The three perspectives on the quantum-gravity problem and their implications for the fate of Lorentz symmetry[1]

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ABSTRACT

Each approach to the quantum-gravity problem originates from expertise in one or another area of theoretical physics. The particle-physics perspective encourages one to attempt to reproduce in quantum gravity as much as possible of the successes of the Standard Model of particle physics, and therefore, as done in String Theory, the core features of quantum gravity are described in terms of graviton-like exchange in a background classical spacetime. From the general-relativity perspective it is natural to renounce to any reference to a background spacetime, and to describe spacetime in a way that takes into account the in-principle limitations of measurements. The Loop Quantum Gravity approach and the approaches based on noncommutative geometry originate from this general-relativity perspective. The condensed-matter perspective, which has been adopted in a few recent quantum-gravity proposals, naturally leads to scenarios in which some familiar properties of spacetime are only emergent, just like, for example, some emergent collective degrees of freedom are relevant to the description of certain physical systems only near a critical point. Both from the general-relativity perspective and from the condensed-matter perspective it is natural to explore the possibility that quantum gravity might have significant implications for the fate of Lorentz symmetry in the Planckian regime. From the particle-physics perspective there is instead no obvious reason to renounce to exact Lorentz symmetry, although ("spontaneous") Lorentz symmetry breaking is of course possible. A fast-growing phenomenological programme looking for Planck-scale departures from Lorentz symmetry can contribute to this ongoing debate.

[1]Based on invited seminars given at *Perspectives on Quantum Gravity: a tribute to John Stachel* (Boston, March 6-7, 2003) and *Tenth Marcel Grossmann Meeting on General Relativity* (Rio de Janeiro, July 20-26, 2003).

1 Preliminaries

1.1 Lorentz symmetry and the three perspectives on the Quantum Gravity problem

Each quantum-gravity research line can be connected with one of three perspectives on the problem: the particle-physics perspective, the general-relativity perspective and the condensed-matter perspective.

From a particle-physics perspective it is natural to attempt to reproduce as much as possible the successes of the Standard Model of particle physics. One is tempted to see gravity simply as one more gauge interaction. Among the quantum-gravity open issues the failure of perturbative renormalization in naive quantum-gravity is perceived as most significant. From this particle-physics perspective a natural solution of the quantum gravity problem would be String-Theory-like: a quantum gravity whose core features are essentially described in terms of graviton-like exchange in a background classical spacetime.

The general-relativity perspective naturally leads to reject the use of a background spacetime, and this is widely acknowledged [1, 2]. Although less publicized, there is also growing awareness of the fact that the development of general relativity relied heavily on the careful consideration of the in-principle limitations that measurement procedures can encounter. Think for example of the limitations that the speed-of-light limit imposes on certain setups for clock synchronization and of the contexts in which it is impossible to distinguish between a constant acceleration and the presence of a gravitational field. In light of the various arguments (some briefly reviewed later in these notes) suggesting that, whenever both quantum mechanics and general relativity are taken into account, there should be an in-principle limitation to the localization of a spacetime point (an event), the general-relativity perspective invites one to renounce to any direct reference to a classical spacetime [3, 4, 5, 6, 7]. Indeed this requirement that spacetime be described as fundamentally nonclassical ("fundamentally quantum"), that the in-principle measurability limitations be reflected by the adoption of a corresponding measurability-limited description of spacetime, is another element of intuition which is guiding quantum-gravity research from the general-relativity perspective. This naturally leads us to consider certain types of discretized spacetimes, as in the Loop Quantum Gravity approach, or noncommutative spacetimes. Loop Quantum Gravity is also a background-independent approach and therefore combines both elements of the general-relativity perspective[a]. Noncommutative spacetimes could be introduced in a background-independent way, as in preliminary attempts reported in Ref. [3] and follow-up work, but in most studies, for simplicity, noncommutative spacetimes are adopted as background spacetimes [8, 9, 10, 11, 12] (leading to an approach which in a sense can be seen as originating from a hybrid of the particle-physics perspective and the general-relativity perspective).

The third possibility is a condensed-matter perspective (see, *e.g.*, the research programs of Refs. [13] and [14]) on the quantum-gravity problem, in which some of the

[a]Although it must be noted that this is actually achieved, so far, at the price of some possibly concerning compromises. For example, as stressed by John Stachel and others, one could be concerned of the fact that most of the Loop-Quantum-Gravity results are obtained preserving only 3D (space) diffeomorphisms.

familiar properties of spacetime are only emergent. Condensed-matter theorists are used to describe some of the degrees of freedom that are measured in the laboratory as collective excitations within a theoretical framework whose primary description is given in terms of much different, and often practically unaccessible, fundamental degrees of freedom. Close to a critical point some symmetries arise for the collective-excitations theory, which do not carry the significance of fundamental symmetries, and are in fact lost as soon as the theory is probed somewhat away from the critical point. Notably, some familiar systems are known to exhibit special-relativistic invariance in certain limits, even though, at a more fundamental level, they are described in terms of a non-relativistic theory. For a rather general class of fermionic systems one finds [13] that at low energies, as a Fermi point is approached, fermions gradually become chiral Weyl fermions, while bosonic collective modes of the vacuum transform into gauge fields and gravity.

Clearly from the (relatively new) condensed-matter perspective on the quantum-gravity problem it is natural to see the familiar classical continuous Lorentz symmetry only as an approximate (emergent) symmetry. Results obtained over the last few years (which are partly reviewed later in these notes) allow us to formulate a similar expectation from the general-relativity perspective. Loop quantum gravity and other discretized-spacetime quantum-gravity approaches appear to require some departures, governed by the Planck scale, from the familiar (continuous) Lorentz symmetry. And in the study of noncommutative spacetimes some Planck-scale departures from Lorentz symmetry might be inevitable, since (at least in a large majority of noncommutative spacetimes) a Lie algebra is not even the appropriate language for the description of the symmetries of a noncommutative spacetime (one must resort to the richer structure of Hopf algebras).

From the particle-physics perspective there is instead no obvious reason to renounce to exact Lorentz symmetry. Minkowski classical spacetime is an admissible background spacetime, and in classical Minkowski there cannot be any *a priori* obstruction for classical Lorentz symmetry. Still, a breakup of Lorentz symmetry, in the sense of spontaneous symmetry breaking, is of course possible. This possibility has been studied extensively [10, 15] over the last few years, particularly in String Theory, which is the most mature quantum-gravity approach that emerged from the particle-physics perspective.

1.2 What do we know about quantum-gravity?

The theory debate clearly is a confrontation between very different perspectives on the quantum-gravity problem. If we had any robust information on quantum gravity certainly at least some of these ideas would have been proven to fail. But after more than 70 years [16] of work on the "quantum-gravity problem" there is still not a single measured number whose interpretation requires advocating "quantum gravity".

I have so far mentioned the quantum-gravity problem as if it was a well-established and familiar concept, but it is perhaps useful to give here an intuitive characterization of this problem. The quantum-gravity problem is sometimes described as a sort of "human discomfort", as a problem pertaining to the achievement of a more satisfactory philosophical worldview. For example, as motivation for research in quantum gravity it is sometimes stated that "quantum theory" (in an appropriate generalized sense) has turned out to be relevant for the description of measurement results in all other branches of fundamental physics, and we therefore must assume that it will eventually be relevant also for spacetime/gravity physics. Analogously (and amounting to the same thing), it is sometimes stated that it is unsatisfactory to have on one side our present unified quantum-field-theory description of electromagnetic, weak and strong forces and on the other side gravity which is still described in a very different way. These "human discomforts" do not of course define a scientific problem, but actually there is, as emphasized by some, a well-defined scientific problem which can be naturally called "quantum-gravity problem".

The scientific problem that can be reasonably called "quantum-gravity problem" is actually the problem of producing numbers (predictions), in a logically-consistent way, for situations in which both gravity effects and particle-physics quantum-field-theory effects cannot be neglected. For example, although we are presently (and for the foreseeable future) unable to set up and observe collisions between two electrons each with energy of, say, $10^{50} eV$, our present theories provide no obstruction for the analysis of such high-energy collisions, but are unable to produce a logically consistent number for, say, the probability that such a collision would result in two muons with certain energies and momenta. The problem, as I shall try to point out later in these notes, resides in the fact that quantum field theory implicitly assumes that gravity effects can be neglected. When the gravity effects are so large that (from the field-theory perspective) space geometry evolves significantly on very short time scales, field theory cannot be consistently applied[b]. Similarly, field theory runs into trouble when gravity effects are strong enough to admit the emergence of spacetime singularities (*e.g.* black holes). We are able to get "numbers" out of quantum field theory in contexts in which there is a curved static (or slowly varying) nonsingular space, but fast-varying and/or singular space geometries are untreatable.

One might argue that $10^{50} eV$ electrons should be the least of our concerns, since we are never going to be able to produce and/or observe them, but first of all in cosmology there are some numbers we should produce that depend on very early times after the big bang (where we have reason to believe that particles with extremely high energies were abundant), and, secondly, the fact that our theories fail to produce numbers in some contexts which those same theories describe as accessible (in principle) makes us concerned in general about the robustness of these theories. Since we know that new elements would have to be introduced in our theories for the description of collisions between $10^{50} eV$ electrons (or for a justification of an in-principle exclusion of such collisions from the list of processes that can occur in Nature), it is natural then to wonder whether those new elements can affect also some of the contexts in which our present theories do provide us an apparently acceptable prediction. In some cases the issues we encounter in analyzing, say, collisions among $10^{50} eV$ electrons might bring to the surface some issues that could also modify more ordinary (but still untested) predictions produced by our theories.

[b] Here the reader should keep in mind that general relativity governs self-consistently the spacetime dynamics in terms of (and together with) the particle dynamics, but particle dynamics is only defined asymptotically, in the S-matrix sense, in quantum field theory. During a collision process the, say, electrons involved are not following any trajectories. We can associate to them some (however fuzzy) trajectories only asymptotically, much before and much after the collision. If one tries to apply general relativity to the formally-classical trajectories that appear in the path integral formulation of quantum mechanics, the problem becomes anyway ill defined (and affected by unremovable divergences) if the energies of the particles are high enough to induce significant geometrodynamics.

There is a very natural explanation for our lack of insight on this quantum-gravity problem. One of the few (perhaps the only) robust hint we have about quantum gravity is that the energy scale at which the particle-physics quantum-field-theory description starts to appear inadequate is the Planck scale $E_p \sim 10^{28} eV$. For particles of those energies and higher the fact that the Standard Model of particle physics ignores gravitational effects is clearly unsatisfactory. And usually the scale that sets the break point of an effective low-energy theory is also the scale that sets the magnitude of the new effects to be expected going beyond the effective low-energy theory. It is therefore reasonable to expect[c] that "quantum-gravity corrections" to our low-energy predictions would be very small, with their magnitude set by some power of the ratio between the Planck length ($L_p \sim 10^{-35} m$, which is the inverse the Planck scale $E_p \sim 10^{28} eV$) and the (much bigger) wavelength of the particles involved in the process. So we have good reasons to suspect that the quantum-gravity effects would be very small (and actually they must be typically small, since we have not managed to see them yet). Contemplating the horrifying smallness of the Planck length the quantum-gravity community had reached the conviction (see, *e.g.*, Ref. [18]) that experimental hints could never be obtained. If this was true, if this expectation was really robust, there could not possibly be a "quantum gravity" scientific programme. But, on the basis of results obtained over the last 4 or 5 years, it is now clear that these pessimistic expectations were based on incorrect premises: "quantum-gravity experiments" are possible. Of course, there is no guarantee that they will ever lead to any actual discovery, but it is clearly incorrect to adopt the *a priori* assumption that the search of the tiny Planck-scale effects should be hopeless.

In order to look for quantum-gravity effects it is of course useful to have some guidance from theories. While the long history of quantum-gravity research has not led to experimental facts, it did produce theories that can valuable both for providing some guidance to quantum-gravity experiments and for clarifying some technical difficulties that are encountered in any theory that attempts to incorporate (as appropriate limits) both general relativity and the quantum field theory that describes the Standard Model of particle physics. Results obtained in String Theory provide encouragement for the idea that a theory combining gravity and the Standard Model of particle physics could admit a perturbative treatment (perturbative renormalizability), at least in certain contexts in which it might be appropriate to make reference to a background spacetime. Before these String-Theory results it appeared that "quantum gravity" should in all cases be treated using (to-be-determined) nonperturbative techniques, with obvious associated difficulties. Another example is provided by some results obtained in Loop Quantum Gravity, which provide encouragement for the idea that a truly background-spacetime-independent quantum theory can be constructed. Before this loop-quantum-gravity studies it appeared that there would be a more profound conflict between the background-spacetime independence of general relativity and the fact that quantum field theory assumes from the start a background spacetime.

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[c]Like all expectations not fully confirmed by experiments, also the apparently-robust expectation that quantum-gravity effects be governed by the Planck scale should be challenged, and has been challenged occasionally. In particular, the so-called "theories with large extra dimensions" [17] provide a scenario for an effective increase of the size of the characteristic quantum-gravity length scale (decrease of the quantum-gravity energy scale). These scenarios are not necessarily "natural", but they do justify some prudence concerning the assumptions being made on the characteristic scale of quantum-gravity effects.

Unfortunately, loop quantum gravity is being constructed (so far, pending work in progress), as a fundamentally nonperturbative theory, without access to the tools of perturbative analysis which are so valuable in our efforts to "produce numbers". And equally unfortunate is the fact that for String Theory there is (so far) no genuinely background-independent formulation. But perhaps these weaknesses should not generate too much concern. For the phenomenological aspects of the line of analysis advocated in these notes these two theories and other popular quantum-gravity approaches simply play the role of toy models. The objective being pursued is the one of finding the first experimental fact (or even the first few experimental facts) about the quantum-gravity problem. And we need some guidance. Where should we look? The toy models can provide inspiration. Even if neither of them ended up providing the full solution of the quantum-gravity problem, it is still rather plausible that they might have managed to capture some genuine feature of the correct theory. Experiments should tell us if this is the case.

1.3 Quantum Gravity Phenomenology

The most difficult aspect of the search of experimental hints relevant for the quantum-gravity problem is the smallness of the effects that one would naturally expect to be induced by a quantum gravity. A key point for this "Quantum Gravity Phenomenology" [19] is that we actually are familiar with ways to gain sensitivity to very small effects. For example, our understanding of brownian motion is based on the fact that the collective result of a large number of tiny microscopic effects eventually leads to observably large macroscopic effects. Similarly, our present very accurate bounds on the possibility of a difference in the masses of the K^0 and $\bar{K}^0$ neutral kaons (relevant for studies of CPT symmetry) are at a level of precision (better than $\Delta m_K < 10^{-18} m_K$) which can only be achieved thanks to the fact that some signatures associated with a $K^0/\bar{K}^0$ mass difference are actually amplified by a large ordinary-physics number present in the relevant physical contexts (the ratio between the average mass of neutral kaons and the difference in mass of the short-living and long-living neutral-kaon weak-interactions eigenstates: $m_K \sim 10^{15}[m_{K_L} - m_{K_S}]$).

Now that quantum-gravity phenomenology has grown into a research area involving some twenty research groups around the world, it is amusing to compare quantum-gravity reviews and grandunification reviews written in the early 1990s. The quantum-gravity reviews considered physics characterized by the scale $10^{28} eV$ and were claiming that experiments could never set useful constraints, and simultaneously the grandunification reviews went into detailed explanations of how certain grandunification pictures were being rules out by data on proton stability. The prediction of proton decay within certain grandunification theories (theories providing a unified description of electroweak and strong particle-physics interactions) is really a small effect, suppressed by the fourth power of the ratio between the mass of the proton and the grandunification scale, which is only three orders of magnitude smaller than the Planck scale ($E_{gut} \sim 10^{25} eV$). In spite of this horrifying suppression, of order $[m_{proton}/E_{gut}]^4 \sim 10^{-64}$, with a simple idea we have managed to acquire a remarkable sensitivity to the possible new effect: the proton lifetime predicted by grandunification theories is of order $10^{39} s$ and "quite a few" generations of physicists should invest their entire lifetimes staring at a single proton before its decay, but by managing to keep under observation a large number of protons (think for example of a situation in which 10^{33} protons are monitored) our sensitivity to proton decay is significantly increased. In that context the number of protons is the (ordinary-physics) dimensionless quantity that works as "amplifier" of the new-physics effect.

