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Letter of Intent

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Tests of CPT and Time Reversal Invariance at LEAR

by

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Tests of CPT and Time Reversal Invariance

1. Introduction

It is well known that the weak interaction is not invariant under change conjugation, C, and the parity operation, P, and there is a clear violation of CP-invariance in the K° decays and, some would say, in the particle/anti-particle asymmetry of the universe. There is no experimental evidence for a failure of CPT invariance, i.e. a difference of particle and anti-particle masses and lifetimes, and powerful theoretical prejudice against a failure of CPT: Any field theory which satisfies Lorentz invariance, hermiticity, local commutativity, and the spin/statistics relations must be CPT-invariant. It is relatively easy to incorporate failure of C- and P- and CP-invariance into field theories but no one has yet devised a theoretical structure which is not CPT-invariant.

These theoretical considerations do not guarantee CPT-invariance (perhaps the physical world is not exactly described by field theories) but they do imply that a failure of T-invariance should be associated with a failure of CP-invariance. In the unique K° case the unitarity theorem of Bell and Steinberger[2] allows the experimental CP parameters to be related to two complex numbers ε and Δ which are such that ε represents a failure of CP- and T-invariance but not of CPT, and Δ represents a failure of CP- and CPT-invariance but not of T, (see Appendices I and II). The existing measurements require a non-zero value of ε and are consistent with Δ=0, although the limits which can be placed on a CPT-violation are not particularly severe compared with the known CP-violation. Specifically the unitarity theorem yields the result

$$\Delta + \tfrac{1}{2}\,\mathrm{Re}\, z_{\ell} = \{(-0.02\pm0.18)+(0.03\pm0.17)i\}\times10^{-3}$$

where z_{ℓ} represents a possible (CPT-violation) difference between the partial widths for K° and $\overline{K^o} \to \pi\ell\nu$ (see Appendix II). Expressed in terms of the difference of masses and widths of K° and $\bar{K}$° the limits on the validity of

CPT-invariance are

$$\left|\frac{M-\bar{M}}{M_L-M_S}\right| < 0.07,\ \left|\frac{\Gamma-\bar{\Gamma}}{\Gamma_S-\Gamma_L}\right| < 0.07$$

$$|M-\bar{M}| < 3\times10^{-7}\,eV,\ |\Gamma-\bar{\Gamma}| < 6\times10^{-7}\,eV$$

where the poor experimental limit Re $z_\ell<0.07$ has been used. None of the other particle/anti-particle mass differences (or magnetic moment differences) can hope to get anywhere near the limit imposed by the K° case as the others involve magnitudes characteristic of the strong interaction (electro-magnetic interaction). The most interesting of the weak decay lifetime comparisons is K^+ v. K^- which has yielded,

$$\left|\frac{\Gamma_+-\Gamma_-}{\Gamma_++\Gamma_-}\right| < 1.5\times10^{-3}$$

$$|\Gamma_+-\Gamma_-| < 1.5\times10^{-10}\,eV$$

the measurement[4] being limited by systematic errors. Other lifetime differences are more accurate, $\pi^{\pm}(<10^{-3})$ and $\mu^{\pm}(<10^{-4})$, but are not so interesting as they do not involve $\Delta S=1$ transitions in which all the CP trouble occurs. (There appears to be no practical way of detecting a small CPT and T-violating, CP-conserving effect[7]).

The observation of CP-violation in K° decays suggested that T-violation might be demonstrated by (a) the existence of a neutron dipole moment at the level of 10^{-23} cm (10^{-13}cm for the neutron radius x 10^{-7} for the weak interaction x 10^{-3} for the CP/T violation), and (b) a transverse polarization in the $K_{\mu3}$ decay at the level of a few parts per thousand. Despite much effort and ingenuity neither the dipole moment[3] ($<6\times10^{-25}$e cm) nor the polarization[14] ($<9\times10^{-3}$) has yet revealed a failure of time reversal invariance.

There are strong reasons for expecting CPT-invariance to be valid and T-invariance to be violated, but the former has not been verified below the level of a few parts per thousand (which is characteristic of the CP-violation) and the latter has not been directly demonstrated. It is proposed to attempt to rectify both of these deficiencies by precise comparison measurements on kaons and anti-kaons using stopping anti-protons from LEAR as a symmetric source practically free of systematic errors. The particular cases that will be considered are

(i) a comparison of the K^+ and K^- lifetimes aiming at an accuracy of 10^{-4} i.e. an order of magnitude improvement on the present CPT limit, and an order of magnitude below the CP parameters,

(ii) a comparison of $K^o \to \pi e \nu$ and $\bar{K}^o \to \pi e \nu$ with a view to (a) demonstrating a failure of time reversal invariance and (b) determining the parameter Re y_ℓ (itself a CPT test) and thus improving the limits on the differences of $K^o/\bar{K}^o$ masses and widths that can be derived from the unitarity theorem to about 10^{-3}. The theoretical basis for the T-invariance measurement is due to Kabir(10) and outlined in Appendix III: By $\Delta Q = \Delta S$ the observation $\pi^- e^+ \nu$ ($\pi^+ e^- \bar{\nu}$) identifies the decaying particle as a K^o ($\bar{K}^o$) and thus permits a comparison of the rates $K^o \to \bar{K}^o$ and $\bar{K}^o \to K^o$ if the initial particle/anti-particle state is known. These rates are expected to differ by 6.5×10^{-3} if CPT but not T is valid. The parameter Re y_ℓ describes a possible difference of the partial widths for $K^o_{\ell 3}$ and $\bar{K}^o_{\ell 3}$ and is determined by a comparison of the rates for an initial K^o ($\bar{K}^o$) decaying to $\pi^- e^+ \nu$ ($\pi^+ \ell \bar{\nu}$) for $t \to 0$. (see Appendix VI). By contrast the T-invariance measurement is a comparison of the rates for an <u>initial</u> K^o ($\bar{K}^o$) decaying to $\pi^+ \ell^- \bar{\nu}$ ($\pi^- \ell^+ \nu$) i.e. the opposite signs of electron change.

In passing it is noted that it may also be possible to improve the limit on the $\Delta Q = -\Delta S$ parameter x_ℓ (see Appendix III) and the CP-parameter η_{+-o} (see

Appendix IV), but not to an extent that is particularly exciting.

2. Experimental Prospects

The low momentum anti-proton beam from LEAR has a rate of $10^6 \bar{p}\, s^{-1}$ in a tiny phase space, $\sim 10\pi$ mm x mm and 0.1% in momentum. For the 100 MeV/c operation which is planned it should be possible to concentrate all the $\bar{p}$ stops into a volume of liquid hydrogen substantially smaller than $1 mm^3$ thereby effectively creating a point source of annihilation products. This source must yield precisely equal rates of kaons and anti-kaons, providing that the anti-protons annihilate in hydrogen. (The annihilation of $\bar{p}n$ can yield $K^- K^\circ$ but not $K^+ \bar{K}^\circ$). The annihilation channels of interest are

$$\bar{p}p \rightarrow K^+ K^- \quad 0.1\%$$

and

$$\bar{p}p \rightarrow K^\pm K^\circ \pi^\mp \quad 0.3\%$$

nearly all the other annihilations leading to multiple pion events, and yielding on average about three charged pions and two neutral pions.

All of the measurements being considered require a magnetic field and wire chambers to determine for each particle the sign of the change and the momentum. It is proposed to use an axially symmetric magnet with the $\bar{p}$ beam entering through one pole and detectors covering the polar angle range $45° < \theta < 135°$ and more or less the full 2π in the azimuth i.e. a solid angle of ½ to 2/3 x 4π, depending on the chamber frames. The solid angle is important as the kaon rates are not large and, in the case of the K_{e3} measurement, there are four charged particles to be detected, $\bar{p}p \rightarrow K^\pm K^\circ \pi^\mp$, $K^\circ \rightarrow \pi^\pm e\, \nu$. It is also important that the detectors should be symmetric and uniform, although it is intended to average out small non-uniformities by periodically reversing the magnetic field which is equivalent to changing particles into antiparticles if the $\bar{p}$ beam is undisturbed by the reversal.

Ultimately the errors in the comparison measurements should come from the counting statistics and the interactions of the hadrons in the material of the detectors. The difference of ionization of positive and negative particles is negligible but nuclear interactions in chambers could generate assymmetric of $\sim 1 \times 10^{-3}$ for K and $\bar{K}$ and also for π^+ and π^-. Calculated corrections based on supplementary measurements would leave an uncertainty of $\sim 5 \times 10^{-5}$.

The statistical limit is based on $10^{12}\bar{p}$, nominally $10^6 \bar{p}\ s^{-1}$ for 10 days. In the case of the K^+ v. K^- lifetime comparison this would yield 10^8 events $K^{\pm}$+($K^{\mp}$ decaying) and a standard deviation for $(\tau_+ - \tau_-)/(\tau_+ + \tau_-)$ of 10^{-4}. A one-day measurement with $3 \times 10^5 \bar{p}\ s^{-1}$ for 8 hours would give a 0.1% result and, being free of any appreciable systematic uncertainties, this would be at least as good as the present CPT limit.

For the $K^o_{\ell 3}$ time reversal demonstration the rates are lower and the interaction uncertainties negligible compared with the statistical error. A $10^{12}\bar{p}$ measurement is expected to yield an assymetry of 0.65±0.1%, where the error is a standard deviation. The minimum measurement necessary to show a time reversal violation would be 10 days of 8 hours at $3 \times 10^5 \bar{p}\ s^{-1}$ for a two standard deviation effect.

While the main features of the K^+ v. K^- and $K_{\ell 3}$ measurements are similar the details are substantially different and it would not be possible to run the two measurements simultaneously. The K^+ v. K^- measurement requires a magnetic field concentrated at the $\bar{p}p$ source, while $K_{\ell 3}$ requires in addition a magnetic field outside the decay volume to identify the sign of the electron charge. The features common to both experiments include the beam, the liquid hydrogen target, the immediate surroundings of the target, and most of the peripheral equipment.

It must also be noted that whereas the K^+ v. K^- measurement is well defined in terms of hardware and time-scale of preparation, $K_{\ell 3}$ is both physically and technically a much larger project requiring substantial development work particularly with regard to electron identification. Thus K^+ v. K^- amounts to an immediate and definite proposal; $K_{\ell 3}$ to a longer term conditional proposal.

3. K^+ v. K^- Lifetimes

The basis of the measurement is the identification of a collinear pair of charged particles of about the right momentum close to the source and the detection of one of, but not both of, the particles in chambers at some distance from the source. Figures 1 and 2 give a schematic view of the arrangement of magnet* and chambers.

