Non-linear corrections to the DGLAP equations, looking for the saturation limits*

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Abstract
The effects of the first nonlinear corrections to the DGLAP equations are studied in light of the HERA data. Saturation limits are determined in the DGLAP+GLRMQ approach for the free proton and for the Pb nucleus.

1. Introduction

Parton distribution functions (PDF) of the free proton, $f_i(x, Q^2)$, are needed for the calculation of the cross sections of hard processes in hadronic collisions. Once they are determined at certain initial scale Q_0^2, the DGLAP equations [1] describe well their scale evolution at large scales. Based on the global fits to the available data several different parametrizations of PDF have been obtained [2, 3, 4]. The older PDF sets do not describe adequately the recent HERA data [5] on the structure function F_2 at the perturbative scales Q^2 at small x. In the analysis of newer PDF sets, such as CTEQ6 [4] and MRST2001 [2], these data have been taken into account. However, difficulties arise when fitting both small and large scale data simultaneously. In the MRST set, the entire H1 data set [5] has been used in the analysis, leading to a good average fit at all scales, but at the expense of allowing for a negative NLO gluon distribution at small x and $Q^2 \lesssim 1$ GeV^2. In the CTEQ6 set only the large scale ($Q^2 > 4$ GeV^2) data have been included, giving a good fit at large Q^2, but leaving the fit at small-x and small Q^2 ($Q^2 < 4$ GeV^2) region worse. Moreover, the gluon distribution at the values of small x and $Q^2 \lesssim 1.69$ GeV^2 has been set to zero.

These problems are interesting as they can be signs of a new QCD phenomenon: at small values of momentum fraction x and scales Q^2, gluon recombination terms, which lead to nonlinear corrections to the evolution equations, can become significant. First of these nonlinear terms have been calculated by Gribov, Levin and Ryskin [6], and, Mueller and Qiu [7]. In the following these correction terms shall be referred to as GLRMQ terms for short. With the modifications, the evolution equations become [7]

$$\frac{\partial xg(x,Q^2)}{\partial \log Q^2} = \left.\frac{\partial xg(x,Q^2)}{\partial \log Q^2}\right|_{\mathrm{DGLAP}} - \frac{9\pi}{2}\frac{\alpha_s^2}{Q^2}\int_x^1 \frac{dy}{y} y^2 G^{(2)}(y,Q^2), \tag{1}$$

$$\frac{\partial x\bar{q}(x,Q^2)}{\partial \log Q^2} = \left.\frac{\partial x\bar{q}(x,Q^2)}{\partial \log Q^2}\right|_{\mathrm{DGLAP}} - \frac{3\pi}{20}\frac{\alpha_s^2}{Q^2} x^2 G^{(2)}(x,Q^2) + \ldots G_{\mathrm{HT}}, \tag{2}$$

where the two-gluon density can be modelled as $x^2G^{(2)}(x, Q^2) = \frac{1}{\pi R^2}[xg(x, Q^2)]^2$, with the radius of the proton $R = 1$ fm. The higher dimensional gluon term G_{HT} [7] is here assumed to be zero. The effects of the nonlinear corrections to the DGLAP evolution of the PDF of the free proton were studied in [8] in view of the recent H1 data; the results are discussed below.

2. The analysis

The goal of the analysis in [8] was (1) to possibly improve the (LO) fit of the calculated $F_2(x, Q^2)$ to the H1 data [5] at small Q^2, while (2) at the same time maintain the good fit at large Q^2, and finally (3) to study the interdependence between the initial distributions and the evolution.