We should therefore focus our attention [19] on experiments which have something to do with spacetime structure and that host an ordinary-physics dimensionless quantity large enough that it could amplify the extremely small effects we are hoping to discover. The amplifier can be the number of small effects contributing to the observed signal (as in brownian motion and in proton-stability studies) or some other dimensionless ordinary-physics number (as in studies of a possible difference in the masses of the K^0 and $\bar{K}^0$).

Using these general guidelines, a few quantum-gravity research lines have matured over these past few years. Later in these notes I will focus on studies of the fate of Lorentz symmetry in quantum spacetime, emphasizing the relevance for observations of gamma rays in astrophysics [20, 21], the relevance for the analysis of the cosmic-ray spectrum [22, 23, 24, 25, 26], and the relevance for certain observations involving particle decays [27, 28].

Concerning laser-interferometric tests of Planck-scale effects I will only comment on the ones [29] that are directly relevant for the study of the fate of Lorentz symmetry in quantum spacetime. I will not discuss "spacetime foam" studies based on laser-interferometry, on which there is a growing literature (see, *e.g.*, Refs. [30, 31, 32]).

Similarly I will not discuss here matter-interferometric limits on Planck-scale effects and limits on Planck-scale effects obtained using the mentioned sensitivity to new physics that one finds naturally in the neutral-kaon system. Matter interferometers and the neutral-kaon system were among the first contexts to be considered from the perspective of Planck-scale effects (see, *e.g.*, Refs. [33, 34, 35, 36, 37, 38]), but in these contexts there is not much discussion of possible implications of Planck-scale departures from Lorentz symmetry, and actually the connection with Planck-scale/quantum-gravity physics remains rather indirect[d]. Both the analysis of matter interferometers and of the neutral-kaon system appear to require a proper understanding of Planck-scale-induced decoherence [36, 38, 39], something which we are still unable to perform satisfactorily even in the simplest quantum spacetimes (such as the simplest noncommutative spacetimes). One must therefore rely on general parametrizations, whose connection with quantum-gravity theories is rather indirect. Similarly, the analysis of the neutral-kaon system requires an understanding of the fate of CPT symmetry in quantum spacetime, and this too is something which we are unable to do rigorously even in the simplest quantum spacetimes [8, 40].

Finally, to give a tentative complete list of quantum-gravity-phenomenology topics which I will not discuss, I should stress that I intend to focus here on the possibility of a "genuinely quantum" spacetime. I will discuss (at least intuitively) the difference between a quantum gravity with a classical spacetime (background) and a quantum gravity with a genuinely quantum spacetime. Interesting ideas about the interplay between gravity and quantum mechanics which do not require a genuinely quantum spacetime can be found in Refs. [41, 42, 43, 44, 45, 46, 47, 48].

[d] This is perhaps the reason why the early studies reported in Refs. [33, 34, 35, 36, 37, 38] did not manage to generate the interest of a significant portion of the quantum-gravity community.

1.4 A key issue: should we adopt a fundamentally quantum spacetime?

As I already stressed it is rather obvious that from the particle-physics perspective one would not expect any departures from Lorentz symmetry and on the contrary from the condensed-matter perspective Lorentz symmetry is naturally seen only as an approximate symmetry. It is instead less obvious what one should expect for the fate of Lorentz symmetry in quantum-gravity approaches based on the general-relativity perspective, and in fact some key insight (leading to the expectation that departures from Lorentz symmetry are usually present) has been gained only very recently, mostly in the study of loop quantum gravity and certain noncommutative spacetimes.

A key point that needs to be clarified when approaching the quantum-gravity problem from the general-relativity perspective is whether or not one should adopt a "genuinely quantum" spacetime. This concept will not be defined rigorously here, but combining various points and remarks in these notes the reader should get an intuitive picture of this concept. A genuinely quantum spacetime is essentially a spacetime in which an event (a spacetime point) cannot be sharply localized. I will use Loop Quantum Gravity (the present understanding of Loop Quantum Gravity) and certain noncommutative spacetimes as examples of genuinely quantum spacetimes. In the case in which one might be able to introduce coordinates for the event, in a quantum spacetime it must be impossible to determine (in the sense of measurement) all of the coordinates of an event.

I shall use the familiar relativistic quantum field theory as an example in which spacetime is not fundamentally quantum. The position of a generic particle cannot be sharply determined in relativistic quantum field theory, but it is possible to determine sharply the position of a particle with infinite mass. This infinite-mass limit gives operative meaning to the classical spacetime background in which we describe relativistic quantum field theory. I shall argue that if it was not for this infinite-mass limit, if there was any incompatibility between the infinite-mass limit and the logical structure of relativistic quantum field theory, it would have been impossible to make reference to a classical background spacetime. But there is no incompatibility between the infinite-mass limit and the logical structure of relativistic quantum field theory, so we do have a classical spacetime in that context.

I will stress that the infinite-mass limit is evidently troublesome once gravity is taken into account, and I will argue, summarizing the evidence that emerged in several studies, that a theory that truly admits both general relativity and quantum mechanics as appropriate limits must renounce to any reference to a classical spacetime. Such a theory is automatically incompatible with the possibility of localizing sharply a spacetime point.

This point is relevant for the fate of Lorentz symmetry in quantum gravity. I will review results obtained over the last 3 or 4 years which suggest that, if spacetime is "quantum" in the sense of noncommutativity or discreteness, the familiar (classical/Lie-algebra, continuous) Lorentz symmetry naturally ends up being only an approximate symmetry of the relevant "flat-spacetime limit" of quantum gravity.

1.5 Outline

These notes are composed of various sections and each section is (nearly) self-contained. Only in rare cases there is a direct reference in a given section to a previous section, but through the combination of the points made in the different sections I am trying to provide the different elements of a certain view of the quantum-gravity problem.

The next section is an aside on the hypothesis of a "genuinely quantum" spacetime. I will argue that there should be an absolute limit on the localization of an event in quantum gravity, and that this fact should invite us to renounce to any reference to a non-physical classical spacetime.

In Section 3 I comment on how different approaches to the quantum-gravity problem describe the fate of Lorentz symmetry in quantum gravity.

Section 4 focuses on the fate of Lorentz symmetry in discretized spacetimes, a topic on which some insight can be gained on the basis of some rather general considerations, even without the guidance of a specific quantum-gravity theory.

In Section 5 I review some recent proposals for testing scenarios for Planck-scale departures from ordinary (classical, continuous) Lorentz symmetry.

Some closing remarks are in Section 6.

2 Aside on the hypothesis of a genuinely quantum spacetime

2.1 Classical spacetime and localization

The concept of a classical spacetime is appropriate in physics (operatively meaningful) when the theory of interest allows to localize sharply a spacetime point. This statement is intended in the same sense that a classical concept of angular momentum is only appropriate when the angular-momentum vector (all of its components) can be sharply measured. In 19th century physics angular momentum was a classical concept. In our modern theories we acknowledge the experimental fact that there are limitations on the measurability of the angular-momentum vector (one cannot measure all of its components simultaneously) and therefore we describe angular momentum using a nonclassical formalism (the one of noncommuting operators) which captures this measurability limitations.

The consistency between the measurability limits established by the formalism and the in-principle measurability limits that affect measurement procedures is a key requirement for a physical theory. This important issue usually takes center stage in the physics literature only when a major "scientific revolution" challenges our understanding of the physical world. In the course of such a "revolution" it is natural to question the logical consistency of the novel theoretical frameworks which are being proposed. Once these logical-consistency issues have been settled, and substantial experimental support for the new theory has been obtained, the focus shifts toward computational matters: one is comfortable with the logical structure of the new theory and with the fact that the new theory has some relevance for the description of Nature, and therefore precise calculations and accurate experiments become the top priority. For example, this natural sequence of steps for the development of new theories is easily recognized in the development of the "relativity revolution" and of the "quantum-theory revolution".

While the limited scope of these notes does not allow me to describe rigorously the issues that are to be considered in measurability analysis (and its role in establishing the logical consistency of a formalism), the interested reader can find a careful discussion in the literature, especially the literature reporting the debate (among Einstein, Peierls, Bohr, Rosenfeld and others) on the measurability of the electromagnetic fields in quantum electrodynamics (see Ref. [49] and references therein). At first these measurability studies appeared to expose an in-principle limitation on the measurability of electromagnetic fields, and this was of serious concern since a measurability limit would have implied an inadequacy of quantum electrodynamics. Quantum electrodynamics makes direct reference to the electromagnetic fields and describes them as sharply measurable (although the sharp measurement of a quantum field can only be achieved at the cost of loosing all information on a conjugate field). Eventually, it was clarified by Bohr and Rosenfeld [49] that there is no limitation on the measurability of the electromagnetic fields, and this opened the way for the wide adoption of quantum electrodynamics.

There are several other examples of the importance of these studies of in-principle measurability limits, and of the necessity that the theoretical framework reproduces faithfully the measurability limits. In understanding the replacement of absolute time by a relative time a key role is played by the analysis of how an absolute maximum velocity limits the synchronization of certain pairs of clocks. In combing quantum mechanics with special relativity one must adopt quantum field theory, rather than a relativistic version of quantum mechanics itself, because the position of a particle of finite (*i.e.* non-infinite) mass cannot be sharply determined through a logically consistent measurement procedure.

In "quantum gravity", a theory that admits both quantum field theory and general relativity as appropriate limits, is it legitimate to adopt a classical spacetime? or is it instead necessary to adopt a nonclassical description of spacetime? I will argue that spacetime is fundamentally nonclassical in quantum gravity. And I will argue that adopting a classical spacetime for quantum gravity is problematic just in the same sense that a naive relativistic formulation of quantum mechanics is problematic (and needs to be replaced by quantum field theory). The relativistic formulation of quantum mechanics can appear to make formal sense up to a certain point, but eventually one discovers some inconsistencies (negative energy states...) and these inconsistencies are easily traced back to the fact that quantum mechanics assumes that the position of a particle can be sharply measured, whereas any procedure that combines the uncertainty principle and special relativity cannot possibly provide a sharp measurement of the particle position. I conjecture that analogously any quantum-gravity theory that assumes a classical spacetime will eventually turn out to be inconsistent or incomplete, because of the failure to provide a logically-consistent description of the measurability limits (obtained by combining the uncertainty principle with general relativity) on the localization of a spacetime point.

2.2 Spacetime in classical mechanics and in nonrelativistic quantum mechanics

Of course, spacetime is classical in classical mechanics. This has a precise operative meaning which I shall not discuss here, since the intuitive picture of a classical spacetime will suffice for the purposes of these notes.

Spacetime is also classical in nonrelativistic quantum mechanics. Quantum mechanics introduces an absolute limit on the (simultaneous) measurability of pairs of conjugate observables, but each observable can still be sharply measured (at the cost of loosing all information on a conjugate observable). Notably the space coordinates of a particle are independent observables (not conjugate to one another) so they can all be measured sharply at once. Quantum mechanics indeed makes direct reference to a classical background spacetime. This classical spacetime can be endowed with proper operative meaning by imagining a dense array of pointlike synchronized clocks. The clocks mark the time variable (really an external variable in quantum mechanics) and the (sharply-measurable) position of the clocks give physical meaning to the space coordinates.

I will argue that the concept of infinite-mass point particles[e] plays a crucial role in spacetime measurability analysis in relativistic quantum field theory. Within nonrelativistic quantum mechanics the role of infinite-mass point particles is more subtle, and I am not even completely sure that such particles are necessary. Still an infinite-mass limit might be hidden in the discussion of the dense array of pointlike synchronized clocks, providing the reference frame. If the clocks had finite mass one should worry about uncertainties in the time evolution of the reference frame, due to the fact that position and velocity cannot be both sharp if the mass is finite. So it appears that the classical spacetime background of nonrelativistic quantum mechanics acquires proper operative meaning only in the limit of infinite mass of the particles that provide identity to the spacetime points. This is of course not troublesome since ordinary quantum mechanics neglects gravitational effects, and therefore its logical consistency can rely on the idealization of a physical reference frame constituted of infinitely massive point particles.

2.3 Spacetime in relativistic quantum field theory

As discussed in the previous subection, the uncertainty principle coexists with Galileo relativity (which describes the symmetries of nonrelativistic quantum mechanics) in such a way that a physically meaningful classical spacetime can be introduced. The formalism of quantum mechanics assumes and requires a classical spacetime since it describes, in terms of the wave function, the probability that a particle be found at time t in the (sharply-defined) space point (x, y, z).

There is no (special-)relativistic version of quantum mechanics because the interplay of the uncertainty principle and special relativity does not allow one to consider the probability that a particle be found at time t in the (sharply-defined) space point (x, y, z). At time t the particle can only be localized with an accuracy set by its Compton wavelength. This can be seen by considering a localization procedure as something which ultimately must involve an interaction between a probe and the particle under study. In order for the localization to achieve δx accuracy the probe must carry at least energy $1/\delta x$ (so that the probe itself is confined to a region of size δx), but if this energy $1/\delta x$ is higher than the mass of the particle being studied/measured additional copies of the original particle could be produced (in a special-relativistic framework) as a result of the measurement procedure. This is of course incompatible with the idea of using the procedure for the measurement of the position of a given particle. The position of a particle of mass m and small velocity/momentum cannot be measured

[e] Of course, in analyses aimed at defining operatively certain physical entities the concept of infinite-mass particle can only be introduced in the sense of a limiting procedure. For example, in referring to a dense array of infinite-mass synchronized clocks one is really thinking of a limiting procedure in which heavier and heavier clocks are used.

with better accuracy than $\delta x \sim 1/m$. More generally the position of a particle of energy E cannot be measured with better accuracy than $\delta x \sim 1/E$.

The fact that the position of a particle with finite (non-infinite) mass cannot be sharply determined imposes that instead of a relativistic quantum mechanics we resort to (relativistic) quantum field theory. It is noteworthy that in quantum field theory it is legitimate to make reference to a classical spacetime, and indeed quantum field theory does make reference to a classical spacetime. In fact, quantum field theory (which again ignores gravity) is perfectly compatible with the introduction of infinite-mass point particles, and these can provide a classical spacetime (a classical reference frame) exactly in the same sense already discussed above for nonrelativistic quantum mechanics. For an infinite-mass particle the combination of the uncertainty principle and special relativity does not introduce a limit on position measurement (the compton wavelength is 0). The (in-priciple) presence of a reference frame constituted of a network of infinite-mass particles allows us to refer to a classical spacetime. In that classical spacetime the positions of finite-mass particles cannot however be sharply determined.

In summary, both ordinary (nonrelativistic) quantum mechanics and (relativistic) quantum field theory do refer to a classical spacetime in a logically consistent way. At least in quantum field theory (but, in the sense discussed in the preceding subsection, also in nonrelativistic quantum mechanics) the classical spacetime can be operatively described in terms of a limiting procedure in which spacetime points are marked by point particles of larger and larger mass, with the sharp localization of a spacetime point accessible as the infinite-mass limit of this procedure. It is therefore possible to combine special relativity and the uncertainty principle while preserving a physically meaningful classical spacetime.

This is no longer possible when gravity is present: combining general relativity and quantum mechanics one finds that on the one hand sharp localization would still require an infinite mass point particle, but on the other hand, since general relativity imposes to treat mass as gravitational charge, the infinite-mass limit is evidently incompatible with the localization measurement procedure.