There are two central region M.W. chambers,

C4: Cylindrical, 35mm diameter, 44mm long,
single planes 128 wires parallel to the $\bar{p}$-beam axis.

C10: Four 10cm x 10cm square single plane chambers,
each with 64 wires perpendicular to the beam axis,

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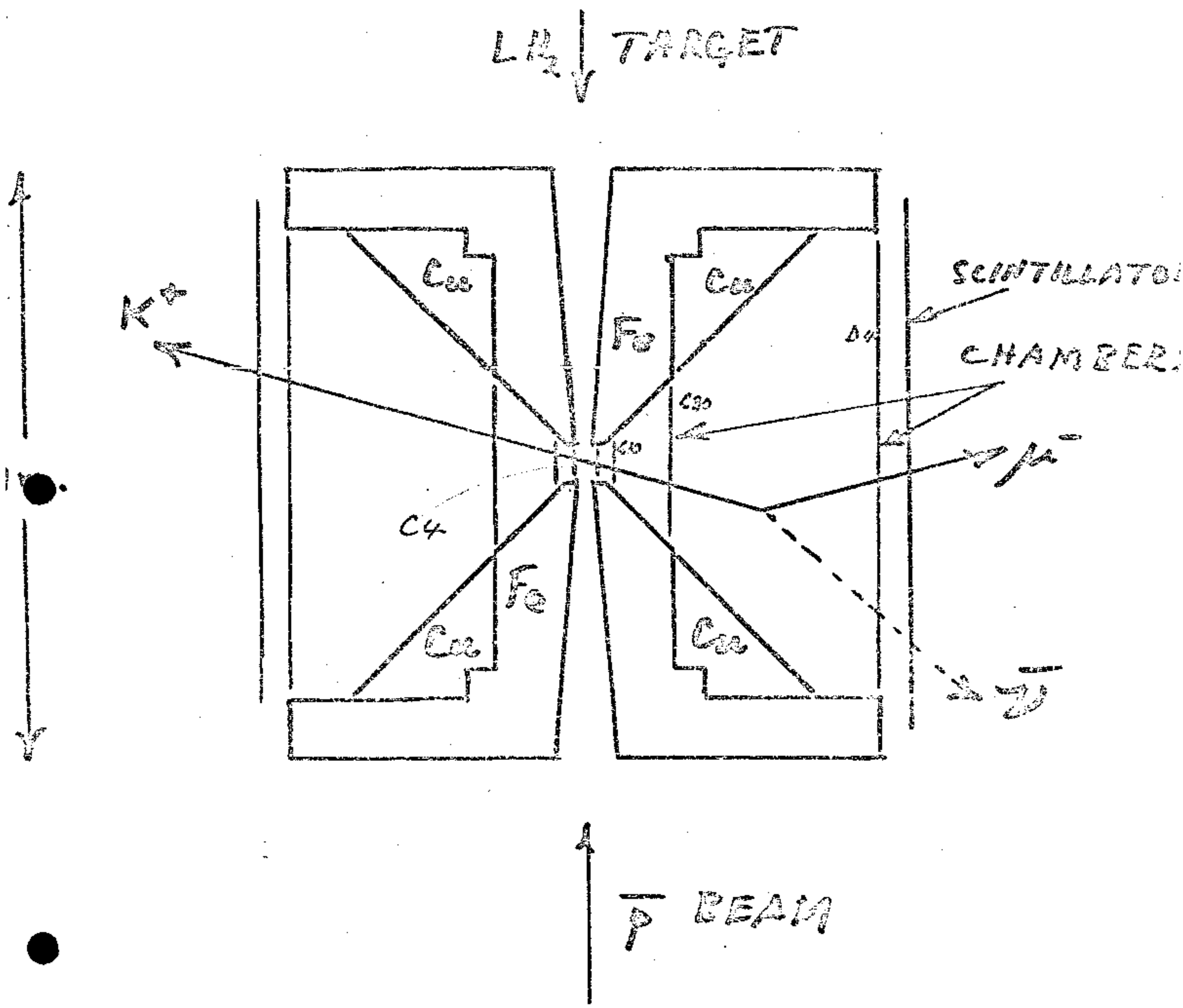
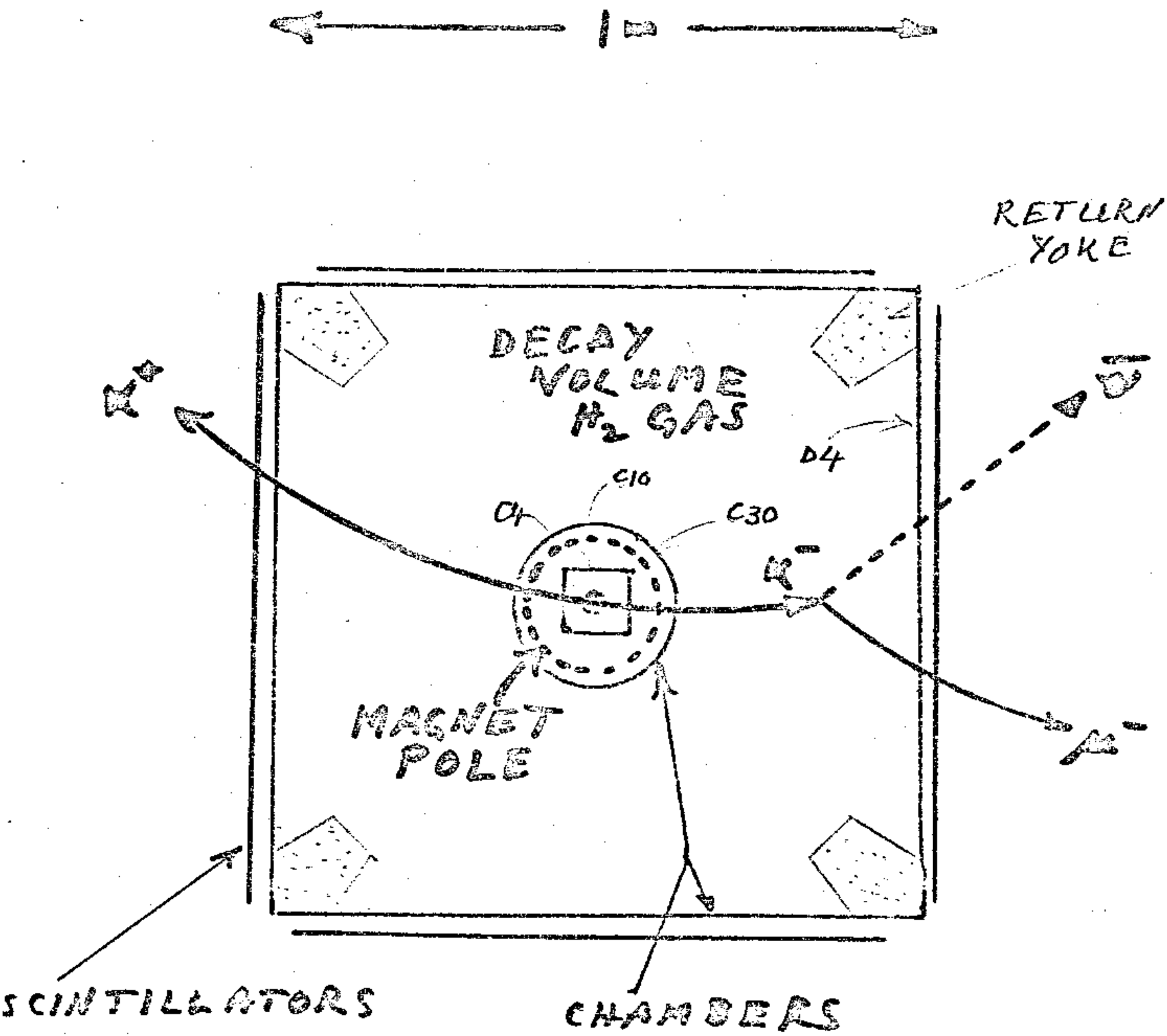
*The magnet is quite modest — ~3 tons and 1 m^3 — and it is planned to mount it on rails so that it can be moved in and out of the beam easily.



MAGNET FOR $\bar{p}p \to K^{\pm} K^{\mp}$

$\Delta\Omega \simeq \frac{1}{2} \times 4\pi$

Fig 1



MAGNET + COUNTERS FOR
$\bar{p}p \rightarrow K^{\pm}K^{\mp}$
$\quad\quad\quad\quad\quad\quad\llcorner\rightarrow$ DECAY

forming a box around the liquid hydrogen target.

The electronic arrangement has to provide hard-wired declustering and a collinear trigger defined as a coincidence of diametrically opposite wires (±1 wire) on both C4 and C10. This is the primary trigger: It is not possible to use scintillator or cherenkov counters in the trigger because of the interaction bias imposed by thick materials. The events which satisfy the collinearity trigger include K^+K^- pairs (500 per $10^6\bar{p}$), collinear $\pi^+\pi^-$ pairs (1500 per $10^6\bar{p}$), and spatially randon $\pi\pi$ pairs ($\sim$1000 per $10^6\bar{p}$). The hits on C4 and C10 together with the hits on the other chambers,

C30: Cylindrical, 30cm diameter, 30cm long, single plane, 1024 wires parallel to the $\bar{p}$-beam axis.

D4: Four 80cm x 100cm drift chambers, each with three planes of double anode wires on a 50mm pitch, two planes with wires parallel to the $\bar{p}$-beam and the third perpendular

are encoded and read out to a programmeable trigger processor which accepts the event subject to the condition:

(a) two and only two hits on C4 and C10

(b) two and only two hits on C30

(c) spacing of hits on C30 consistent with a pair of 800 MeV/c particles

(d) one and only one set of hits on D4 consistent with one member of the pair of particles recorded by C4 and C10 and with the momentum 800 MeV/c.

Conditions (a), (b) and (c) effectively suppress the spatial random triggers.

Condition (d) suppresses the true collinear $\pi^+\pi^-$ pairs and the non-decaying K^+K^- pairs: Only 2% of the $\pi^+\pi^-$ pairs suffer a decay within the volume of the apparatus and the decay muon hardly deviates from the pion direction. It appears that condition (d) can be satisfied using the wire numbers (25mm cell) only without decoding the drift-time digitization. The 20% of the K^+K^- pairs which suffer a decay usually yield just one charged particle but the direction is not closely correlated with the $K^\pm$ direction.

