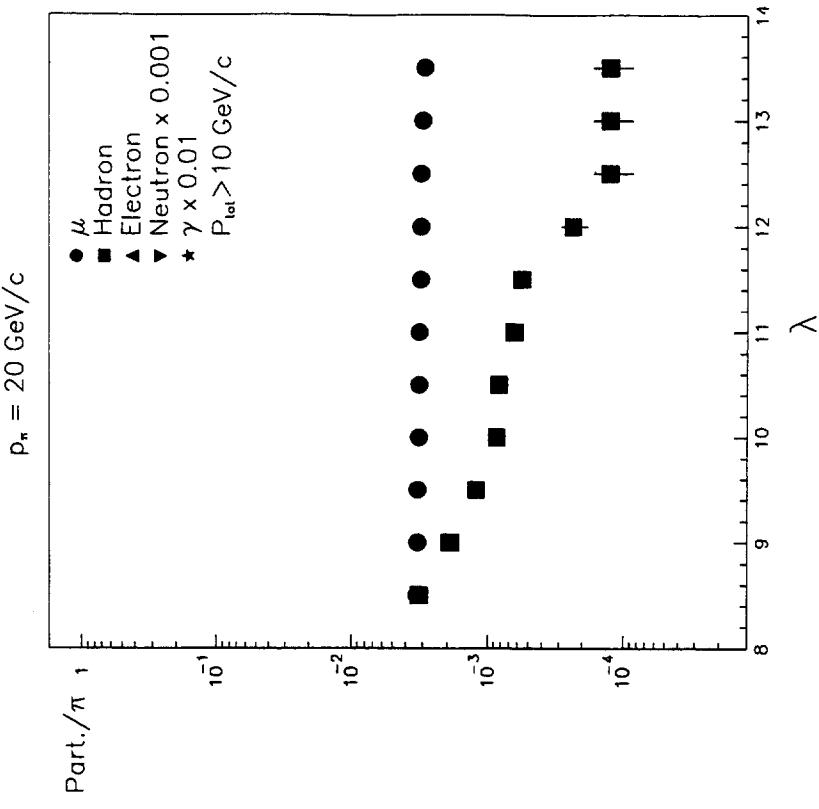
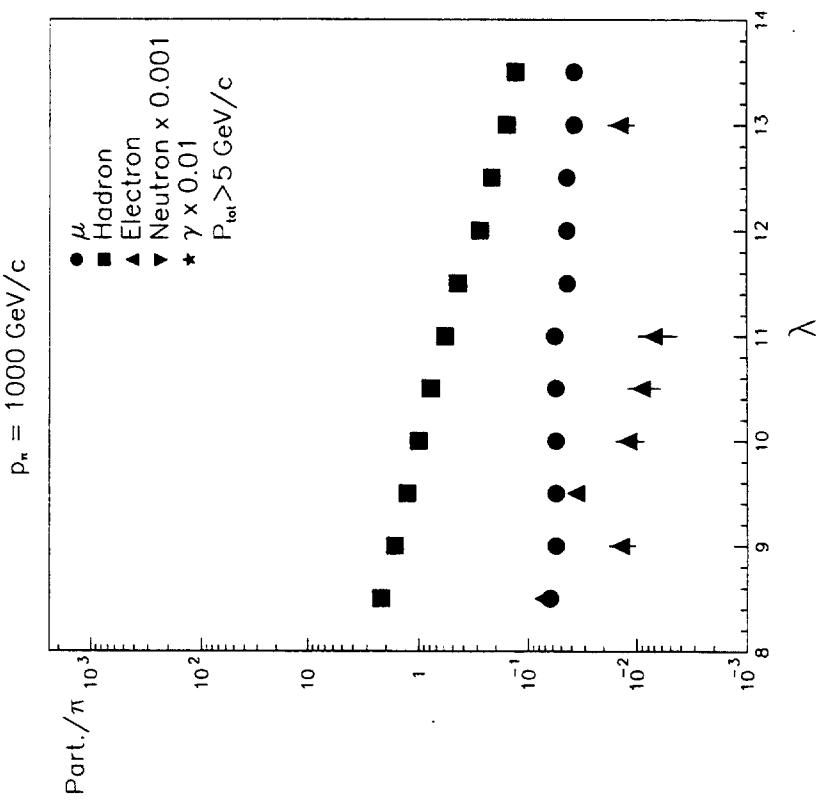


Figure 15: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 1000 GeV/c. A momentum cut of 5 GeV/c has been placed on the particles.

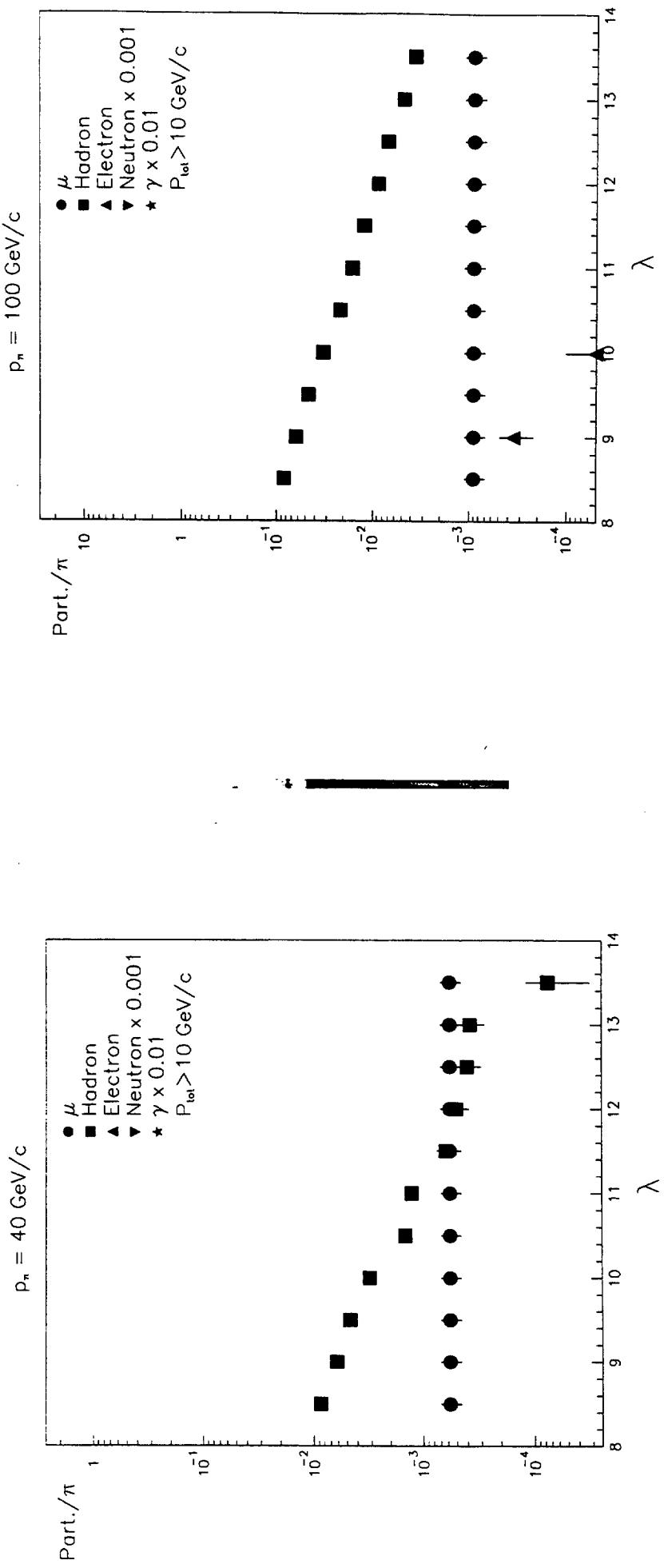


Figure 17: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 40 GeV/c. A momentum cut of 10 GeV/c has been placed on the particles.

Figure 18: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 100 GeV/c. A momentum cut of 10 GeV/c has been placed on the particles.

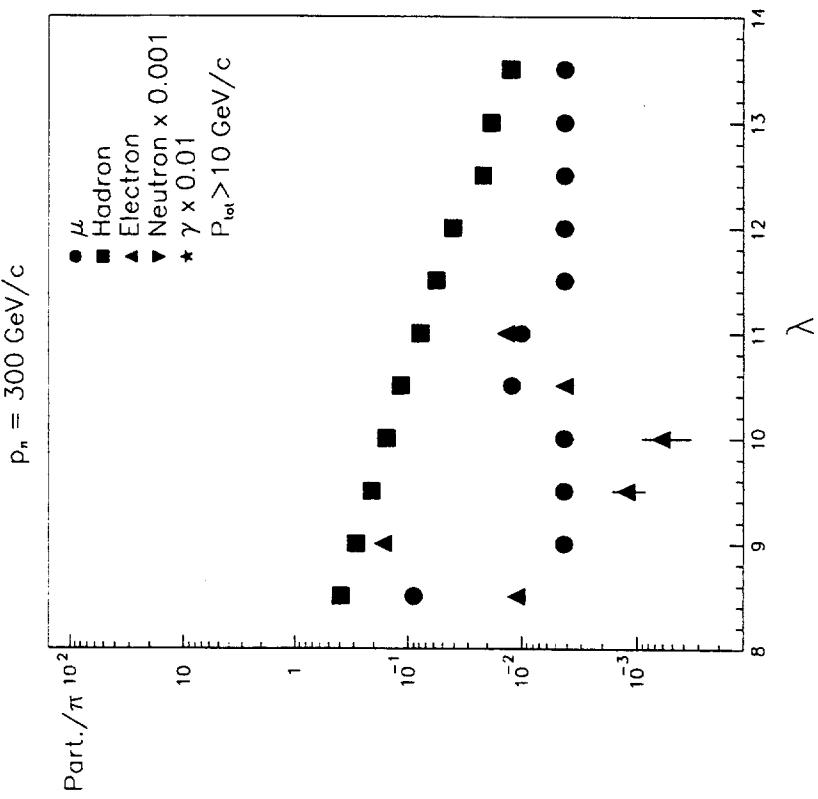


Figure 19: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 300 GeV/c. A momentum cut of 10 GeV/c has been placed on the particles.

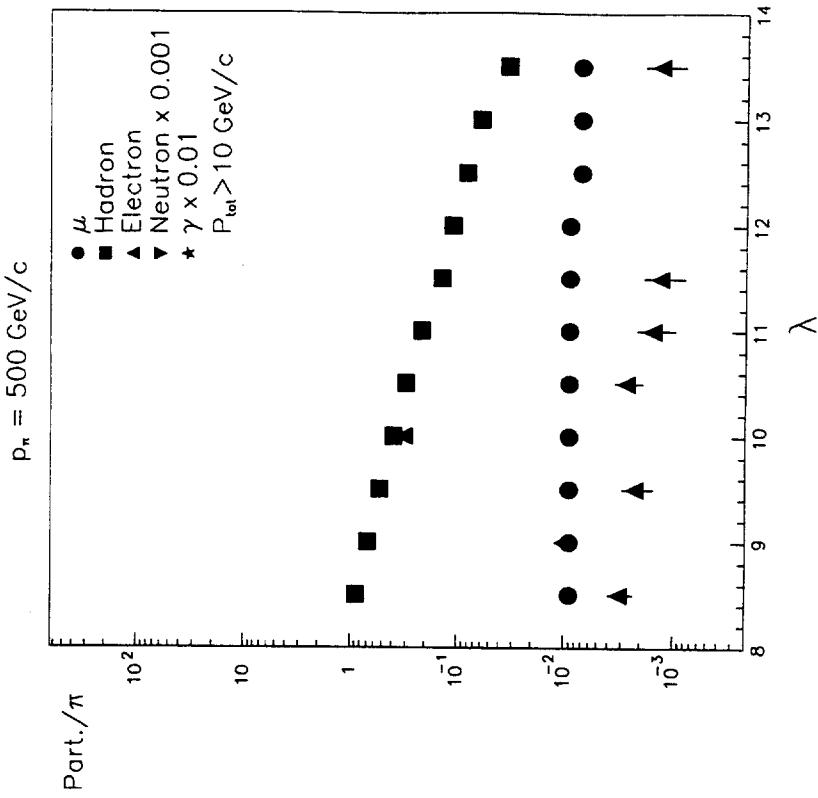


Figure 20: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 500 GeV/c. A momentum cut of 10 GeV/c has been placed on the particles.

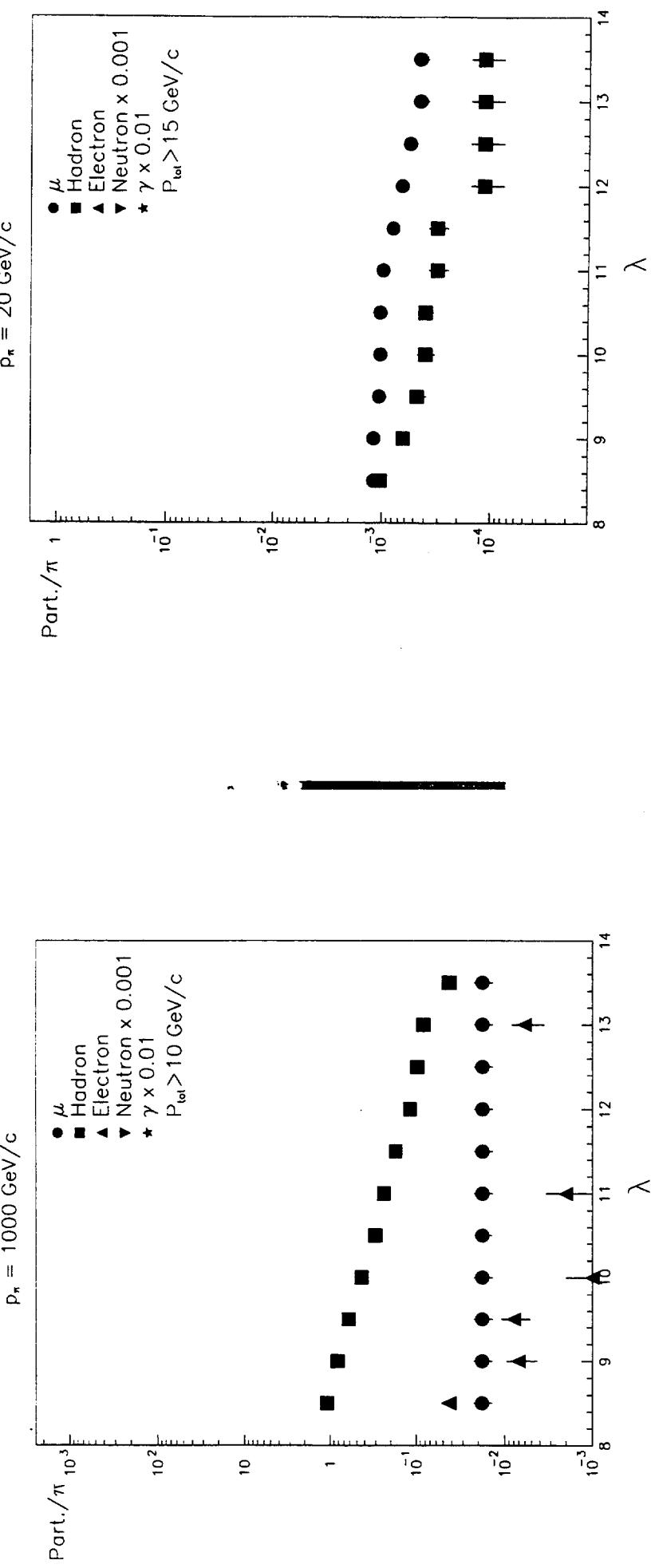


Figure 21: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 1000 GeV/c. A momentum cut of 10 GeV/c has been placed on the particles.

Figure 22: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 20 GeV/c. A momentum cut of 15 GeV/c has been placed on the particles.

