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TRANSPORT AND SPIN EFFECTS IN HOMOGENEOUS MAGNETIC SUPERLATTICE

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Abstract

Homogeneous semiconductors under spacially periodic external magnetic fields exhibit spin-band splitting and displacements, more clearly defined than in diluted magnetic semiconductor superlattices. We study the influence of the geometrical parameters and the spin-field interaction on the electronic transport properties. We show that by varying the external magnetic field, one can easily block the transmission of either the spin-up or the spin-down electrons.

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The theoretical and the experimental research on multilayer and heterostructures of meso and nanoscopic dimensions have been profuse since the pioneering work of Tsu and Esaki [1,2]. In the last years, the transport properties of spin-polarized electrons through magnetic multilayer structures have become of great interest [3,4], and a rapid transition from basic research of Giant Magnetic Resonance effects [5–7] to applied research followed. Different types of systems are currently in use [8–10]. One of the major obstacles in producing these systems has been the low solubility of magnetic elements. It is then appealing to study alternative systems possessing the desired properties but still easier to produce and control. For this purpose, a system in which the spin-field interaction and the phase coherence are present is needed. These requirements, responsible for the Giant Magneto Resistance and the underlying band structure, are convened if a charged spin-1/2 particle moves in a spacially periodic magnetic field. In this communication we consider a homogeneous 2D transport media transformed in a *homogeneous* magnetic superlattice when the external magnetic field is assumed partially or totally blocked on alternating stripes. We study the transport properties within the transfer matrix approach. We solve the Schrödinger equation for the spin-up and spin-down uncoupled channels, and evaluate the superlattice scattering amplitudes. Using compact formulas of the theory of finite periodic systems [11], the characteristic transport properties and the spin band displacements are easily studied in terms of standard special functions. A simple analysis of the physical quantities, as functions of the magnetic field and the geometrical parameters, leads to interesting features that may be of practical importance.

Let us consider an electron moving along a 2D wave guide of transversal width w and alternating stripes WSWSWSW... along the z-axis. In this superlattice we denote by W a region exposed to a "weaker" or zero magnetic field and by S a region under a comparatively "stronger" magnetic field $B_{\rm s}\ \hat{\imath}$. As shown in figure 1, the lengths of these regions are $l_{\rm w}$ and $l_{\rm s}$, respectively. A single-cell, with length $l_{\rm c} = l_{\rm w} + l_{\rm s}$, can be chosen in different ways but we prefer the structure $\mathrm{W}^{1/2}\mathrm{S}\mathrm{W}^{1/2}$, where $\mathrm{W}^{1/2}$ means half stripe of W.

For each of the homogeneous regions of the superlattice, W and S, the Schrödinger equation

$$\begin{array}{l}\left[\frac{1}{2m}\left(-i\hbar\nabla - \frac{e}{c}\mathbf{A}_\eta\right)^2 - \frac{e\hbar}{2m}\mathbf{B}_\eta\cdot\vec{\sigma}\right. \\ \left.+V - E\right]\Psi\left(y,z\right) = 0, \qquad\qquad \eta = \mathrm{w,s}\end{array} \tag{1}$$

contains the confining hard wall potential

$$V = \left\{\begin{array}{ll} 0 & \text{for } 0 < y < w \\ \infty & \text{otherwise}\end{array}\right. . \tag{2}$$

In the Landau gauge, with $\mathbf{A}_\eta = (0, -B_\eta z, 0)$, and assuming the linear combination

$$\Psi = \sum_{r=1}^{N}\varphi_r\left(z\right)\sin\frac{r\pi}{w}y, \tag{3}$$

where the sum runs over the N propagating modes with transverse momenta $k_{\perp s}(= s\pi/w)$, such that $k_N^2 = \frac{2m^*}{\hbar^2}E - k_{\perp N}^2 \geq 0$, the Schrödinger equation becomes the system of coupled equations [12]

$$H_s\varphi_s\left(z\right) - \sum_{r=1}^{N}\left[U_{sr}\left(z\right) - \frac{2iz}{l_{B_\eta}^2}I\left(s,r\right)\right]\varphi_r\left(z\right) = 0. \tag{4}$$

H_s is the single mode Hamiltonian

$$H_s = -k_{\perp s}^2 + \frac{\partial^2}{\partial z^2} - \frac{z^2}{l_{B_\eta}^4} + \frac{e}{\hbar c} B_\eta \sigma_z + \frac{2m}{\hbar^2} E \tag{5}$$

with $l_{B_\eta} (= \sqrt{\hbar c / e B_\eta})$ as the magnetic length. $U_{sr}(z)$ and $I(s,r)$ are respectively the matrix elements

$$U_{sr}(z) = \frac{4m^*}{\hbar^2 w} \int_0^w dy V(y,z) \sin\left(\frac{r\pi}{w} y\right) \sin\left(\frac{s\pi}{w} y\right) \tag{6}$$

and

$$I(s,r) = \begin{cases} 0 & \text{for } r+s \text{ even} \\ \frac{4sr}{w(r^2-s^2)} & \text{for } r+s \text{ odd} \end{cases} . \tag{7}$$

