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Techniques for NNLO Higgs production in the Standard Model and the MSSM

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New techniques developed in connection with the NNLO corrections to the Higgs production rate at hadron colliders and some recent applications are reviewed.

1 Introduction

The NLO corrections for the dominant Higgs production mechanism at hadron colliders, $gg \to H$, amount to about 70% and suffer from rather large scale uncertainties.[1,2,3] The need for the evaluation of the NNLO cross section has resulted in promising new calculational techniques. The first part of this talk will briefly review these techniques. In the second part, we will discuss a recent application, namely the NNLO cross section for MSSM Higgs production in bottom quark fusion.

2 Techniques for Higgs production at NNLO

The first calculation of the NNLO prediction for the cross section $\sigma(pp \to H + X)$ used the classic approach of computing the amplitudes for virtual and real corrections, squaring them, and integrating over the final state phase space. The two-loop virtual amplitude for $gg \to H$ was evaluated[4] using the method of Baikov and Smirnov[5] that maps the occuring integrals to the well-known class of three-loop propagator-type integrals.[6] The phase space integration for the two-loop virtual terms is trivial, resulting in $\delta(1-x)$, where $x = M_H^2/\hat{s}$ and $\hat{s}$ is the partonic center of mass (c.m.) energy.

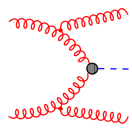
The one-loop amplitude for the radiation of a single massless parton has to be interfered with the corresponding tree-level expression, and integrated over the two-particle phase space. Both loop and phase space integration can be performed analytically.

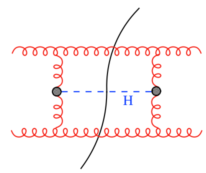
This leaves us with the tree-level contributions for the radiation of two massless partons. The squared amplitude can be obtained straightforwardly with the help of `FORM`.[7] In the first approach that we are going to describe, the phase space integrals were evaluated in terms of an expansion in $(1 - x)$. The leading term is called the *soft approximation*[8,9] and is formally of order $(1 - x)^{-1}$, where the associated divergence as $x \to 1$ is parameterized in terms of the distributions $\delta(1 - x)$ and $[\ln^n(1 - x)/(1 - x)]_+$. The higher orders in $(1 - x)$ can be obtained by a systematic expansion of the squared amplitude and the phase space measure.[10] The crucial point is that, independent of the degree of this expansion, one always ends up with the same type of integrals. This classifies the procedure as an algorithm, which can be fully automated.

Integrating the resulting expansion of the partonic cross section over the parton densities, one observes that higher orders in $(1-x)$ are numerically irrelevant, and the resulting hadronic cross section is phenomenologically equivalent to the result derived from the exact partonic cross section.[10] On the other hand, one can make an ansatz of a sum of polylogarithms with unknown coefficients, expand it in $(1-x)$, and compare the result with the expansion obtained for the partonic cross section.[11] Given that this expansion is known to sufficiently high (but finite!) orders, this determines the coefficients of the polylogarithms and thus the exact result for the partonic cross section.[11,12]

Clearly, the method of phase-space expansion is a bottom-up approach: starting at the soft approximation, one can successively improve the accuracy of the result by including higher orders in $(1-x)$, until a sufficient number of terms is known to invert the series and arrive at the exact result.

A second method to obtain the NNLO result for the Higgs production rate has been developed by Anastasiou and Melnikov[13]. According to the Cutkosky rules, one can write, for example,

$$\int \mathrm{dPS} \Big|$$ 

$$\Big|^2 =$$ 

(1)

where the initial and final states of the diagram on the r.h.s. are identical. *Without cuts*, this diagram would be a double-box diagram with two external scales, $\hat{s}$ and M_H. Such diagrams can be evaluated with the help of the general algorithms that have been developed within the last few years in the context of $2 \to 2$ scattering amplitudes at NNLO (see, e.g., a recent review[14]). The crucial observation of Anastasiou and Melnikov[13] was that these algorithms are directly applicable to *cut diagrams* of the kind shown on the r.h.s. of Eq. (1).[a] In this way, the partonic cross section for Higgs production was derived in closed form[13]. Needless to say that a re-expansion of this closed expression recovered the expressions derived through phase-space expansion[10]. In turn, the result obtained by inverting the expansion[11,12] confirmed the closed expression. The NNLO results for the production of a pseudo-scalar Higgs were obtained independently and simultaneously in both approaches[12,16]. Meanwhile, the NNLO results for both scalar and pseudo-scalar Higgs production have been re-confirmed using the analoguous approach that was applied to the NNLO Drell-Yan calculation.[17]

The phenomenological implications of the NNLO are significant and have been discussed extensively in the literature.[10,13,17] They shall not be repeated here due to space limitations.

3 Higgs production in bottom quark fusion

The production rate for a SM Higgs boson being under good theoretical control, one may ask to which extent these results are applicable also for Higgs boson production in supersymmetric models. For simplicity, we will restrict ourselves to the Minimal Supersymmetric Standard Model (MSSM) in this talk.

The answer is that the NNLO production rate of a neutral scalar Higgs boson in the MSSM can be inferred directly from the known MSSM prediction in only a rather restricted parameter space, in particular for small $\tan\beta$ and large squark masses. Also the production of a pseudo-scalar Higgs boson has been evaluated at NNLO[12,16,17] within these restrictions.