The net result is ~ 70 $K^\pm$ (K decaying) events per $10^6\bar{p}$ plus a few events due to chamber inefficiencies to be recorded for subsequent analysis. If the magnetic field is cylindrically symmetric the analysis would be particularly simple as momenta and charge signs can be determined by look up table without any tracking procedure.

The main purpose of the scintillators, four hodoscopes each of 10 strips with two photo-multipliers per strip, which stand outside the D4 chambers in Fig. 2 is to provide a timing signal for the drift chamber time-digitization. The scintillators are also required for setting up and calibrating the drift chambers with cosmic rays.

The data relating to the difference between K^+ and K^- interactions in chambers and other materials is given in Appendix V. On average a $K^\pm$ travels about 0.6m between the source and the D4 chambers which is 10% of the decay length at 800 MeV/c. In fact it is only the region between and including C30 and the first plane of D4 which is important for interactions: the trigger requires that the K^+K^- pair survives as far as C30, and interactions after the first plane of D4 are reduced in importance by a factor equal to the inefficiency of a D4 plane (a measured 3%).

It is planned to construct the single plane C30 with 6μ mylar windows supported by Kevlar threads, with a total mass of 3 mg/cm^2. It is estimated from the interaction data of Appendix V that the difference between K^- and K^+ for particles which record on C30 and interact in C30 is about 1×10^{-5}.

The mass of a D4 chamber between the entry window and the first anode plane is about 5mg/cm^2, giving an estimated difference between K^+ and K^- for particles which interact in D4 but do not record on the first plane of 4×10^{-5}.

It is proposed to measure the K^+/K^- interaction difference by introducing into the apparatus of Figs. 2 and 3 a sheet of material with the same composition as a chamber but 10^3 as massive, placed (a) immediately after C30, and (b) immediately before D4. There should then be a difference between the rates for K^+ (K^- "decay") and K (K^+ "decay") of several per cent, which difference can be measured to an accuracy of a few per cent. Interpretted with the help of simulation calculations the net uncertainty due to interactions in C30 and D4 should not exceed 3×10^{-6}. As the dimension of the apparatus is about one tenth of a $K^\pm$ decay length, the error in the lifetime difference would be $\sim 3 \times 10^{-5}$.

The space between C30 and D4 is filled with H_2 gas at 1 atmosphere in which the $K^\pm$ interactions will cause a difference of 7×10^{-5}. The $K^\pm p$ cross-sections are known to $\sim$ 5% accuracy and the appropriate correction will leave an uncertainty of 3.5×10^{-5} in the K^+/K^- lifetime difference.

The net uncertainty due to interactions is $\surd(3 + 3.5) \times 10^{-5}$, which is below, but not far below, the statistical error of 10^{-4} which might reasonably be achieved.

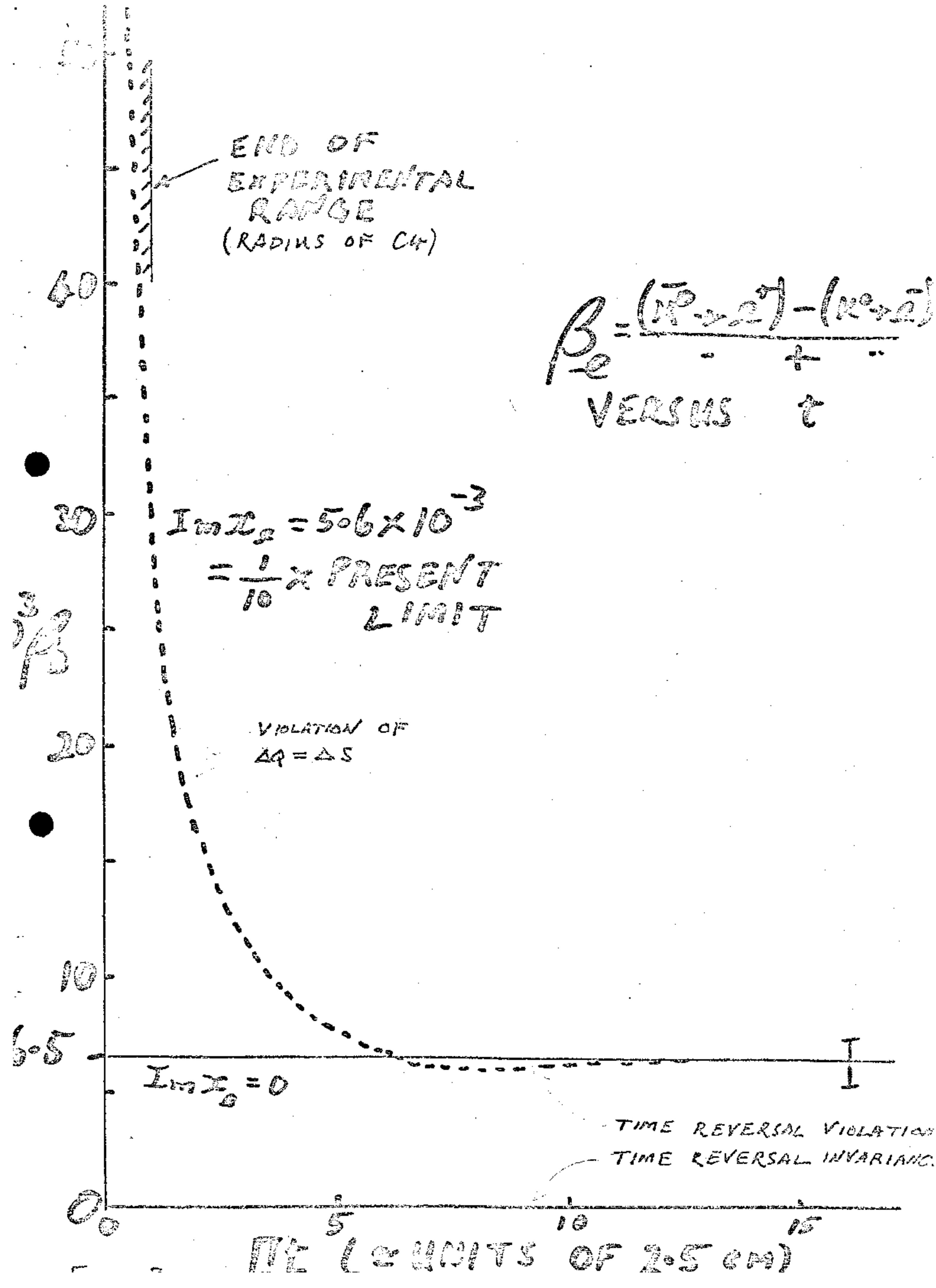
4. $K^o_{\ell 3}$ v. $\bar{K}^o_{\ell 3}$

The source of neutral kaons is $\bar{p}p \to K^{\pm}\pi^{\mp}K^o$, 3×10^{-3} per $\bar{p}$, where the K^+ (K^-) identifies $\bar{K}^o$ (K^o) by strangeness conservation, and the decays to $\pi^{\pm}e^{\mp}\nu$ can be distinguished by the charge of the electron. Thus all from rates

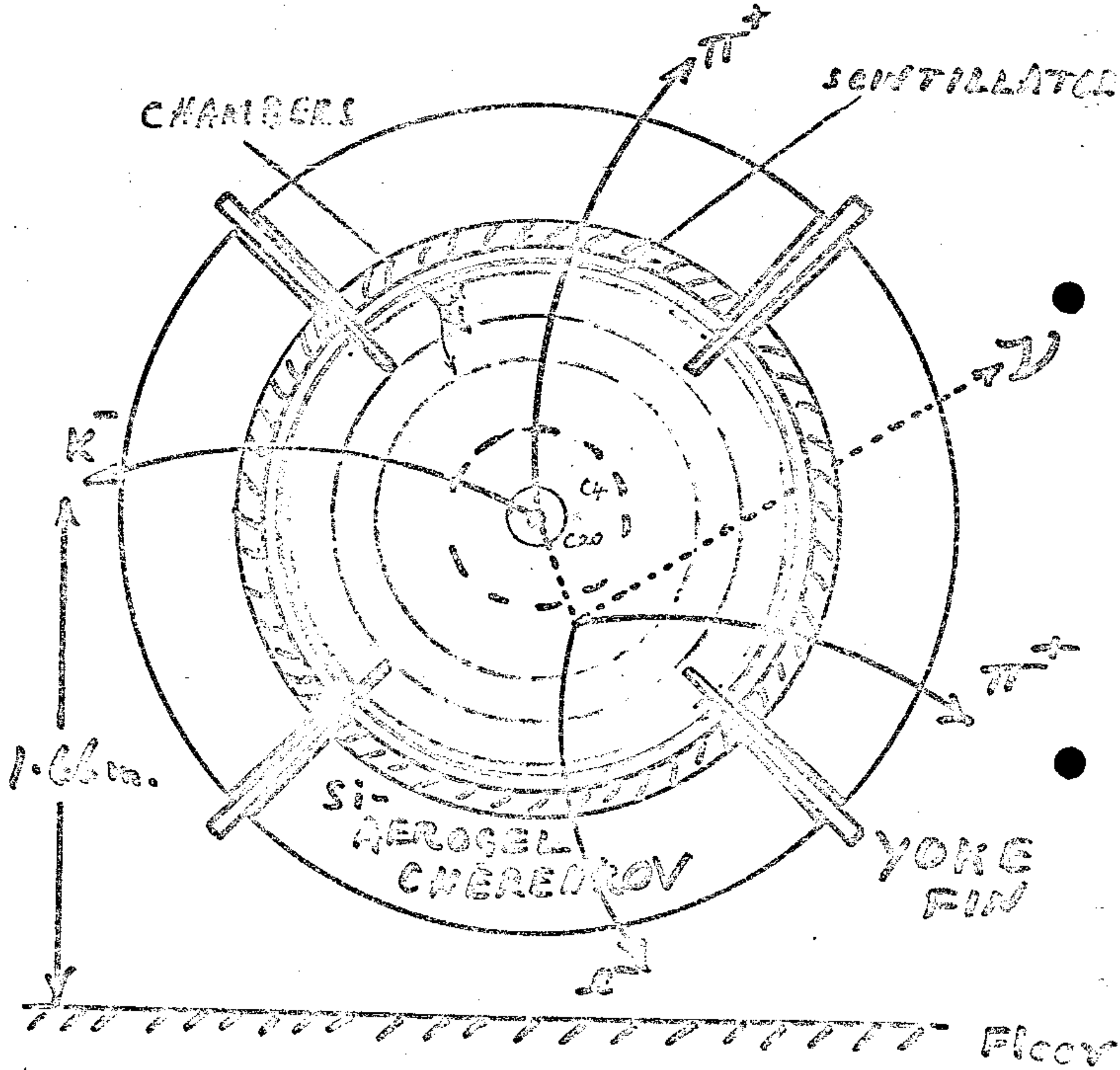
$$K^o \to e^+,\ K^o e^-,\ \bar{K}^o \to e^+,\ \bar{K}^o \to e^-$$

can be measured and the various ratios of Appendix III calculated. The main interest is the time reversal test β_e which is the difference between the rates for an (initial K^o) $\to e^-$ and an (initial $\bar{K}^o$) $\to e^+$. Providing only that $\Delta Q=\Delta S$ is satisfied then βe is expected to be independent of time, i.e. of the point of decay of $K^o \to \pi \ell \nu$ and of the momentum of the K^o. If however there is in fact a small amplitude for $\Delta Q=-\Delta S$ then βe will depend on time in the manner of Fig. 3 which is drawn for a typical K^o momentum of 500 MeV/c and assumes CPT-invariance.

