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Production of muon pairs in annihilation of high-energy positrons with resting electrons

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Abstract

A Monte Carlo generator for the electromagnetic production of muon pairs by annihilation of high-energy positrons with atomic electrons is described. The computer code is designed as a standard electromagnetic process for GEANT4. The formulas and algorithms are described and illustrated in detail.

The code has been applied to study muon production by high energy positrons in a spoiler. The results are presented and compared to muon pair production by gamma conversion.

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1 Introduction

The Monte Carlo simulation of muon production in high energy electromagnetic showers is important for the design of collimation, the final focus region and detectors of linear colliders. The main electromagnetic muon production process, the conversion of high energy photons into a pair of muons has been described recently [1]. It has meanwhile been fully implemented into Geant4 [2] and is part of the standard distribution package.

Here we describe another process, the positron annihilation with atomic electrons into muons.

2 Total cross section

The annihilation of positrons and target electrons with the creation of muon pairs in the final state ($\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-$) may give an appreciable contribution to the total number of muons produced in high-energy electromagnetic cascades. The threshold positron energy in the laboratory system for this process with the target electron at rest is

$$E_{\mathrm{th}} = 2m_\mu^2/m_e - m_e \approx 43.69\,\mathrm{GeV}\,, \tag{1}$$

where m_μ and m_e are the muon and electron masses, respectively. The total cross section of the process on the electron is [3]

$$\sigma = \frac{\pi\, r_\mu^2}{3}\,\xi\left(1+\frac{\xi}{2}\right)\sqrt{1-\xi}\,, \tag{2}$$

where $r_\mu = r_e\, m_e/m_\mu$ is the classical muon radius, $\xi = E_{\mathrm{th}}/E$, and E is the total positron energy in the laboratory frame. In Eq. 2, approximations are made that utilize the inequality $m_e^2 \ll m_\mu^2$.

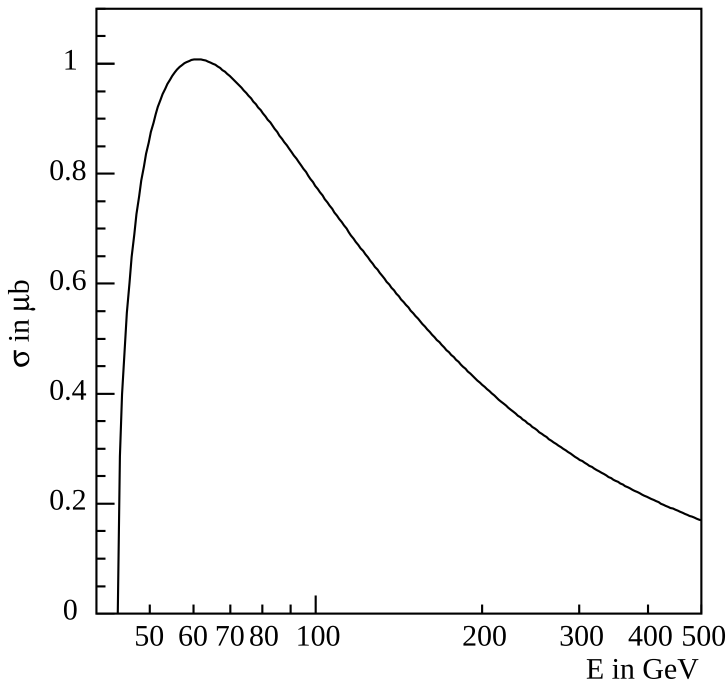

Figure 1: Total cross section for the process $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-$ as function of the positron energy E in the laboratory system.

The cross section as function of the positron energy E is shown in Fig.1. It has a maximum at $E = 1.396\,E_{\mathrm{th}}$ (and not at $E \approx 1.7E_{\mathrm{th}}$, as written in [3], [4]). The maximum value of the cross section is $\sigma_{\max} = 0.5426\,r_\mu^2 = 1.008\,\mu\mathrm{b}$.

3 Sampling of energies and angles

It is convenient to simulate the muon kinematic parameters in the centre-of-mass (c.m.) system, and then to convert into the laboratory frame.