Important physical information can be obtained even if we restrict our analysis to energies below the second channel threshold $E < E_2 = \frac{2\hbar^2\pi^2}{m^* w^2}$, and neglect the contribution from the evanescent modes contributions. Since, experimentally, it is not easy to completely block the magnetic fields, we consider non-zero the magnetic fields in general. As usual, it is convenient to introduce the function

$$\varphi(z) = e^{-z^2/2l_{B_\eta}^2} \psi(z) \tag{8}$$

which leads us to the confluent hypergeometric equation

$$\left(\frac{\partial^2}{\partial z^2} - \frac{2z}{l_{B_\eta}^2} \frac{\partial}{\partial z}\right) \psi(z) + \frac{2\mathbf{b}_\eta}{l_{B_\eta}^2} \psi(z) = 0 \tag{9}$$

with $\mathbf{b}_\eta = \left(k_1^2 l_{B_\eta}^2 - 1 + \sigma_z\right)/2$ and $k_1^2 = \frac{2m^*}{\hbar^2} E - k_{\perp 1}^2$. Notice that for each propagating mode we have the spin-up and the spin-down physical channels.

Inside a stripe, under a stronger or weaker magnetic field, the solution of the hypergeometric equation is given by

$$\begin{aligned} \psi(z) = {}_1F_1\left(\frac{-\mathbf{b}_\eta}{2}; \frac{1}{2}; \frac{(z-z_i)^2}{l_{B_\eta}^2}\right) \psi(z_i) + \\ (z-z_i)\ {}_1F_1\left(\frac{1-\mathbf{b}_\eta}{2}; \frac{3}{2}; \frac{(z-z_i)^2}{l_{B_\eta}^2}\right) \psi'(z_i), \end{aligned} \tag{10}$$

which is clearly invariant under the gauge transformation $\mathbf{A}_\eta \to \mathbf{A}_\eta + (0, B_\eta l_c, 0)$. To calculate the superlattice physical quantities, we now work within the transfer matrix approach where as shown in reference [11], it is sufficient to find the solution of a single cell. In the one channel limit and for uncoupled spin-channels the time reversal violating term vanishes. The two channel transfer matrix $\mathbf{M}_\eta$ connecting wave vectors between two points z_1 and z_2 inside or at the border of any region η has the structure

$$\mathbf{M}_\eta(l_\eta) = \begin{pmatrix} \alpha_\eta & \beta_\eta \\ \beta_\eta^* & \alpha_\eta^* \end{pmatrix} \tag{11}$$

with

$$\begin{aligned} \alpha_\eta &= \tfrac{1}{2}\left[\mathbf{f} + \mathbf{g}' - i\left(\mathbf{f}'/k - \mathbf{g}k\right)\right], \\ \beta_\eta &= \tfrac{1}{2}\left[\mathbf{f} - \mathbf{g}' - i\left(\mathbf{f}'/k + \mathbf{g}k\right)\right]. \end{aligned}$$

Here

$$\mathbf{f} = {}_1F_1\left(\frac{-\mathbf{b}_\eta}{2};\frac{1}{2};\frac{l_\eta^2}{l_{B_\eta}^2}\right)e^{-l_\eta^2/2l_{B_\eta}^2}$$
$$\mathbf{g} = l_\eta\ {}_1F_1\left(\frac{1-\mathbf{b}_\eta}{2};\frac{3}{2};\frac{l_\eta^2}{l_{B_\eta}^2}\right)e^{-l_\eta^2/2l_{B_\eta}^2}$$

and $\mathbf{f}'$, $\mathbf{g}'$ are the corresponding derivatives with respect to z.

With the multiplicative property of the transfer matrices, the single-cell $\mathrm{W}^{1/2}\mathrm{SW}^{1/2}$ transfer matrix $\mathbf{M}_1$ is given by

$$\mathbf{M}_1 = \mathbf{M}_\mathrm{w}\left(z_{i+1}, z_\mathrm{s}\right)\mathbf{M}_\mathrm{s}\left(z_\mathrm{s}, z_\mathrm{w}\right)\mathbf{M}_\mathrm{w}\left(z_\mathrm{w}, z_i\right) = \begin{pmatrix}\alpha_1 & \beta_1\\ \beta_1^* & \alpha_1^*\end{pmatrix} \tag{12}$$

with $z_\mathrm{w} = z_i + l_\mathrm{w}/2$, $z_\mathrm{s} = z_i + l_\mathrm{w}/2 + l_\mathrm{s}$. Using the transmission amplitude formula [11]

$$t_n = \frac{t_1^*}{t_1^* p_n - p_{n-1}}, \tag{13}$$

where p_n is the Chebyshev polynomial of the second kind of order n evaluated at $Tr\mathbf{M}_1/2$, a variety of interesting transport properties, implicit in the band structure behavior, can be found if we vary the relevant superlattice parameters and the magnetic fields.