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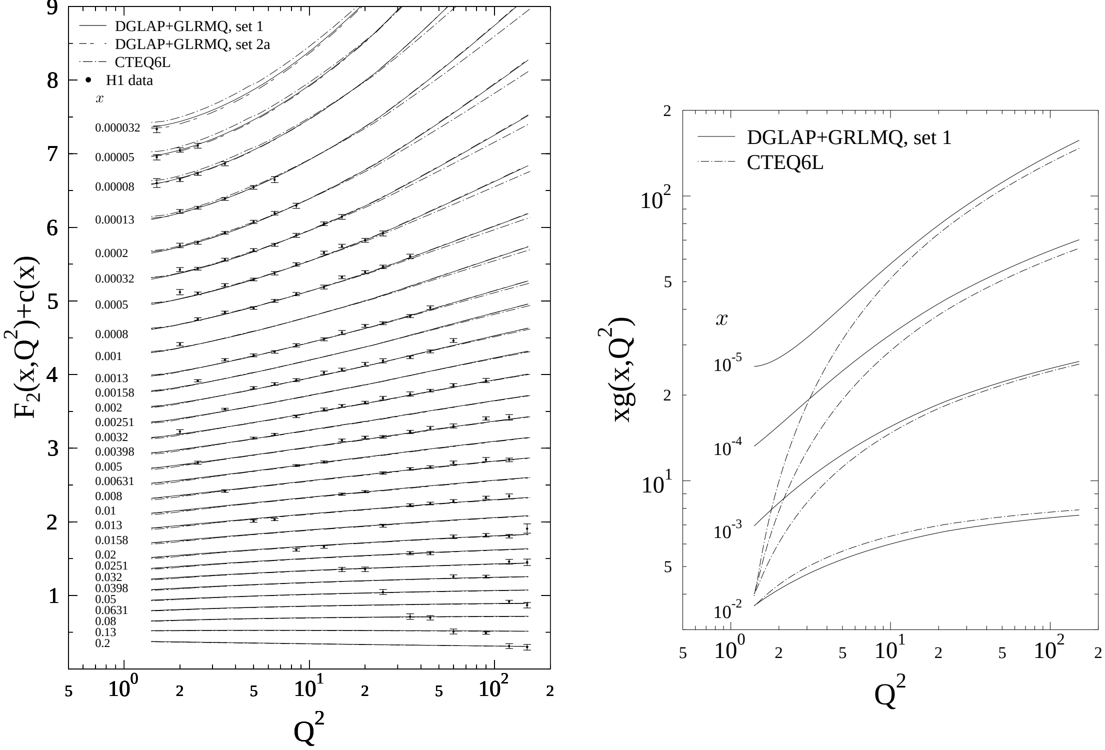

Fig. 1: **Left:** $F_2(x, Q^2)$ calculated using CTEQ6L [4] (dotted-dashed) and the DGLAP+GLRMQ results with set 1 (solid) and set 2a (double dashed) [8], compared with the H1 data [5]. **Right:** The Q^2 dependence of the gluon distribution function at fixed x, from set 1 evolved with DGLAP+GLRMQ (solid), and directly from CTEQ6L (dotted-dashed).

In CTEQ6L a good fit to the H1 data is obtained (see Fig. 1) with a flat small-x gluon distribution at $Q^2 \sim 1.4$ GeV^2. As can be seen from Eqs. (1-2), the GLRMQ corrections slow down the scale evolution. Now one may ask whether the H1 data can be reproduced equally well with different initial conditions (i.e. assuming larger initial gluon distributions) and the GLRMQ corrections included in the evolution. This question has been studied in [8] by generating three new sets of initial distributions using DGLAP + GLRMQ evolved CTEQ5 [3] and CTEQ6 distributions as guidelines. The initial scale was chosen to be $Q_0^2 = 1.4$ GeV^2, slightly below the smallest scale of the data points. The modified distributions at Q_0^2 were constructed piecewise from CTEQ5L and CTEQ6L distributions evolved down from Q^2 = 3 and 10 GeV^2 (CTEQ5L) and Q^2 = 5 GeV^2 (CTEQ6L). A power law form was used in the small-x region to tune the initial distributions until a good agreement with the H1 data was found.

The difference between the three sets in [8] is that in set 1 there is still a nonzero charm distribution at $Q_0^2 = 1.4$ GeV^2, which is slightly below the charm mass treshold, taken to be $m_c = 1.3$ GeV in CTEQ6. In sets 2 the charm distribution has been removed at the initial scale and the resulting deficit in F_2 has been compensated by slightly increasing the other sea quarks at small x. Moreover, the effect of the charm was studied by using different mass tresholds: $m_\mathrm{c} = 1.3$ GeV in set 2a whereas in set 2b it is $m_\mathrm{c} = \sqrt{1.4}$ GeV, i.e. charm begins to evolve immediately from the initial scale.

The results from the DGLAP+GLRMQ evolution with the new initial distributions are shown in Figs. 1. The left panel shows the comparison between the H1 data and the (LO) structure function $F_2(x, Q^2) = \sum_i e_i^2 x[q_i(x, Q^2) + \bar{q}_i(x, Q^2)]$ calculated from set 1 (solid lines), set 2a (double dashed) and the CTEQ6L parametrization (dotted-dashed lines). As can be seen, the results are very similar, which shows that with modified initial conditions and DGLAP+GLRMQ evolution, one obtains as good or even a better fit to the HERA data ($\chi/N = 1.13$, 1.17, 0.88 for the sets 1, 2a, 2b, correspondingly) as with the CTEQ6L distributions ($\chi/N = 1.32$).