2.4 Spacetime in quantum gravity: events marked by collisions involving massless point particles or closed strings

The idea of an absolute limit on localization has a very long tradition in quantum-gravity research [16]. Some representative studies of various realizations of this idea can be found in Refs. [3, 4, 5, 6, 50, 51]. The simplest argument is found in Refs. [50, 51]. It can be summarized by viewing again a localization procedure as something which ultimately must involve an interaction between a probe and the "target" particle under study. Assuming that the probe is a massless particle (or a massive particle in the relativistic regime, with velocity high enough to neglect the mass) one is led straightforwardly to a limit on localization which is set by the Planck length L_p.

Again a key point is that in order for the localization procedure to achieve δx accuracy the probe must carry at least energy $1/\delta x$. However, taking now into account gravity we see that it is necessary to require $\delta x \geq L_p$. In fact, a source of localization uncertainty comes from the uncertainties in the gravitational interaction between the probe and the target. It suffices to consider these uncertainties in a small region, of size ϵ, around the collision, *i.e.* the stage of the procedure when the probe and the target are at distances of order ϵ. And we consider $\epsilon \sim \delta x$ since in order for the collision to be localized with accuracy δx the probe and the target must eventually come to be at least as close as δx. The gravitational energy stored in the system during this stage of the collision, which lasts a time of order ϵ (since the probe is massless/relativistic), is of order[f] $U \sim L_p^2 ME/\epsilon$, where M is the mass of the target particle and E is the energy of the probe. As a result of the uncertainty in the probe's energy, $\delta E \sim 1/\delta x$, this gravitational energy is also uncertain by an amount of order $\delta U \sim L_p^2 M \delta E/\epsilon$. Consequently when the probe-target distance is of order ϵ the probe momentum is uncertain by an amount $\delta p_\gamma \sim L_p^2 M \delta E/\epsilon$ and by momentum conservation also the momentum of the target particle has the same uncertainty $\delta p_M \sim L_p^2 M \delta E/\epsilon$. There is therefore a time interval of order ϵ around the time of the collision in which the velocity of the target particle is uncertain by an amount $\delta v_M \sim L_p^2 \delta E/\epsilon$. One concludes that the position of the collision cannot be established with better accuracy than $\delta x' \sim \delta v_M \epsilon \sim L_p^2 \delta E$. This indeed leads to the conclusion $\delta x \geq L_p$, since $\delta E \sim 1/\delta x$. Concerning the uncertainty in the time of the collision one easily finds (the probe is relativistic) $\delta t \sim \delta x \geq L_p$. I also observe, in preparation for a point I shall articulate later in this section, that $\delta x \delta t \geq L_p^2$.

There has been some interest in generalizing this analysis to the case in which the probe is a closed string rather than a photon. It is useful to observe that (even though a dedicated study is still missing) in light of the related findings reported in Ref. [52] one expects only one relevant difference between ordinary (point-like) massless probes and closed-string probes: closed-string probes have the property that their size increases with their momentum in such a way that localization is limited to $\delta x \geq L_s$, where L_s is the string length, and (since the analysis in Ref. [52] requires $L_s > L_p$) this is consistent with the general expectation $\delta x \geq L_p$.

2.5 Spacetime in quantum gravity: events marked by collisions of neutral nonrelativistic particles

The localization limit $\delta x \geq L_p$ is widely accepted within the quantum-gravity community. I stress that this localization limit relies on two crucial ingredients: the nature and strength of the gravitational interactions and the fact that a massless particle with energy uncertainty δE has position uncertainty $1/\delta E$. I observe that, while in practice localization procedures always rely on massless (or anyway relativistic probes), according to the Bohr-Rosenfeld line of analysis [49] (which is really the definitive work on the role of measurability analyses in the logical structure of a physical theory) it is necessary to wonder whether the probes that turn out to be useful for practical reasons are the ones conceptually best suited for the task of localization. Moreover, one should not only search among the probes we find to be available in Nature: any type of ("gedanken") probe that is consistent with the conceptual/formal structure of the theory should be considered [49]. It is plausible that the correct quantum-gravity

[f] For simplicity, the analysis of the uncertainties in the gravitational energy stored in the system composed by the photon and the target particle is here discussed within Newtonian gravity. As shown in Ref. [50], the estimate obtained using Newtonian gravity turns out to be correct (using general relativity one obtains the same estimate, after a somewhat more lengthy analysis).

would predict its constituents, but at present, since we are still uncertain about the structure of the correct theory, any discussion of a general localization limitation must be accompanied by a very general analysis of possible probes.

In order to establish a localization limit of more general validity one should in particular consider the possibility of using non-relativistic neutral probes, where "neutral" here indicates that they only carry the familiar gravitational (mass/energy) charge, and only interact gravitationally.

For neutral non-relativistic probes the generalization of the analysis reviewed in the preceding section is rather straightforward. Let me consider the event of collision between a probe of velocity $V_P \ll 1$ and mass M_P (mass of the probe) and a target particle of mass M_T. The source of localization uncertainty due to the uncertainties in the gravitational interaction between the probe and the target is also easily analyzed in the case of non-relativistic neutral probe. Again it suffices to consider these uncertainties in the stage of the procedure when the probe and the target have distances of a certain (arbitrarily chosen but small) order ϵ. The gravitational energy stored in the system during this stage of the collision, which lasts a time of order ϵ/V_P, is of order $U \sim L_p^2 M_T E_P/\epsilon$, where E_P is the probe's energy. As a result of the uncertainty $M_P V_P \delta V_P$ in the probe energy this gravitational energy is also uncertain[g] by an amount of order $\delta U \sim L_p^2 M_T M_P V_P \delta V_P/\epsilon$. Consequently when the probe-target distance is of order ϵ the probe momentum is uncertain by an amount $L_p^2 M_T M_P \delta V_P/\epsilon$ and by momentum conservation also the momentum of the target has the same uncertainty. There is therefore a time interval of order ϵ/V_P around the time of the collision in which the velocity of the target is uncertain by an amount $\delta V_T \sim L_p^2 M_P \delta V_P/\epsilon$, leading to a target-position uncertainty $\delta x_T \sim L_p^2 M_P \delta V_P$. Correspondingly the probe-position uncertainty is of order $\delta x_P \sim 1/(M_P \delta V_P)$. Since the overall localization uncertainty will receive contributions both from δx_T and δx_P, $\delta x \geq \delta x_T + \delta x_P$, one concludes that $\delta x \geq L_p/\sqrt{V_P}$. Following analogous reasoning for the uncertainty in the time of the event one finds $\delta t \geq L_p/\sqrt{V_P^3}$. Since consistency with the hypothesis of using a nonrelativistic probe requires $V_P \ll 1$, we can safely conclude that there is nothing to be gained by substituting massless probes with neutral nonrelativistic probes. While using a massless probe one finds $\delta x \geq L_p$, the use of a neutral nonrelativistic probe leads to an even more severe limit on localization $\delta x \geq L_p/\sqrt{V_P}$. I also observe that, while with a massless probe we found $\delta t \delta x \geq L_p^2$, using neutral nonrelativistic probes the localizzation procedure is even more strictly limited: $\delta t \delta x \geq L_p^2/V_P^2 \gg L_p^2$.

[g] Note that one can also estimate δU as $\delta U \sim [L_p^2 M_T M_P/(\epsilon + \delta x_0)] - [L_p^2 M_T M_P/\epsilon]$, thereby obtaining the result $\delta V_T \geq L_p^2 M_P \delta x_0/(V_P \epsilon^2)$. This would allow to conclude that there is a time interval of order ϵ/V_P around the time of the collision in which the velocity of the target is uncertain by an amount $\delta V_T \sim L_p^2 M_P \delta V_P/\epsilon$ and the velocity of the probe is (see above) uncertain by at least $\delta V_P \geq M_P^{-1} V_P^{-1}$. From these uncertainties one would again be able to conclude that the position of the collision can only be established with accuracy worse than L_p.

2.6 Spacetime in quantum gravity: events marked by D-particle collisions

As stressed above there is a quantum-gravity intuition that assumes a minimum localization uncertainty $\delta x \geq L_p$. I have also stressed that this intuition is based on an argument which is not fully robust, since it relies on the assumption that relativistic probes carrying only gravitational charge ("neutral") should achieve the best localization of a spacetime point. In the preceding subsection I removed one of these assumptions by considering a nonrelativistic probe (still neutral), and found that this cannot be used to improve on $\delta x \geq L_p$. In this subsection I intend to remove the other key assumption: I want to explore the possibility that probes carrying other charges (in addition to the gravitational) might lead to a better localization. My conclusion will be positive: certain types of charged particles can be used to obtain an improved (although still limited) localization. This might have rather profound implications, since it modifies the traditional quantum-gravity intuition favouring $\delta x \geq L_p$.

My observation actually relies on well-established results. In the study of the String Theory a new type of spacetime probe was encountered, the so-called D-particles. I shall not enter into a detailed description of this new probes, which however the interested reader can easily find in the literature (see, *e.g.*, Refs. [53, 54, 55, 56]). One key point is that the underlying supersymmetry of the theoretical framework imposes that D-particles, besides carrying gravitation charge, also carry another charge associated (in an appropriate sense) to the gravitational charge through supersymmetry. The end result is that [56] as long as the distance d between two D-particles is greater than $\sqrt{v}L_s$ (denoting with v the relative velocity and L_s the string length) the energy stored in a two-D-particle system can be described as $U \sim -L_s^6 v^4/d^7$, up to an (here irrelevant) overall numerical factor of order 1. This energy law replaces the corresponding Newton energy law that applies to uncharged particles. Another key property of D-particles is the relation between the D-particle mass M_D the string coupling g_s and the string length: $M_D \sim g_s^{-1} L_s^{-1}$. For $g_s \ll 1$ D-particles with appropriately small relative velocity can be basically treated as ordinary very-weakly-interacting point particles up to distances as small as $g_s^{1/3} L_s$, without encountering comparatively large relative-position quantum uncertainties.

On the basis of these facts in Refs. [55, 56] it was argued, within an analysis of D-particle scattering, that the point of collision between two D-particles can be localized with uncertainty as low as $\delta x \sim g_s^{1/3} L_s$ (but not lower). Since in the relevant theoretical framework $g_s \ll 1$ and the (10-dimensional) Planck length is $L_p \sim g_s^{1/4} L_s$, this result suggests that a certain level of subPlanckian localization accuracy is achievable.

Indeed proceeding in complete analogy with the discussion presented in the previous two subsections, and taking into account the mentioned D-particle properties, I find that the limit on localization using a D-particle probe (and target) is $\delta x \geq g_s^{1/3} L_s \sim g_s^{1/12} L_p$. I do observe however that such a level of spatial localization can only be achieved at the cost of a rather poor level of temporal localization. In fact, I find (again proceeding in complete analogy with the discussion presented in the previous two subsections) that the event is temporally localized with uncertainty $\delta t \geq g_s^{-1/3} L_s \geq g_s^{-7/12} L_p$, and, since $g_s \ll 1$, this amounts to an uncertainty limit which is significantly larger than the usually expected Planck-scale limit.

The example of D-particles suggests that the usual quantum-gravity expectation that $\delta x \geq L_p$ and $\delta t \geq L_p$ should separately hold might be incorrect. It is not obvious that D-particles should be included in the analysis. The fact that they have emerged in String Theory is of course not sufficient to conclude that they should be part of the correct quantum gravity. But, in the spirit of the Bohr-Rosenfeld ideas, D-particles might have to be considered even if they turned out not to exist in Nature. And actually we are free to look for other types of formal descriptions of probes which might achieve an even better localization. On the basis of this observation, I argue that a satisfactory conclusive analysis of the quantum-gravity limit on localization is still missing. The fact that the infinite-mass limit is no longer available in quantum gravity implies that some localization limit must hold, but additional studies are needed in order to establish the exact form of the limit.

In this respect I want to venture formulating a conjecture: for all types of probes that can be introduced in a logically consistent manner in quantum gravity the following measurability limit will apply $\delta x \delta t \geq L_p^2$. So I argue that $\delta x \geq L_p$ and $\delta t \geq L_p$ might not have to hold simultaneously, but for the product of the two uncertainties one should always find[h] $\delta x \delta t \geq L_p^2$. This is actually verified in all the contexts I considered, including the D-particle context, where the relation[i] $\delta x \delta t \geq L_s^2 \gg L_p^2$ holds.

With respect to this conjecture one possible concern could come from the fact that the analyses that inspired it are (to a large extent) 1+1-dimensional. Perhaps a relation of the type $\delta x \delta y \delta z \delta t \geq L_p^4$ should also be considered. It is tempting[j] to think of a localization limit which basically states that a given event can only be localized within a four-volume of a certain fixed size (*e.g.* L_p^4). This would also ease concerns about the covariance of the measurability limit (a four-volume is a rotation/boost invariant).

3 Spacetime and Lorentz symmetry in some quantum-gravity approaches

3.1 The three possibilities for the fate of Lorentz symmetry in quantum gravity

There are three possibilities for the fate of Lorentz symmetry in quantum gravity: Lorentz symmetry remains unmodified (exact ordinary Lorentz symmetry in the flat-spacetime limit), Lorentz symmetry is broken, Lorentz symmetry is deformed.

[h] An uncertainty limit even more significant than the $\delta x \delta t \geq L_p^2$ might emerge in contexts in which there is quantum-gravity-induced decoherence, and finds support in various versions of the Salecker-Wigner inspired quantum-gravity measurability bounds [5, 6].

[i] Yoneya (see, *e.g.* Ref. [57]) has proposed arguments in favour of the general validity in String Theory of an uncertainty relation $\delta x \delta t \geq L_s^2$. The arguments adopted by Yoneya do not appear to be fully in the spirit of more traditional measurability analyses (and therefore it would be important to find additional evidence in support of $\delta x \delta t \geq L_s^2$), but it is nonetheless noteworthy that an uncertainty relation of the type $\delta x \delta t \geq L_s^2$, derived in perturbative String-Theory frameworks with $L_s > L_p$, would be again consistent with my more general conjecture $\delta x \delta t \geq L_p^2$.

[j] John Stachel has often expressed a similar intuition.

It is of course not difficult to characterize the case in which Lorentz symmetry is preserved. In the flat-spacetime limit of quantum gravity the familiar relations between observations done by different inertial observers should emerge. If a given length is at rest and has value L for observer O, an observer O' boosted with velocity V (along the direction defined by the interval being measured) with respect to O must attribute to that length the value $L' = \sqrt{1 - V^2} L$. For massless particles all observers agree on the dispersion relation $E = p$.

We are also all familiar with the concept of "broken Lorentz symmetry" that is being encountered and discussed in some quantum-gravity research lines. It is completely analogous to the familiar situation in which the presence of a background selects a preferred class of inertial observers. This is reflected, for example, in the fact that the dispersion relation for light travelling in water, in certain crystals, and in other media is modified. Of course, the existence of crystals is fully compatible with a theoretical framework that is fundamentally Lorentz invariant, but in presence of the crystal the Lorentz invariance is manifest only when different observers take into account the different form taken by the tensors that characterize the crystal (or other background/medium) in their respective reference systems. If the observers only take into account the transformation rules for the energy-momentum of the particles involved in a process the results are not the ones predicted by Lorentz symmetry. In particular, the dispersion relation between energy and momentum of a particle depends on the background (and therefore takes different form in different frames since the background tensors take different form in different frames).