Although we want to emphasize the spin-field interaction effects on the spin band structure, we shall also analyze the total transmission coefficient behavior as a function of the geometrical parameters l_w, l_s and w. In all of the figures shown below, the n-cell transmission coefficients are plotted with solid lines while broken lines represent the single-cell transmission coefficients and. In all cases, only the indicated parameter that is modified whilst the others are kept fixed. Since the evolution of the band structure associated with the changes in the geometrical parameters is the same for spin-up and spin-down electrons, and because of lack of space, we shall only show the effects on the spin-down bands.

In figure 2, the geometrical parameters are kept fixed while the magnetic length l_{B_s} is varied. As expected from the Zeeman term in the Hamiltonian, a blue shift results by increasing the magnetic field (or decreasing the magnetic length). Since the phase coherence responsible for the gap formation (which is related to the Giant Magneto Resistance effect) depends on the difference $B_\mathrm{s} - B_\mathrm{w}$, a corresponding increase or decrease of the gap widths is observed, with an opposite behavior for the band width.

In figures 3a)-c), the effects of the geometrical-lengths on the band structure are shown. In 3a), it is evident that the only effect that results by varying the wave-guide width w is an overall energy displacement of the band structure. On the other hand, when the length l_s or l_w is changed, a different behavior is found. In these cases (see figures 3b) and 3c)), the band and the energy level densities increase by increasing the lengths. In the limit l_s/w, $l_\mathrm{w}/w \gg 1$ the band widths tend to zero and become equidistant, with a separation of the order of $\hbar\omega_c$, where $\omega_c = \frac{eB}{cm}$ is the cyclotronic frequency. This can, after some algebra, be analytically shown.

Spin-field interaction effects on the electron transport properties can be summarized as follows. The spin bands for spin-up and spin-down electrons, in figure 4, are nearly the same but displaced in energy one from the other. This is the band splitting effect. A careful observation of the bands exhibits also differences in the peak-valley. The displacement of the spin bands is sensitive to small changes in the physical parameters and in the external magnetic field. This is an important property that should be explored experimentally. In general, when the magnetic field is increased, the evolution of the spin-up band structure is toward lower energies while for spin-down electrons is toward higher energies, similar to the energy level behavior in atomic and 0-D systems. This behavior is apparent in the transmission coefficients shown in figure 4. As found for diluted magnetic semiconductors, the energy bands for both spin-up and spin-down electrons can be brought into coincidence or be displaced one from the other when the magnetic field is varied. Hence, for a fixed electron energy we pass from a magnetic superlattice where spin-down electrons are transmitted, to a magnetic superlattice where the transmission is for spin-up electrons (see figures 3a,c) at E_1), which is the well known spin polarization effect. Similarly, one can find conditions where both types of electrons can pass through with the same energy (see for example 3b) at E_2).

In this work we have studied the effects of spin on the transport properties of spin-half particles when some of the magnetic superlattice parameters and external field are changed. We found that choosing appropriately the values of the external magnetic field, the blocking fraction (defined by the relative difference $(B_{\rm s}-B_{\rm w})/B_{\rm s}$) and the geometrical extension on which the fields act, one can induce important blue or red shifts of the band structure. These evolutions have also been shown to depend on the spin-projections. Working out the uncoupled two channel case, analytical solutions in terms of standard special functions were obtained and the numerical calculations became rather simple.

We conclude that the homogeneous magnetic superlattices are well-suited candidates to test quantum coherence phenomena and the spin-dependent transport properties.

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[12] The contribution of the evanescent modes is small and neglected here.

FIGURE CAPTIONS

FIG. 1. A 2-D homogeneous magnetic superlattice. Regions under a weak tranverse magnetic field B_{w} alternate with regions under a stronger magnetic field B_{s}.

FIG. 2. Band structure shifts induced by changing the magnetic field B_{s}, as indicated, while B_{w} is kept fixed (equal to $0mT$).

FIG. 3. Effects of the geometrical parameters on the band structure of spin-down electrons. In 3a) an almost rigid displacement is produced when only the tranverse width w is changed and the remaining parameters kept fixed. In 3b) and 3c) the lengths l_{s} and l_{w} are, respectively, varied. In the limit l_{s}/w, $l_{\mathrm{w}}/w \gg 1$ the band widths tend to zero and become equidistant, with a separation of the order of $\hbar\omega_c$, where $\omega_c = \frac{eB}{cm}$ is the cyclotronic frequency.

FIG. 4. The transmission coefficients and the relative positions of the bands and gaps induced by varying the magnetic field B_{w} on a superlattice with 9 cells and $B_{\mathrm{s}} = 2T$, for spin-up and spin-down electrons. The spin-up and spin-down bands move in oposite directions. Under certain conditions the bands may be made to coincide. At E_1 and magnetic fields as in a) and c), respectively, either the spin-up or the spin-down electrons are transmitted. In b) and for an energy E_2) both types of electrons can pass through.