The evolution of the gluon distribution functions in the DGLAP+GLRMQ and DGLAP cases is illustrated more explicitly in the right panel of Fig. 1, in which the absolute distributions for fixed x are plotted as a function of Q^2 for set 1 and for CTEQ6L. The figure shows how the differences which are large at initial scale vanish during the evolution due to the GLRMQ effects. At scales $Q^2 \gtrsim 4$ GeV^2 the GLRMQ corrections fade out rapidly and the DGLAP terms dominate the evolution.

3. Saturation

The DGLAP+GLRMQ approach also offers a way to study the gluon saturation limits. For each x in the small-x region, the saturation scale $Q^2_{\rm sat}$ can be defined as the value of the scale Q^2 where the DGLAP and GLRMQ terms in the nonlinear evolution equation become equal, $\frac{\partial xg(x,Q^2)}{\partial \log Q^2}|_{Q^2=Q^2_{\rm sat}(x)} = 0$. The region of applicability of the DGLAP+GLRMQ is at $Q^2 > Q^2_{\rm sat}(x)$ where the linear DGLAP part dominates the evolution. In the saturation region, at $Q^2 < Q^2_{\rm sat}(x)$, the GLRMQ terms dominate, and all nonlinear terms become important.

In order to find the saturation scales $Q^2_{\rm sat}(x)$ for the free proton, the obtained initial distributions (set 1) at $Q_0^2 = 1.4$ GeV^2 have to be evolved downwards in scale using the DGLAP+GLRMQ equations. As discussed in [8], since only the first correction term has been taken into account, the gluon distribution near the saturation region should be considered as an upper limit. Consequently, the obtained saturation scale is an upper limit as well. The result is shown in Fig. 2 (asterisks). The saturation line for the free proton from the geometric saturation model by Golec-Biernat and Wüsthoff (G-BW) [11] is also plotted (dashed line) for comparison. It is interesting to note that although the DGLAP+GLRMQ and G-BW approaches are very different, the slopes of the curves are very similar at the smallest values of x.

Saturation scales for nuclei can also be determined in a similar manner. For a nucleus A, the two-gluon density can be modelled as $x^2G^{(2)}(x,Q^2) = \frac{A}{\pi R_A^2}[xg(x,Q^2)]^2$, i.e. the effect of the correction is enhanced by a factor of $A^{1/3}$. Now a first estimate for the saturation limit can be obtained by starting the downwards evolution at high enough scales, $Q^2 = 100\ldots 200$ GeV^2, where the GLRMQ terms are negligible. The result, which similarly to the proton case is an upper limit, is shown for Pb in Fig. 2 (dots). The effect of the nuclear modifications was also studied by applying the EKS98 [10] parametrization at the high starting scale. As a result, the saturation scales $Q^2_{\rm sat}(x)$ are somewhat reduced, as shown in Fig. 2 (crosses). The saturation limit obtained for a Pb nucleus by Armesto in a Glauberized geometric saturation model [12] is shown (dotted-dashed) for comparison. Again, despite of the differences between the approaches, the slopes of the curves are strikingly similar.

For further studies and for more accurate estimates of $Q^2_{\rm sat}(x)$ in the DGLAP+GLRMQ approach, a full global fit analysis for the nuclear parton distribution functions should be performed, along the same lines as in EKRS [13, 10] and in HKM [14].

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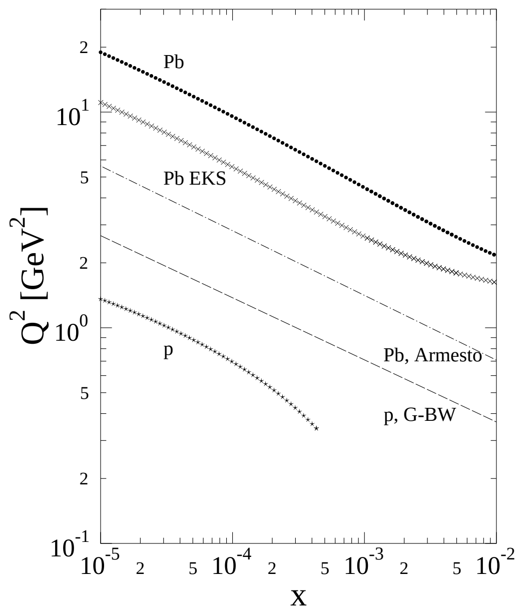

Fig. 2: The gluon saturation limits in the DGLAP+GLRMQ approach for proton (asterisks) and Pb ($A = 208$), with (crosses) and without (dots) nuclear modifications [8]. The saturation line for the proton from the geometric saturation model [11] (dashed line), and for Pb from [12] (dotted-dashed) are also plotted.

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