The arrangement of magnet and detectors is shown schematically in Figs. 4 and 5. This is not a firm design and an alternative yoke arrangement in the form of a cylinder just outside the silica aerogel (hatched in Fig. 4) is being considered; the yoke cylinder would be pierced with holes for the Cherenkov photomultipliers and would serve as a magnetic shield. What is firm is the geometric acceptance of $45^o<\theta<135^o$ and $0<\phi<180^o$, $\Delta\Omega \simeq (2/3)4\pi$, the cylindrical large-gap magnet with >0.1 Tesla at 50cm radius, the three cylindrical chambers



END OF EXPERIMENTAL RANGE
(RADIUS OF CH)
VERSUS t
Im x = 5·6 × 10^-3
= 1/10 × PRESENT LIMIT
VIOLATION OF
ΔQ = ΔS
Im x = 0
TIME REVERSAL VIOLATION
TIME REVERSAL INVARIANC
40
30
20
10
6·5
0
5
10
15
(= UNITS OF 2·5 cm)



MAGNET + COUNTERS FOR

$\bar{p}p \rightarrow K^0 K^{\pm} \pi^{\mp}$

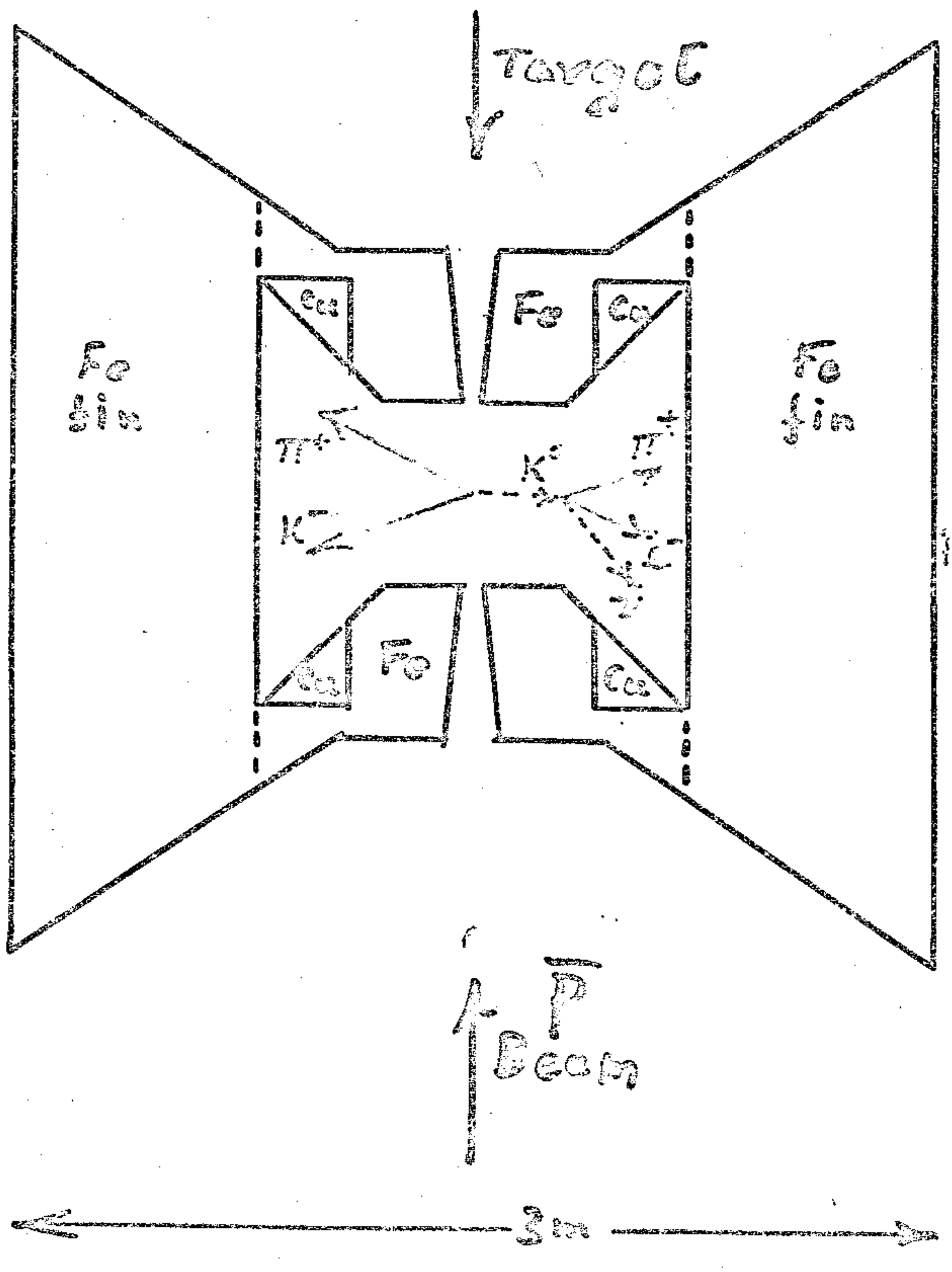
$\hookrightarrow \mu^{\pm} \nu \pi^{\mp}$

Fig 4

Magnet for $\bar{p}p \to K^0 K^{\pm} \pi^{\mp}$

$\hookrightarrow e^{\pm} \nu \pi^{\mp}$

$\Delta\Omega \sim \frac{2}{\sqrt{10}} 4\pi$



(the outer chamber being two dimensional) between 50 and 80cm from the source for determining momenta and the charge signs, and some electron identification system (scintillators and Si-a ogel Cherenkov in Fig. 4). The central region, shown as a dot in Fig. 4, includes a tiny liquid hydrogen target, a single plane cylindrical M.W. chamber of 3.5cm diameter and 128 wires (C4, as in section 3. K^+ v. K^-), and a single plane cylindrical M.W. chamber of 20cm diameter and 256 wires (C20).

The primary trigger requirement for a good event $\bar{p}p \to K^{\pm}\pi K^o$, $K^o \to \pi e \nu$, is

- two charged particles near the source i.e. two hits on C4 on non-adjacent wires,
- four charged particles in the outer detectors, beyond 50cm from the source,
- one and only one signal from the electron detector

At the zeroth level of approximation of perfectly efficient chambers, an ideal threshold Cherenkov for electrons, and no scattering or photon interactions, there is no background; the only neutral particle of appreciable decay length is the K°. In practice there are a multiplicity of background trigger sources but their sum is less than the 100 triggers per second that might reasonably be recorded for analysis, and does not swamp the 10 good triggers per second expected for $10^6 \bar{p}\ s^{-1}$.

It is necessary to identify a K° by means of the missing mass reconstructed from the $K^{\pm}\pi$ momenta. For a cylindrically symmetric magnetic field and a point source this is a very quick calculation as no tracking is required and could in principle be carried out on-line. Given the K° momentum from this calculation the momenta of the charged decay products $\pi^{\pm}e^{\mp}$ determine the neutrino missing mass and hence identify the K_{e3} decay without the need of an electron detector, but apart from decays which occur within the 20cm diameter chamber C20 there is not enough magnetic deflection to distinguish $\pi^{\pm} e^{\mp} \nu$ from $\pi^{\pm} \mu^{\mp} \nu$ or $\pi^{+}\pi^{-}\pi^{o}$ decays. It is of course the decays close to the source which are interesting for the parameters x_{e} and η_{+-o}. (see Appendices II and III).

The scintillators of Fig. 4, between the chambers and the electron identifier, are intended to provide a timing signal and to provide a record of the approximate spatial positions of charged particle tracks with good time resolution. Whether or not the scintillators can be included in the trigger depends on a determination of the K^{+}/K^{-} and π^{+}/π^{-} bias imposed by interactions. It will have been noticed that the trigger requires the observation of the K° decay products and that the comparison of decay rates depends absolutely on the assumptions that (a) the production channels $\bar{p}p \to K^{+}\bar{K}^{o}\pi^{-}$ and $\bar{p}p \to K^{-}K^{o}\pi^{+}$, are exactly equal, (b) the detection of $K^{+}\pi^{-}$ and $K^{-}\pi^{+}$ is free of bias, and (c) the detection of the decay particles $\pi^{+}e^{-}$ and $\pi^{-}e^{+}$ is also free of bias. The alternative of identifying $K^{\pm}$ independant of the K° decay and normalizing is not considered feasible because of the broad range of $K^{\pm}$ momenta (o to 750 MeV/c), the overlap both in terms of momenta and velocity with the much more numerous pions, and the interactions of both π and K in a necessarily elaborate detector. However even with an ideal detector for the K^{O} decay electron it will probably be necessary to obtain supplementary information concerning the $K^{\pm} \pi^{\mp}$ because of the sign ambiguity which can occur when reconstructing the K^{O} missing mass from similar $K^{\pm}$ and $\pi^{\mp}$ momenta. The most promising prospect is time of flight: typically there is a 2 ns difference in time between the arrival of $\pi^{\pm}$ and $K^{\mp}$ at the scintillators, i.e. a 4 ns difference between the alternative interpretations, $K^+\pi^-$ and $K^-\pi^+$, of a particular event. It should be added that a false interpretation would dilute the time reversal signal but would not create a false assymetry, providing the interactions in the scintillators can be controlled using the kinematically unambiguous events.