While the case in which Lorentz symmetry is preserved and the case in which Lorentz symmetry is broken are familiar, the third possibility recently explored in the quantum-gravity literature, the case of deformed Lorentz symmetry introduced in Ref. [58], is rather new and it might be useful to describe it here intuitively. It is the idea that in quantum gravity it might be appropriate to introduce a second observer-independent scale, a large-energy/small-length scale, possibly related to the Planck scale. It would amount to another step of the same type of the one that connects Galilei Relativity and Einstein's Special Relativity: whereas in Galilei Relativity the description of rotation/boost transformations does not involve any invariant/observer-independent scale, the observer-independent speed-of-light scale "c" is encoded in the Lorentz rotation/boost transformations (which can be viewed as a c-deformation of the Galileo rotation/boost transformations), and similarly in the case of a deformed Lorentz symmetry of the type introduced in Ref. [58] there are two scales encoded in the rotation/boost transformations between inertial observers (observers which are still indistinguishable, there is no preferred observer). In addition to the familiar observer-independent velocity scale c, there is a second, length (or inverse-momentum), observer-independent scale λ.

In order to provide additional intuition for the concept of deformed Lorentz symmetry let me consider the particular much-studied case in which the deformation involves a new dispersion relation $m^2 = f(E, p; \lambda)$ with $f(E, p; \lambda) \rightarrow E^2 - p^2$ in the limit $\lambda \rightarrow 0$ (see, *e.g.*, Refs. [58, 59, 60] and, for some related follow-up work in cosmology, also see Ref. [61]). A modified dispersion relation can also emerge (and commonly emerges) when Lorentz symmetry is broken, but of course the role of the modified dispersion relation in the formalism is very different in the two cases: when Lorentz symmetry is broken the modified dispersion relation reflects properties of a background/medium and the laws of boost/rotation transformation between inertial observers are not modified, while when Lorentz symmetry is deformed the modified dispersion relation reflects the properties of some new laws of boost/rotation transformation between inertial observers. This comparison provides an invitation to consider again the analogy with the transition from Galilei Relativity to Special Relativity. In Galilei Relativity, which does not have any relativistic-invariant scale, the dispersion relation is written as $E = p^2/(2m)$ (whose structure fulfills the requirements of dimensional analysis without the need for dimensionful coefficients). As experimental evidence in favour of Maxwell equations started to grow, the fact that those equations involve a special velocity scale appeared to require (since it was assumed that the validity of the Galilei transformations should not be questioned) the introduction of a preferred class of inertial observers, *i.e.* the "ether" background. Special Relativity introduces the first observer-independent scale, the velocity scale c, its dispersion relation takes the form $E^2 = c^2p^2 + c^4m^2$ (in which c plays a crucial role for what concerns dimensional analysis), and the presence of c in Maxwell's equations is now understood not as a manifestation of the existence of a preferred class of inertial observers but rather as a manifestation of the necessity to deform the Galilei transformations (the Lorentz transformations are a dimensionful deformation of the Galilei transformations). Analogously in some recent quantum-gravity research there has been some interest (see later in these notes) in dispersion relations of the type $c^4m^2 = E^2 - c^2\vec{p}^2 + f(E, \vec{p}^2; E_p)$ and the fact that these dispersion relations involve an absolute energy scale, E_p, has led to the assumption that a preferred class of inertial observers should be introduced in the relevant quantum-gravity scenarios. But, as I stressed in the papers proposing physical theories with deformed Lorentz symmetry [58], this assumption is not necessarily correct: a modified dispersion relation involving two dimensionful scales might be a manifestation of new laws of transformation between inertial observers, rather than a manifestation of Lorentz-symmetry breaking.

This concludes my brief non-technical review of the concepts of preserved Lorentz symmetry, broken Lorentz symmetry, and deformed Lorentz symmetry. Before closing this subsection I must however introduce some terminology, which might be useful to readers interested in finding additional reading material on this subject. Relativistic theories based on deformed Lorentz transformations, with two relativistic-invariant scales, are sometimes called "doubly-special relativity" theories [58, 59, 60]. In describing a framework with broken Lorentz symmetry the fact that there is still full invariance under transformations taking into account of both the background and the particles energy/momentum is sometimes called "invariance under observer Lorentz transformations" [62]. And the fact that in a framework with broken Lorentz symmetry there are departures from Lorentz symmetry, if one does not take into account the laws of transformation of the backgrounds (and simply transforms the energy-momentum of the particles involved in a process), is sometimes described as a "lack of invariance under particle Lorentz transformations" [62].

3.2 Aside on the hypothesis of the Planck scale as a relativistic invariant

In quantum-gravity proposals it is very common that the Planck scale (or some related scale, like the string length) acquires a special role. One then must understand what are the implications of the presence of the Planck scale for Lorentz symmetry. The analysis is often rather difficult, since the formalisms used in quantum-gravity research have a very rich structure. As partial guidance for this type of studies one can resort to some useful analogies. The four topics on which I want to comment briefly are: the possibility of a maximum-velocity scale and its implications for Galilei transformations, the possibility of a minimum-wavelength scale and its implications for Lorentz transformations, the possibility of angular-momentum discretization scale (in the sense of ordinary nonrelativistic quantum mechanics) and its implications for rotation transformations, and the possibility of a maximum-acceleration scale and its implications for Lorentz transformations.

The Galilei transformations act on velocities, and therefore the presence of a maximum-velocity scale naturally invites one to consider departures from Galilei symmetry. If different inertial observers attribute different values to the maximum-velocity scale one should find that Galilei symmetry is broken (ether). If the maximum-velocity scale takes the same value for all inertial observers then Galilei transformation must be replaced by new laws of transformation between inertial observers (Special Relativity), in which the maximum-velocity scale should appear explicitly in the formulas that governs the transformations between different inertial observers (in fact, c does appear in the Lorentz transformation laws).

The Lorentz transformations act on wavelengths, and therefore the presence of a minimum-wavelength scale would naturally invite one to consider departures from Lorentz symmetry. If different inertial observers attribute different values to the minimum-wavelength scale one should find that Lorentz symmetry is broken ("quantum-gravity ether"). If the minimum-wavelength scale takes the same value for all inertial observers then Lorentz transformations must be replaced by new laws of transformation between inertial observers (doubly special relativity [58]), in which the minimum-wavelength scale should appear explicitly in the formulas that governs the transformations between different inertial observers (in fact, the minimum-wavelength scale does appear in the deformed transformation laws, in the doubly-special-relativity frameworks that do assume a minimum wavelength [58]).

Space-rotation transformations do act on the angular-momentum vector, but, as discussed more carefully in the later Section 5, the type of $\hbar$-discretization of angular momentum introduced in ordinary nonrelativistic quantum mechanics is not affected by the discretization scale [63, 64]. Therefore this angular-momentum discretization scale $\hbar$ does not require departures from space-rotation symmetry. In the formulas describing space-rotation transformations within ordinary nonrelativistic quantum mechanics the angular-momentum discretization scale $\hbar$ does not appear.

The Lorentz transformations do not act on accelerations, and therefore the presence of a maximum-acceleration scale would not encourage one to consider departures from Lorentz symmetry [65]. There is no reason for a maximum-velocity scale to appear in the formulas that govern the transformations between different inertial observers.

Clearly all of these scales can be introduced as invariants of some relevant symmetry transformations, but it is also clear that the nature of these invariants is somewhat different. I would like to tentatively propose the terminologies "trivial invariant" and "nontrivial invariant". The maximum-velocity scale is a nontrivial invariant of Lorentz boost transformations. And similarly a maximum-wavelength scale would be a nontrivial invariant of suitable doubly-special-relativity (deformed Lorentz) boost transformations. The angular-momentum-discretization scale encountered in ordinary nonrelativistic quantum mechanics is a trivial invariant of space-rotation transformations. And similarly a maximum-acceleration scale would be a trivial invariant of Lorentz boost transformations.

3.3 Fuzzyness and Lorentz symmetry

The description of different possibilities for the fate of Lorentz symmetry in quantum gravity was (intentionally) rather vague in the previous subsections. To provide a more precise characterization one should in particular describe how the symmetries are realized on (ensembles of) measurements. In particular, a point which appears to be often overlooked in the literature concerns the fact that symmetries should also govern the formulas that describe quantum uncertainty relations, and if quantum gravity introduces new uncertainty relations the rotation/boost transformations will have to be applied also to the uncertainty relations, so that the overall picture is consistent with the symmetry principles.

I will try to give an intuitive description of this concept in this subsection, by discussing a specific context. Consider a source which emits "simultaneously" a large number of photons (massless particles), and the photons are such that their energies $E \pm \delta E$ are contained in a certain wide range $E_0 - \Delta E \leq E \leq E_0 + \Delta E$, the range ΔE being much larger than the average uncertainties δE. With "simultaneously" here one of course must mean a level of simultaneity which is at least compatible with what we already know about ordinary quantum mechanics: since the time of emission is uncertain, $\delta t \sim 1/\delta E$, the simultaneity cannot be better than $1/\delta E$. One class of predictions coming from (deformed, broken or preserved) Lorentz symmetry concerns the average times of arrival at a detector. Unmodified Lorentz symmetry imposes that the average time of arrival is independent of energy, $t^f_{E_1} = t^f_{E_2}$ for any E_1 and E_2. In a deformed-Lorentz-symmetry or broken-Lorentz-symmetry scenario one could have instead a certain dependence of the average time of arrival on the energy (also see later Section 5).

Setting aside the analysis of the average arrival times (for which the implications of a given symmetry scenario are easily seen), let us consider the uncertainty in the arrival times. An interesting hypothesis is that quantum gravity might affect these uncertainties. Perhaps a group of particles emitted "simultaneously" with energy E_1 and energy uncertainty δE would reach the detector after a journey of time duration T with a time-of-arrival uncertainty $\delta t^f = F(E_1, T, \delta t^i; L_p)$, where F is some function that describes possible energy dependence, time-of-travel dependence and dependence on the initial time-of-emission uncertainty. The actual form of the function F should somehow reflect the symmetries of the theory (in particular, the admissible forms for the function F are different depending on whether there is exact classical Lorenz symmetry, deformed Lorentz symmetry or broken Lorentz symmetry).

3.4 Spacetime and Lorentz symmetry in String Theory

String Theory is the most mature quantum-gravity approach coming from the particle-physics perspective. As such it of course attempts to reproduce as much as possible the successes of quantum field theory, with gravity seen (to a large extent) simply as one more gauge interaction. Although the introduction of extended objects (strings, branes, ...) leads to subtle elements on novelty, in String Theory the core features of quantum gravity are described in terms of graviton-like exchange in a background classical spacetime.

Indeed String Theory does not lead to spacetime quantization, at least in the sense that its background spacetime has been so far described as completely classical. However, this point is not fully settled: it has been shown that String Theory eventually leads to the emergence of fundamental limitations on the localization of a spacetime event, which are not yet formalized in a fully satisfactory manner [66], and this might be in conflict with the assumption of a classical background spacetime. The Bohr-Rosenfeld consistency criteria are not yet satisfied: one adopts a background spacetime which can be classical, but then the theory itself tells us that the localization of a spacetime point is affected by a fundamental limitation. Clearly the description of spacetime in String Theory is still being developed, and requires the analysis of some subtle points. This logical inconsistency probably tells us that the classical spacetime background cannot be anything else but a formal tool, void of operative meaning, which should be eventually replaced by a physically meaningful spacetime picture in which no classical-spacetime idealization is assumed.

If eventually there will be a formulation of String Theory in a background spacetime that is truly quantum, it is likely (on the basis of the observations reported in these notes) that Lorentz symmetry will then not be an exact symmetry of the theory. If instead somehow a classical spacetime background can be meaningfully adopted, of course then there would be no *a priori* reason to conjecture departures from Lorentz symmetry: classical Minkowski spacetime would naturally be an acceptable background, and a theory in the Minkowski background can be easily formulated in Lorentz-invariant manner. Still , it is noteworthy that, even assuming that it makes sense to consider a classical background spacetime, the fate of Lorentz symmetry in String Theory is somewhat uncertain: it has been found that under appropriate conditions (a vacuum expectation value for certain tensor fields) Lorentz symmetry is broken in the sense I described above. In these cases String Theory admits description (in the effective-theory sense) in terms of field theory in a noncommutative spacetime [15] with most of the studies focusing on the possibility that the emerging noncommutative spacetime is "canonical". Canonical noncommutative spacetimes are discussed in Subsection 3.4 and I postpone to that subsection a discussion of some features of the relevant Lorentz-symmetry breaking.

In summary in String Theory (as presently formulated, admitting classical backgrounds) it is natural to expect that Lorentz symmetry be preserved. In some cases (when certain suitable background/"external" fields are introduced) this fundamentally Lorentz-invariant theory can experience Lorentz-symmetry breaking. There has been so far no significant interest or results on deformation of Lorentz symmetry in String Theory (see, however, Refs. [67]).

3.5 Spacetime and Lorentz symmetry in Loop Quantum Gravity

Loop Quantum Gravity is the most mature approach to the quantum-gravity problem that originates from the general-relativity perspective. As in the case of String Theory, it must be stressed that the understanding of this rich formalism is still in progress. As presently understood, Loop Quantum Gravity predicts an inherently discretized spacetime [68], and this occurs in a rather compelling way: it is not that one introduces by hand an *a priori* discrete background spacetime; it is rather a case in which a fully background-independent analysis ultimately leads, by a sort of self-consistency, to the emergence of spacetime discretization. There has been much discussion recently, prompted by the studies [20, 69, 70], of the possibility that this discretization might lead to broken Lorentz symmetry. Although there are cases in which a discretization is compatible with the presence of continuous classical symmetries [71, 63, 64], it is of course natural, when adopting a discretized spacetime, to put Lorentz symmetry under careful scrutiny. Arguments presented in Refs. [69, 70, 72], support the idea of broken Lorentz symmetry in Loop Quantum Gravity.

Moreover, very recently Smolin, Starodubtsev and I proposed [73] (also see the follow-up study in Ref. [74]) a mechanism such that Loop Quantum Gravity would be described at the most fundamental level as a theory that in the flat-spacetime limit admits deformed Lorentz symmetry. Our argument originates from the role that certain quantum symmetry groups have in the Loop-Quantum-Gravity description of spacetime with a cosmological constant, and observing that in the flat-spacetime limit (the limit of vanishing cosmological constant) these quantum groups might not contract to a classical Lie algebra, but rather contract to a quantum Hopf algebra.

In summary in Loop Quantum Gravity the study of the fate of Lorentz is still at a preliminary stage. All three possibilities are still being explored: Lorentz-symmetry preserved, broken or deformed. It is noteworthy however that until 3 or 4 years ago there was a nearly general consensus that Loop Quantum Gravity would preserve Lorentz symmetry, whereas presently the intuition of a majority of experts has shifted toward the possibility that Lorentz symmetry be broken or deformed[k].

3.6 On the fate of Lorentz symmetry in canonical noncommutative spacetime

There has been much recent interest in flat noncommutative spacetimes, as possible quantum versions of Minkowski spacetime. Most of the work has focused on various parts of the two-tensor parameter space

$$[x_\mu, x_\nu] = i\theta_{\mu\nu} + i\gamma^{\beta}_{\mu\nu} x_\beta \ . \tag{1}$$

The assumption that the commutators of spacetime coordinates would depend on the coordinates at most linearly is usually adopted for simplicity, but it also captures a very general intuition: assuming that the Planck scale governs noncommutativity (and therefore noncommutativity should disappear in the formal $L_p \rightarrow 0$ limit) and assuming that the commutators do not involve singular, $1/x^n$, terms one actually cannot write anything more general than

$$[x_\mu, x_\nu] = iL_p^2 Q_{\mu\nu} + iL_p C^{\beta}_{\mu\nu} x_\beta \ , \tag{2}$$

where now the tensors, Q and C, are dimensionless.