In other regions of parameter space, virtual contributions of sypersymmetric particles such as squarks may become important.[18] On the other hand, large values of $\tan\beta$ enhance the Yukawa

[a] Let us remark that the meaning of "cut" in this context is not restricted to Cutkosky-cuts. For example, the cut lines can be initial rather than final states (which leads to the method used to originally derive the virtual terms[5,4]), or they can be restricted to a particular kinematic configuration.[15]

coupling of bottom quarks. Thus, the $gg\phi$ coupling may have a significant contribution from virtual bottom quarks (ϕ denotes any of the neutral Higgs bosons in the MSSM). In this case, the NNLO corrections are much harder to evaluate, because the effective-Lagrangian approach of the top quark case is not expected to work.

The main focus here shall be another effect of an enhanced bottom Yukawa coupling, namely the increased rate of associated production of a Higgs boson with a bottom–anti-bottom quark pair. There has been an on-going discussion concerning the proper description of this process.[19,20,21,22] *A priori*, the leading order contribution is, of course, the tree-level process $gg \to \phi b\bar{b}$, where ϕ denotes any of the MSSM Higgs bosons h, H, A. However, when evaluating the total rate for this process, the integration over small bottom-p_T leads to collinear logarithms of the form $l_b \equiv \ln(m_b^2/M^2)$, where M is a scale of the order of the Higgs boson mass. Since $m_b \ll M_\phi \sim M$, these logarithms should be resummed. This is achieved by introducing bottom quark densities and making the process $b\bar{b} \to \phi$ the leading order contribution. Schematically, one can write the total cross section in the bottom density approach as follows:

$$\sigma(pp \to H + X) = \sum_{n=0}^{\infty} (\alpha_s l_b)^{n+2} \left[c_{n0} + c_{n1}\frac{1}{l_b} + \frac{1}{l_b^2}\left(c_{n2} + \alpha_s d_{n3} + \alpha_s^2\, d_{n4} + \ldots\right)\right] . \qquad (2)$$

This equation is to be understood as follows: First of all, one should note that the c_{ni} and d_{ni} are not obtained individually for each n, because the sum over n is implicit in the parton densities. Including only the leading order process $b\bar{b} \to \phi$, one obtains the contribution from the c_{n0}. The NLO diagrams contribute terms of order $1/l_b$ (e.g., $gb \to b\phi$) and of order α_s ($b\bar{b} \to \phi$ at 1-loop) with respect to the leading term. Both are contained in the c_{n1}. At N^{n}LO, for $n \geq 2$, we have terms of order α_s^{n-k}/l_b^k w.r.t. LO, where $k = 0, 1, 2$. The reason why there can be only two inverse powers of l_b comes from the fact that there are only two initial state partons. Looking at Eq. (2), it becomes clear why the NNLO plays an exceptional role in this process: It comprises all terms at leading order in α_s.

The calculation of the process[b] $pp \to (b\bar{b})\phi$ in the bottom density approach proceeds in complete analogy to, say, Drell-Yan production of virtual photons. Technical details of the calculation can be found elsewhere.[23]

Fig. 1 (a) shows the factorization scale dependence of the cross section at LO, NLO, and NNLO at the LHC, for a Higgs mass of $M_H = 120\,$GeV at the LHC. We notice several intriguing features: First, in contrast to the LO and NLO result, the NNLO curve has a clearly distinguished point where the derivative is zero, *i.e.* where the sensitivity to μ_F is minimal. Second, the NNLO corrections are zero at a point where the NLO corrections are small. Third, this point is close to the point of least sensitivity. And fourth, all these points are compatible with a previous estimate[20,22,21] of the "natural" factorization scale for this process of around $\mu_F = M_H/4$. For the Tevatron, the overall picture is essentially the same[23]. These features nicely demonstrate the self-consistency of the bottom density approach and support the general considerations concerning the proper choice of the factorization scale for heavy quark partons.[20,22,21]

Fig. 1 (b) shows the total cross section for $pp \to (b\bar{b})H + X$ as a function of the Higgs boson mass up to NNLO, for two different values of the factorization scale, indicating the theoretical uncertainty (the renormalization scale dependence can be neglected).

Conclusions. Recent progress in the evaluation of radiative corrections has led to NNLO predictions for Higgs production at hadron colliders, both in the SM and the MSSM. The results are very stable with respect to scale variations and indicate a well-behaved perturbative series.

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[b] We adopt the notation $(b\bar{b})\phi$ in order to indicate that the bottom quarks may be produced at small transverse momenta and thus escape detection.

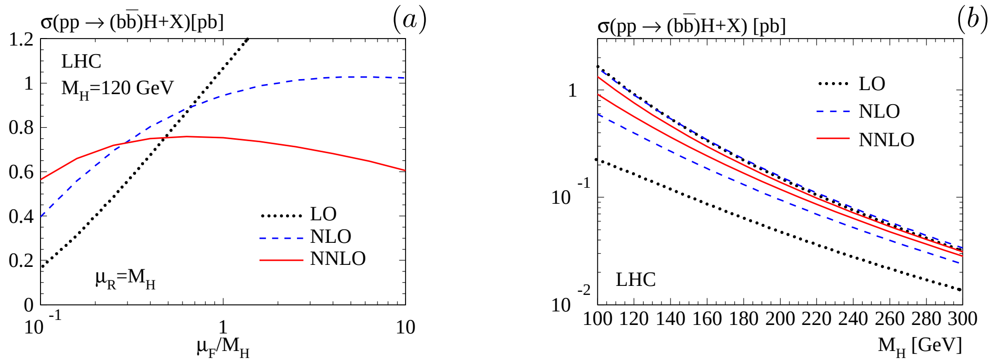

Figure 1:
(a) Factorization scale dependence of the cross section for $pp \to (b\bar{b})H + X$ ($\tan\beta = 1$); (b) Total cross section for $pp \to (b\bar{b})H + X$ — upper/lower line: $\mu_F = 0.7 M_H / \mu_F = 0.1 M_H$ [$\mu_R = M_H$].

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