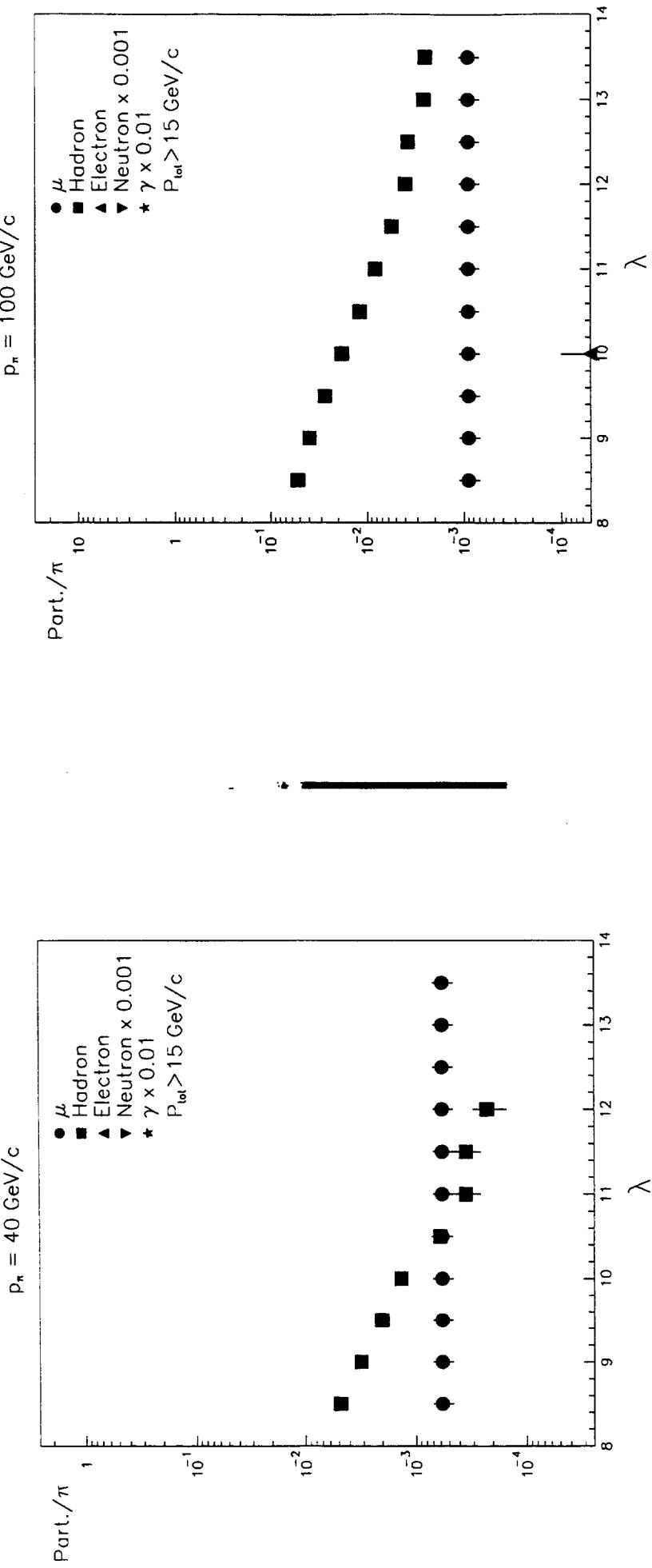


Figure 23: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of  $40 \text{ GeV}/c$ . A momentum cut of  $15 \text{ GeV}/c$  has been placed on the particles.

Figure 24: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of  $100 \text{ GeV}/c$ . A momentum cut of  $15 \text{ GeV}/c$  has been placed on the particles.

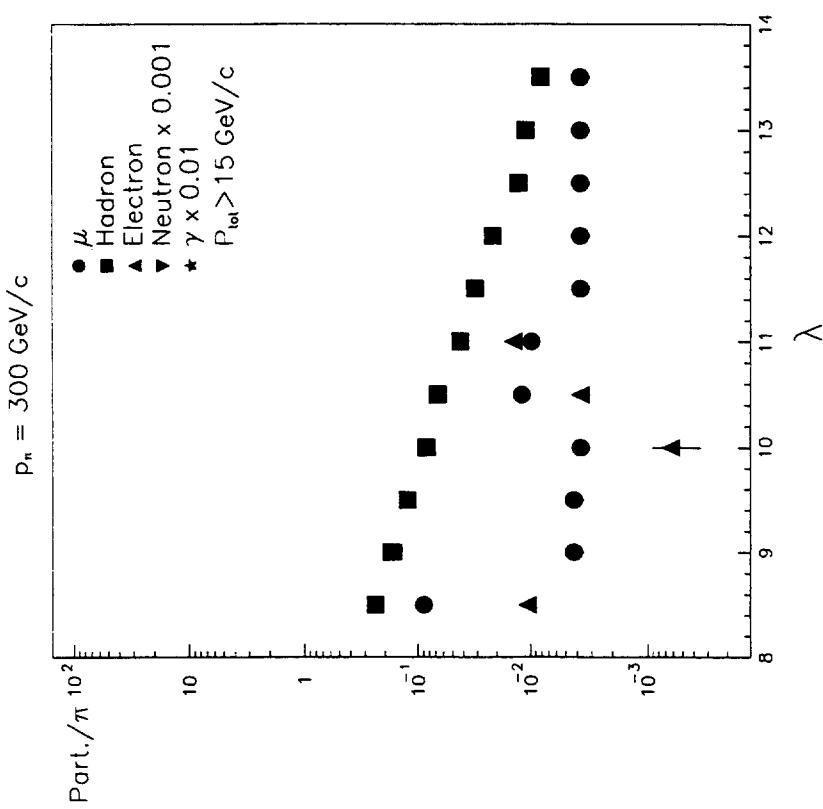


Figure 25: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 300 GeV/c. A momentum cut of 15 GeV/c has been placed on the particles.

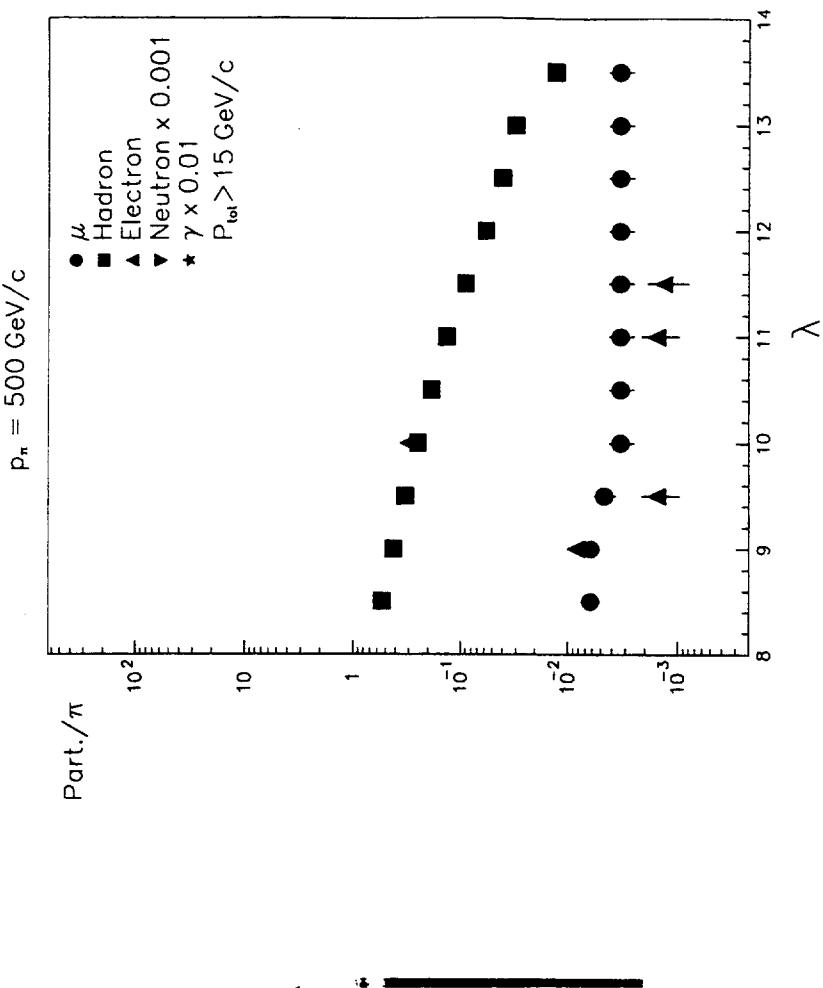


Figure 26: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 500 GeV/c. A momentum cut of 15 GeV/c has been placed on the particles.

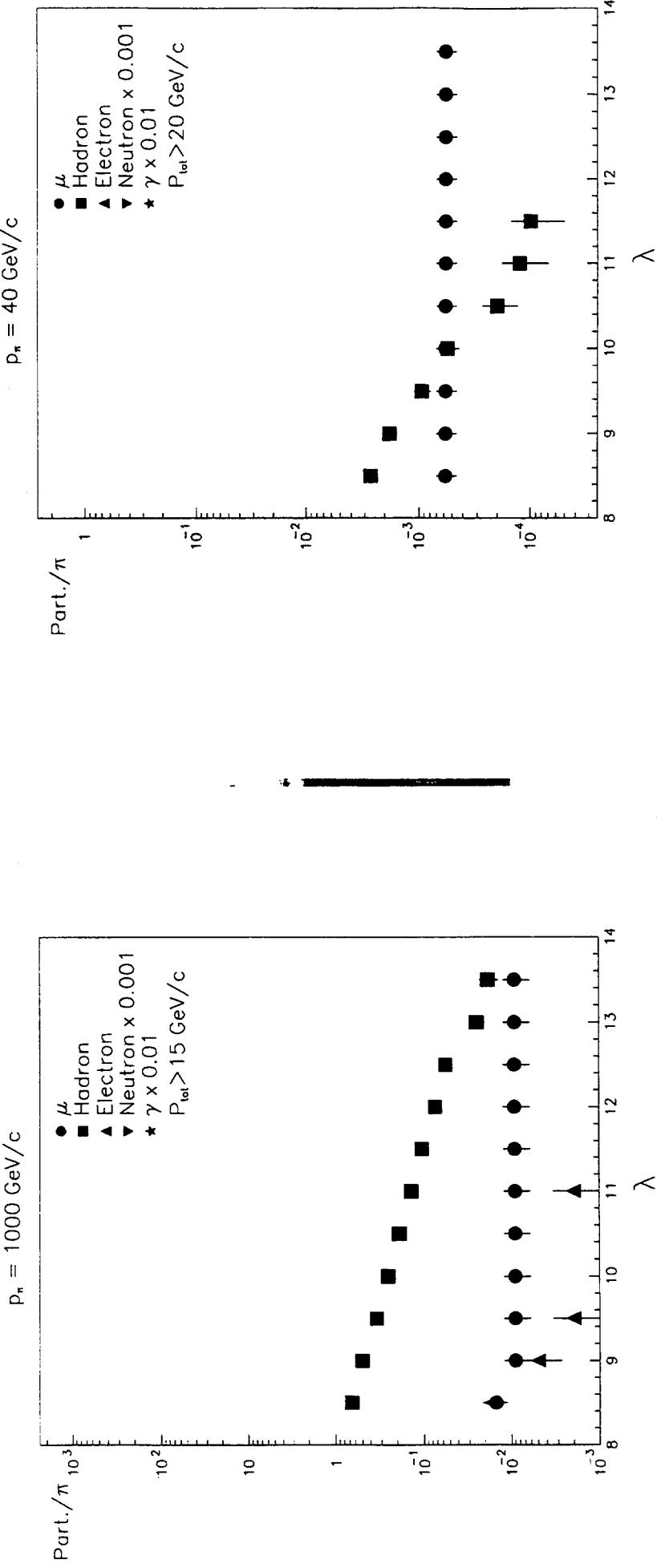


Figure 27: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 1000 GeV/c. A momentum cut of 15 GeV/c has been placed on the particles.

Figure 28: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 40 GeV/c. A momentum cut of 20 GeV/c has been placed on the particles.

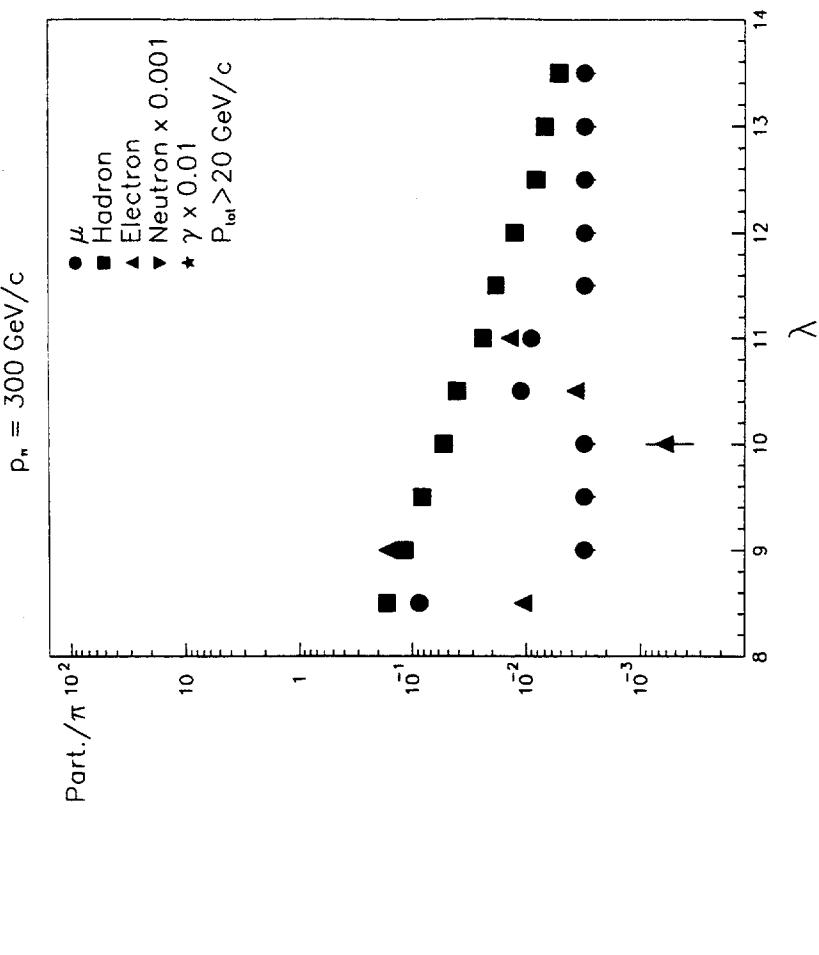
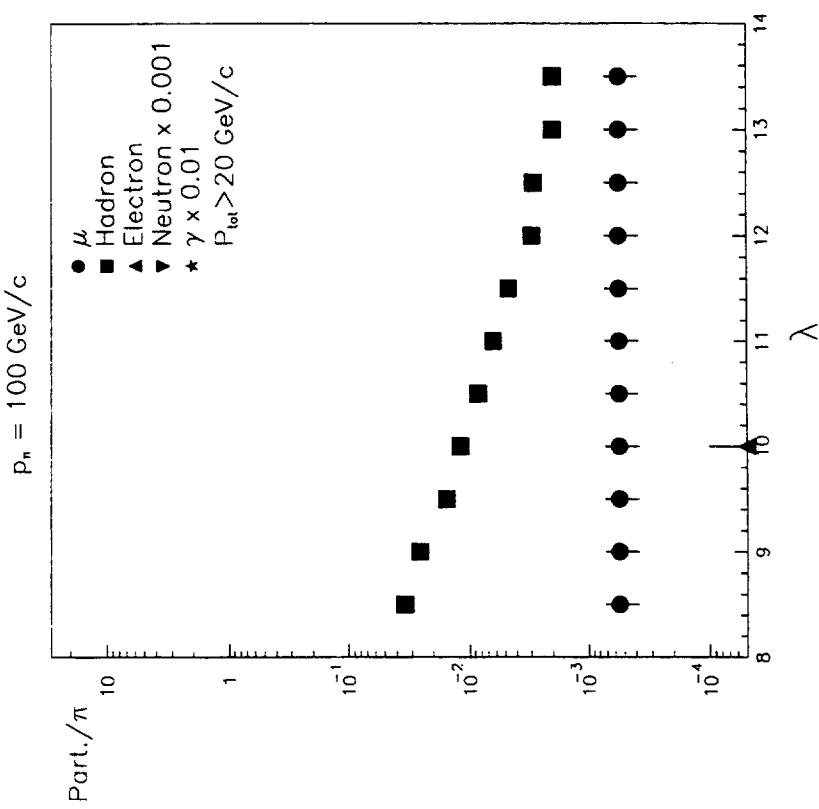


Figure 29: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 100 GeV/c. A momentum cut of 20 GeV/c has been placed on the particles.