The evolution of a neutral K system characterized by time-dependent amplitudes a and $\bar{a}$ for the $|K\rangle$ and $|\bar{K}\rangle$ components, respectively, is given by

$$-\frac{d}{dt}\begin{pmatrix} a \\ \bar{a} \end{pmatrix} = (iM + \tfrac{1}{2}\Gamma)\begin{pmatrix} a \\ \bar{a} \end{pmatrix},$$

where M and Γ are each Hermitian matrices, and t is the time measured in the rest system of the K meson. Expressed in terms of their elements the matrices are

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{pmatrix} \text{ and } \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix}.$$

The matrix $iM + \frac{1}{2}\Gamma$ has eigenvalues $\gamma_S = iM_S + \frac{1}{2}\Gamma_S$ and $\gamma_L = iM_L + \frac{1}{2}\Gamma_L$. We define small parameters

$$\varepsilon = (-\operatorname{Im} M_{12} + i\operatorname{Im}\Gamma_{12}/2)/(\gamma_S - \gamma_L)$$

and

$$\Delta = [i(M_{11} - M_{22}) + (\Gamma_{11} - \Gamma_{22})/2]/[2(\gamma_S - \gamma_L)].$$

We can then express the eigenvectors as

$$|K_S\rangle = \frac{1}{\sqrt{2}}\,\frac{1}{(1+|\varepsilon+\Delta|^2)^{1/2}}\,[(1+\varepsilon+\Delta)|K\rangle + (1-\varepsilon-\Delta)|\bar{K}\rangle]$$

and

$$|K_L\rangle = \frac{1}{\sqrt{2}}\,\frac{1}{(1+|\varepsilon-\Delta|^2)^{1/2}}\,[(1+\varepsilon-\Delta)|K\rangle - (1-\varepsilon+\Delta)|\bar{K}\rangle].$$

The parameter ε represents a CP violation with T nonconservation. The parameter Δ represents a CP violation with CPT nonconservation.

If we form a state $|K(t)\rangle$ which is an arbitrary superposition of $|K_S\rangle$ and $|K_L\rangle$ with amplitudes a_S and a_L at $t=0$, we can compute its norm $\langle K(t)|K(t)\rangle$ as a function of time. At $t=0$ by conservation of probability we have the relation

$$-\frac{d}{dt}\langle K(t)|K(t)\rangle\Big|_{t=0} = \sum_f |a_S \operatorname{am}(K_S \to f) + a_L \operatorname{am}(K_L \to f)|^2,$$

where f represents the set of final states. Explicit evaluation of the expression gives

$$[-i(M_S - M_L) + (\Gamma_S + \Gamma_L)/2]\langle K_S|K_L\rangle = \sum_f [\operatorname{am}(K_S \to f)]^*[\operatorname{am}(K_L \to f)].$$

A number of definitions and a particular phase convention are used. We define $\tilde{\Delta} = \Delta - (A_0 - \bar{A}_0)/(A_0 + \bar{A}_0)$, where A_0 and $\bar{A}_0$ are the standing-wave amplitudes for K and $\bar{K}$, respectively, to decay to the $I=0$ state of two pions. A_0 and $\bar{A}_0$ are chosen real and define the phase convention used in the analysis. From the experimental parameters we define

$$\varepsilon_0 = \tfrac{2}{3}\eta_{+-} + \tfrac{1}{3}\eta_{00} \quad\text{and}\quad \varepsilon_2 = \tfrac{1}{3}\sqrt{2}\,(\eta_{+-} - \eta_{00}),$$

and

$$\alpha(f) = (1/\Gamma_S)[\operatorname{am}(K_S \to f)]^*[\operatorname{am}(K_L \to f)].$$

With these definitions we find to a good approximation that

$$(-i\Delta M/\Gamma_S + \tfrac{1}{2})(2\operatorname{Re}\varepsilon - 2i\operatorname{Im}\tilde{\Delta}) = \varepsilon_0 + \sum_f \alpha(f) \qquad \text{(A1)}$$

and

$$\varepsilon - \tilde{\Delta} = \varepsilon_0 . \qquad \text{(A2)}$$

The sum over f, which now excludes the $I=0$ π-π state, consists of the following terms:

$$\alpha(\pi\pi, I=2) = A_2/A_0\, e^{i(\delta_2-\delta_0)}\varepsilon_2^*$$

$$\alpha(\pi^0\pi^0\pi^0) = [\Gamma(K_L \to \pi^0\pi^0\pi^0)/\Gamma_S]\eta_{000}^*,$$

$$\alpha(\pi\mu\nu) = [\Gamma(K_L \to \pi\mu\nu)/\Gamma_S]2i\operatorname{Im} x_\mu,$$

and

$$\alpha(\pi e\nu) = [\Gamma(K_L \to \pi e\nu)/\Gamma_S]2i\operatorname{Im} x_e,$$

where

$$\eta_{+-0} = \operatorname{am}(K_S \to \pi^+\pi^-\pi^0)/\operatorname{am}(K_L \to \pi^+\pi^-\pi^0),$$

$$\eta_{000} = \operatorname{am}(K_S \to \pi^0\pi^0\pi^0)/\operatorname{am}(K_L \to \pi^0\pi^0\pi^0),$$

and x_l is the ratio, $\operatorname{am}(\Delta Q = -\Delta S)/\operatorname{am}(\Delta Q = \Delta S)$, for $K \to \pi l \nu_l$. The quantities η_{+-0} and η_{000} are CP-violating ratios. (The final state $\pi^+\pi^-\pi^0$ can be CP even or odd. Here we refer only to the odd state.) The measurements of η_{000} and η_{+-0} are not at present very accurate and are consistent with zero. If we use the experimental limits which exist (Particle Data Group, 1980), we find

$$\operatorname{Re}\alpha = \operatorname{Re}\sum_f \alpha(f) = (0.14 \pm 0.19)\times 10^{-3}$$

and

$$\operatorname{Im}\alpha = \operatorname{Im}\sum_f \alpha(f) = (-0.19 \pm 0.25)\times 10^{-3}.$$

The equations (A1) and (A2) take a very simple form if we resolve the components of ε and $\tilde{\Delta}$ parallel and perpendicular to the direction which makes an angle ϕ_n with the real axis, where

$$\phi_n = \tan^{-1}\left(-\frac{2(M_S - M_L)}{(\Gamma_S - \Gamma_L)}\right).$$

We then find

$$\varepsilon_\parallel = \varepsilon_{0_\parallel} + \cos\phi_n \operatorname{Re}\alpha,$$

$$\varepsilon_\perp = -\cos\phi_n \operatorname{Im}\alpha,$$

$$\tilde{\Delta}_\parallel = \cos\phi_n \operatorname{Re}\alpha,$$

and

$$\tilde{\Delta}_\perp = -\varepsilon_{0_\perp} - \cos\phi_n \operatorname{Im}\alpha.$$

The experimental values of $\varepsilon_{0_\parallel}$ and $\varepsilon_{0_\perp}$ are, respectively, $(2.27 \pm 0.03)\times 10^{-3}$ and $(0.16 \pm 0.09)\times 10^{-3}$. We then find

$$\varepsilon_\parallel = (2.37 \pm 0.19)\times 10^{-3},$$

$$\varepsilon_\perp = (0.14 \pm 0.18)\times 10^{-3},$$

$$\tilde{\Delta}_\parallel = (0.10 \pm 0.14)\times 10^{-3},$$

and

$$\tilde{\Delta}_\perp = (-0.02 \pm 0.20)\times 10^{-3}.$$

Within the present experimental limits we find that all the measurements are consistent with T violation and CPT conservation. In particular, we see that the limit on $\varepsilon_\perp$ is very small, so that we cannot expect ϕ_{+-} and φ_{00} to differ greatly from ϕ_n. Further, if the values of η_{000}, η_{+-0}, x_e, and x_μ were $<10^{-2}$, then we would find $|\varepsilon_\perp| \leq 10^{-5}$. Such an expectation is reasonable if the strength of the CP violation is roughly the same in all modes.

James W. Cronin:

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Appendix I.

Appendix II: The Unitarity Relation[2]

In the notation of Cronin[1] (Appendix I) the general state of a neutral kaon is

$$| K(t)\rangle = a_s | K_s\rangle e^{-\gamma_s t} + a_L | K_L\rangle e^{-\gamma_L t}$$

which at t = 0 decays at a rate

$$\{-\frac{d}{dt} \langle K(t) | K(t)\rangle\}_{t=0} = \Gamma_s | a_s |^2 + \Gamma_L |a_L|^2 + [i(M_s - M_L)$$

$$+ \tfrac{1}{2}(\Gamma_s + \Gamma_L)][a_s a_L^* \langle K_L|K_s\rangle]$$

$$+ [-i(M_s - M_L) + \tfrac{1}{2}(\Gamma_s + \Gamma_L)][a_s^* a_L \langle K_s|K_L\rangle]$$

which, if probability is conserved, is equal to the transition rate at t = 0,

$$\sum_f | a_s \langle f|T|K_s\rangle + a_L \langle f|T|K_L\rangle|^2$$

Now $\Gamma_s = \sum_f |\langle f|T|K_s\rangle|^2$ and $\Gamma_L = \sum_f |\langle f|T|K_L\rangle|^2$ which leaves the unitarity relation

$$[-i(M_s - M_L) + \tfrac{1}{2}(\Gamma_s + \Gamma_L)] \langle K_s|K_L\rangle$$

$$= \sum_f \langle f|T|K_s\rangle^* \langle f|T|K_L\rangle$$

where the density of states factors have been absorbed into the normalisation of the final states $|f\rangle$.

Expanding $|K_s\rangle$ and $|K_L\rangle$ in terms of the orthogonal states $|K_o\rangle$ and $|\bar{K}_o\rangle$ gives

$$\langle K_s \mid K_L\rangle = 2\mathrm{Re}\varepsilon - 2\, i\mathrm{Im}\,\Delta$$

which agrees with Cronin (eqn. (A1)) since Im Δ = Im $\widetilde{\Delta}$. The interesting parameter is Δ representing a T-invariant and CP- and CPT- non-invariant interaction and we would like to evaluate Δ in terms of experimentally measured quantities, without further assumptions.

The lepton assymetry δ_ℓ for the decays $K^\circ, \bar{K}^\circ \to \ell^\pm \pi\, \nu$ is given by

$$\begin{aligned}\delta_\ell &= 2\mathrm{Re}(\varepsilon-\Delta)-\mathrm{Re}\; y_\ell + \mathrm{Re}(\bar{x}_\ell - x_\ell)\\ &= 2\mathrm{Re}(\varepsilon-\Delta)-\mathrm{Re}\; Z_\ell\end{aligned}$$

as derived in Appendix III.