Most authors actually consider two particular limits [75]: the "canonical noncommutative spacetimes", with $\gamma^{\beta}_{\mu\nu} = 0$,

$$[x_\mu, x_\nu] = i\theta_{\mu\nu} \tag{3}$$

[k]Actually it is of course conceivable that Lorentz symmetry be deformed and broken. This would mean that at the fundamental level the laws of transformation between inertial observers are described *a la* doubly-special relativity [58] (deformed Lorentz symmetry), and then, for example, some tensor fields acquire a vacuum expectation value.

and the "Lie-algebra noncommutative spacetimes", with $\theta_{\mu\nu} = 0$,

$$[x_\mu, x_\nu] = i\gamma^\beta_{\mu\nu} x_\beta \ . \tag{4}$$

Let me start by discussing briefly the fate of Lorentz symmetry in canonical noncommutative spacetimes. An intuitive characterization can be obtained by looking at wave exponentials. The Fourier theory in canonical noncommutative spacetime is based [75] on simple wave exponentials $e^{ip^\mu x_\mu}$ and from the $[x_\mu, x_\nu] = i\theta_{\mu\nu}$ noncommutativity relations one finds that

$$e^{ip^\mu x_\mu} e^{ik^\nu x_\nu} = e^{-\frac{i}{2} p^\mu \theta_{\mu\nu} k^\nu} e^{i(p+k)^\mu x_\mu} \ , \tag{5}$$

i.e. the Fourier parameters p_μ and k_μ combine just as usual, with the only new ingredient of the overall phase factor that depends on $\theta_{\mu\nu}$. The fact that momenta combine in the usual way reflects the fact that the transformation rules for energy-momentum from one (inertial) observer to another are still the familiar, undeformed, Lorentz transformation rules. However, the product of wave exponentials depends on $p^\mu \theta_{\mu\nu} k^\nu$, it depends on the "orientation" of the energy-momentum vectors p^μ and k^ν with respect to the $\theta_{\mu\nu}$ tensor. This is a first indication that in these canonical noncommutative spacetimes there is Lorentz symmetry breaking. The $\theta_{\mu\nu}$ tensor plays the role of a background that identifies a preferred class of inertial observers[l]. Different particles are affected by the presence of this background in different ways, leading to the emergence of different dispersion relations, as shown by the results [9, 10, 11, 12] of the study of field theories in canonical noncommutative spacetimes.

3.7 On the fate of Lorentz symmetry in κ-Minkowski noncommutative spacetime

In canonical noncommutative spacetimes Lorentz symmetry is "broken" and there is growing evidence that Lorentz symmetry breaking occurs for most choices of the tensors θ and γ. It is at this point clear, in light of several recent results, that the only way to preserve Lorentz symmetry is the choice $\theta = 0 = \gamma$, *i.e.* the case in which there is no noncommutativity and one is back to the familiar classical commutative Minkowski spacetime. When noncommutativity is present Lorentz symmetry is usually broken, but recent results suggest that for some special choices of the tensors θ and γ Lorentz symmetry might be deformed, rather than broken. In particular, this appears to be the case for the Lie-algebra κ-Minkowski [8, 76, 77, 78, 79, 80] noncommutative spacetime $(l, m = 1, 2, 3)$

$$[x_m, t] = \frac{i}{\kappa} x_m \ , \qquad [x_m, x_l] = 0 \ . \tag{6}$$

[l] Note that these remarks apply to canonical noncommutative spacetimes as studied in the most recent (often String-Theory inspired) literature, in which $\theta_{\mu\nu}$ is indeed simply a tensor (for a given observer, an antisymmetric matrix of numbers). I should stress however that the earliest studies of canonical noncommutative spacetimes (see Ref. [3] and follow-up work) considered a $\theta_{\mu\nu}$ with richer mathematical properties, notably with nontrivial algebra relations with the spacetime coordinates. In that earlier, and more ambitious, setup it is not obvious that Lorentz symmetry is broken: the fate of Lorentz symmetry depends on the properties attributed to $\theta_{\mu\nu}$.

κ-Minkowski is a Lie-algebra spacetime that clearly enjoys classical space-rotation symmetry; moreover, at least in the Hopf-algebra sense discussed in Ref. [79], κ-Minkowski is invariant under noncommutative translations. Since I am focusing here on Lorentz symmetry, it is particularly noteworthy that in κ-Minkowski boost transformations are necessarily modified [79]. A first hint of this comes from the necessity of a deformed law of composition of momenta, encoded in the so-called coproduct (a standard structure for a Hopf algebra). One can see this clearly by considering the Fourier tranform. It turns out [8, 78, 81] that in the κ-Minkowski case the correct formulation of the Fourier theory requires a suitable ordering prescription for wave exponentials. From

$$: e^{ik^\mu x_\mu} :\equiv e^{ik^m x_m} e^{ik^0 x_0} \ , \tag{7}$$

as a result of $[x_m, t] = ix_m/\kappa$ (and $[x_m, x_l] = 0$), it follows that the wave exponentials combine in a nontrivial way:

$$(: e^{ip^\mu x_\mu} :)(: e^{ik^\nu x_\nu} :) =: e^{i(p\dot{+}k)^\mu x_\mu} : \quad . \tag{8}$$

The notation "$\dot{+}$" here introduced reflects the behaviour of the mentioned "coproduct" composition of momenta in κ-Minkowski spacetime:

$$p_\mu \dot{+} k_\mu \equiv \delta_{\mu,0}(p_0 + k_0) + (1 - \delta_{\mu,0})(p_\mu + e^{\lambda p_0} k_\mu) \ . \tag{9}$$

As argued in Refs. [58] the nonlinearity of the law of composition of momenta might require an absolute (observer-independent) momentum scale, just like upon introducing a nonlinear law of composition of velocities one must introduce the absolute observer-independent scale of velocity c. The inverse of the noncommutativity scale λ should play the role of this absolute momentum scale. This invites one to consider the possibility [58] that the transformation laws for energy-momentum between different observers would have two invariants, c and λ.

It is not yet fully established whether κ-Minkowski can be the basis for physical theories with deformed Lorentz symmetry, but very recent works provide encouragement for this idea [58, 59]. In work that preceded Refs. [58], some examples of Hopf algebras, the so-called κ-Poincaré algebras, which could describe deformed infinitesimal symmetry transformations for κ-Minkowski, had been worked out [77], but it was believed (on the basis of a few attempts [82]) that these algebra structures would not be compatible with a genuine symmetry group of finite transformations. In Refs. [58] it was proposed that one should look for deformed transformation laws that form a genuine group and it was shown that one example of the κ-Poincaré Hopf algebras previously considered in the mathematical literature did allow for the emergence of a group of finite transformations of the energy-momentum of a particle (while the same is not true for other examples of these Hopf algebras). That result amounts to proving that the mathematics of κ-Poincaré Hopf algebras (and therefore possibly κ-Minkowski) can meaningfully describe the one-particle sector of a physical theory in a way that involves deformed Lorentz symmetry. But it is still unclear whether some κ-Poincaré Hopf algebras can be used to construct a theory which genuinely enjoys deformed Lorentz symmetry throughout, including multiparticle systems.

The recipe adopted in the κ-Poincaré literature for the description of two-particle systems relies on a law of composition of momenta obtained through the coproduct sum

(9), and an action of boosts on the composed momenta induced by the action on each of the momenta entering the composition. This has been adopted in the κ-Poincaré literature even very recently [83], not withstanding the new deformed-Lorentz-symmetry perspective proposed in Ref. [58]. From a deformed-Lorentz-symmetry perspective this κ-Poincaré description of two-particle systems is not acceptable; in fact, for a particle-producing collision process $a + b \rightarrow c + d$ laws of the type $(p_a \dot{+} p_b)^\mu = (p_c \dot{+} p_d)^\mu$ are inconsistent with the relevant laws of transformation for the momenta of the four particles. The condition $(p_a \dot{+} p_b)^\mu = (p_c \dot{+} p_d)^\mu$ can be imposed in a given inertial frame but it will then be violated in other inertial frames (*i.e.* $(p_a \dot{+} p_b)^\mu - (p_c \dot{+} p_d)^\mu = 0 \rightarrow (p'_a \dot{+} p'_b)^\mu - (p'_c \dot{+} p'_d)^\mu \neq 0$).

So, in summary, in κ-Minkowski spacetime there are definitely some departures from Lorentz symmetry, and it appears likely that these departures could be codified within the deformed-Lorentz-symmetry (doubly-special relativity) scenario proposed in Ref. [58], but more work is needed to fully establish the role of deformed rotation/boost transformations.

4 Aside on Lorentz symmetry in discrete spacetimes

4.1 Introduction and summary of this Section

Now that I have provided a general picture of the fate of Lorentz symmetry in quantum gravity, I am basically ready to consider possible experimental tests that could establish which of these different pictures is correct. This is discussed in the next section. Before that, I thought it might be appropriate to devote a few pages to an aside on one of the key topics of debate in the quantum-gravity literature: does spacetime discretization automatically imply a broken Lorentz symmetry?

It is rather natural for quantum-gravity research to consider Planck-scale discretization, and this is one of the reasons for the interest in possible departures from Lorentz symmetry. In fact, most types of spacetime discretizations would be clearly incompatible with the presence of an exact continuous (Lorentz) symmetry. While this is true, the assumption that ordinary Lorentz symmetry be only an approximate symmetry in discrete-spacetime pictures is often made too simplistically: some quantum-gravity papers rely on the assumption that in any discretized space it would not be possible to realize continuous symmetries (and from that it is concluded that spacetime discretization would necessarily be in conflict with continuous Lorentz symmetry). But clearly it is not true that by introducing some element of discretization in a space one must necessarily renounce to the presence of continuous symmetries. There is an example in physics with which we are all familiar: angular momentum is discretized in ordinary (nonrelativistic) quantum mechanics but the theory is still consistent with invariance under space rotations.

While most forms of discretization are incompatible with continuous symmetries, some discretizations are compatible with continuous symmetries. It is therefore not possible to assume *a priori* that any scenario for spacetime discretization considered in the quantum-gravity literature should lead to departures from Lorentz symmetry. The fate of Lorentz symmetry should be examined carefully in any specific discretization scenario.

In the remainder of this section I make some observations which could be useful for the analysis of the implications for Lorentz symmetry of popular ideas about a quantum-gravity discretization of the spectrum of the observables length, area, 3-volume, and 4-volume. I start with some comments on the somewhat analogous situation involving space-rotation symmetry and angular-momentum discretization. I then consider the possible implications for Lorentz symmetry of discretization of length, area, 3-volume, and 4-volume. In the closing subsection I argue that these observations might be relevant for the analysis of the fate of Lorentz symmetry in the Loop Quantum Gravity approach.

4.2 Space-rotation symmetry in classical and ordinary (non-relativistic) quantum mechanics

For the purposes of these notes it is sufficient to focus on the implications of space-rotation symmetry for the angular momentum 3-vector. When an observer measures one or more components of the angular momentum of a classical system some facts can immediately be deduced about how that same angular momentum appears to a second observer[m], an observer whose reference axes are rotated with respect to the ones of the first observer. Let us call (x, y, z) the axes of the first observer O and (x', y', z') the axes of the second observer O'. If O measures all three components of the angular momentum, along the (x, y, z) axes, everything can be said about all of the components of that angular momentum along the (x', y', z') axes of O'. The triads (L_x, L_y, L_z) and $(L_{x'}, L_{y'}, L_{z'})$ are of course different but they are related by a simple rule of transformation (a space-rotation transformation). Similarly, if O measures the modulus of the angular momentum vector everything can be said about how that modulus appears to a second observer: the value of the modulus is the same for both observers. However, if O measures only the x component of the angular momentum it is still not possible to predict any of the components $L_{x'}, L_{y'}, L_{z'}$ that are most meaningful for O'.

Space-rotation symmetry transformations are crucial for the objectivity ("reality") of the angular-momentum vector. Each observer characterizes this vector by three (real, dimensionful) measured numbers. Each of these numbers is to be seen as the projection of the objective vector $\vec{L}$ along one of the axes of the observer, and, of

[m]These remarks, which concern how the same physical process is described by different observers, characterize passive space-rotation symmetry transformations. The active transformations instead connect different processes observed by a single observer. For example, in a world with space-rotation symmetry a collection of systems prepared in a way that does not break that symmetry will have to enjoy, as an esemble, the same properties along any given direction (*e.g.* the average result of measurements of L_x should coincide with the average result of measurements of L_y). Another example of manifestation of space-rotation symmetry within the class of processes observed by a single observer is the fact that the total angular momentum of an isolated system does not change in time (space-rotation symmetry imposes a constraint on the physical processes observed by a single observer by disallowing processes in which the total angular momentum of an isolated system is not a constant of time evolution).

course, in turn these axes must be physically identified by the observer. For example, an observer may choose as "x axis" the direction of a certain magnetic field (another vector), and in that case a crucial role is played by the fact that both in measurement and in theory one can meaningfully consider the projection $\vec{L}\cdot\vec{B}$. The observable simply denoted by "L_x" in the formalism inevitably corresponds physically to an observable obtained from two objective vectors, the angular-momentum vector $\vec{L}$ and a second vector such as $\vec{B}$. When $\vec{B}$ is known one can set up a measurement procedure for $L_x \equiv \vec{L}\cdot\vec{B}$. Knowledge of the three components L_x, L_y, L_z (*i.e.* of the projections along a triplet of orthogonal directions $\vec{B}^{(i)}$,$\vec{B}^{(j)}$,$\vec{B}^{(k)}$) is sufficient for predicting the component along any other given direction. But the knowledge of a single component, L_x, is not sufficient to determine how the angular-momentum vector projects along some other direction.

This observation is rather crucial in understanding how space-rotation symmetry, a classical continuous symmetry, can be maintained as a symmetry of systems in ordinary nonrelativistic quantum mechanics, in which angular momentum is "discretized". One might, at first sight, be skeptical that some rules of mechanics that discretize angular momentum could enjoy a continuous symmetry, but more careful reasoning quickly leads to the conclusion that there is no *a priori* contradiction between discretization and a continuous symmetry. In fact, the type of discretization of angular momentum which emerges in ordinary non-relativistic quantum mechanics is fully consistent with classical space-rotation symmetry.

It will be proper [64] to speak of classical symmetries of a quantum theory whenever all the measurements that the quantum theory allows are still subject to the rules imposed by the classical symmetry. Certain measurements that are allowed in classical mechanics are no longer allowed in quantum mechanics, but on those measurements that are still allowed at the quantum level the symmetry criteria can fail or succeed just as in classical mechanics. It is actually easy to verify that the presence of classical space-rotation symmetry is perfectly compatible with the principles of ordinary nonrelativistic quantum mechanics.

Just as in classical mechanics, in quantum mechanics the information that O obtains by measuring the square-modulus L^2 of the angular momentum is sufficient to establish how that square-modulus appears to a second observer: the value of the modulus is the same for both observers. It happens to be the case that the values of L^2 are constrained by quantum mechanics on a discrete spectrum (while all real positive values are allowed in classical mechanics), but this of course does not represent an obstruction for the action of the continuous symmetry on invariants, such as L^2.

When O measures the x component, L_x, of the angular momentum it is still not possible to predict the value of any of the components of that angular momentum along the (x', y', z') axes of O'. This is true at the quantum level just as much as it is true at the classical level. The fact that quantum mechanics constrains the values of the observable L_x on a discrete spectrum is compatible with continuous space-rotation symmetry, simply because the symmetry does not make predictions relevant for the single measurement of L_x.

In classical physics space-rotation symmetry also governs the relation between the triple sharp measurement (L_x, L_y, L_z) made by O and the corresponding measurement of $(L_{x'}, L_{y'}, L_{z'})$ made by O'. This statement is neither true nor false in quantum mechanics. In fact, quantum mechanics excludes[n] the possibility of simultaneous classical/sharp measurement of all components of angular momentum. This prediction of the classical symmetry is, in a sense, not verifiable in ordinary quantum mechanics, but it would be improper to say that it fails.