Figure 30: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 300 GeV/c. A momentum cut of 20 GeV/c has been placed on the particles.

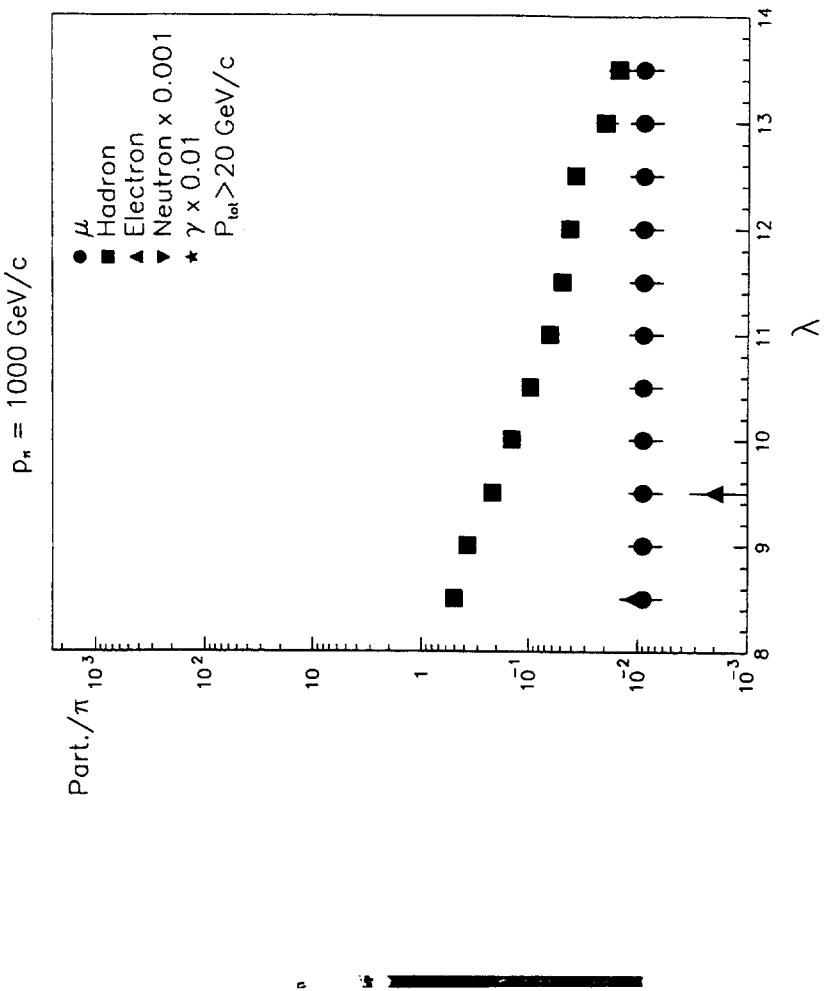
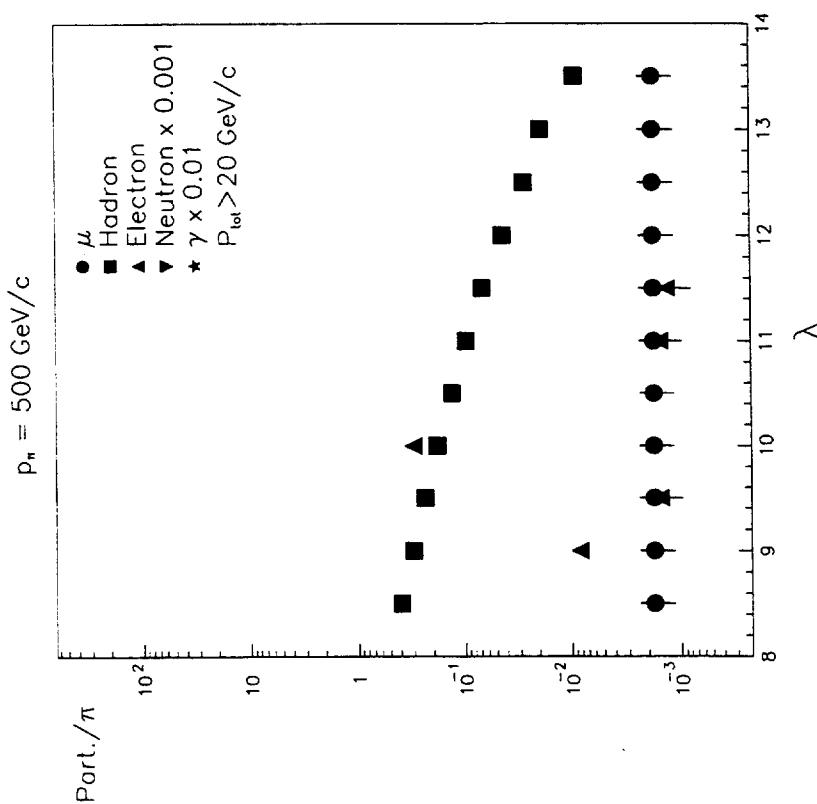


Figure 31: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 500 GeV/c. A momentum cut of 20 GeV/c has been placed on the particles.

Figure 32: Number of particles vs interaction length for various classes of punch through particles coming from pions having momenta of 1000 GeV/c. A momentum cut of 20 GeV/c has been placed on the particles.

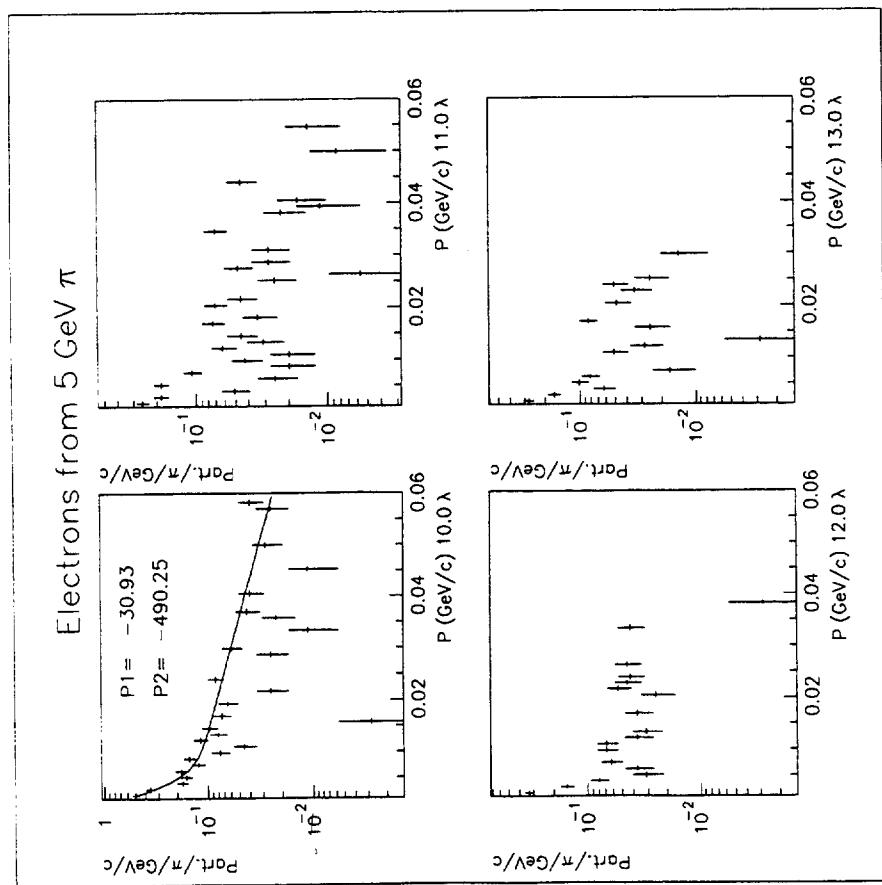


Figure 33: Momentum spectra of electrons coming from pions having momenta of 5 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as P1 and P2.

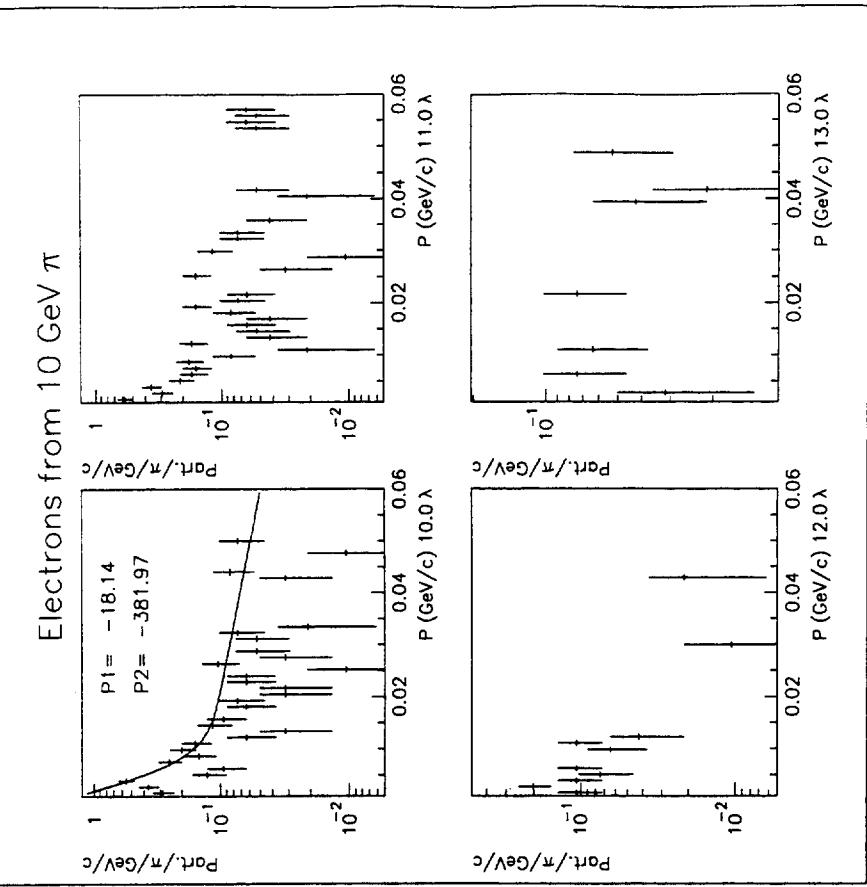


Figure 34: Momentum spectra of electrons coming from pions having momenta of 10 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as P1 and P2.

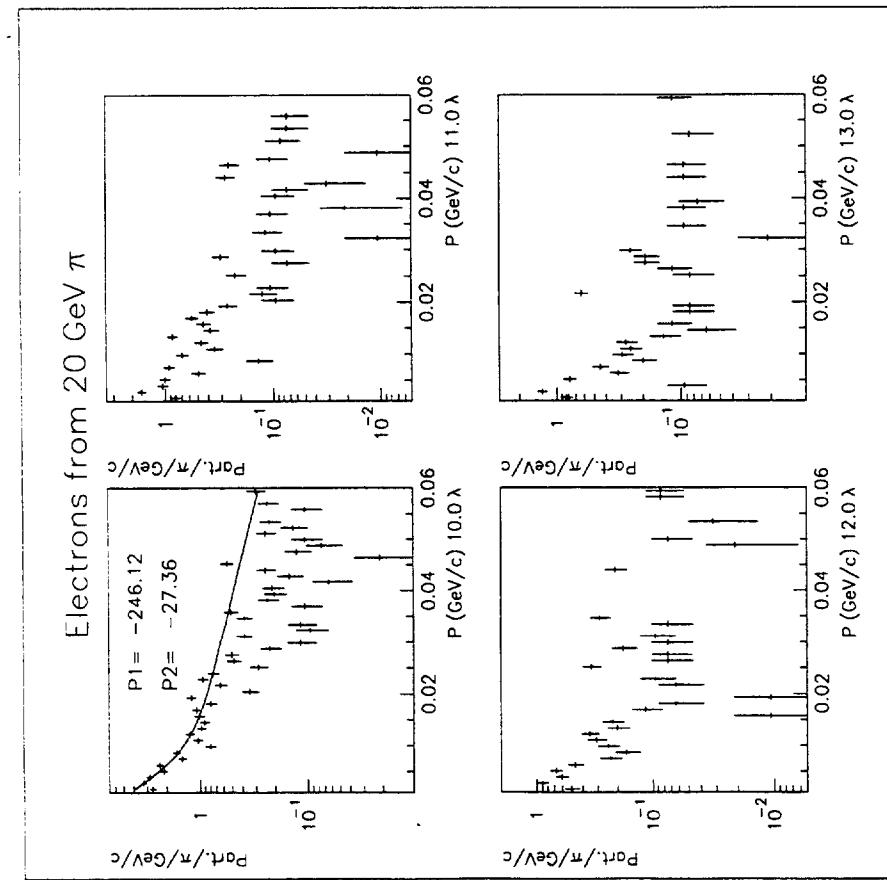


Figure 35: Momentum spectra of electrons coming from pions having momenta of 20 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as  $P_1$  and  $P_2$ .

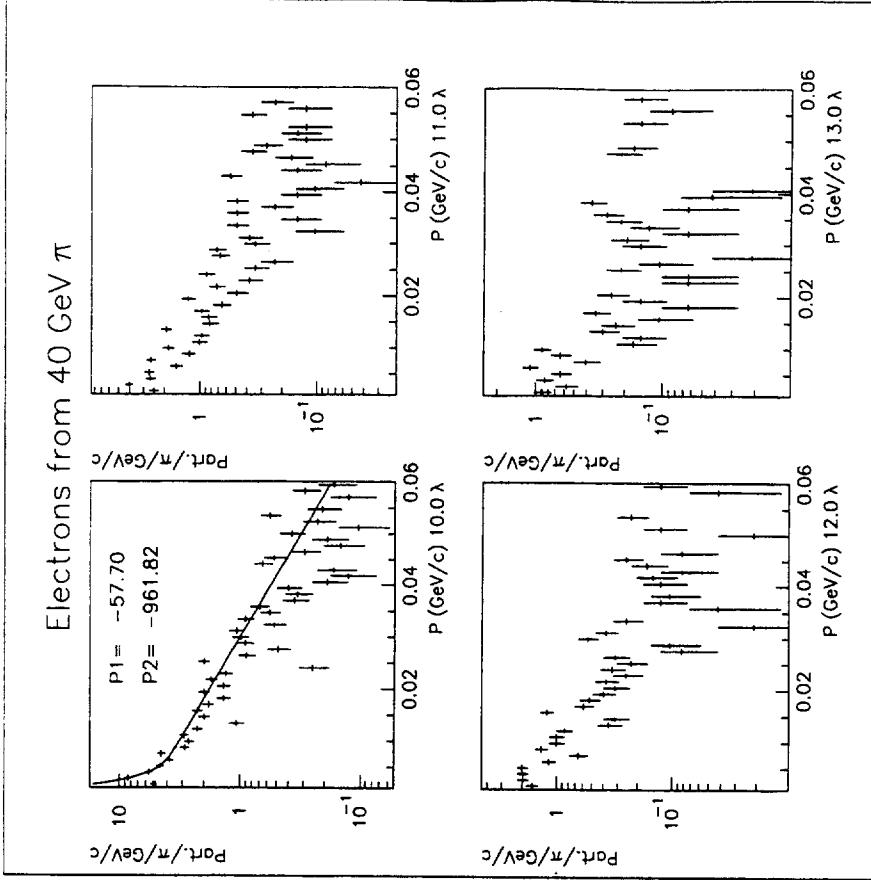


Figure 36: Momentum spectra of electrons coming from pions having momenta of 40 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as  $P_1$  and  $P_2$ .