We can thus write

$$\begin{aligned}\langle K_s|K_L\rangle &= 2\mathrm{Re}\varepsilon - 2\, i\mathrm{Im}\,\Delta\\ &= \delta_\ell + 2\mathrm{Re}\Delta - 2\, i\mathrm{Im}\Delta + \mathrm{Re}\; Z_\ell\\ &= \delta_\ell + 2\,\Delta^* + \mathrm{Re}\; Z_\ell\end{aligned}$$

Following Cronin the unitarity relation for Δ can now be expressed in terms of measured parameters:

$$\begin{aligned}[-i(M_s-M_L) + \tfrac{1}{2}(\Gamma_s+\Gamma_L)]\; [\delta_\ell + 2\Delta^* + \mathrm{Re}\; Z_\ell]\\ = \eta_{+-}\;\Gamma(K_s \to \pi^+\pi^-) + \eta_{oo}\;\Gamma(K_s \to \pi^o\pi^o)\\ + \eta^*_{+-o}\;\Gamma(K_L \to \pi^-\pi^+\pi^o) + \eta^*_{ooo}\;\Gamma(K_L \to \pi^o\pi^o\pi^o)\\ + (\delta_e + 2\Delta^* + x^*_e - \bar{x}_e)\;\Gamma(K_L \to \pi e\nu)\\ + (\delta_\mu + 2\Delta^* + x^*_\mu - \bar{x}_\mu)\;\Gamma(K_L \to \pi\mu\nu)\end{aligned}$$

which, using the values of the parameters tabulated by the Particle Data

Group, yields

$$\Delta + \tfrac{1}{2}\mathrm{Re}\, Z_\ell = \{(-0.02 \pm 0.18) + (0.03 \pm 0.17)i\} \times 10^{-3}$$

and $\varepsilon = \{(1.63 \pm 0.19) + (1.72 \pm 0.18)i\} \times 10^{-3}$.

Using the approximation $(M_L - M_s) = (\Gamma_s - \Gamma_L)/2$ we obtain

$$\frac{M_{11} - M_{22}}{M_L - M_s} = 2\,(\mathrm{Im}\Delta - \mathrm{Re}\Delta) = (0.10 \pm 0.50) \times 10^{-3} + \mathrm{Re}\, Z_\ell$$

and $$\frac{\Gamma_{11} - \Gamma_{22}}{\Gamma_s - \Gamma_L} = 2(\mathrm{Re}\Delta + \mathrm{Im}\Delta) = (0.02 \pm 0.50) \times 10^{-3} - \mathrm{Re}\, Z_\ell$$

where $M_{11} - M_{22}$ and $\Gamma_{11} - \Gamma_{22}$ are respectively the differences of masses and widths of K° and $\bar{K}^\circ$. The two standard deviation limit for T-invariance and CP- and CPT- violation is then

$$\frac{M_{11} - M_{22}}{M_L - M_s} < 72 \times 10^{-3}$$

$$\frac{\Gamma_{11} - \Gamma_{22}}{\Gamma_s - \Gamma_L} < 72 \times 10^{-3}$$

where the experimental limit $|\mathrm{Re}\, Z_\ell| < 72 \times 10^{-3}$ has been used.+ The parameter $Z_\ell = y_\ell + x_\ell - \bar{x}_\ell$ represents a possible difference of the partial widths of K° and $\bar{K}^\circ$ for decay to $\pi\ell\nu$, and makes the greatest contribution by far to the limit that can be imposed on Δ and hence the validity of CPT invariance in terms of the mass and width differences of K° and $\bar{K}^\circ$. However it should be stressed that all of the experimental evidence is consistent with CPT invariance as is shown by Cronin in Appendix I .

+ See Appendix VI

Appendix III: Ratios for $K^o \to \pi l \nu$

It follows from the formulation of Appendix I that a neutral kaon which at t=o is in a state $\psi(t=o) = |K^o\rangle$ develops in time as

$$\psi(t) = \tfrac{1}{2}\{(1 + 2\Delta)|K^o\rangle + (1 - 2\varepsilon)|\overline{K}^o\rangle\}e^{-\gamma_S t}$$

$$+ \tfrac{1}{2}\{(1 - 2\Delta)|K^o\rangle - (1 - 2\varepsilon)|\overline{K}^o\rangle\}e^{-\gamma_L t}$$

Similarly a state $\overline{\psi}$ (t=o) = $|\overline{K}^o\rangle$ develops,

$$\overline{\psi}(t) = \tfrac{1}{2}\{(1 + 2\varepsilon)|K^o\rangle + (1 - 2\Delta)|\overline{K}^o\rangle\} e^{-\gamma_S t}$$

$$- \tfrac{1}{2}\{(1 + 2\varepsilon)|K^o\rangle - (1 + 2\Delta)|\overline{K}^o\rangle\}e^{-\gamma_L t}$$

In the notation of Gjesdal, et.al.[9] modified to allow for a possible CPT violation ($f \neq \bar{f}$)

$$\langle\pi^- \ell^+ \nu|T|K^o\rangle = f \qquad \langle\pi^- \ell^+ \nu|T|\overline{K}^o\rangle = g$$

$$\langle\pi^+ \ell^- \nu|T|K^o\rangle = \bar{g}^* \qquad \langle\pi^+ \ell^- \nu|T|\overline{K}^o\rangle = \bar{f}^*$$

and the small parameter $x_\ell = g/f$ measures the amplitude for the $\Delta S = -\Delta Q$ transitions relative to the $\Delta S = \Delta Q$ transitions.

The decay rates for ψ and $\bar{\psi}$ to $\ell^\pm$ are (dropping a factor of 1/4) proportional to:

$$\begin{array}{lcc} & \psi & \bar{\psi} \\ \ell^+: & |f(1+2\Delta+x_\ell)e^{-\gamma_S t}+f(1-2\Delta-x_\ell)e^{-\gamma_L t}|^2; & |f(1+2\varepsilon+x_\ell)e^{-\gamma_S t}-f(1+2\varepsilon-x_\ell)e^{-\gamma_L t}|^2 \\ \ell^-: & |\bar{f}^*(1-2\varepsilon+\bar{x}_\ell^*)e^{-\gamma_S t}-\bar{f}^*(1-2\varepsilon-\bar{x}_\ell^*)e^{-\gamma_L t}|^2; & |\bar{f}^*(1-2\Delta+\bar{x}_\ell^*)e^{-\gamma_S t}+\bar{f}^*(1+2\Delta-\bar{x}_\ell^*)e^{-\gamma_L t}|^2 \end{array}$$

For $\gamma_S t \gg 1$ and dropping the factor $|e^{-\gamma_L t}|^2$ we have the decay rates

$$\begin{array}{lcc} & \psi & \bar{\psi} \\ \ell^+: & |f|^2(1 - 4\mathrm{Re}\Delta - 2\mathrm{Re}\, x_\ell) & |f|^2(1 + 4\mathrm{Re}\varepsilon - 2\mathrm{Re}\, x_\ell) \\ \ell^-: & |\bar{f}|^2(1 - 4\mathrm{Re}\varepsilon - 2\mathrm{Re}\, \bar{x}_\ell) & |\bar{f}|^2(1 + 4\mathrm{Re}\Delta - 2\mathrm{Re}\, \bar{x}_\ell) \end{array}$$

We can construct six differences for $\gamma_s t \gg 1$ for particles which at t=o are explicitly K° or $\bar{K}^\circ$. Putting $\bar{f} = f(1 + y_\ell)$ we have,

$$\delta_\ell = \frac{(K^\circ \to \ell^+) - (K^\circ \to \ell^-)}{(K^\circ \to \ell^+) + (K^\circ \to \ell^-)} = 2\ \mathrm{Re}\ (\varepsilon - \Delta) - \mathrm{Re}\ y_\ell + \mathrm{Re}(\bar{x}_\ell - x_\ell)$$

$$\bar{\delta}_\ell = \frac{(\bar{K}^\circ \to \ell^+) - (\bar{K}^\circ \to \ell^-)}{(\bar{K}^\circ \to \ell^+) + (\bar{K}^\circ \to \ell^-)} = 2\ \mathrm{Re}\ (\varepsilon - \Delta) - \mathrm{Re}\ y_\ell + \mathrm{Re}\ (\bar{x}_\ell - x_\ell)$$

$$\beta_\ell = \frac{(\bar{K}^\circ \to \ell^+) - (K^\circ \to \ell^-)}{(\bar{K}^\circ \to \ell^+) + (K^\circ \to \ell^-)} = 4\ \mathrm{Re}\ \varepsilon - \mathrm{Re}\ y_\ell + \mathrm{Re}\ (\bar{x}_\ell - x_\ell)$$

$$\alpha_\ell = \frac{(\bar{K}^\circ \to \ell^-) - (K^\circ \to \ell^+)}{(\bar{K}^\circ \to \ell^-) + (K^\circ \to \ell^+)} = 4\ \mathrm{Re}\ \Delta + \mathrm{Re}\ y_\ell - \mathrm{Re}(\bar{x}_\ell - x_\ell)$$

$$\gamma_\ell^+ = \frac{(\bar{K}^\circ \to \ell^+) - (K^\circ \to \ell^+)}{(\bar{K}^\circ \to \ell^+) + (K^\circ \to \ell^+)} = 2\ \mathrm{Re}\ (\varepsilon + \Delta)$$

$$\gamma_\ell^- = \frac{(\bar{K}^\circ \to \ell^-) - (K^\circ \to \ell^-)}{(\bar{K}^\circ \to \ell^-) + (K^\circ \to \ell^-)} = 2\ \mathrm{Re}\ (\varepsilon + \Delta)$$

δ_ℓ has been measured[9] for $e^\pm$ and $\mu^\pm$ yielding an average value of $(3.30 \pm 0.12) \times 10^{-3}$ and exhibiting the time dependence expected.

α_ℓ is expected to be zero and the unitarity relation (Appendix II) imposes an upper limit of $|\alpha_\ell|$ $< 1.52 \times 10^{-3}$. A non-zero value of α_ℓ would indicate a failure of CPT invariance.