In summary, in quantum mechanics there is a specific type of "discretization of angular momentum" which only affects measurements of space-rotation invariants and measurements, such as the measurement of a single component L_x of angular momentum, on which space-rotation symmetry makes no predictions at all (space-rotation symmetry is not such that one can predict the value of $L_{x'}$ on the basis of the measurement of L_x). Since space-rotation symmetry does govern the relation between the triple measurement (L_x, L_y, L_z) made by O and the corresponding measurement of $(L_{x'}, L_{y'}, L_{z'})$ made by O', it is crucial (for the compatibility betweeen quantum mechanics and space-rotation symmetry) that according to quantum mechanics the measurement of one component, say L_x, introduces (in general) a significant uncertainty concerning L_y and L_z. If some theory (clearly very different from quantum mechanics) allowed the simultaneous sharp measurement of L_x, L_y, L_z and predicted discrete spectra for them, then the classical continuous space-rotation symmetry would inevitably fail to apply.

These observations clarify the deep connection between discretization and noncommutativity. In a space-rotation-invariant theory discretization of the spectrum of L_x requires noncommutativity of L_x with L_y and L_z. In traditional textbooks the relation between discretization and noncommutativity is only stressed at the level of formalism (as a property of the operators we use to formalize the properties of the relevant measurements), but here I have considered a direct relation between discretization of measurement results and noncommutativity of observables (intended as an obstruction for the simultaneous sharp measurement) in the context of a theory which, like ordinary quantum mechanics, is compatible with space-rotation symmetry.

We only need one last (but very important) test before concluding that the discretization of L_x in quantum mechanics is truly compatible with space-rotation symmetry. This is connected with the fact that, as I stressed, discretization of L_x requires, in presence of space-rotation symmetry, that whenever L_x is sharply measured the other components L_y and L_z are affected by significant uncertainty. There is a risk here of a logical inconsistency: one must verify that (at least some of) the procedures that are suitable for the sharp measurement of L_x are not such that they require sharp information on L_y and L_z. Even this test is successful: one can indeed measure sharply L_x without using any knowledge of L_y and L_z. For example, the Stern-Gerlach setup[o] measures sharply L_x without using any knowledge of L_y and L_z. Since also this final

[n] Of course, only the properties of generic eigenstates are of interest here. The fact that one could have an eigenstate with $L_x = L_y = L_z = 0$, in the special case $L^2 = 0$, has no implications for my argument. Also note that the condition $L_x = L_y = L_z = 0$ does not involve the discretization scale $\hbar$ and is space-rotation invariant both at the classical and the quantum level ($L_x = L_y = L_z = 0 \rightarrow L_{x'} = L_{y'} = L_{z'} = 0$).

[o] The Stern-Gerlach setup realizes physically the projection of the vector $\vec{L}$ along the direction of a magnetic field $\vec{B}$. It provides the measurement of L_x in terms of a primary measurement which is a measurement of a corresponding coordinate of the point of arrival of the particle on a screen. The value of the measured coordinate is insensitive (even in classical physics) on the value of L_y and L_z.

logical-consistency test is successful, I conclude that the type of discretization of angular momentum which is realized in ordinary nonrelativistic quantum mechanics is fully compatible with classical continuous space-rotation symmetry.

I will later make use of the criteria introduced in this subsection for some considerations on the possibility that some form of discretization of lengths, areas, 3-volumes and 4-volumes might be compatible with the presence of classical continuous Lorentz symmetry. The analogy is very close, but there are some important differences. A key observation arises there at the last stage of analysis, the one that here required us to verify that one could actually measure sharply L_x without using any knowledge of L_y and L_z.

4.3 Discretization of lengths and Lorentz symmetry

I now want to explore the possibility that discretization of lengths might be compatible with ordinary (classical and continous) Lorentz symmetry. In setting up this analysis it is useful to start by considering a very simple procedure for the measurement of lengths, and examine this procedure ignoring, for the moment, all the measurability limits imposed by quantum mechanics (I therefore examine the measurement procedure as if it was carried through in classical mechanics). Let us consider a ruler with extremities marked A and C on a given spaceship O (the inertial frame O). In order to measure the length of the ruler, $L = AC$, one places a mirror at C and sends a light signal from A to C, which eventually returns at A after reflection by the mirror at C. The length of the ruler will be obtained from the measurement of the time needed for the two-way journey $A \to C \to A$. This time of travel is measured by a "light-clock of size d", *i.e.* another light beam is bounced back and forth between point A and a point B located at a distance d from A in the direction orthogonal to AC (if A and C lie on the x axis, B has the same x coordinate as A). A clock "tick" corresponds to each event of return of the light-clock beam at A. For the clock to be useful the distance $d = AB$ must be known very accurately, and in order to measure L accurately it must be that $d \ll L$.

I am assuming that the 3-point system A, B, C, is in rigid motion with respect to the observer on the spaceship (with respect to the origin of the inertial frame O), but it is useful not to assume that the points A, B, C are at rest. A possible dependence of the measurement result on the velocity of the A-B-C system will in fact play an important role in some observations reported later on. Let us therefore introduce a velocity V, which is the velocity of the A-B-C system with respect to the observer on the spaceship; specifically, let us take this common velocity of the points A, B, C as a 3-vector of modulus V, directed along the AC direction (along the x axis), pointing away from A and toward C.

The velocity V is already relevant at the level of establishing the calibration of the light-clock. Since the AB light-clock is moving with velocity V with respect to O, the observer O sees the trajectory of the light-clock light beam as a "zig-zag" between the moving points A and B. [For example, when bounced back from B toward A the light beam, according to observer O, goes in an oblique direction, and while the light beam progresses toward A, the point A keeps moving with velocity V.] We conclude that, according to observer O, each tick of the light-clock corresponds to a time $\tau_V = 2\,\overline{AC}/\sqrt{c^2 - V^2} = 2\,d/\sqrt{c^2 - V^2}$. The dependence on V of the light-clock tick time is easily understood as a manifestation of time dilatation: if the light-clock is at rest ($V = 0$) each tick corresponds to a time $\tau_0 = 2\,d/c$, while for $0 < V < c$ the light-clock tick corresponds to $2\,d/\sqrt{c^2 - V^2} > \tau_0$.

The velocity V also enters in the relation between the measured time (the time for the two-way $A \to C \to A$ journey) and the sought length L of the AC ruler. Since the AC ruler is moving with velocity V, the two parts of the two-way journey of the probe are of different length. For the first part of the journey of the probe the fact that the ruler is moving causes an increase of the duration of the probe's trip toward the next extremity of the ruler, while for the second part of the journey the distance is effectively shortened by the motion of the ruler. The first part of the journey ($A \to C$) requires that the probe travel a distance $cL/(c-V)$, while for the second part ($C \to A$) the distance is $cL/(c+V)$. The relation between the time T (given in the "number of ticks" form $N\tau_V$) needed by the probe for its two-way journey and the length L of the AC ruler is

$$T = N\tau_V = \frac{L}{c-V} + \frac{L}{c+V} = \frac{2cL}{c^2 - V^2} \;, \tag{10}$$

i.e.

$$L = \frac{T(c^2 - V^2)}{2c} = \frac{N\tau_V(c^2 - V^2)}{2c} = Nd\sqrt{1 - \frac{V^2}{c^2}} \;, \tag{11}$$

Here the V dependence is a manifestation of the FitsGerald-Lorentz length contraction. If the ruler is at rest ($V = 0$) its length L is given by the number of ticks of the clock multiplied by the size of the light-clock, $L = Nd$, while for $0 < V < c$ one finds a contracted length of the ruler $Nd\sqrt{1 - V^2/c^2} < Nd$.

Let us now consider a second spaceship/observer O' moving with velocity V_0 with respect to O. Of course the measurement procedure just described from the O perspective can be simultaneously witnessed by O'. However, O' must attribute a different calibration to the light-clock, since for O' the velocity of the A-B-C system is $V' = (V + V_0)/(1 + VV_0/c^2)$. It is easy to verify that for O' each tick of the light-clock amounts to $2\,d/\sqrt{c^2 - V'^2}$. And according to O' the result of the measurement procedure, the fact that the two-way journey of the probe takes N ticks of the light-clock, leads to the conclusion that the ruler has length $L' = Nd\sqrt{1 - V'^2/c^2}$.

In summary, a x-axis boost corresponding to relative OO' velocity V_0 is such that a ruler with x-axis velocity V and length L for O is, for O', a ruler with velocity $V' = (V+V_0)/(1+VV_0/c^2)$ and length $L' = L\sqrt{(c^2 - V'^2)/(c^2 - V^2)}$. The way in which a Lorentz boost transforms the length L into the length L' depends on the velocity of the ruler. It is also important to notice that the measurement procedure for L, at least as here setup in terms of a primary time measurement, can only be successful if the velocity of the ruler is known (from Eq.(11) we see that $L = f(T;V) = T(c^2 - V^2)/(2c)$).

Let us first focus on the fact that the way in which a Lorentz boost transforms the length L into the length L' depends on the velocity of the ruler. This can be viewed in analogy with the fact that a space-rotation around the z axis transforms the x-axis component of angular momentum, L_x, into the x'-axis component, $L_{x'}$, in a way that depends on the value of the y-axis component L_y: $L_{x'} = \cos(\alpha)L_x + \sin(\alpha)L_y$. As emphasized in the previous subsection, discretization of the L_x ($L_{x'}$) spectrum can be compatible with invariance under arbitrary continuous α-angle rotation only if the instances in which L_x does take a sharp (discrete) value are such that L_y is affected by an irreducible uncertainty. This is the case in ordinary quantum mechanics, where the L_x and L_y observables do not commute. Analogously, in a quantum-gravity theory in which the spectrum of lengths is discrete compatibility with Lorentz symmetry requires [64] that the length observable does not commute with the velocity observable, *i.e.* it requires that the instances in which the length of the ruler L does take a sharp (discrete) value are such that the velocity V of the ruler is affected by an irreducible uncertainty. This follows from the fact that a Lorentz boost by a velocity V_0 leads to a transformation $\{V, L\} \to \{V', L'\} \equiv \{(V+V_0)/(1+VV_0/c^2), L\sqrt{(c^2 - V'^2)/(c^2 - V^2)}\}$. The fact that a Lorentz boost transforms the length L into the length L' in a way that is continuous and depends on the velocity of the ruler leads to a necessary condition, $[L, V] \neq 0$, for length discretization to be compatible with ordinary Lorentz symmetry.

However, even in theories in which $[L, V] \neq 0$ it might still not be possible to achieve a logically-consistent scheme for the compatibility of length discretization with Lorentz symmetry. An obstruction is suggested from the observation that, as emphasized above, a sharp measurement of the length of the ruler L requires that the velocity of the ruler is known exactly. In ordinary quantum mechanics L_x discretization is compatible with space-rotation symmetry because sharp measurements of L_x introduce a large uncertainty in the measurement of L_y. Since information on L_y is not needed for the completion of L_x measurement procedures (as in the Stern-Gerlach example) it is perfectly logical to contemplate contexts in which L_x is sharply measured while L_y is affected by an irreducible uncertainty. Now we have seen that length discretization could be compatible with Lorentz symmetry only if a sharp measurement of L introduces a large uncertainty in the measurement of V. But since sharp information on V is needed for the completion of a sharp L measurement procedure it is puzzling to contemplate contexts in which L is sharply measured while V is affected by an irreducible uncertainty.

This obstruction represents a serious challenge for the idea of discrete lengths introduced compatibly with ordinary Lorentz symmetry. The obstruction cannot be eliminated within the length-measurement procedure here adopted: any uncertainty in the velocity of the ruler would lead to at least some uncertainty in measurement of the length of the ruler (therefore creating a conflict with hypothesis of sharp measurement of L). Perhaps one should consider other length-measurement procedures, but the reader will easily verify that all the commonly considered length-measurement procedures do require sharp knowledge of the velocity of the ruler in order to achieve a sharp measurement of its length. Moreover, from a conceptual perspective it is puzzling to consider the possibility that only some very special length-measurement procedures could achieve sharp results. In fact, we usually refer to the "length of the ruler" as if it was an intrinsic property of the ruler, verifiable with any of a large choice of possible equivalent measurement procedures. If we must consider two classes of length measurements, a "class A" that can achieve sharp measurement and a "class B" that cannot, we would be more properly thinking of two different observables[p], one sharply measurable and one with fuzzy properties.

[p] An observable is properly introduced through a specific measurement procedure. We can attribute several measurement procedures to "the same" observable only when these procedures give the same results (allowing us to abstract the concept of length as an intrinsic property of the ruler, rather than a property of a specific measurement procedure applied to the ruler).

Some of these conceptual issues should be studied in the future, and the results may affect the perspective here advocated, but in the meantime it appears necessary to take seriously the obstruction here encountered. While waiting for studies of alternative length-measurement procedures (length-measurement procedures that somehow do not require knowledge of the speed of the ruler), it is proper to assume that we do not have a logically-consistent scenario for introducing length discretization in a way that is compatible with ordinary Lorentz symmetry.

4.4 On discretization of area, 3-volume and 4-volume

The conclusions drawn in the previous subsection for what concerns length discretization are also applicable to area discretization and 3-volume discretization. The action of a Lorentz boost on a given area (3-volume) depends on the velocity of the surface (3-dimensional object) whose area (3-volume) is being measured. And a sharp measurement of an area (3-volume) requires that the velocity of the surface (3-dimensional object) is simultaneously known sharply. This observation can be easily verified by considering the area of the triangle defined by 3 mirrors. That area can be measured in terms of a time-of-travel procedure analogous to the one described in the previous section.

For areas there has been also much discussion [84] of the possibility to measure the area of a metal plate using an electromagnetic device that keeps a second metal plate at a small distance d and measures the capacity C of the capacitor formed by the two plates. The primary measurement would be the capacity, and the sought area would be evaluated through the relation[q] $A = dC/\epsilon_0$. Also in the case of this area-measurement procedure it is necessary to assume that one can measure accurately the velocity[r] of the (metallic) surface whose area is being measured. In fact it is necessary to make sure that the two surfaces that compose the capacitor are parallel (constant distance d) and that they be centered with respect to one another. If the second surface (the one that belongs to the measuring device) is much larger than the surface whose area is being measured one should be concerned about "boundary effects" since the formula $A = dC/\epsilon_0$ actually assume a highly symmetric configuration (it strictly applies to infinite parallel metallic plates). If the two surfaces are roughly of the same size any relative velocity would of course affect the capacity.

The situation for area ad 3-volume therefore appears to be completely analogous to the one more carefully described here for what concerns lengths. Area discretization, intended as the existence of "states" in which area is sharp (no uncertainty) and only allowed to take discrete values, could be compatible with ordinary Lorentz symmetry

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[q]The presence of ϵ_0 reflects the simplifying assumption that the measurement be perfomed in absolute vacuum. This simplification does not affect the validity of my remarks.

[r]Although it is rather marginal with respect to the line of analysis advocated here, I should stress that in this area-measurement procedure based on capacity measurement it is necessary to measure the distance between the plates: if d is not known sharply then the relation between C and A becomes fuzzy and the discretization of A may become unobservable. Assuming that all area measurements rely on some distance/length measurement one would conclude that there are inevitable logical inconsistencies in any attempt to construct a theory in which areas can be sharply measured but lengths cannot be sharply measured. It is therefore a high priority for future research on these topics to establish whether it is possible to devise an area-measurement procedure that does not rely on any length measurements.

only in a theory in which the sharp measurement of the area requires an irreducible uncertainty in the measurement of the velocity of the surface whose area is being measured. But actually, since in the measurement procedures so far considered the sharp measurement of areas requires an equally sharp knowledge of the velocity of the surface, there appears to be a logical obstruction for the idea of a discretization of area that is consistent with ordinary Lorentz symmetry. Clearly an analogous argument applies to 3-volume discretization.