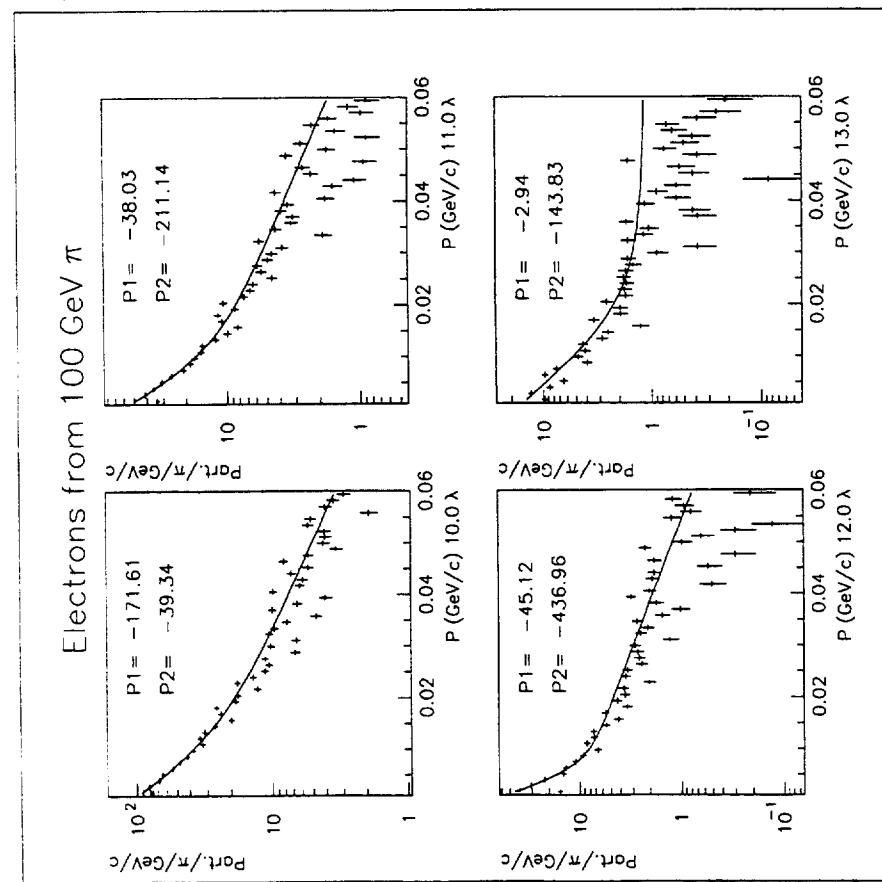


Figure 37: Momentum spectra of electrons coming from pions having momenta of 100 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as P1 and P2.

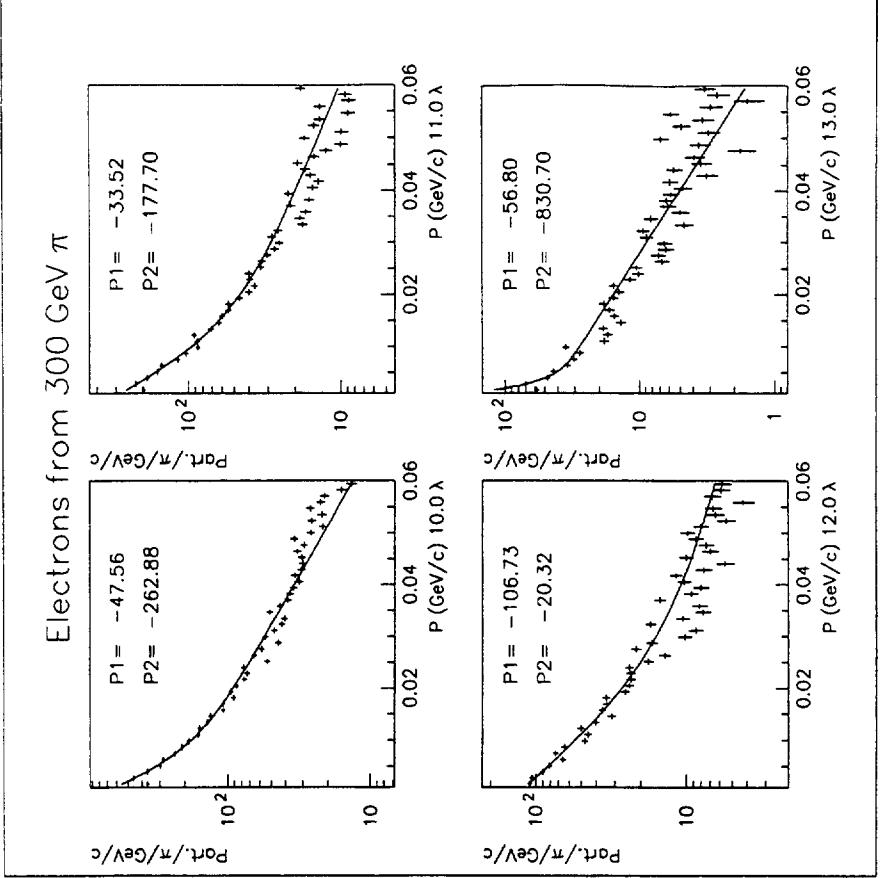


Figure 38: Momentum spectra of electrons coming from pions having momenta of 300 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as P1 and P2.

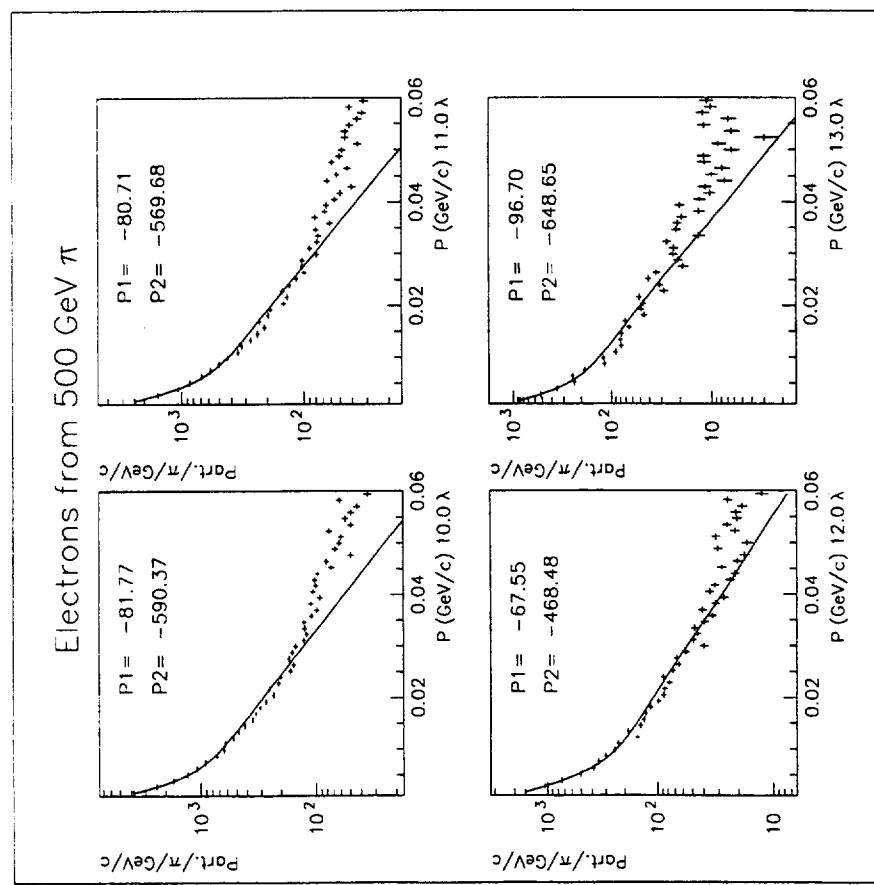


Figure 39: Momentum spectra of electrons coming from pions having momenta of 500 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as P1 and P2.

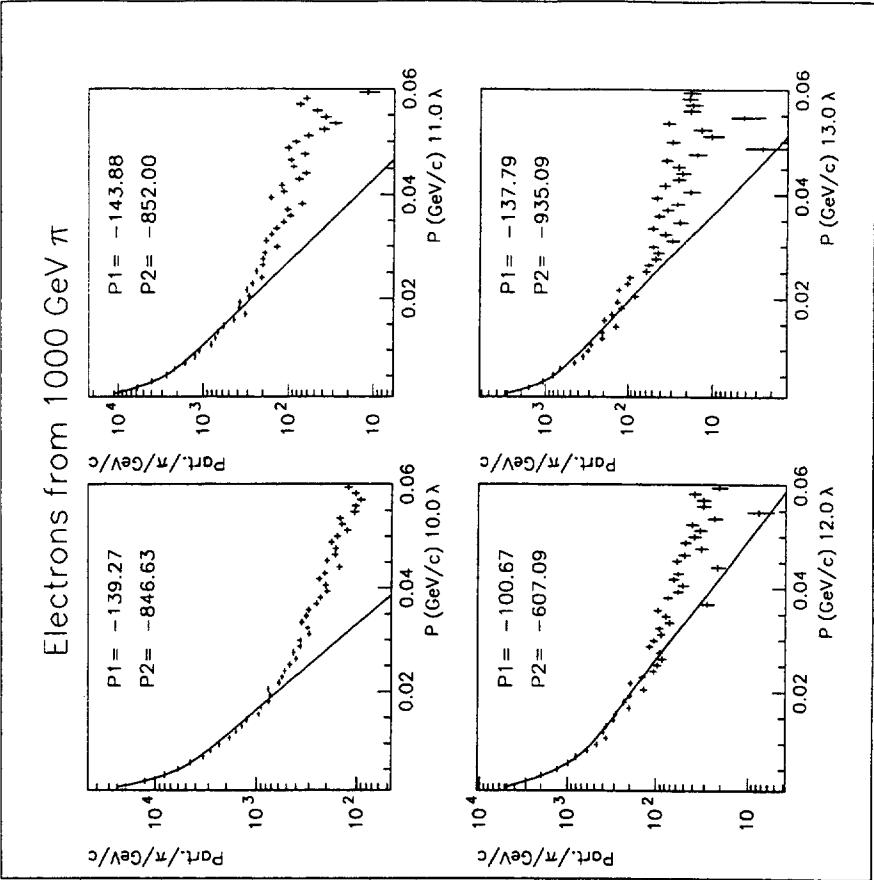


Figure 40: Momentum spectra of electrons coming from pions having momenta of 1000 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as P1 and P2.

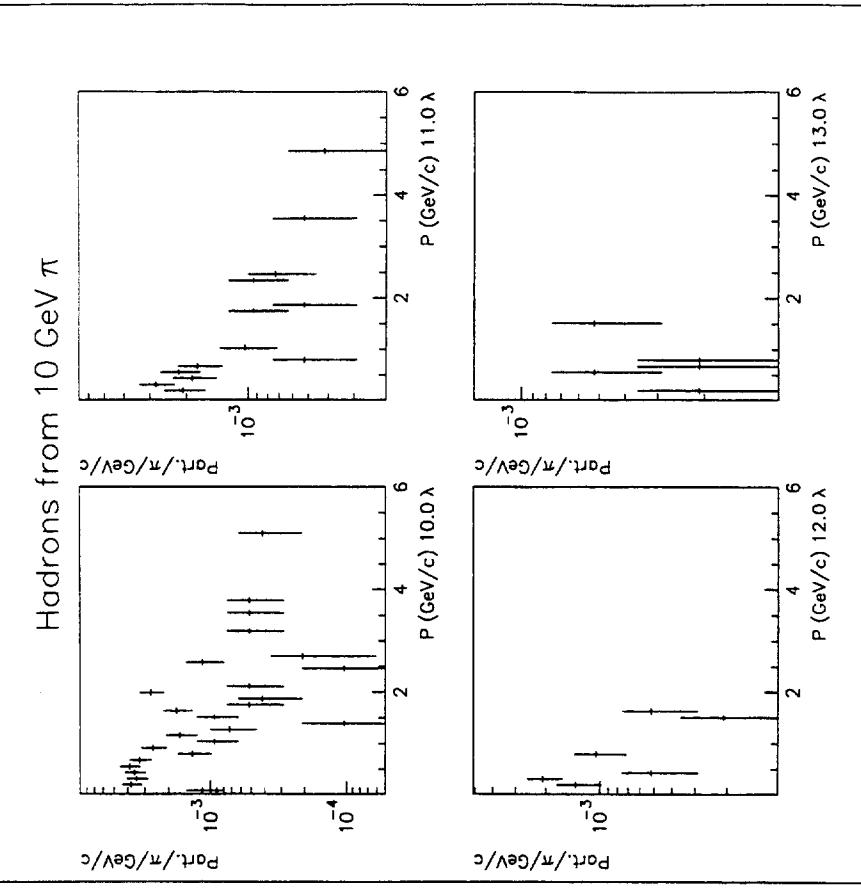


Figure 42: Momentum spectra of hadrons coming from pions having momenta of 10 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths.

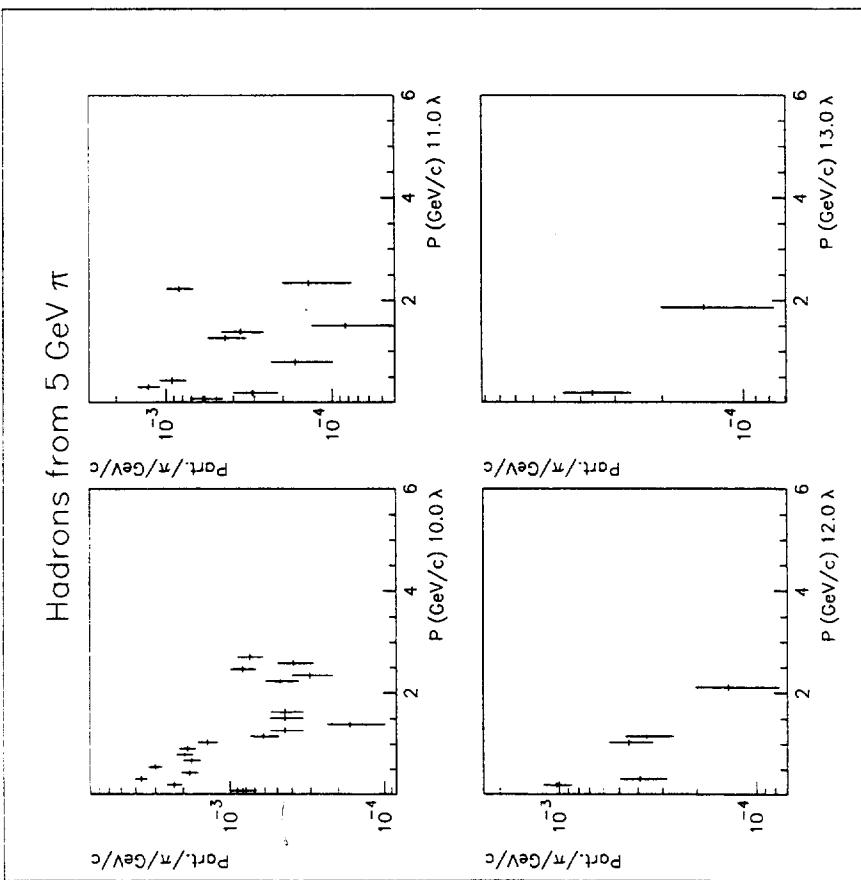


Figure 41: Momentum spectra of hadrons coming from pions having momenta of 5 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths.