The prediction for $\beta_\ell = 4\ \mathrm{Re}\ \varepsilon = (6.52 \pm 0.76) \times 10^{-3}$ and, providing $\bar{x}_\ell$ $x_\ell = 0$, this ratio should be independent of t as was pointed out by Kabir[10] many years ago. With the full time dependence and $x_\ell = \bar{x}_\ell$,

$$\beta_\ell = 4\ \mathrm{Re}\ \varepsilon + 4(\mathrm{Im}\ x_\ell) \left\{ \frac{\sin\ (M_L - M_S)\ t}{e^{\frac{1}{2}(\Gamma_S - \Gamma_L)t} + e^{-\frac{1}{2}(\Gamma_S - \Gamma_L)t} - 2\cos\ (M_L - M_S)t} \right\} - \mathrm{Re}\ y_\ell$$

The coefficient of 4 Im x_ℓ is ~ 1 for $\Gamma_s t = 1$, ~ 0.2 for $\Gamma_s t = 3$, varies as $\sim(\Gamma_s t)^{-1}$ for small t, and rapidly becomes small for large t. The present limit for Im x_ℓ and Re y_ℓ and

$$|\mathrm{Im}\ x_\ell| < 56 \times 10^{-3},\ |\mathrm{Re}\ y_\ell| < 72 \times 10^{-3},$$

which for an observation in the region $1 < \Gamma_s t < 3$ would give

$$\beta_\ell = 6.5 \times 10^{-3} \pm (<100 \times 10^{-3}) \pm (< \quad \mp 2 \times 10^{-3})$$

which is a large effect in a small region. If $y_\ell = 0$ as assumed in ref.12, then a non-zero value of β_ℓ for $\Gamma_s t > 5$ is an unambiguous demonstration of a violation of time reversal invariance, since β_ℓ depends on ε but not on Δ. Conversely a non-zero value of α_ℓ would indicate a failure of CPT- but not T-invariance. It is noted that α_ℓ, β_ℓ and γ_ℓ are subject to the same experimental bias due to the $K^\circ/\bar{K}^\circ$ and K^-/K^+ interactions, but that δ_ℓ and $\bar{\delta}_\ell$ are independent of this bias but sensitive to the $\pi^\pm$ interaction bias.

A value of $\beta_\ell \neq o$ means that the rate for $K^\circ \to \bar{K}^\circ$ is not equal to the rate for $\bar{K}^\circ \to K^\circ$. Kabir(10) illustrates the time reversal significance by considering the rate for the double reaction,

$$K^- p \to \bar{K}^\circ n \quad ; \quad \bar{K}^\circ \to K^\circ \quad ; \quad K^\circ p \to K^+ n$$

and the inverse

$$K^+ n \to K^\circ p \quad ; \quad K^\circ \to \bar{K}^\circ \quad ; \quad \bar{K}^\circ n \to K^- p$$

If $K^\circ \rightleftarrows \bar{K}^\circ$ were not equal then there would be a failure of detailed balance and therefore of time reversal invariance.

It will have been noticed that the discussion above differs from conventional wisdom to the extent that we have not assumed $f = \bar{f}$, i.e. $\Gamma(K^\circ \to \pi^- \ell^+ \nu) = \Gamma(\bar{K}^\circ \to \pi^+ \ell^- \nu)$. This equality follows in the first order of perturbation theory from CPT- invariance[2,13] and it is believed that the final state Coulomb interaction is not important, but we have not been able to find a good theoretical reason for putting $f = \bar{f}$ without assuming CPT. Experimentally the relative magnitudes of the partial widths for K° and $\bar{K}^\circ$ are not at all well determined which leaves a large unknown parameter Re y_ℓ in the interpretation.

Appendix IV: The $K^\circ/\bar{K}^\circ$ sensitivity to η_{+-o}

In the notation of Appendix I a neutral kaon which at t=o is in a state $\psi(t=o) = |K^\circ\rangle$ develops in time as

$$\psi(t) = \frac{1}{\sqrt{2}}(1 - \varepsilon + \Delta)|K_S\rangle e^{-\gamma_S t} + \frac{1}{\sqrt{2}}(1 - \varepsilon - \Delta)|K_L\rangle e^{-\gamma_L t}$$

and similarly $\bar{\psi}$ (t=o) = $|\bar{K}^\circ\rangle$ develops as

$$\bar{\psi}(t) = \frac{1}{\sqrt{2}}(1 + \varepsilon - \Delta)|K_S\rangle e^{-\gamma_S t} - \frac{1}{\sqrt{2}}(1 + \varepsilon + \Delta)|K_L\rangle e^{-\gamma_L t}$$

Absorbing the density of states factor in the final state normalization the decay rate $R(t)_{+-o}$ for the state $\psi(t)$ to $\pi^+\pi^-\pi^o$ is

$$R(t)_{+-o} = |\langle\pi^+\pi^-\pi^o|T|\psi(t)\rangle|^2$$

and similarly for $\bar{\psi}(t)$

$$\bar{R}(t)_{+-o} = |\langle\pi^+\pi^-\pi^o|T|\bar{\psi}(t)\rangle|^2$$

Defining as usual

$$\langle\pi^+\pi^-\pi^o|T|K_S\rangle = \eta_{+-o}\langle\pi^+\pi^-\pi^o|T|K_L\rangle$$

we obtain

$$\gamma_\pi = \left(\frac{\bar{R} - R}{\bar{R} + R}\right)_{+-o} \approx 2\,\mathrm{Re}\,(\varepsilon + \Delta) - 2\,(\mathrm{Re}\,\eta_{+-o}\cos z - \mathrm{Im}\,\eta_{+-o}\sin z)e^{-z}$$

where $z = \frac{1}{2}\Gamma_S t$ and the approximation $M_L - M_S \approx \frac{1}{2}(\Gamma_S - \Gamma_L) \approx \frac{1}{2}\Gamma_S$ has been used. The present limit on η_{+-o} is $|\eta_{+-o}|^2 < 0.12$ which is interpreted as Re or Im $\eta_{+-o} < 0.3$. Thus there is a possibility of a quite gross effect due to η_{+-o} at small z.

Experimentally it may be difficult[+] to distinguish the decay branch

$$K_L \rightarrow \pi^+ \pi^- \pi^0 \qquad 12\%$$

from $K_L \rightarrow \pi^\pm \mu^\mp \nu \qquad 27\%$

for neutral kaons of $\sim$500 MeV/c. However for large z the $\pi\mu\nu$ assymetry

$$\gamma_\ell = \tfrac{1}{2}(\gamma_\ell^+ + \gamma_\ell^-)$$

$$= 2 \text{ Re } (\varepsilon + \Delta), \text{ see Appendix III}$$

which is $= \left(\frac{\bar{R} - R}{\bar{R} + R}\right)_{+-0}$ above.

For arbitrary z

$$\gamma_\ell = 2 \text{ Re}\varepsilon \,(1 - \cos z/\cosh z) + 2 \text{ Re } \Delta \tanh z$$
$$+ 2 \text{ Im } \Delta \sin z/\cosh z$$

if the $\Delta Q = -\Delta s$ amplitude $x_\ell = 0$.

As $z \rightarrow 0$, $\gamma_\ell \rightarrow 0$; and as $z \rightarrow \infty$, $\gamma_\ell \rightarrow 2 \text{ Re } (\varepsilon + \Delta)$.

For the sake of simplicity (and because it is probably true) we consider $\Delta = 0$ as well as $x_\ell = 0$, and calculate the assymetry ratio for $K^\circ/\bar{K}^\circ$ decaying to $\pi^+\pi^-\pi^0$ and unresolved $\pi^\pm\mu^\mp\nu$ i.e. two charged particles detected, neither of which triggers the electron identifier and which together do not have an invariant mass of M_K. (A small, sign independent, inefficiency of the electron identification is merely equivalent to a slight change in the $\pi\mu\nu$ branching ratio). We define the inclusive ratio as

$$\gamma_{\pi\nu} = \frac{\bar{R}_\pi + \bar{R}_\mu - R_\pi - R_\mu}{\bar{R}_\pi + \bar{R}_\mu + R_\pi + R_\mu}$$

where the subscript π stands for $\pi^+\pi^-\pi^0$ and μ for $\pi^\pm \mu^\mp \nu$. Now

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[+]This is an excessively pessimistic assumption. For most events it should be possible to decide between a neutral missing mass of 135 MeV, π°, and zero, ν.

$$\bar{R}_\pi + R_\pi = |\langle\pi^+\pi^-\pi^o|T|K_L\rangle|^2 \, e^{-\Gamma_L t}$$

$$\bar{R}_\pi - R_\pi = \gamma_\pi(\bar{R}_\pi + R_\pi)$$

$$\bar{R}_\mu + R_\mu = |f|^2(e^{-\Gamma_S t} + e^{-\Gamma_L t}) = |\langle\pi^-\mu^+\nu|T|K^o\rangle|^2(e^{-\Gamma_S t} + e^{-\Gamma_L t})$$

$$\bar{R}_\mu - R_\mu = \gamma_\mu(\bar{R}_\mu + R_\mu)$$

Since $\Gamma_S >> \Gamma_L$ we can write in terms of $z = \frac{1}{2}\Gamma_S t$

$$\gamma_{\pi\mu} = \frac{\gamma_\pi(B_\pi/B_\mu)e^{+z} + 2\gamma_\ell \cosh z}{(B_\pi/B_\mu)e^z + 2\cosh z}$$

where $B_\pi/B_\mu = 0.124/0.271 = 0.46$ is the ratio of the K_L branching ratios to $\pi^+\pi^-\pi^o$ and to $\pi^\pm \mu^\mp \nu$. For large z

$$\gamma_{\pi\mu} = \frac{\gamma_\pi(B_\pi/B_\mu) + \gamma_\ell}{(B_\pi/B_\mu) + 1} = 2 \text{ Re } \varepsilon \simeq 3.3 \times 10^{-3}$$

For $z << 1$ $\quad \gamma_\ell \simeq 2z^2 \text{ Re } \varepsilon$

and $\gamma_\pi \simeq 2 \text{ Re } \varepsilon - 2(1-z)(\text{Re } \eta - z \text{ Im } \eta)$

$$\gamma_{\pi\mu} = \frac{\frac{B_\pi}{B_\mu}\{2 \text{ Re } \varepsilon - 2(1-z)(\text{Re } \eta - z \text{ Im } \eta)\}(1+z) + 4z^2 \text{ Re } \varepsilon}{(B_\pi/B_\mu)(1+z) + 2}$$

$$\rightarrow \frac{2(B_\pi/B_\mu) \text{ Re } (\varepsilon - \eta)}{(B_\pi/B_\mu) + 2} \qquad \text{as } z \rightarrow 0$$

$$= 0.37 \text{ Re } (\varepsilon - \eta)$$

There is an experimentally accessible region, roughly $0.5 < z < 1$, i.e. one to two K_S decay lengths, where an $|\eta| >> |\varepsilon|$ would show up very clearly:-

$$\gamma_{\pi\mu} \sim 3.1 \times 10^{-3} + 100 \times 10^{-3} (\text{Im } \eta - \text{Re } \eta)$$

in the region of interest. Statistically it might be possible to put a limit on $\gamma_{\pi\mu}$ of $<3 \times 10^{-3}$ and hence on $(\text{Im } \eta - \text{Re } \eta) < 0.03$, an order of magnitude improvement on the present limit. Note that if $\eta = \eta_{+-o} \simeq \eta_{+-}$ as expected there is no prospect of detecting any effect since not only is η_{+-} small but $\text{Im } \eta_{+-} - \text{Re}\eta_{+-} \simeq 0$.