The energies of all particles are the same in the c.m. frame and equal to

$$E_{\rm cm} = \sqrt{\frac{1}{2}\, m_e (E + m_e)}\,. \qquad (3)$$

The muon momenta in the c.m. frame are $P_{\rm cm} = \sqrt{E_{\rm cm}^2 - m_\mu^2}$. Let us denote as $x = \cos\theta_{\rm cm}$ the cosine of the angle between c.m. momenta of μ^+ and e^+. From the differential cross section (e.g., [4]) it is easy to derive that, apart from normalization, the distribution in x is described by

$$f(x)\, dx = (1 + \xi + x^2\,(1 - \xi))\, dx\,, \quad -1 \le x \le 1\,. \qquad (4)$$

The value of this function is contained in the interval $(1 + \xi) \le f(x) \le 2$. The generation of x is straightforward using the rejection technique. Fig. 2 shows both generated and analytic distributions.

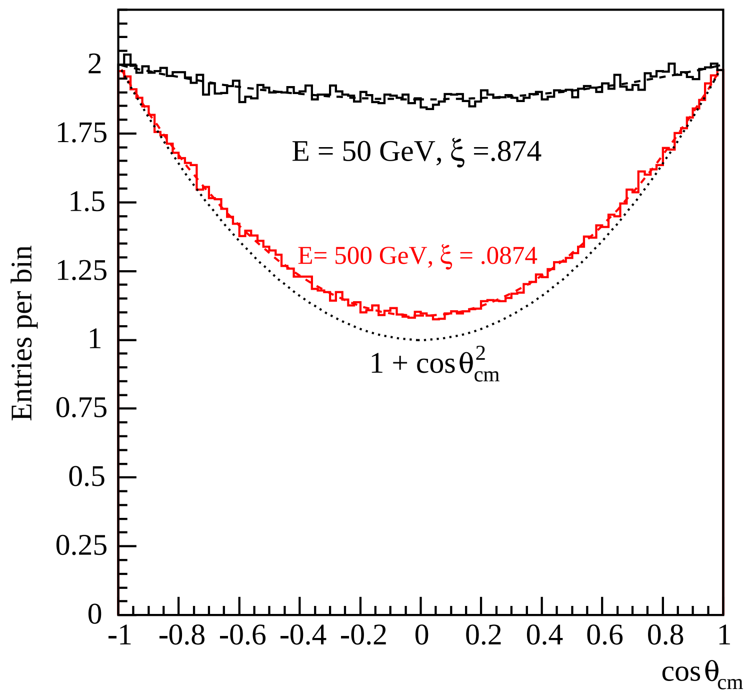

Figure 2: Generated histograms with 10^6 entries each and expected $\cos\theta_{\rm cm}$ distributions (dashed lines) at $E = 50$ and 500 GeV positron energy (in the Lab.). The asymptotic $1 + \cos\theta_{\rm cm}^2$ distribution valid for $E \to \infty$ is shown as dotted line.

The transverse momenta of the μ^+ and μ^- particles are the same, both in the c.m. and the lab frame, and their absolute values are equal to

$$P_\perp = P_{\rm cm}\, \sin\theta_{\rm cm} = P_{\rm cm}\, \sqrt{1 - x^2}\,. \qquad (5)$$

The energies and longitudinal components of the muon momenta in the laboratory system may be obtained by means of Lorentz transformation. Let us denote the velocity and Lorentz factor of the centre-of-mass in the laboratory frame as

$$\beta = \sqrt{\frac{E - m_e}{E + m_e}}\,, \quad \gamma \equiv \frac{1}{\sqrt{1 - \beta^2}} = \sqrt{\frac{E + m_e}{2m_e}} = \frac{E_{\rm cm}}{m_e}\,. \qquad (6)$$

We then obtain the laboratory energies and longitudinal components of momenta for positive and negative muons as:

$$E_+ = \gamma\,(E_{\rm cm} + x\,\beta\,P_{\rm cm})\,, \quad P_{+_\parallel} = \gamma\,(\beta E_{\rm cm} + x\,P_{\rm cm})\,, \tag{7}$$

$$E_- = \gamma\,(E_{\rm cm} - x\,\beta\,P_{\rm cm})\,, \quad P_{-_\parallel} = \gamma\,(\beta E_{\rm cm} - x\,P_{\rm cm})\,. \tag{8}$$

Finally, for the vectors of the muon momenta we get:

$$\mathbf{P}_+ = (+P_\perp \cos\varphi, +P_\perp \sin\varphi, P_{+_\parallel})\,, \tag{9}$$

$$\mathbf{P}_- = (-P_\perp \cos\varphi, -P_\perp \sin\varphi, P_{-_\parallel})\,, \tag{10}$$

where φ is a random azimuth angle chosen between 0 and $2\,\pi$. The z-axis is directed along the momentum of initial positron in lab frame.