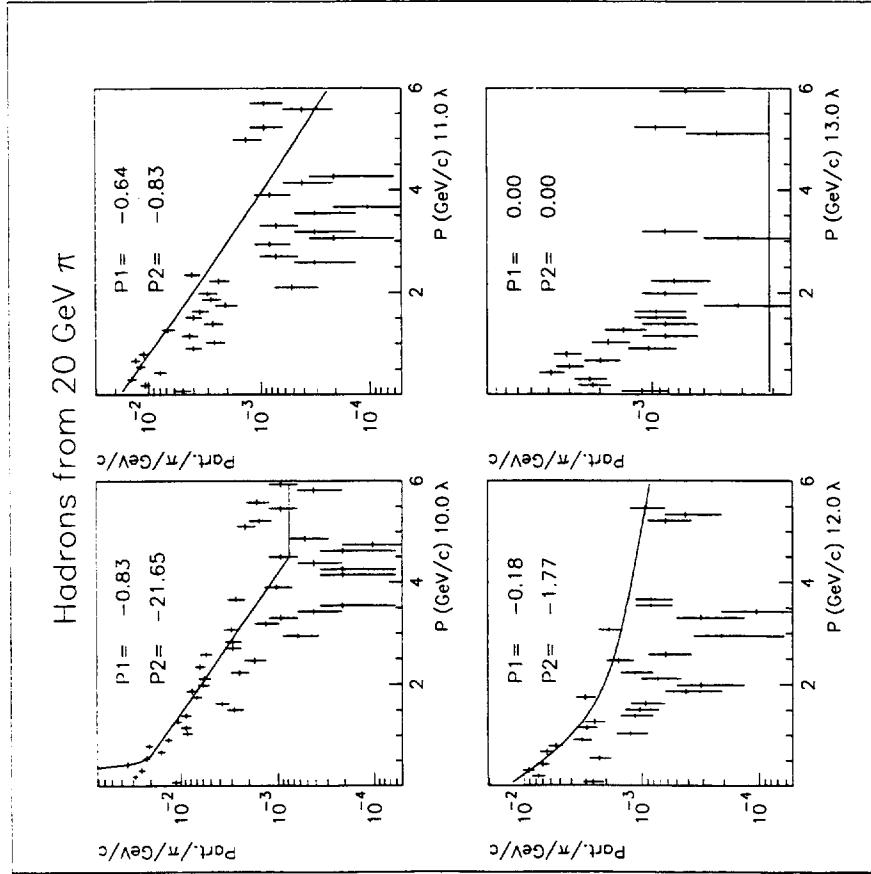


Figure 43: Momentum spectra of hadrons coming from pions having momenta of 20 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as  $P_1$  and  $P_2$ .

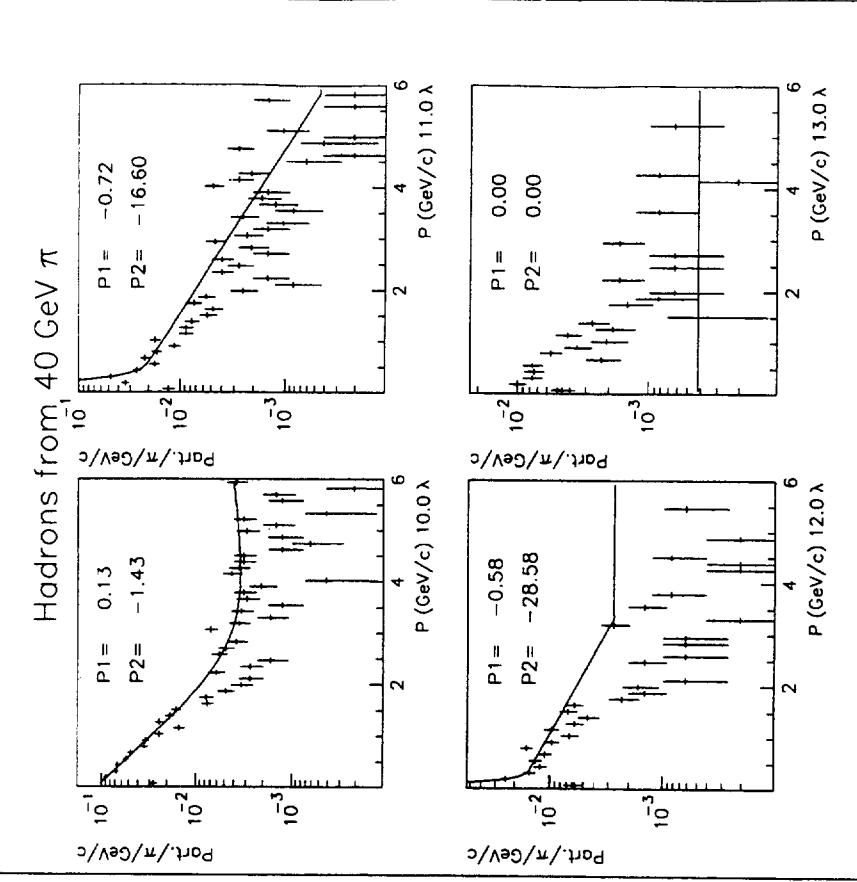


Figure 44: Momentum spectra of hadrons coming from pions having momenta of 40 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as  $P_1$  and  $P_2$ .

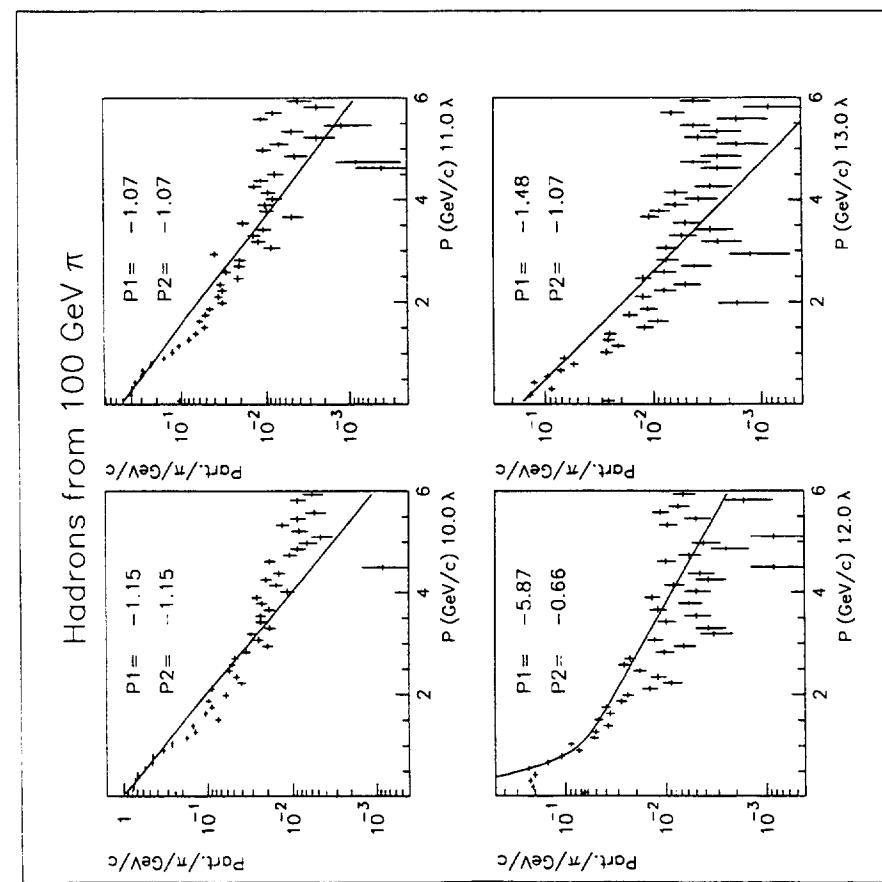


Figure 45: Momentum spectra of hadrons coming from pions having momenta of 100 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as P1 and P2.

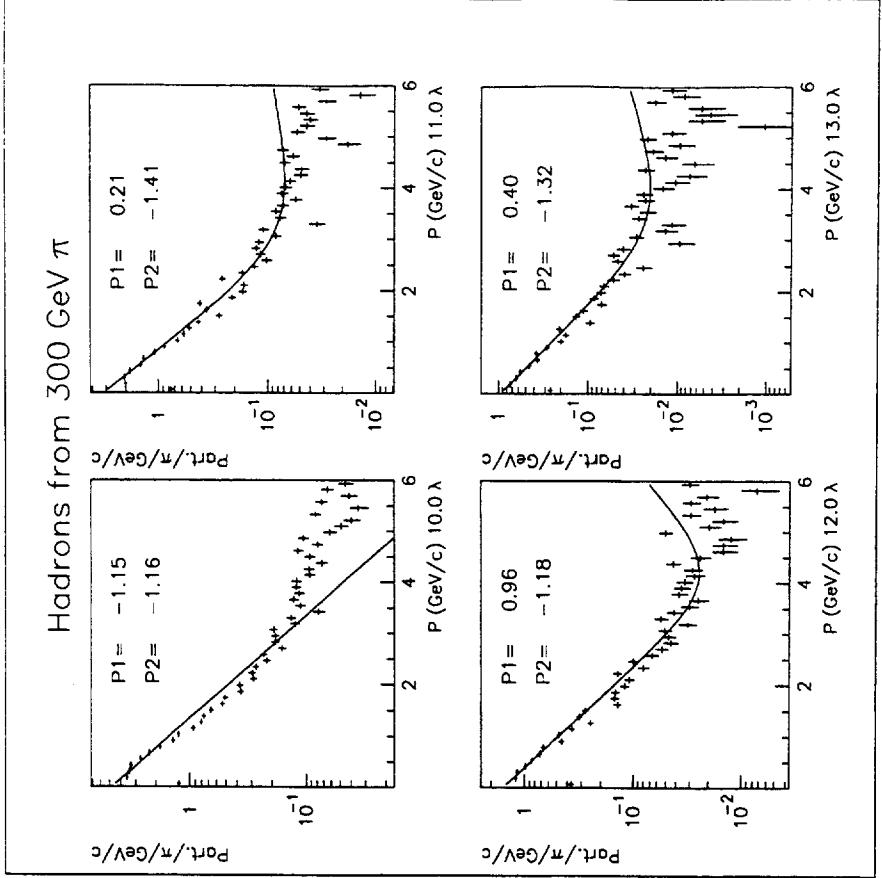


Figure 46: Momentum spectra of hadrons coming from pions having momenta of 300 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as P1 and P2.

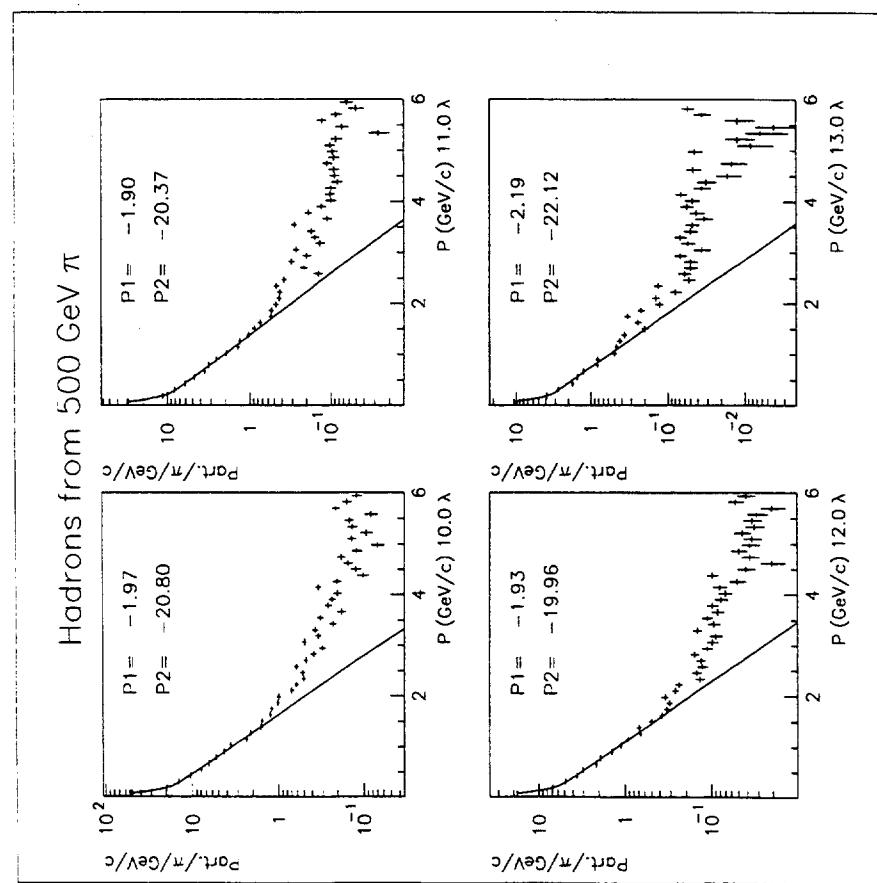


Figure 47: Momentum spectra of hadrons coming from pions having momenta of 500 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as P1 and P2.

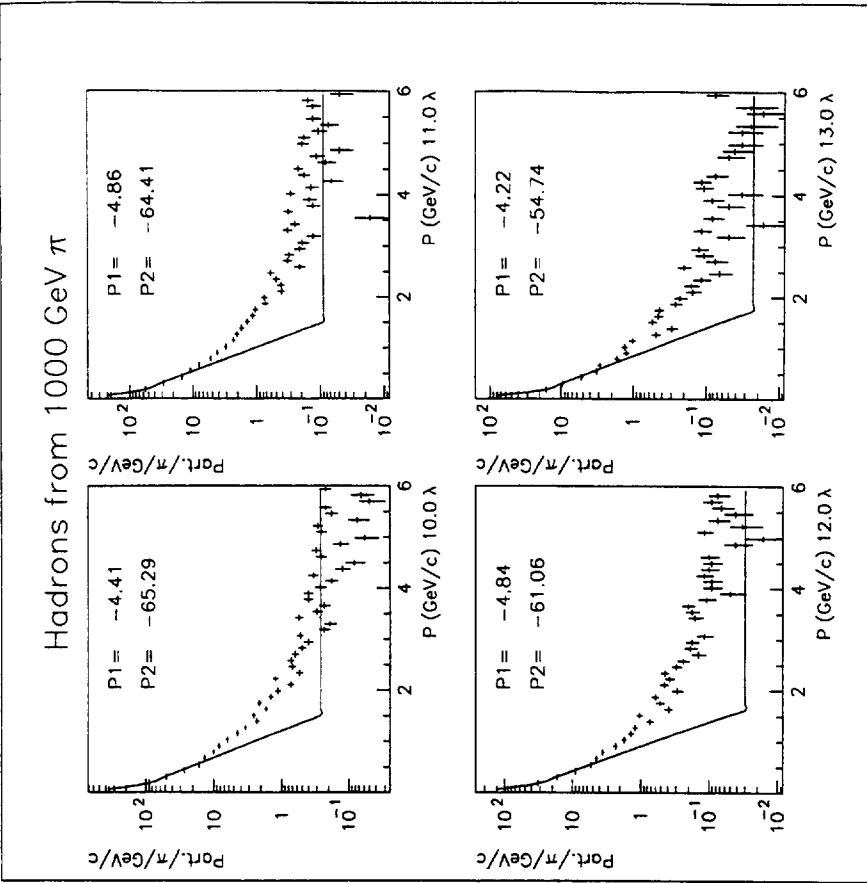


Figure 48: Momentum spectra of hadrons coming from pions having momenta of 1000 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as P1 and P2.