Appendix V: $K^{\pm}$ Nuclear Interactions

The decay volume is made up of four pyramids based on the drift chambers and truncated by the MW chambers. A kaon which "disappears" while in this volume will be regarded as decaying so that all nuclear interactions must be understood very well. Ideally the decay volume would be evacuated but that is not a practical proposition with thin window chambers.

At 800 MeV/c the $K^{\pm}$ N cross-sections are as follows:-

K^+p	σ_T^+	=	13 mb	$\simeq \sigma_{Elast.}$
K^+n	σ_T^+	=	16 mb	
K^-p	σ_T^-	=	40 mb	
	$\sigma^-_{Elast.}$	=	20 mb	
K^-n	σ_T^-	=	30 mb	
	$\sigma_{Elast.}$	=	15 mb	

$$\text{and } \therefore \text{ for Kp : } \sigma_T^- - \sigma_T^+ = 27 \text{ mb}, \quad \text{and Kn : } \sigma_T^- - \sigma_T^+ = 14 \text{ mb} \quad \Big\} \text{ Av.} \sim 20 \text{ mb},$$

most of the difference being inelastic. Ignoring Glauber shadowing gives the following estimates for the interaction loss, $\sim \sigma_T$, of $K^{\pm}$ in various materials, which should be compared with the $K^{\pm}$ decay rate of 10% per 0.6m.

Material	$K^+ \times 10^5$	$K^- \times 10^5$	$(K^- - K^+) \times 10^5$
Chambers per mg/cm^2	0.87	2.10	1.23
- M.W. chamber (2 plane, normal incidence)	5.3	13	7.5
- small drift (2 plane, normal incidence)	16.5	40	23
- large drift (2 plane, normal incidence)	23	55	32
Air per 0.6m. (74 mg/cm^2)	64	155	91
Chamber gas per 0.6m. (90 mg/cm^2)	78	188	111
He per 0.6m. (18 mg/cm^2)	9.1	22	13
H_2 per 0.6m. (9 mg/cm^2)	4.1	13	8.5

The right hand column indicates the extent to which nuclear interactions could create a false difference in the K^+ lifetimes; the numbers in this column are equivalent to parts in 10^4 difference for K^+ and K^- lifetimes. Bugg et al. (P.R. 168, 1474, 1968) give σ_T for K^+ ^{12}C=177mb and K^- ^{12}C=338mb, c.f. $6xK^+d$ = 176 mb and $6xK^-d$=408 mb, which suggest that the K^+/K^- differences should be reduced by 35% except for H_2.

Appendix VI: The Inequality of the $K^0_{\ell 3}$ Rates

CPT-invariance requires that the decay rates for $K^0 \to \pi^- \ell^+ \nu$ and $\bar{K}^0 \to \pi^+ \ell^- \bar{\nu}$ must be equal in the first order of perturbation theory (disregarding the Coulomb interaction in the final state). However as the validity of CPT is being questioned it is necessary to introduce an inequality parameter y_ℓ and, pedantically, a difference between the $\Delta s = -\Delta Q$ amplitudes, x_ℓ and $\bar{x}_\ell$, for K^0 and $\bar{K}^0$ decays. (Appendix III). The measured ℓ^+/ℓ^- difference for K_L decays is then

$$\delta_\ell = 2\mathrm{Re}(\varepsilon - \Delta) - \mathrm{Re}y_\ell + \mathrm{Re}(\bar{x}_\ell - x_\ell) = 2\mathrm{Re}(\varepsilon - \Delta) - \mathrm{Re}z_\ell$$

where Δ is directly related to the mass and width differences of K^0 and $\bar{K}^0$, and ε is the CP-violating, CPT-conserving parameter, and we must look to other measurements to determine Δ and y_ℓ (or z_ℓ) separately.

Nothing more can be learnt from $\ell^\pm$ measurements on a single source of neutral kaons, or from the K_L or K_S lifetimes. It is necessary to make a comparison of the rates of $K_{\ell 3}$ derived from a K^0 source and a $\bar{K}^0$ source, but no direct measurements normalized to the source strength have been published. Experimentally the listed $K_{\ell 3}$ decay rates are in fact measurements of the ratio of the number of $K_L \to \pi \ell \nu$ events to the number of $K_S \to \pi^+ \pi^-$ events, and these ratios exist for both K^0 and $\bar{K}^0$ sources. From Appendix III the $K_{\ell 3}$ decay rate per initial K^0 or $\bar{K}^0$ is, for $\Gamma_S t >> 1$, proportional to

$$|f|^2(1-4\mathrm{Re}\Delta-2\mathrm{Re}x_\ell) + |\bar{f}|^2(1-4\mathrm{Re}\varepsilon-2\mathrm{Re}\bar{x}_\ell) \text{ for a } K^0 \text{ source}$$

and

$$|f|^2(1+4\mathrm{Re}\Delta-2\mathrm{Re}x_\ell) + |\bar{f}|^2(1+4\mathrm{Re}\Delta-2\mathrm{Re}\bar{x}_\ell) \text{ for a } \bar{K}^0 \text{ source}$$

For $K_S \to \pi^+ \pi^-$ we expand the initial K^0 or $\bar{K}^0$ in terms of K_S and K_L (see Appendix IV) and obtain a K_S decay rate per initial K^0 on K^0 proportional to

$(1-2\mathrm{Re}\varepsilon+2\mathrm{Re}\Delta)$ for a K^0 source,

and $(1+2\mathrm{Re}\varepsilon-2\mathrm{Re}\Delta)$ for a $\overline{K^0}$ source

Then the ratio $R=(K_{\ell 3})_L/(K_{\pi 2})_S$ is to the first order in the parameters $\varepsilon, \Delta, x_\ell, \bar{x}_\ell$ and y_ℓ defined by $f = f(1+y_\ell)$,

$R = 2-8\mathrm{Re}\Delta-2\mathrm{Re}(x_\ell+\bar{x}_\ell)+2\mathrm{Re}y_\ell$ for a K^0 source,

and $\bar{R} = 2+8\mathrm{Re}\Delta-2\mathrm{Re}(x_\ell+\bar{x}_\ell)+2\mathrm{Re}y_\ell$ for a $\overline{K^0}$ source,

and $\frac{\bar{R}-R}{\bar{R}+R} = 4\mathrm{Re}\Delta$. The ratios R and $\bar{R}$ are assumed to be proportional to the published $K_{\ell 3}$ and $\overline{K^0}_{\ell 3}$ decay rates derived from the measured ratios.

The best of the $K_{\ell 3}$ decay rate measurements are due to Burgun et al[15] for K^0 and Weber et al [16] for $\overline{K^0}$. Both groups used a hydrogen bubble chamber and scanned and analysed in a similar manner, and as the work was done at about the same time presumably the same 'constants' were employed. The source for K^0 was $K^+p\to K^0p\pi^+$ and for $\overline{K^0}$ it was $K^-p\to\overline{K^0}n$. The stated decay rates are

Burgun et al K^0 $\Gamma(K_L\to\pi\ell\nu)=(12.4\pm0.7)\times10^6 s^{-1}$

Weber et al $\overline{K^0}$ $\Gamma(K_L\to\pi\ell\nu)=(13.1\pm1.3)\times10^6 s^{-1}$

and the $\overline{K^0}/K^0$ difference $\div$ sum is 0.027 ± 0.059 from which we obtain

$\mathrm{Re}\Delta = (7\pm15)\times10^{-3}$

and $|\mathrm{Re}\Delta| < 36\times10^{-3}$ at 2S.D..

If all the $K_{\ell 3}$ decay rate measurements, Burgen, Weber, Cho et al[17] and Mann et al[18], had been used the limit would have been reduced to $|\mathrm{Re}\Delta| < 33\times10^{-3}$, but there are some doubts about the consistency of the original measurements. We adopt $|\mathrm{Re}\Delta| < 36\times10^{-3}$.

The expression for Δ_ℓ above gives $\mathrm{Re}Z_\ell = \mathrm{Re}y_\ell+\mathrm{Re}(x_\ell-\bar{x}_\ell)=2\mathrm{Re}\varepsilon-\Delta_\ell-2\mathrm{Re}\Delta$

$$\simeq -2\mathrm{Re}\Delta-(0.1\pm0.4)\times10^{-3}$$

using the unitary analysis of Cronin (Appendix I), and the experimental value of δ_ℓ, from which we obtain

$|\mathrm{Re}Z_\ell| < 72\times10^{-3}$ at 2S.D.

Unitarity (Appendix II) provides a much more stringent limit on

$|\Delta+\frac{1}{2}ReZ_{\ell}|<10^{-3}$, which is itself a test of CPT-invariance, but without a model for CPT-violation it is impossible to say anything at this level of accuracy about the more physical properties, the fractional difference of the partial widths of $K^{0}_{\ell 3}/\overline{K^{0}}_{\ell 3}$ and of the masses and widths of $K^{0}/\overline{K^{0}}$.

The experimental prospects for improving the measurements using $\bar{p}p$ as a symmetric source of K^{0} and $\overline{K^{0}}$ are very favourable indeed. In principle $Re y_{\ell}$ can be determined directly by a comparison of the $K^{0}_{\ell 3}$ and $\overline{K^{0}}_{\ell 3}$ decay rates at t=0, but the useful region of space is < 2 K_S decay lengths from the source and the statistical accuracy would be poor. A better measurement can be obtained by a comparison of $K_L \rightarrow \ell^{\pm}\pi\nu$ rates from a K^{0} source and a $\overline{K^{0}}$ source either normalized against the K^{0} production which yields $2Re(\varepsilon+\Delta)$, or against $K_S \rightarrow \pi^{+}\pi^{-}$ in the manner of the Burgun et al. v. Webber et al. analysis, which yields $4Re\Delta$. It would not be unreasonable to hope to determine $Re\Delta$ with an error of 2×10^{-4}.

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