The maximum and minimum energies of the muons are given by

$$E_{\rm max} \approx \frac{1}{2}\,E\left(1+\sqrt{1-\xi}\right)\,, \tag{11}$$

$$E_{\rm min} \approx \frac{1}{2}\,E\left(1-\sqrt{1-\xi}\right) = \frac{E_{\rm th}}{2\left(1+\sqrt{1-\xi}\right)}\,. \tag{12}$$

The fly-out polar angles of the muons are approximately

$$\theta_+ \approx P_\perp/P_{+_\parallel}, \quad \theta_- \approx P_\perp/P_{-_\parallel}\,; \tag{13}$$

the maximal angle $\theta_{\rm max} \approx \dfrac{m_e}{m_\mu}\sqrt{1-\xi}$ is always small compared to 1.

4 Implementation details, validity

The process is called `AnnihiToMuPair`. The source code has been added to
`$G4INSTALL/source/processes/electromagnetic/standard`
with `src/G4AnnihiToMuPair, include/G4AnnihiToMuPair.hh`.

The process described is assumed to be purely electromagnetic (based on virtual γ exchange). Production with Z-boson exchange and the $\gamma-Z$ interference are neglected. The Z-pole corresponds to a positron energy of $E = M_Z^2/2m_e = 8136\,\mathrm{TeV}$. The validity of the current implementation is therefore restricted to initial positron energies of less than about 1000 TeV.

5 Cross section per atom and comparison with gamma conversion

The cross section of Eq. 2 was given per electron. To obtain the cross section per atom, we have to multiply with Z, the number of electrons per atom. Muon pair production by gamma conversion in the field of the nucleus has a cross section which scales with Z^2. Gamma conversion is therefore more important than annihilation in heavy materials like tungsten [5].

Spoilers for high energy $\mathrm{e^+e^-}$ colliders will have to be constructed with low Z materials like carbon [6], for which the cross section of both processes is rather comparable, see Table 1.

Table 1: Numerical values for the total cross section per atom for different elements. The energy of the incoming particle (γ in the case of pair production and e^+ in the case of annihilation) is 100 GeV.

Element	Z	$\sigma_{\gamma\to\mu+\mu-}$ μb	$\sigma_{e+e-\to\mu+\mu-}$ μb
H	1	0.1921	0.7702
C	6	5.870	4.621
Pb	82	886.4	63.15

6 Application: muon production in a spoiler

The code described here has been used to predict the muon yield from 10^9 positrons and electrons of 250 GeV energy, impacting perpendicular on a graphite spoiler of one radiation length thickness (18.8 cm).

The computing time of the Geant4 runs was reduced (to about 2 hours on a desktop computer) by

- introducing a low energy cutoff of 0.4 GeV below which all particles are locally stopped
- generation of 10^6 (rather than 10^9) showers with 1000 times increased muon production cross sections.

Fig. 3 shows the energy spectra of the positrons in the annihilation and the angle between the muons produced, as observed in the laboratory system. In case of electrons as primary particles, the annihilation is only possible via secondary positrons in the shower (shaded histograms in Fig. 3).

Incident positrons can annihilate directly. In most cases, the positron will already have lost some energy (mainly by bremsstrahlung), but there is also a finite probability (see the high energy peak) that the positron annihilates at nearly maximum energy.

The laboratory angle between the muons in the pairs produced is limited to about 4.8 mrad ($m_e/m_\mu = 4.84 \times 10^{-3}$, see also Section 3).