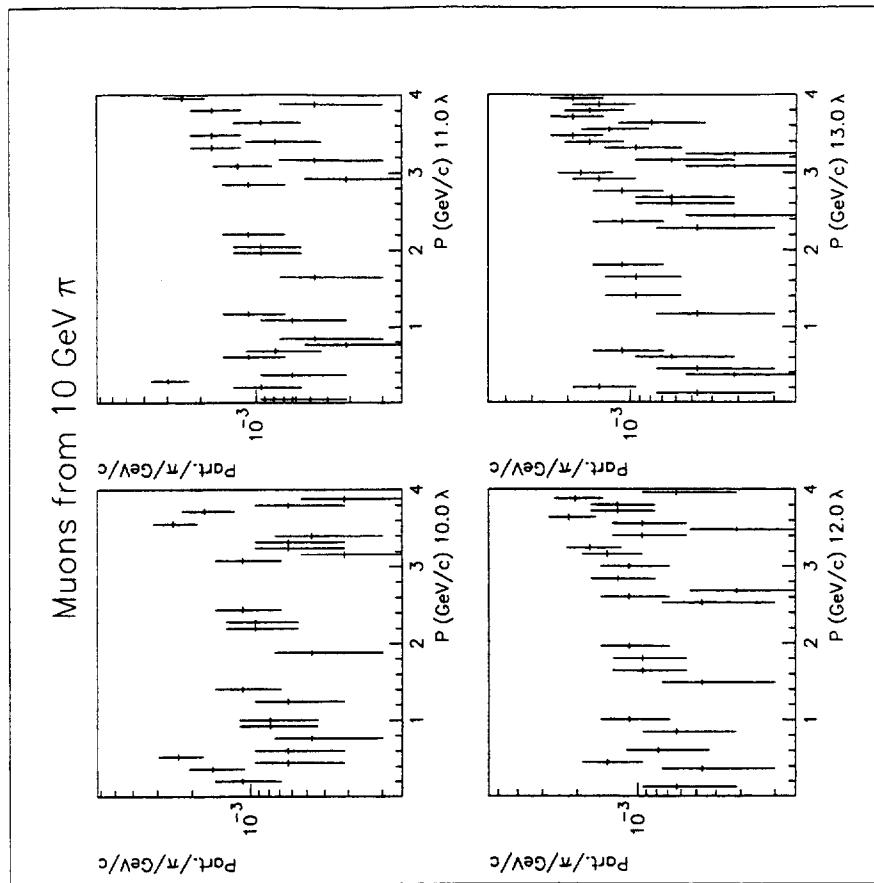


Figure 50: Momentum spectra of muons coming from pions having momenta of 10  $\text{GeV}/c$ . The spectra are shown for 10, 11, 12 and 13 interaction lengths.

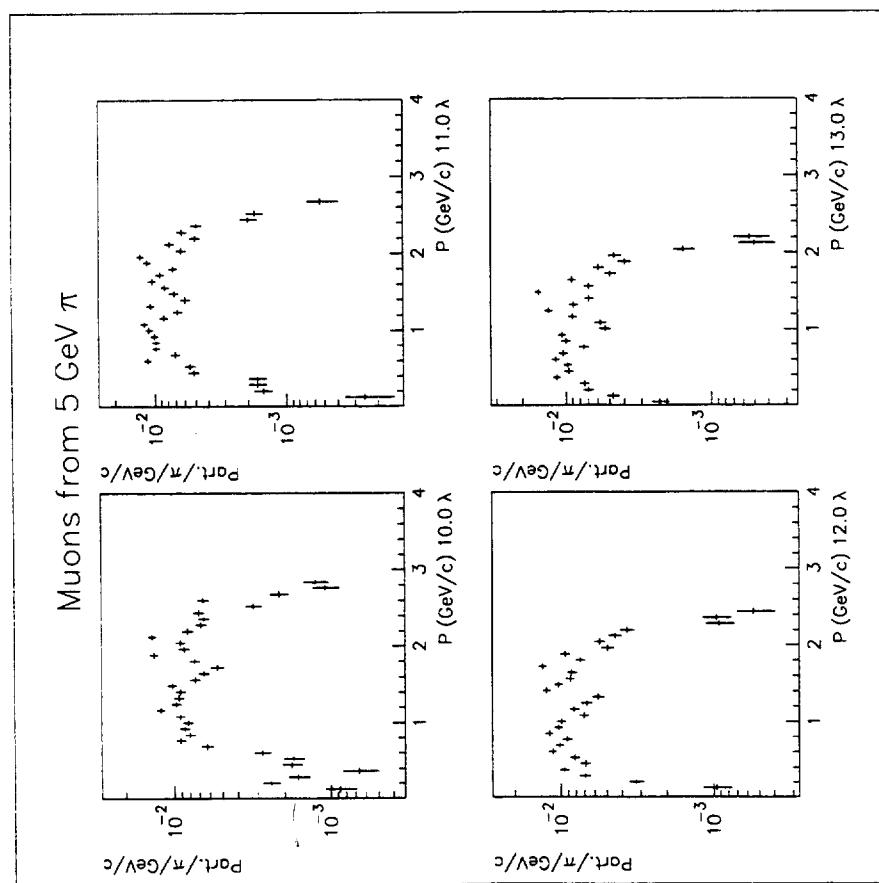


Figure 49: Momentum spectra of muons coming from pions having momenta of 5  $\text{GeV}/c$ . The spectra are shown for 10, 11, 12 and 13 interaction lengths.

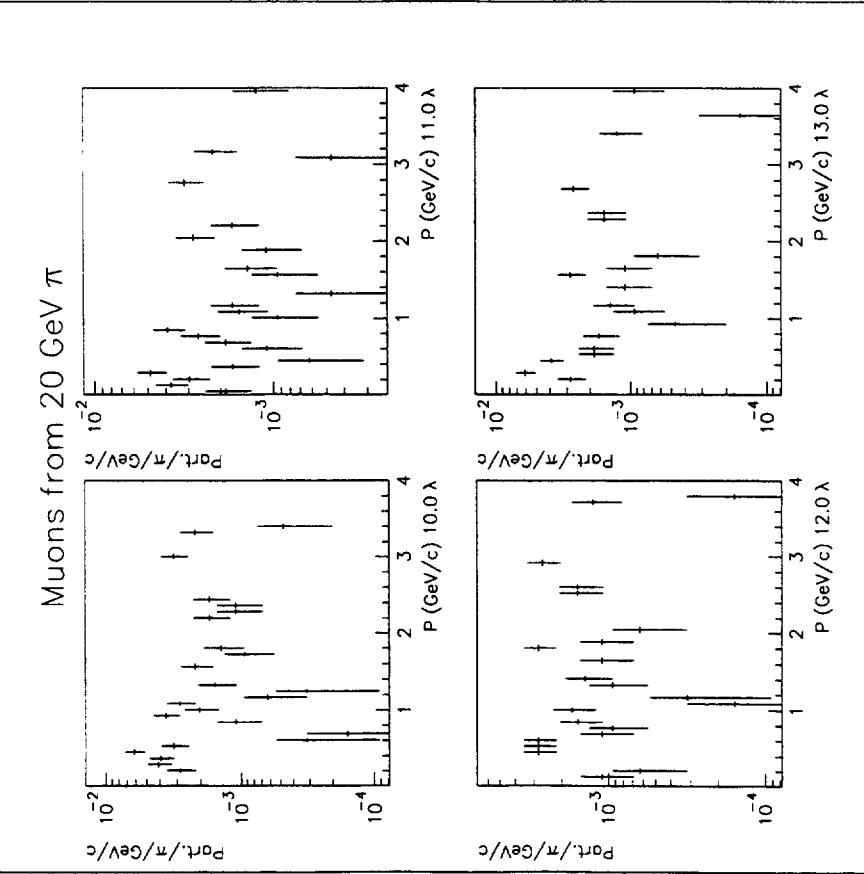


Figure 51: Momentum spectra of muons coming from pions having momenta of 20  $\text{GeV}/c$ . The spectra are shown for 10, 11, 12 and 13 interaction lengths.

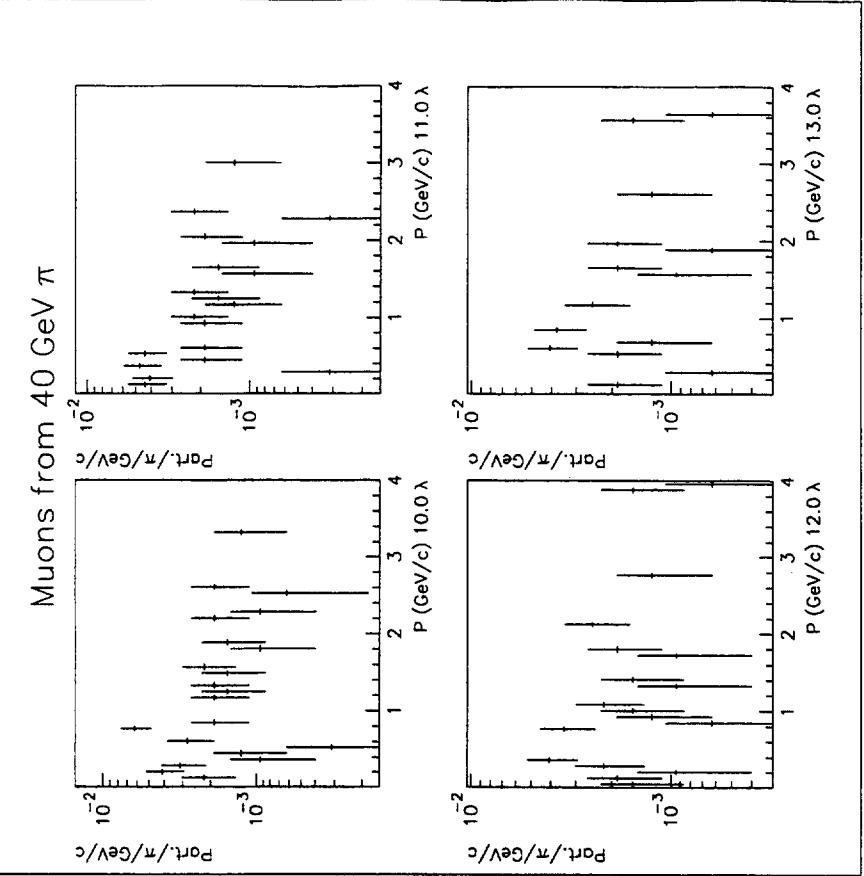


Figure 52: Momentum spectra of muons coming from pions having momenta of 40  $\text{GeV}/c$ . The spectra are shown for 10, 11, 12 and 13 interaction lengths.

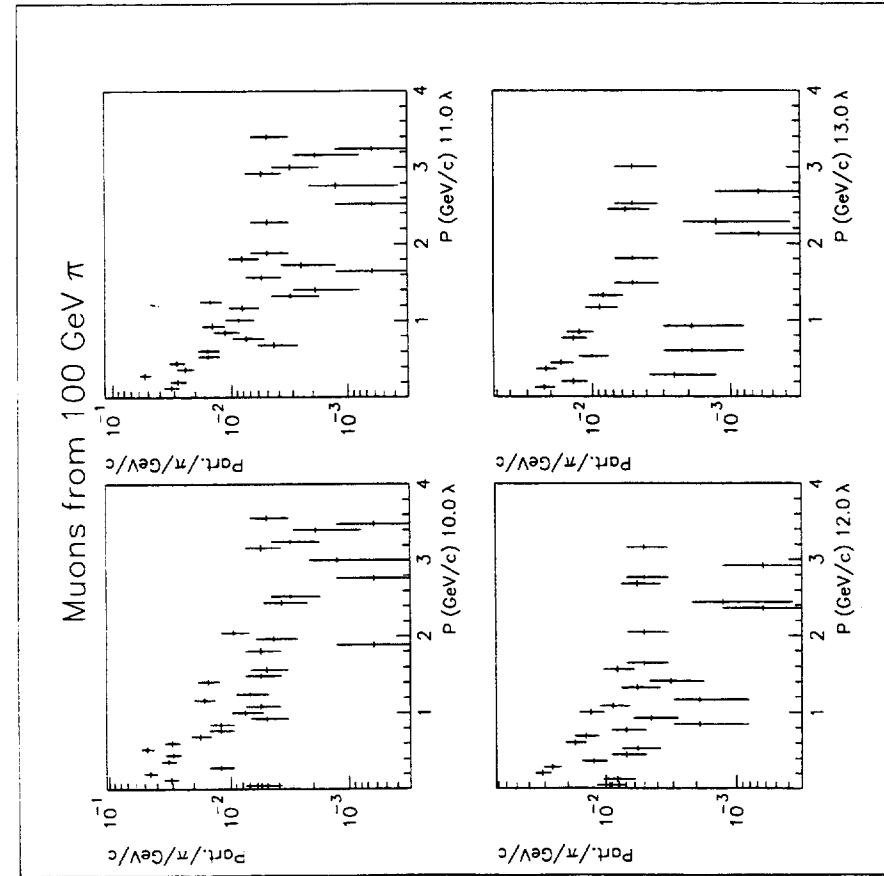


Figure 53: Momentum spectra of muons coming from pions having momenta of 100 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths.

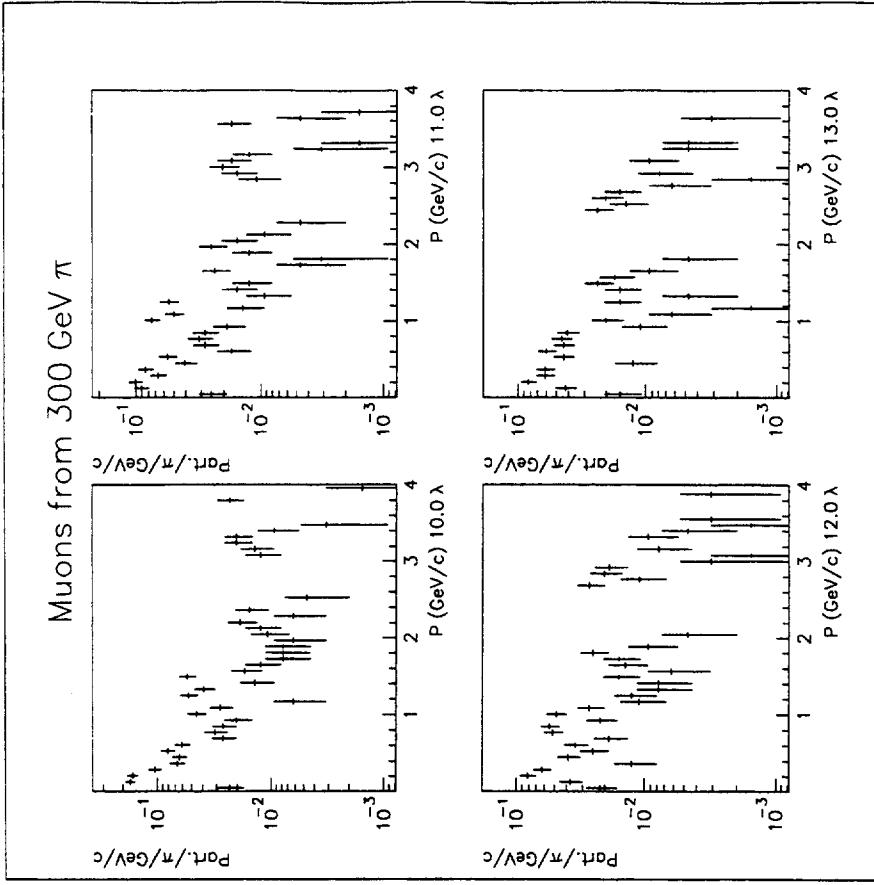


Figure 54: Momentum spectra of muons coming from pions having momenta of 300 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths.