Somewhat different is the case of 4-volumes. A 4-volume is an invariant of Lorentz transformations. There should not be any in-principle obstruction for introducing a discretization of 4-volumes in a way that is compatible with ordinary Lorentz symmetry. I am not really familiar with measurement procedures that allow the measurement of a 4-volume without resorting to separate measurements of, say, a 3-volume and a time interval, but assuming that procedures for the direct measurement of 4-volumes can be devised it appears natural to assume that the outcome of these measurements could be constrained on a discrete spectrum (when the outcome is sharp) and that this could be implemented in a way that is fully compatible with ordinary Lorentz symmetry.

4.5 Lorentz symmetry and the type of discretization of area and 3-volume discussed in the Loop Quantum Gravity literature

As already mentioned earlier in these notes, the present understanding of Loop Quantum Gravity involves a discretization of spacetime. The discretization is such that [68] the spectra of the area observable and of the 3-volume observable are discrete.

Since it is not uncommon (although not necessary either, as stressed above) for a "discretized space" to be affected by departures from the symmetries of the continuum limit, the fact that in the Loop Quantum Gravity literature there has been much discussion of broken Lorentz symmetry scenarios [69, 70, 72] and deformed Lorentz symmetry scenarios [73, 74] could be interpreted as a manifestation of this discretization.

Indeed, following the line of reasoning advocated in this section (and in Ref. [64]) one is led to the conclusion that, while in general discretization of a space does not necessarily imply loss of all continuous symmetries, a discretization of area and 3-volume cannot be introduced compatibly with Lorentz symmetry. Even if one devises a scenario in which the area observable does not commute with the surface-velocity observable, one still should end up finding a logical inconsistency between discretization of area and Lorentz symmetry. The inconsistency, as stressed above, originates from the fact that, when the area observable and the surface-velocity observable do not commute, a sharp measurement of the area observable appears to be impossible, since (at least in the most common area-measurement procedures, some of which have been considered here) information on the surface velocity is needed in order to perform an accurate surface-area measurement.

In giving a physical meaning to the area-discretization results it is commonly stated in the Loop-Quantum-Gravity literature that the flat surface of a table (or similar examples) could only take certain discrete values. In light of the analysis here reported this appears to be inconsistent with an unmodified Lorentz symmetry.

In light of the present preliminary status of the development of Loop Quantum Gravity, it is perhaps useful to stress that the arguments presented here do not completely rule out a compatibility between area discretization and ordinary Lorentz symmetry. In order for my argument to be applicable the area discretization must concerns flat surfaces in a flat spacetime. It seems to me that at present very little is understood of the physical interpretation of Loop-Quantum-Gravity area eigenstates. The familiar Lorentz-transformation formulas for areas (and surface velocity) assume that the underlying spacetime is flat and the surface is flat. Perhaps none of the Loop-Quantum-Gravity area eigenstates would provide a suitable description of this situation (although the example adopted in the Loop-Quantum-Gravity literature, making reference to the flat surface of a table, would suggest it). None of the results obtained in the Loop-Quantum-Gravity literature appears to prove that the area of a flat surface can be measured sharply. Perhaps only certain specific non-flat surfaces can be measured sharply. If the area discretization only concerns such non-flat surfaces then there is no obvious reason for questioning ordinary Lorentz symmetry on the basis of area discretization.

5 Experimental searches of Planck-scale departures from Lorentz symmetry

Studies of the fate of Lorentz symmetry in quantum gravity provide an excellent example of quantum-gravity-phenomenology research line. As discussed in the previous sections, in several (though, of course, not all) approaches to the quantum-gravity problem one finds some evidence of departures from the familiar Lorentz symmetry. Like other effects discussed in the quantum-gravity literature, the ones associated with departures from Lorentz symmetry are very striking from a conceptual perspective. There is a general consensus that some strikingly new effects should be present in quantum gravity, although different intuitions for the quantum gravity problem may lead to favouring one or another of these effects. As mentioned, it was traditionally believed that even such strikingly new effects (certainly leading to characteristic signatures) could not be tested because of their small magnitude, set by the small ratio between the energy of the particles involved and the Planck energy scale. Work on quantum-gravity phenomenology has proven that this old expectation is incorrect. There is of course no guarantee that "quantum-gravity experiments" will ever lead to any actual discovery, but it is clearly incorrect to adopt the *a priori* assumption that the search of the tiny Planck-scale effects should be hopeless. This point is very clearly illustrated in the context of tests of Planck-scale departures from Lorentz symmetry, on which I focus in this section.

Rather than providing a more general discussion, I intend to convey my point in a simple way by focusing on the possible emergence of Planck-scale-modified dispersion relations,

$$E^2 = m^2 + \vec{p}^2 + f(\vec{p}^2, E, m; L_p) \ , \tag{12}$$

which are found in the large majority of quantum-gravity-motivated schemes for deviations from ordinary Lorentz invariance (see, *e.g.*, Refs. [8, 10, 20, 69, 72, 77]).

If the function f is nontrivial[s] and the energy-momentum transformation rules are unmodified (the familiar Lorentz transformations) then clearly f cannot have the exact same structure for all inertial observers. In this case Lorentz symmetry is necessarily "broken", in the sense clarified earlier in these notes, and it is legitimate to assume that, in spite of the deformation of the dispersion relation, the rules for energy-momentum conservation would be undeformed.

If instead f does have the exact same structure for all inertial observers, then necessarily the laws of transformation between observers must be deformed (they cannot be the ordinary Lorentz transformation rules). In this case Lorentz symmetry is deformed, in the sense of the doubly special relativity [58] discussed earlier in these notes. There is no preferred frame. The deformation of the laws of transformation between observers impose that one must also necessarily [58] deform the rules for energy-momentum conservation (these rules are "laws of physics" and must therefore be the same for all inertial observers).

While the case of deformed Lorentz symmetry might exercise a stronger conceptual appeal (since it does not rely on a preferred class of inertial observers), for the purposes of this paper it is sufficient to consider the technically simpler context of broken Lorentz symmetry. Upon admitting a broken Lorentz symmetry it becomes legitimate, for example, to adopt a dispersion relation with leading-order-in-L_p form

$$E^2 \simeq \vec{p}^2 + m^2 - \eta (L_p E)^n \vec{p}^2 \ , \tag{13}$$

without modifying the rules for energy-momentum conservation. In (13) η is a phenomenological parameter of order 1 (and actually, for simplicity, I will often implicitly take $\eta = 1$). n, the lowest power of L_p that leads to a nonvanishing contribution, is model dependent. In any given noncommutative geometry one finds a definite value of n, and it appears to be equally easy [58, 59, 85] to construct noncommutative geometries with $n = 1$ or with $n = 2$. In Loop Quantum Gravity one might typically expect [85] to find $n = 2$, but certain scenarios [69, 86] have been shown[t] to lead to $n = 1$.

I will use this widely-used scheme for Planck-scale Lorentz-symmetry breaking, with dispersion relation (13) and unmodified rules for energy-momentum conservation, to illustrate how a tiny (Planck-length suppressed) effect can be observed in certain experimental contexts. The analysis will also show that the difference between the case $n=1$ and the case $n=2$ is very significant from a phenomenology perspective. Already with $n = 1$, which corresponds to effects that are linearly suppressed by the Planck length, the correction term in Eq. (13) is very small: assuming $\eta \simeq 1$, for particles with energy $E \sim 10^{12} eV$, some of the highest-energy particles we produce in laboratory, it represents only a correction of one part in 10^{16}. Of course, the case $n = 2$ pays the even higher price of quadratic suppression by the Planck length and for $E \sim 10^{12} eV$ its effects are at the 10^{-32} level.

[s]For example, it would be pointless to introduce an $f = L_p^2 [E^2 - \vec{p}^2 - m^2]^2$, since then the dispersion relation (12) would be equivalent to $E^2 = m^2 + \vec{p}^2$.

[t]Note however that the Loop-Quantum-Gravity scenario of Ref. [69] does not exactly lead to the dispersion relation (13): for photons ($m = 0$) Ref. [69] describes a polarization-dependent effect (birefringence).

5.1 Gamma-ray bursts and Planck-scale-induced in-vacuo dispersion

A deformation term of the type $L_p^n E^n p^2$ in the dispersion relation, such as the one in (13), leads to a small energy dependence of the speed of photons of order $L_p^n E^n$ (using the relation $v = dE/dp$). An energy dependence of the speed of photons of order $L_p^n E^n$ is completely negligible (both for $n = 1$ and for $n = 2$) in nearly all physical contexts, but, at least for $n = 1$, it can be significant [20, 21] in the analysis of short-duration gamma-ray bursts that reach us from cosmological distances. For a gamma-ray burst a typical estimate[u] of the time travelled before reaching our Earth detectors is $T \sim 10^{17}s$. Microbursts within a burst can have very short duration, as short as $10^{-4}s$. We therefore have one of the "amplifiers" mentioned in Section 1: the ratio between time travelled by the signal and time structure in the signal is a (conventional-physics) dimensionless quantity of order $\sim 10^{17}/10^{-4} = 10^{21}$. It turns out that this "amplifier" is sufficient to study energy dependence of the speed of photons of order $L_p E$. In fact, some of the photons in these bursts have energies in the $100MeV$ range and higher. For two photons with energy difference of order $\Delta E \sim 100MeV$ an $L_p \Delta E$ speed difference over a time of travel of $10^{17}s$ leads to a relative time-of-arrival delay of order $\Delta t \sim \eta T L_p \Delta E \sim 10^{-3}s$. Such a quantum-gravity-induced time-of-arrival delay could be revealed [20, 21] upon comparison of the structure of the gamma-ray-burst signal in different energy channels.

The next generation of gamma-ray telescopes, such as GLAST [87], will exploit this idea to search for energy dependence of the speed of photons of order $L_p E$.

The same analysis leading to a time-of-arrival difference of order $10^{-3}s$ for the $n = 1$ case, leads of course to a much smaller effect in the case $n = 2$ (the case of quadratic suppression by the Planck length). For $n = 2$ the same analysis leads to a time-of-arrival-difference estimate of order $10^{-23}s$, which is much beyond the sensitivities achievable with GLAST and all foreseeable gamma-ray observatories.

Some access to effects characterized by the $n = 2$ case could be gained by exploiting the fact that, according to current models [88], gamma-ray bursters should also emit a substantial amount of high-energy neutrinos. With advanced planned neutrino observatories, such as ANTARES [89], NEMO [90] and EUSO [91], it should be possible to observe neutrinos with energies between 10^{14} and 10^{19} eV. Models of gamma-ray bursters predict in particular a substantial flux of neutrinos with energies of about 10^{14} or 10^{15} eV. One could, for example, compare the times of arrival of these neutrinos emitted by gamma-ray bursters to the corresponding times of arrival of low-energy photons. For the case $n = 1$ one would predict a huge time-of-arrival difference ($\Delta t \sim 10^4 s$) and even for the case $n = 2$ the time-of-arrival difference could be significant (*e.g.* $\Delta t \sim 10^{-9}s$) and possibly within the reach of observatories that could conceivably be planned for the not-so-distant future.

Current models of gamma-ray bursters also predict some production of neutrinos with energies extending to the $10^{19}eV$ level and higher. For such ultra-energetic neutrinos a comparison of time-of-arrival differences with respect to soft photons also emitted

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[u] Up to 1997 the distances from the gamma-ray bursters to the Earth were not established experimentally. By a suitable analysis of the gamma-ray-burst "afterglow" [20], it is now possible to establish the distance from the gamma-ray bursters to the Earth for a significant portion of all detected bursts. $10^{17}s$ is a rough average of this distance measurements.

by the burster should provide, assuming $n=2$, an even more significant signal, possibly at the level $\Delta t \sim 1s$, which would be comfortably[v] within the realm of timing accuracy of the relevant observatories.

5.2 UHE cosmic rays and Planck-scale-modified thresholds

Let us now consider another significant prediction that comes from adopting the dispersion relation (13). I want to show that also certain types of energy thresholds for particle-production processes may be sensitive to the tiny $L_p^n E^n p^2$ modification of the dispersion relation I am considering. While in-vacuo dispersion, discussed in the preceding subsection, only depends on the deformation of the dispersion relation[w], the effects considered in this subsection also depend on the rules for energy-momentum conservation, which are not modified in the Lorentz-symmetry breaking scenario I am considering.

Let us start by considering a collision between a soft photon of fixed/known energy ϵ and a high-energy photon of energy E. It is useful to review briefly the usual calculation of the E threshold for electron-positron pair production: $\gamma + \gamma \rightarrow e^+ + e^-$. One can optimize the calculation by starting with the observation that the photon-photon invariant evaluated in the lab frame must be equal to (among other things) the electron-positron invariant evaluated in the center-of-mass frame:

$$(E + \epsilon)^2 - (P - p)^2 = 4m_e^2 \ . \tag{14}$$

Using the ordinary special-relativistic dispersion relation, this leads to the "threshold condition"

$$E \geq E_{th} = m_e^2/\epsilon \ . \tag{15}$$

[v] For this strategy relying on ultra-high-energy neutrinos the delicate point is clearly not timing, but rather the statistics (sufficient number of observed neutrinos) needed to establish a robust experimental result. Moreover, it appears necessary to understand gamma-ray bursters well enough to establish whether there are typical at-the-source time delays. For example, if the analysis is based on a time-of-arrival comparison between the first (triggering) photons detected from the burster and the first neutrinos detected from the burster it is necessary to establish that there is no significant at-the-source effect such that the relevant neutrinos and the relevant photons are emitted at significantly different times. The fact that this "time history" of the gamma-ray burst must be understood only with precision of, say, $1s$ (which is a comfortably large time scale with respect to the short time scales present in most gamma-ray bursts) gives us some hope that the needed understanding could be achieved in the not-so-distant future.

[w] The dispersion relation (13) can also be implemented in a doubly special relativity (deformed Lorentz symmetry) scenario [58]. The in-vacuo-dispersion analysis discussed in the preceding subsection applies both to Lorentz-symmetry breaking and Lorentz-symmetry deformation scenarios adopting (13). When (13) is adopted in a Lorentz-symmetry deformation scenario it is necessary [58] to consistently modify the laws of energy-momentum conservation. Therefore the analysis of Planck-scale-modified thresholds discussed in this subsection, which assumes unmodified laws of energy-momentum conservation, does not apply to the scenario in which (13) is adopted in a Lorentz-symmetry deformation scenario. Planck-scale-modified thresholds are present also in the case of Lorentz-symmetry deformation, but there are significant quantitative differences [58].

Notice that in going from (14) to (15), using the ordinary dispersion relation, the leading-order terms of the type E^2,P^2 have cancelled out, leaving behind the much smaller (if $\epsilon \ll E$) term of order $E\epsilon$. This cancellation provides the "amplifier" needed in quantum-gravity phenomenology, which in this case can be identified as E/ϵ. In presence of the Planck-scale departures from Lorentz symmetry the threshold will be significantly modified if $L_p^n E^n p^2$ is comparable to (or greater than) $E\epsilon$. While we normally expect L_p-related effects to become significant when the particles involved have energy $1/L_p$, here for $n = 1$ the effect is already significant when $E \sim (\epsilon/L_p)^{1/2}$, which can be considerably smaller than $1/L_p$ if ϵ is small. Analogously for $n = 2$ the effect starts being significant at $E \sim (\epsilon/L_p^2)^{1/3}$. In fact, adopting the modified dispersion relation (13) and imposing ordinary (unmodified) energy-momentum conservation one finds [25] the modified threshold relation

$$E_{th}\epsilon - \eta L_p^n E_{th}^{2+n} \frac{2^n - 1}{2^{2+n}} \simeq m_e^2 \tag{16}$$

(which again is valid when $E \gg m$ and $E \gg \epsilon$).