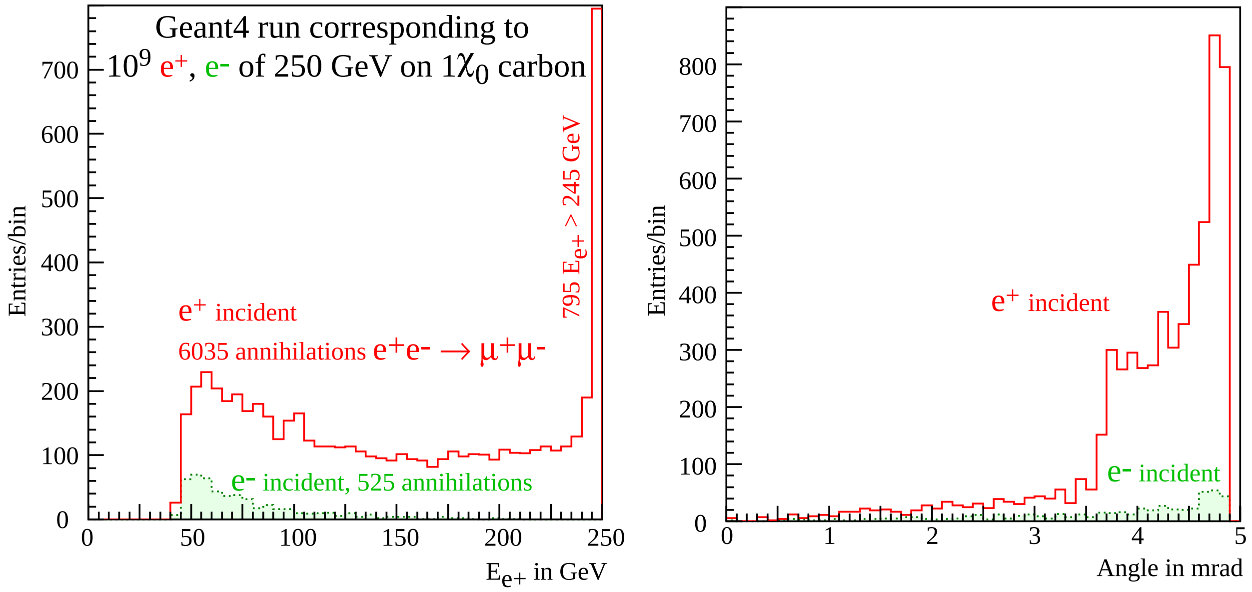

Figure 3: Energy of the positron in the annihilation (left) and angle between the muons produced (right), both in the laboratory frame.

Fig. 4 shows muon energy spectra corresponding to 10^9 positrons or electrons impacting on the spoiler. The muon spectra for gamma conversion with initial positrons or electrons were compatible within statistics (for clarity only the e^+ spectrum is shown). The high energy muon spectrum is dominated by annihilation of primary positrons. Annihilation with initial electrons is only possible via secondary positrons. Table 2 gives actual numbers obtained in the Geant4 runs described here.

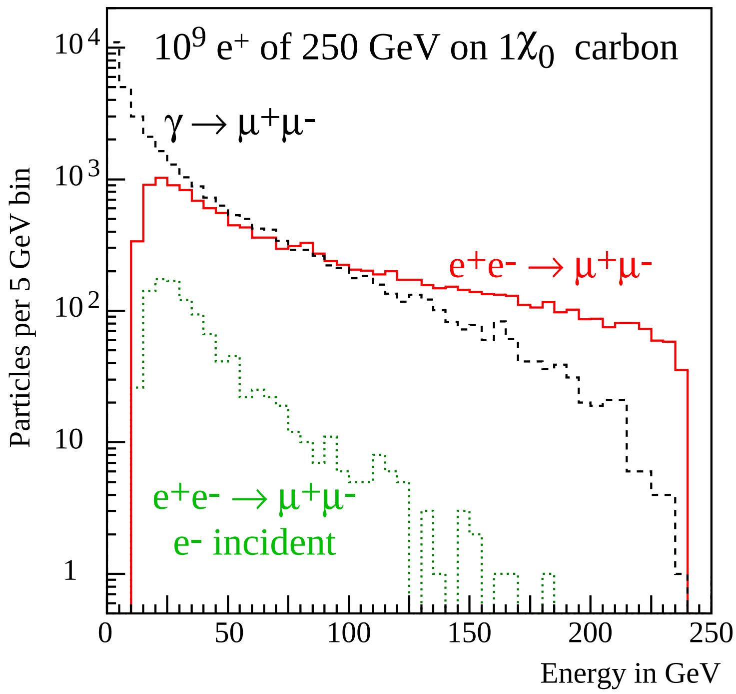

Figure 4: Muon energies in the laboratory system.

Table 2: Number of muons produced by 10^9 positrons or electrons impacting on a 1 radiation length thick graphite spoiler.

Primary Particle	Process	muons, $E > 20$ GeV	muons, $E > 100$ GeV
e+, 250 GeV	$\gamma \to \mu^+\mu^-$	11644	1822
e+, 250 GeV	$e^+e^- \to \mu + \mu-$	10872	3316
e-, 250 GeV	$e^+e^- \to \mu + \mu-$	883	41

Comment

Note. The algorithm could also directly be applied to the simulation of τ-pairs produced in annihilation of positrons with target electrons. The respective threshold energy is $E_{\rm th} = 12.4\,\rm TeV$.

Acknowledgements

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References

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