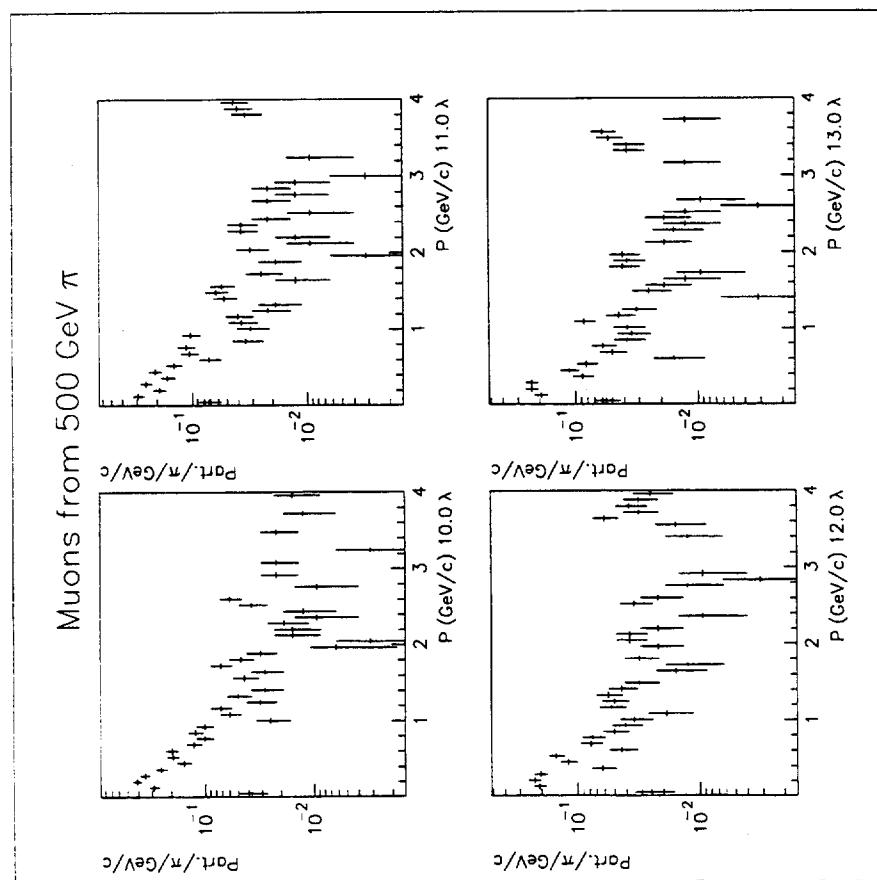


Figure 55: Momentum spectra of muons coming from pions having momenta of 500 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths.

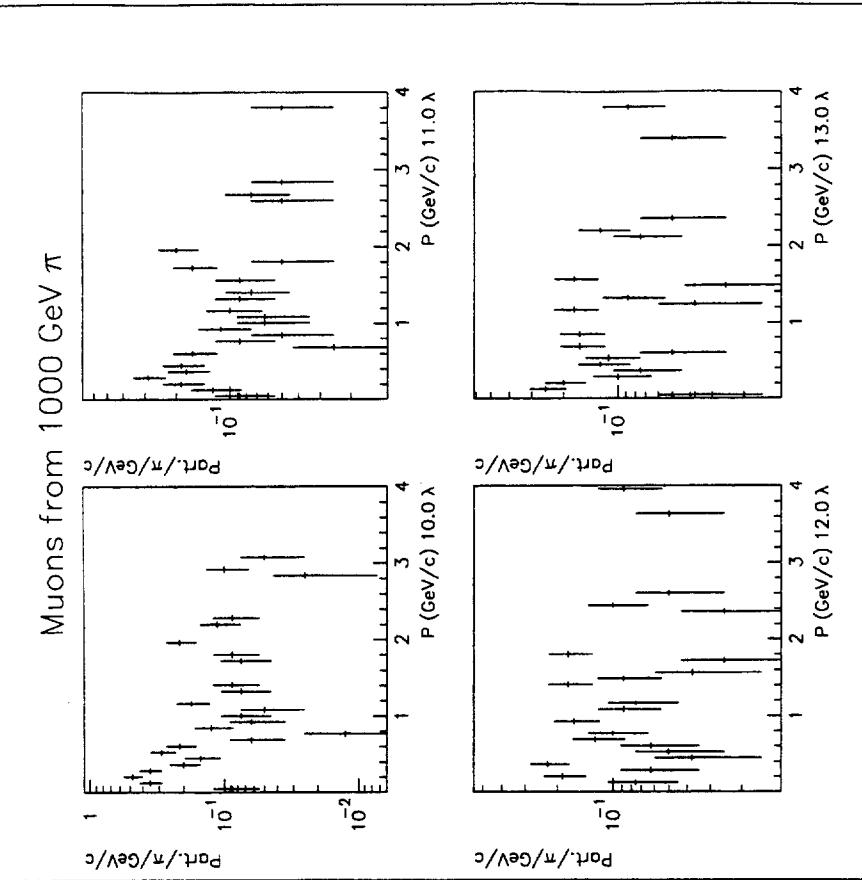


Figure 56: Momentum spectra of muons coming from pions having momenta of 1000 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths.

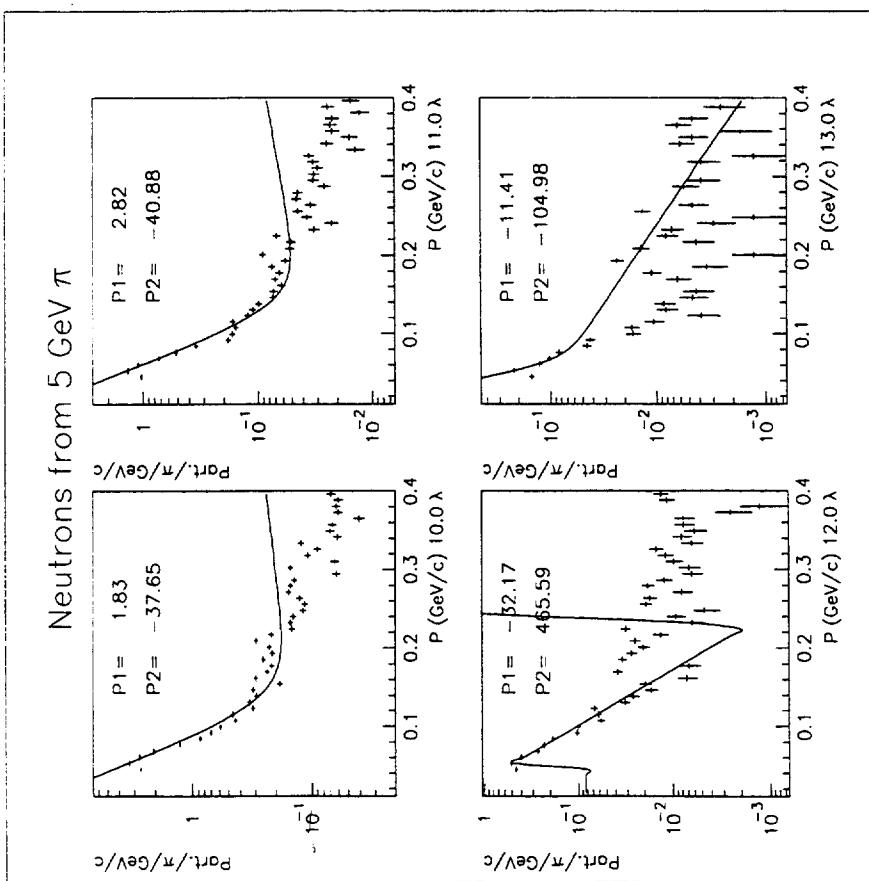


Figure 57: Momentum spectra of neutrons coming from pions having momenta of 5 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as  $P_1$  and  $P_2$ .

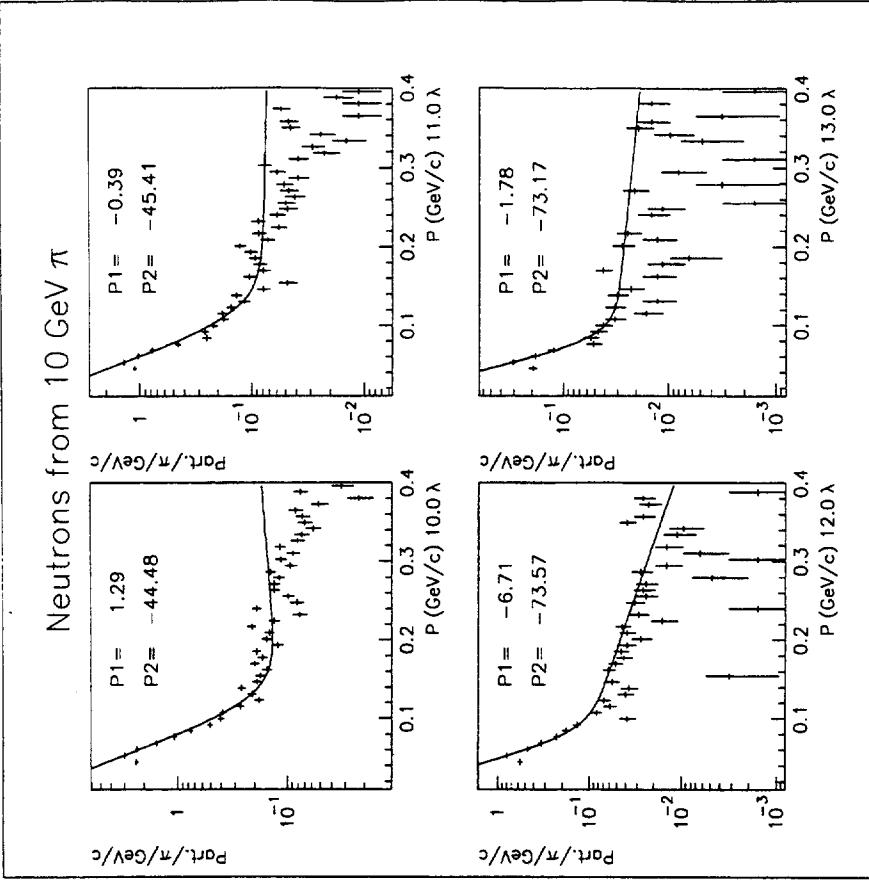


Figure 58: Momentum spectra of neutrons coming from pions having momenta of 10 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as  $P_1$  and  $P_2$ .

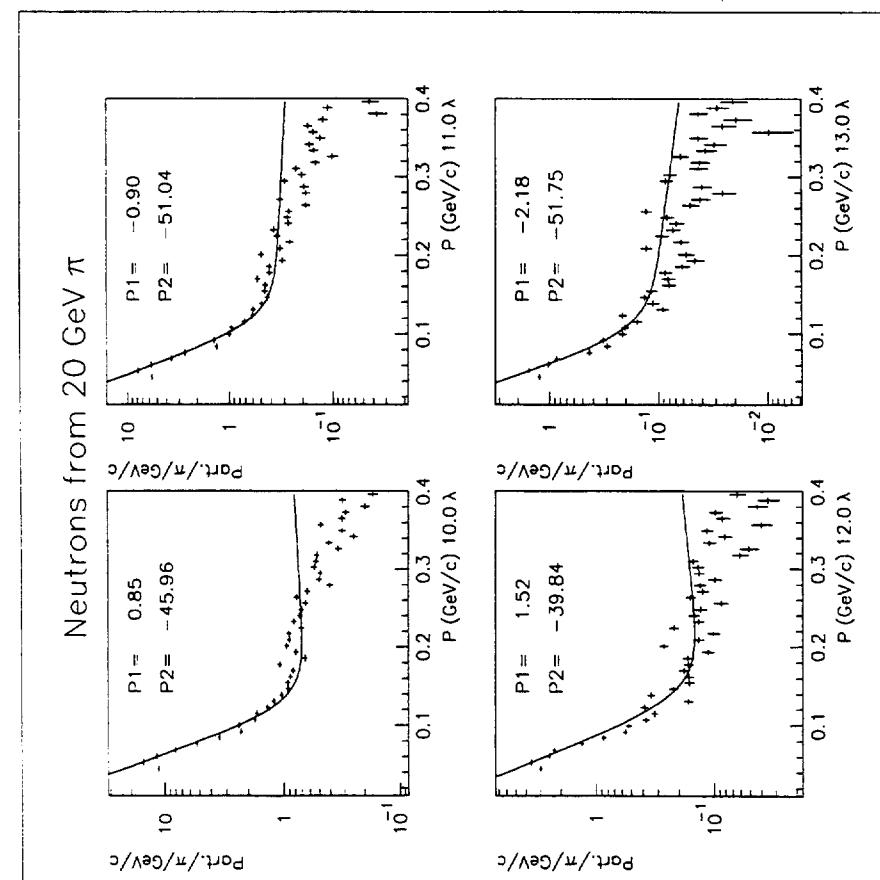


Figure 59: Momentum spectra of neutrons coming from pions having momenta of 20 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as  $P_1$  and  $P_2$ .

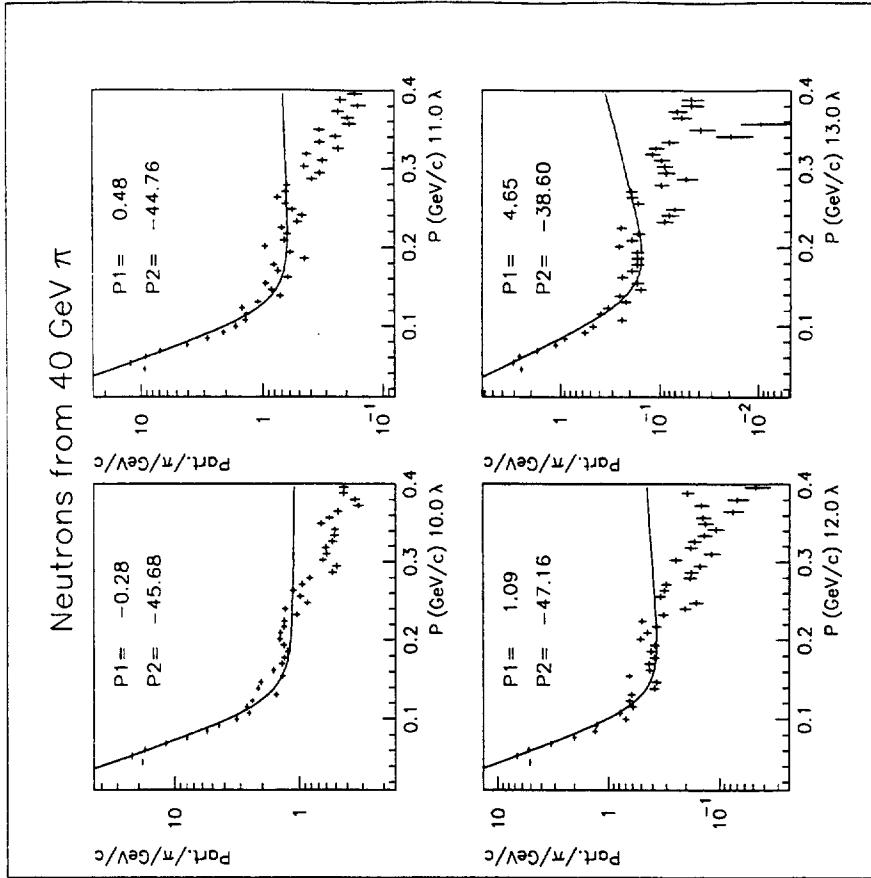


Figure 60: Momentum spectra of neutrons coming from pions having momenta of 40 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as  $P_1$  and  $P_2$ .