Analogous modifications of threshold relations are found for other processes. In particular, the case of photopion production, $p + \gamma \to p + \pi$, also leads to an analogous result in the case in which the incoming proton has high energy E while the incoming photon has energy ϵ such that $\epsilon \ll E$. In fact, adopting the modified dispersion relation (13) and imposing ordinary (unmodified) energy-momentum conservation one finds [25] the modified threshold relation

$$E_{th} \simeq \frac{(m_p + m_\pi)^2 - m_p^2}{4\epsilon} - \eta \frac{L_p^n E_{th}^{2+n}}{4\epsilon} \left(\frac{m_p^{1+n} + m_\pi^{1+n}}{(m_p + m_\pi)^{1+n}} - 1 \right) . \tag{17}$$

This result on the photopion-production threshold is relevant for the analysis of UHE (ultra-high-energy) cosmic rays. A characteristic feature of the expected cosmic-ray spectrum, the so-called "GZK limit", depends on the evaluation of the minimum energy required of a cosmic ray in order to produce pions in collisions with cosmic-microwave-background photons. According to ordinary Lorentz symmetry this threshold energy is $E_{th} \simeq 5 \cdot 10^{19} eV$ and cosmic rays with energy in excess of this value should loose the excess energy through pion production. Strong interest was generated by the observation [22, 23, 24, 25, 26, 92] that the Planck-scale-modified threshold relation (17) leads, for positive η, to a significantly higher estimate of the threshold energy, an upward shift of the GZK limit. This would provide a description of the observations of the high-energy cosmic-ray spectrum reported by AGASA [93], which can be interpreted as an indication of a sizeable upward shift of the GZK limit. Both for the case $n = 1$ and for the case $n = 2$ the Planck-scale-induced upward shift would be large enough [22, 23, 24, 25, 26, 85, 92] for quantitative agreement with the UHE cosmic-ray observations reported by AGASA.

There are other plausible theory explanations for the AGASA "cosmic-ray puzzle", and the experimental side must be further explored, since another cosmic-ray observatory, HIRES, has not confirmed the AGASA results. The situation will become clearer with planned more powerful cosmic-ray observatories, such as the Pierre Auger Observatory, which will soon start taking data. Still, the possibility that in these cosmic-ray studies we might be witnessing the first manifestation of a quantum property of spacetime is of course very exciting; moreover, whether or not they end up being successful in describing cosmic-ray observations, these analyses provide another explicit example of a minute Planck-scale effect that can leave observable traces in actual data. If Auger ends up establishing that UHE cosmic-ray data are fully consistent with ordinary Lorentz symmetry this would result in very significant (Planck-scale) experimental bounds on quantum-spacetime-induced breakup of Lorentz symmetry not only for the case $n = 1$ but also for the case $n = 2$.

In addition to the process $p + \gamma \to p + \pi$ (and its implications for UHE cosmic rays), also the process $\gamma + \gamma \to e^+ + e^-$ has been considered from the point of view of experimental tests. From the result (16) it follows that, if $n = 1$, for $E \sim 10TeV$ and $\epsilon \sim 0.01eV$ the modification of the threshold is significant. These values of E and ϵ are relevant for the observation of multi-TeV photons from certain Blazars [24, 25]. This high-energy photons travel to us from very far and they travel in an environment populated by soft photons, some with energies suitable for acting as targets for the disappearance of the hard photon into an electron-positron pair. Depending on some still-poorly-known properties (such as the density) of the far-infrared soft-photon background the spectrum of multi-TeV photons from certain Blazars carries information that can be used to test the result (16), if η is positive. For $n = 1$ we are very close [26, 94, 95] to the sensitivity necessary to test the Planck-scale effects, but for $n = 2$ the effects are too small for testing in the foreseeable future.

5.3 Planck-scale modified decay amplitudes

The study of certain particle decays provides yet another possibility to test the idea of broken Lorentz symmetry at the Planck scale in the way codified by the model I am using in this section as illustrative example (the phenomenological model that adopts the modified dispersion relation (13), with unmodified laws of energy-momentum conservation). For negative η some particles which are stable at low energies become unstable above a certain energy scale, while for positive η some particles which are unstable at low energies become (nearly) stable at high energies.

Let me start by considering the case of positive η, which is the choice of sign needed in order to push upward the GZK limit for cosmic rays. I consider the simple example of the decay of a pion into two photons, and I focus on the case $n = 1$. Again it is useful to first review the relevant derivation within ordinary relativistic kinematics. One can optimize the calculation by starting with the observation that the photon-photon invariant in the lab frame should be equal to the pion invariant:

$$(E + E')^2 - (\vec{p} + \vec{p}')^2 = m_\pi^2 \ . \tag{18}$$

Using the conventional relativistic dispersion relation this can be easily turned into a relation between the energy E_π of the incoming pion, the opening angle ϕ between the outgoing photons, and the energy E of one of the photons (the energy E' of the second photon is of course not independent; it is given by the difference between the energy of the pion and the energy of the first photon):

$$\cos(\phi) \ = \ \frac{2EE' - m_\pi^2}{2EE'} \ , \tag{19}$$

where indeed $E' = E_\pi - E$. Of course, $\cos(\phi)$ must be ≤ 1, since ϕ must be a real physical angle for all values of E. Note however that typically (unless $E \simeq 0$ or $E \simeq E_\pi$) $m_\pi^2 \ll 2EE' \sim E_\pi^2/2$ and the equation for $\cos(\phi)$ has the form $\cos(\phi) = (2EE' - \Delta)/2EE'$, with $\Delta = m_\pi^2$. So the fact that $\cos(\phi) \leq 1$ for all values of E depends only on the fact that $\Delta > 0$, which is automatically satisfied within ordinary relativistic kinematics through the prediction $\Delta = m_\pi^2$. A new kinematics predicting that $\Delta < 0$ for some values of E would have significant implications. In order to have a negative Δ it is sufficient to introduce a relatively small correction, a correction of order m_π^2. This is what happens in the scheme I am considering. The modified dispersion relation (13) when combined with unmodified energy-momentum conservation, assuming $n = 1$, modifies the relation between ϕ, E_π and E according to the formula [27]

$$\cos(\phi) \simeq \frac{2EE' - m_\pi^2 + 3L_p E_\pi EE'}{2EE' + L_p E_\pi EE'} . \tag{20}$$

This relation shows that at high energies the phase space available to the decay is reduced by the Planck-scale correction: for given value of E_π certain values of E that would normally be accessible to the decay are no longer accessible (they would require $cos\theta > 1$). This effect starts to be noticeable at pion energies of order $(m_\pi^2/L_p)^{1/3} \sim 10^{15} eV$, but only very gradually (at first only a small portion of the available phase space is excluded).

As observed in Refs. [96, 27] this prediction can be tested through its implications for the longitudinal development of the air showers produced by interaction of high-energy cosmic-rays with the atmosphere. The pion lifetime is in fact a key factor in determining the longitudinal development of these air showers. Remarkably, certain puzzling features have been reported in analyses [96] of the longitudinal development of these air showers, and a possible explanation could be provided [27] by the type of high-energy pion stability that the Planck-scale effects can induce. Independently of whether or not this preliminary experimental encouragement is confirmed by more refined data on pion decay, it is important for the line of argument here presented that this scheme for the analysis of pion stability is another example of a Planck-scale effect that can become significant in processes involving particles with energies well below the Planck scale.

A interesting result is found also in the case of negative η: some particles which are stable at low energies become unstable at high energies. A much studied example is "photon instability": the process $\gamma \rightarrow e^+ + e^-$ would be allowed at high energies if one adopts the modified dispersion relation (13) and unmodified laws of energy-momentum conservation. The process $\gamma \rightarrow e^+ + e^-$ can be analyzed in close analogy with the previously discussed process $\gamma + \gamma \rightarrow e^+ + e^-$. Assuming $n = 1$, one finds that the process $\gamma \rightarrow e^+ + e^-$ is allowed when the photon energy is higher than $(m_e^2/L_p)^{1/3} \sim 10^{13} eV$. Observations in astrophysics appear to be in conflict [26, 97] with this prediction, and therefore the case with $n = 1$ and negative η is ruled out experimentally. More evidence that Planck-scale effects can be tested (so much so that some possibilities are being ruled out).

5.4 Interferometry and Planck-scale-induced in-vacuo dispersion

In discussions of possible experimental tests of Planck-scale (quantum-gravity) modifications of Lorentz symmetry it is commonly assumed that such tests should rely exclusively on astrophysics, as in the examples discussed so far in this section. However, Lämmerzahl and I recently observed [29] that in the foreseeable future (perhaps a not-so-distant future) Planck-scale modifications of Lorentz symmetry could be tested in the controlled laboratory setup of modern laser interferometers (LIGO/VIRGO-type ground interferometers or LISA-type space interferometers). Our observation is based on the idea of operating such an interferometer with two different frequencies[x], perhaps obtained from a single laser beam by use of a "frequency doubler" (see *e.g.* [98]).

I here just want to discuss a rough and simple-minded estimate of the magnitude of the effect, within a specific interferometric setup. The interested reader can find a more realistic analysis, and descriptions of other interferometric setups in Ref. [29]. Once again I will adopt as illustrative example of Planck-scale departures from Lorentz symmetry the one codified in the modified dispersion relation (13). I will only consider the case $n = 1$, *i.e.* departures from the standard dispersion relation that are only linearly suppressed by the Planck length. The even smaller effects that one encounters in the case $n = 2$ are clearly beyond the reach of the interferometric studies considered in Ref. [29].

Let me start by considering an interferometer with two orthogonal arms, respectively of length L and L'. L and L' are kept distinct because the signal turns out to be proportional to $|L - L'|$. Before entering the interferometer, a monochromatic wave with frequency ω goes through a frequency doubler. Both emerging beams, of frequencies ω and 2ω, are then split into a part that goes through the arm of length L and a part that goes through the arm of length L'. When the beams are finally back (after reflection by mirrors) at the point where the interference patterns are formed, one then has access to two interference patterns: one interference pattern combines two waves of frequency ω and the other interference pattern is formed combining analogously two waves of frequencies 2ω. In an idealized setup (ignoring for example a possible wavelength dependence in beam-mirror interactions) the observed intensities would be governed by

$$I_\omega \propto \frac{1}{2}\left(1 + \cos\phi_\omega\right)\,, \quad \phi_\omega = k(L' - L)\,, \tag{21}$$

$$I_{2\omega} \propto \frac{1}{2}\left(1 + \cos\phi_{2\omega}\right)\,, \quad \phi_{2\omega} = k'(L' - L)\,, \tag{22}$$

where k' is the wavelength associated with the doubled frequency 2ω.

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[x] Modern interferometers achieve remarkable accuracies also thanks to an optimization of all experimental devices for response to light of a single frequency. The requirement of operating with light at two different frequencies is certainly a challenge for the realization of interferometric setups of the type proposed in Ref. [29]. This and other practical concerns are not discussed here. The interested reader can find a preliminary discussion of these challenges in Ref. [29].

With the ordinary unmodified dispersion relation, one has that $k' = 2k$, but in the case of our Planck-scale-deformed dispersion relation

$$k' \simeq 2k + \eta \frac{k^2}{\omega_p} \,, \tag{23}$$

where ω_p is the Planck frequency ($\omega_p \sim E_p$).

One can then rewrite $\phi_{2\omega} - \phi_\omega$, from (21) and (22), using again the Planck-scale-deformed dispersion relation

$$\phi_{2\omega} - \phi_\omega = \omega \left(1 + \frac{3}{2}\eta \frac{\omega}{\omega_p}\right)(L' - L) \,. \tag{24}$$

This phase-difference relation characterizes a key difference between the standard dispersion relation and the Planck-scale-deformed dispersion relation in the interferometric setup here considered. In the case of the standard classical-spacetime dispersion relation one expects a specific type of correlations, which follow straightforwardly from $k' = 2k$, between the values of I_ω and $I_{2\omega}$ for given values of $L' - L$. For example, clearly one expects that the intensity $I_{2\omega}$ of the wave at frequency 2ω has a maximum whenever $L' - L = 2j\pi/\omega$ (with j any integer number), and that correspondingly the intensity I_ω of the wave at frequency ω has either a maximum or a minimum. One therefore predicts, without any need to establish the value of j, that the configurations in which there is a maximum of $I_{2\omega}$ must also be configurations in which there is a maximum or a minimum of I_ω. The Planck-scale-modified dispersion relation modifies this prediction: for example, as codified by Eq. (24), the modified dispersion relation predicts that configurations in which there is a maximum of $I_{2\omega}$ should be such that I_ω is in the neighborhood but not exactly at one of its maximum/minimum values. More precisely, when $L' - L$ is such that $I_{2\omega}(L' - L)$ is at a maximum value the quantum-gravity effect in (24) predicts that $I_\omega(L' - L)$ should differ from a maximum/minimum value of I_ω as if for being out-of-phase by an amount $(3\eta/2) \cdot (L' - L)\omega^2/\omega_p$.

This type of characteristic feature could be looked for by, for example, taking data at values of $L' - L$ that differ from one another by small (smaller than $1/\omega$) amounts in the neighborhood of a value of $L' - L$ that corresponds, say, to a maximum of $I_{2\omega}$. Perhaps, techniques for the active control of mirrors which are already being used in modern interferometers might be adapted for this task, and the development of dedicated techniques does not appear beyond our reach. In Ref. [29] we reported a simple-minded comparison of the magnitude of the Planck-scale-induced phase difference to the phase sensitivity of LIGO/VIRGO-type or LISA-type interferometers. That comparison provided some encouragement for the idea of performing these tests in the not-so-distant future, although several technical challenges are to be overcome before any attempt of an actual realization of this type of interferometric setup.

6 Closing remarks

There is perhaps something to be learned from looking at quantum-gravity research in the way I here advocated: exploring the connection between a given quantum-gravity approach and the perspective that generated it. This exercise appears to suggest that what one finds in a given quantum-gravity approach might be more directly connected with the perspective which has been adopted, rather than with something intrinsic in the quantum-gravity problem. Many features of quantum-gravity approaches that originate from the particle-physics perspective, such a String Theory, simply reflect the intuition that one develops working with the Standard Model of particle physics. Analogously quantum-gravity approaches that originate from the general-relativity perspective or from the condensed-matter perspective carry a strong trace of intuition developed in working in those fields.

The expectations concerning the fate of Lorentz symmetry in quantum gravity appear to be a natural way to discriminate between the different perspectives. As I stressed here, it is rather obvious that a particle-physics perspective should lead to quantum-gravity approaches in which there is no *a priori* reason for departures from ordinary Lorentz symmetry. And it is equally obvious that instead from the condensed-matter perspective Lorenz symmetry should only emerge as an approximate symmetry. Somewhat more subtle are the indications of the general-relativity perspective for the fate of Lorentz symmetry, and therefore I devoted a significant portion of these notes to the general-relativity perspective. Results obtained in recent years, some of which I reviewed here, suggest that also from the general-relativity perspective some departures from Lorentz symmetry might naturally emerge. If one adopts a description based fundamentally on noncommutative geometry (as some aspects of the general-relativity perspective could invite us to do) departures from Lorentz symmetry are really very natural, perhaps inevitable. And there is now growing evidence that also when the general-relativity perspective leads to discretized-spacetime approaches, as in Loop Quantum Gravity, departures from Lorentz symmetry are naturally encountered. I here also presented additional observations, of more general validity, that favour the presence of departures from Lorentz symmetry in approaches based on the general-relativity perspective.

Tests of possible Planck-scale departures from Lorentz symmetry, besides giving us a chance of finding the first experimental facts about the quantum-gravity realm, are therefore also a way to check which one of these three perspectives should be favoured. Remarkably, as discussed in Section 6, there will be several opportunities in these coming years for experimental searches of Planck-scale departures from Lorentz symmetry.

In choosing a title for these notes I ended up adopting one suggesting that there are only three possible perspectives on the quantum-gravity problem ("**The** three perspectives on the quantum-gravity problem"), but it is not unlikely that what we really need is the discovery of a novel fourth perspective, which may or may not be based on a combination of the three perspectives here considered.

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