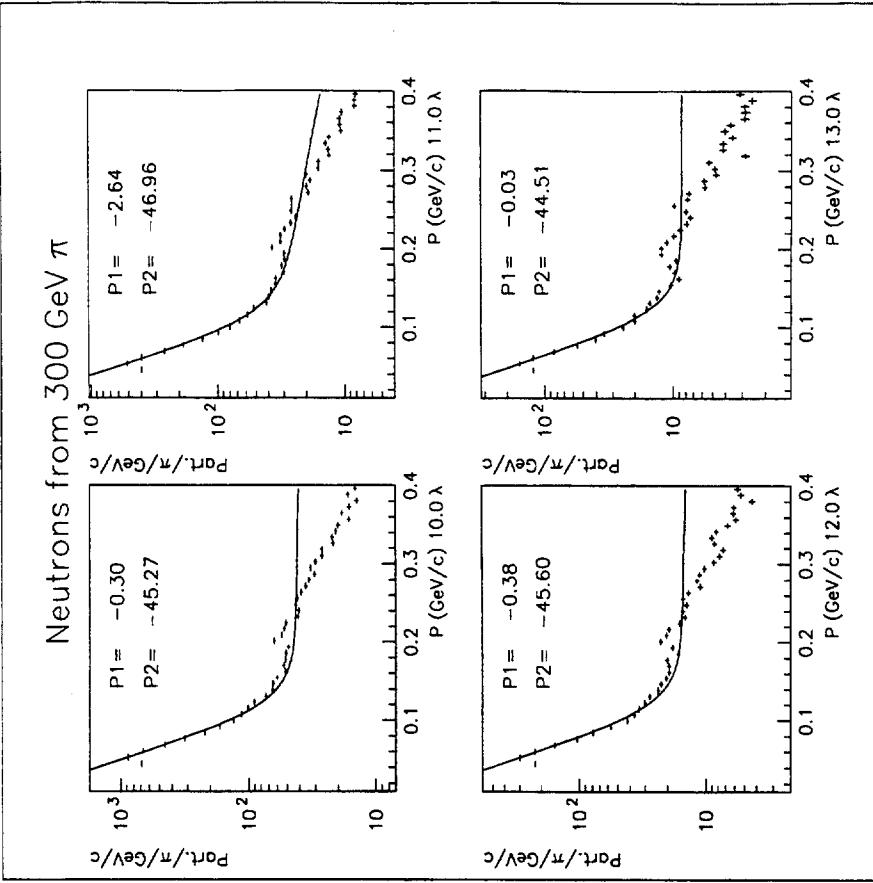


Figure 62: Momentum spectra of neutrons coming from pions having momenta of 300 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as P1 and P2.

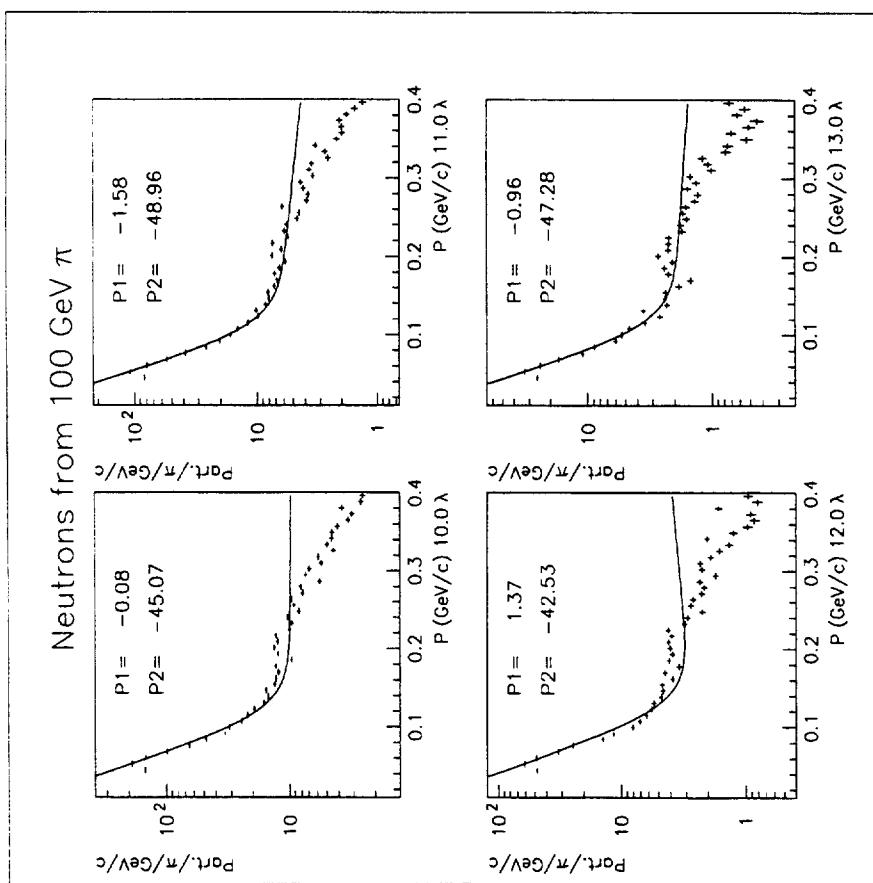


Figure 61: Momentum spectra of neutrons coming from pions having momenta of 100 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as P1 and P2.

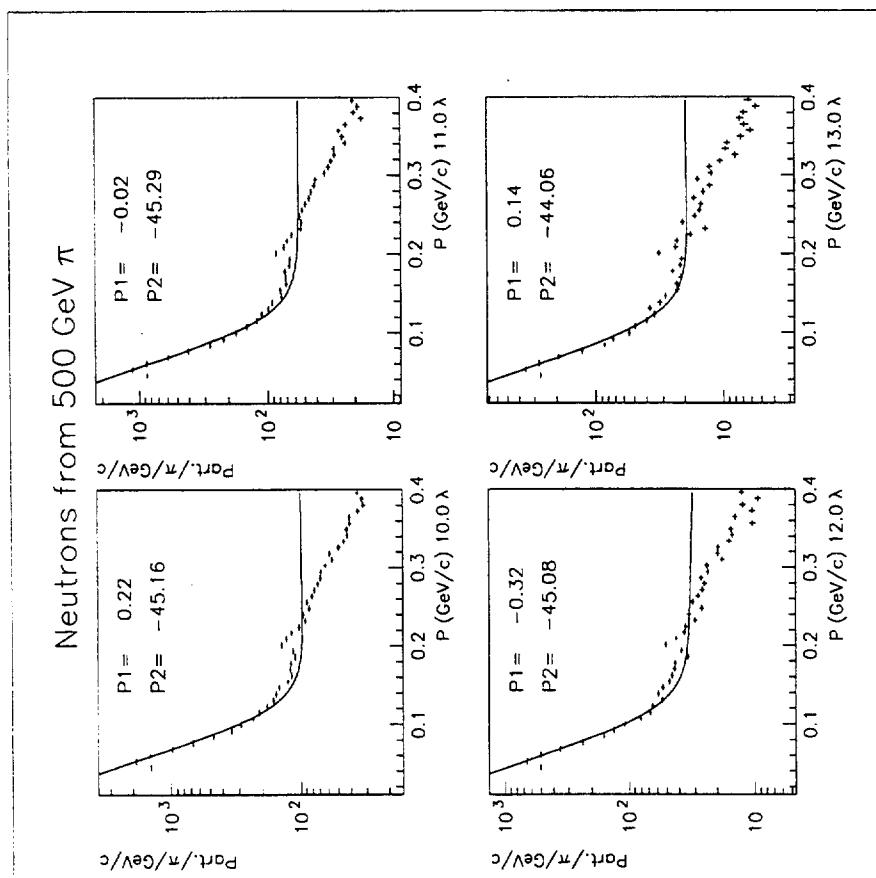


Figure 63: Momentum spectra of neutrons coming from pions having momenta of 500 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as P1 and P2.

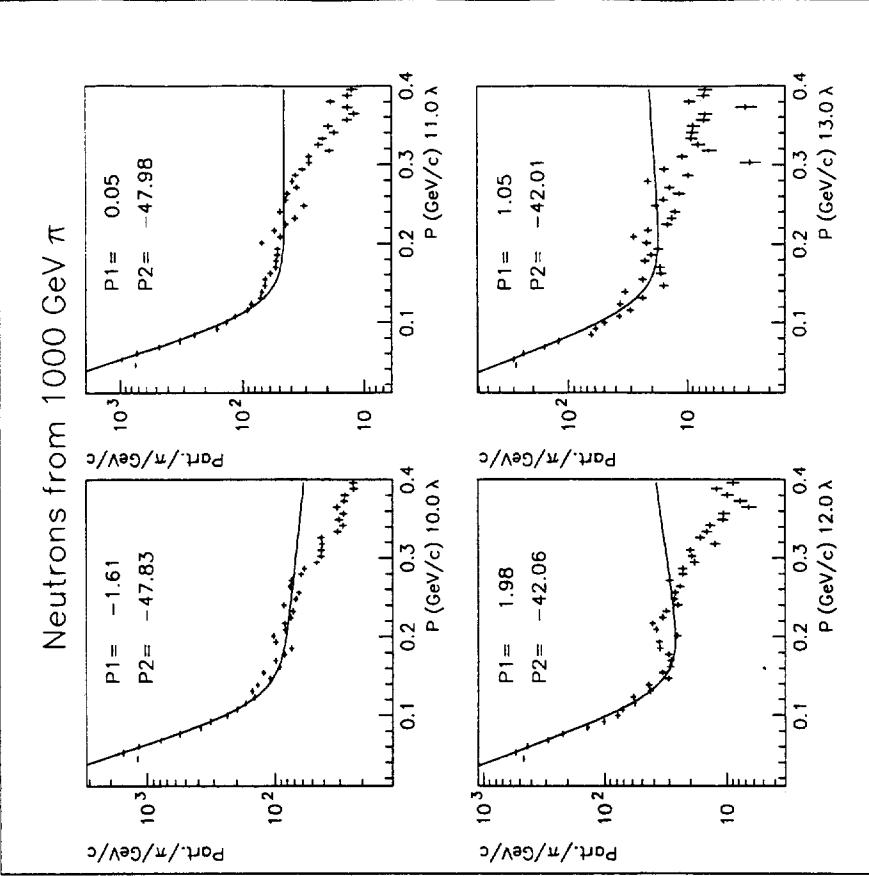


Figure 64: Momentum spectra of neutrons coming from pions having momenta of 1000 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as P1 and P2.

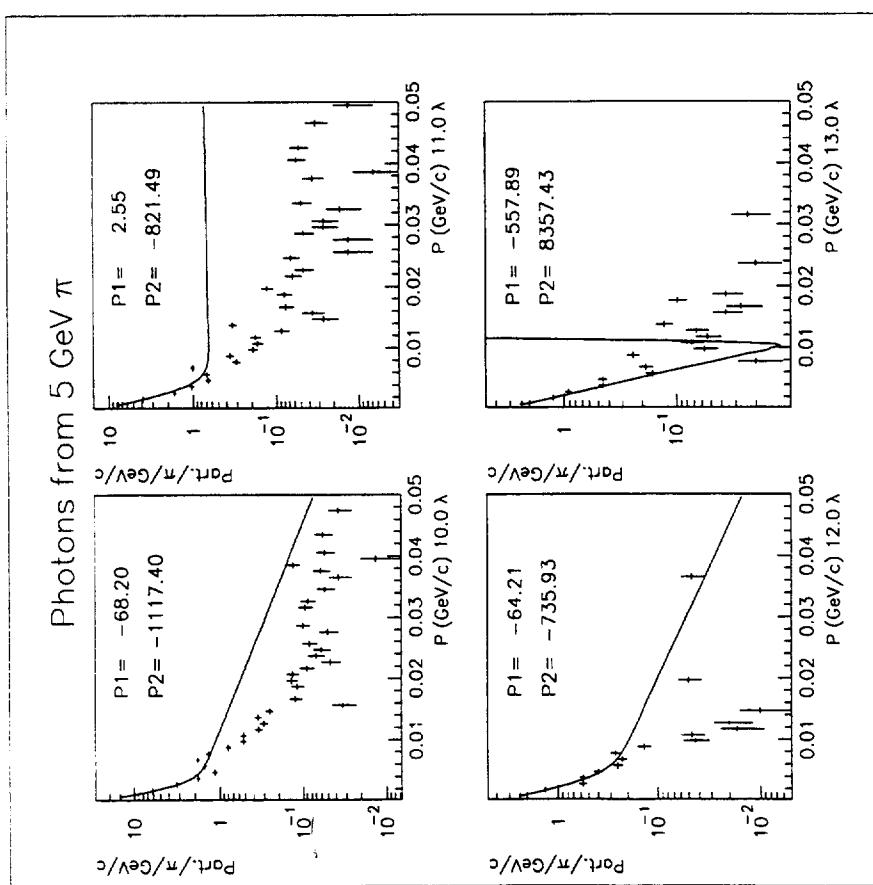


Figure 65: Momentum spectra of photons coming from pions having momenta of 5 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as P1 and P2.

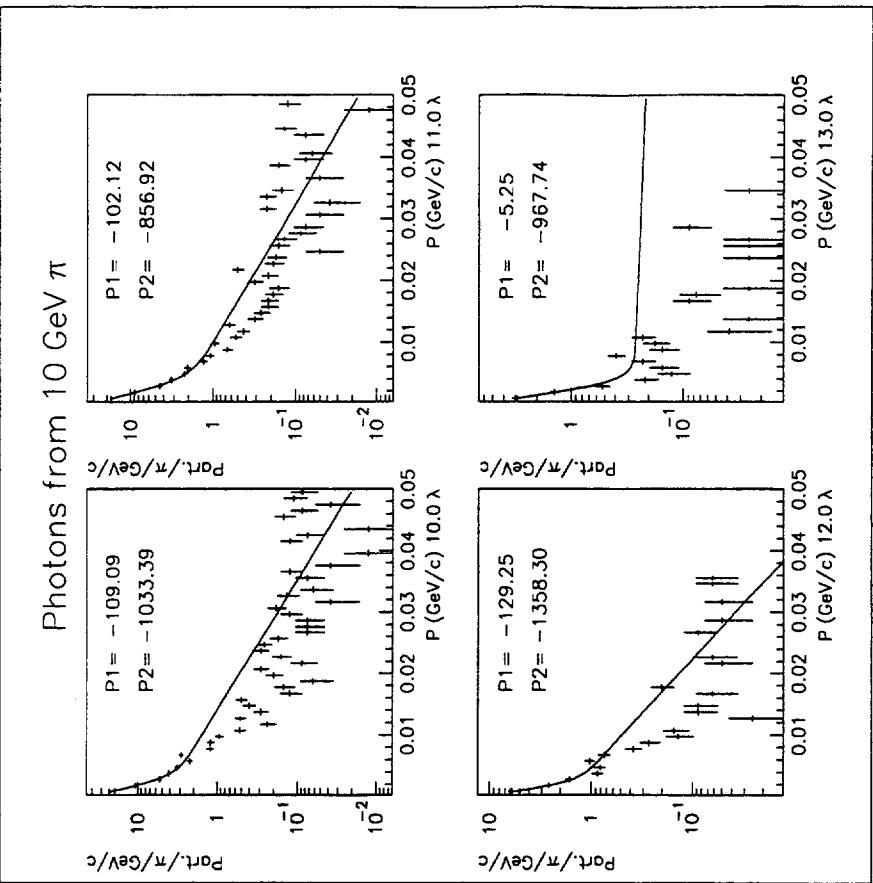


Figure 66: Momentum spectra of photons coming from pions having momenta of 10 GeV/c. The spectra are shown for 10, 11, 12 and 13 interaction lengths. The coefficients of the two fit exponentials are given as P1 and P2.