# Measurement of the Non-Gaussian Component of Two-Particle Correlation Functions

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**Abstract**

Large acceptance and/or high rate heavy ion experiments are increasingly sensitive to small deviations from the Gaussian profile that HBT analysis has until now imposed on the correlation function. A new method which extracts the width of the correlation function via moments is evaluated. It is less biased than the conventional Gaussian fitting method because no a priori assumptions are made with respect to the profile of the correlation function, and a more model independent extraction of the correlation parameters is possible. This new method is used to analyze data from NA49. The sensitivities and limitations of this new method will be discussed along with implications for future measurements.

# 1 Introduction

Intensity interferometry is the only measurement that is able to provide an estimate of the spatial and temporal extension of the system created in a relativistic heavy ion collision. These are important quantities for several reasons. First, they are necessary to deduce the energy density as well as the lifetime of the source, both critical parameters that link theoretical calculation to experimental measurements. Second, these quantities are believed to fix the initial conditions created in such collisions. Although the energy produced in a collision can be measured directly, the volume must be inferred from indirect measurements. Third. the source dimensions are also important because of the pre-disposition that exists in modelling and interpreting the evolution of hadronic observables in terms of statistical and thermodynamic models. As such the spatial and temporal extension of the source is extremely important in trying to fix the equation of state, a long time goal of nuclear physics.

The interpretation of the parameters extracted from correlation functions is model dependent because of the convolution of the spatial and temporal components [1]. However one of the virtues of particle interferometry is that the measurement makes no assumption about the thermodynamic properties of the system. The correlation is an effect driven entirely from quantum statistics and the symmetry properties that wave functions of identical particles possess. Identical boson pairs tend to be preferentially emitted close in momentum space while the opposite holds for fermions. The degree of enhancement in momentum space contains information on the distribution of the separation distances of independent emitting points because of the relation of conjugate variables.

The NA35 and NA49 experiments at CERN-SPS were the first to exploit the statistical power of large acceptance coverage to particle correlation studies in ion-ion collisions [2, 3]. A typical event at NA49 consists of some 1500 charged particles which corresponds to $\sim 10^6$ pairs! This has allowed the construction of correlation functions with insignificant statistical error from ensembles of events. However there still exists rather large systematic uncertainties in the construction and interpretation of the correlation functions themselves. Furthermore as statistical power has increased, deviations from the assumed Gaussian profile have become more evident. Such behavior has been suggested to be a result of real physical effects. These include exotic source shapes emitting patterns of particles like a thin shell [4], or the result of long-lived resonances decaying outside the source smearing the correlation function by introducing long tails [5].

Although the possibility is intriguing that the non-Gaussian character of a correlation function contains physics, there is a valid question as to whether or not the correlation function is sensitive to such effects because of the relatively large and unknown systematic effects introduced in construction of the correlation function. For example, the Coulomb correction is poorly understood and it will be shown that careless treatment of this correction can result in the introduction of a highly non-Gaussian structure into the correlation function.

No matter, whether or not the non-Gaussian component of the correlation function contains real physics, its presence can not be dealt within the present framework of extracting radius parameters from correlation functions. This paper reports analyses from an investigation of a method that is able to extract radius parameters without the need to introduce the bias of a presupposed functional profile. Part 2 will briefly review the derivation of a Gaussian correlator from a probability density function, highlighting the assumptions that result in the use of a Gaussian profile of the source density. Part 3 will describe the methodology of the new formalism. Part 4 will outline the sensitivity of this new method along with some of its limitations and future prospectives as well as the implications of this method.

## 2 Derivation of a Gaussian Correlator

Up until this point in time, the method used to extract radii parameters from correlation functions has exclusively used the fitting of a Gaussian parameterization to the measured data points. This was not only for convenience but also because the precision of measurements before the advent of 158 GeV/A Pb beams at the CERN-SPS did not allow discrimination between different profiles of the correlation function and a Gaussian was thought to be a satisfactory approximation to the data.

The most rigorous theoretical formalism that derives the form of the correlator from the probability density function was developed by Heinz and collaborators [1]. The emission function of the source, $S_i(x_i, p_i)$ which specifies that a particle of type $i$ is emitted from a position $x_i$ with a momentum $p_i$ can be approximated by a Gaussian component $\tilde{S}_i$ and a correction term $\epsilon_i$:

$$S_i(x_i, p_i) = \tilde{S}_i(x_i, p_i) + \epsilon_i(x_i, p_i) \tag{1}$$

The two particle correlation function $C_2$ is defined as a ratio of the two particle coincident yield to that of an uncorrelated system and measures the Fourier transform of the emission function:

$$C_2(p, p^{'}) = \frac{Y(p, p')}{Y(p)Y(p')}_{meas} = 1 + \frac{\int_{source} d^4x d^4x^{'} \; S(x,p) e^{iq\cdot x} S^*(x^{'}, p^{'}) e^{-iq\cdot x^{'}}}{\int_{source} d^4x d^4x^{'} \; S(x,p) S^*(x^{'}, p^{'})} \tag{2}$$

where $q = p - p^{'}$ is the momentum difference between the particle pair. Because of the properties of a Gaussian under Fourier transform, if the emission function is taken to be Gaussian, so to is the correlation function. As such, it can be parameterized as [1]:

$$C_2(q) = 1 + \lambda e^{-(B^{-1})_{\mu\nu} \, q^{\mu} \, q^{\nu}} \tag{3}$$

where **B** is a tensor which characterizes the space-time variances of the emission function:

$$(B^{-1})_{\mu\nu} = < x_\mu x_\nu > - < x_\mu >< x_\nu > \tag{4}$$

Before the large statistical power of current experiments, there was sufficient uncertainty in the correlation functions so that any small scale deviation from Gaussian shape was indiscernible. As statistical power has increased, not only have multi-dimensional parameterization of the correlation function been introduced to study the different orientations of the source (i.e. Pratt-Bertsch [7], Yano-Koonin-Podgoretski [8]), but some limitations in the understanding of the correlation function, most notably with respect to the Coulomb correction have been brought to light. A myriad of models for the Coulomb correction have been recently introduced (i.e. Pratt [9], NA35 [10], Baym–Braun-Munzinger [11], etc.), however, an invariant assumption in all the studies has been that the profile of the correlation function is Gaussian.

Recently, Wiedemann has presented a formalism that allows the quantification of the widths of the correlation function by calculating the n$^{th}$ order "q" moments of the correlation functions rather than relying on a fitting procedure to extract them [6]. This method allows the width of a correlation function to be quantified by its second order moment given by:

$$< q_i q_j >= \frac{\int d^3\mathbf{q}\; q_i\, q_j\, [C(q,K)-1]}{\int d^3\mathbf{q}\, [C(q,K)-1]} = D^{-1}_{ij}(K) \tag{5}$$

where $C(q,K)$ is the correlation function, $D^{-1}_{ij}(K)$ is a tensor which contains the information of the correlation widths. Note that this formalism allows for cross-terms as suggested by Heinz and collaborators [12]. The radii parameters extracted via equation 5 would be identical to those extracted by fitting a Gaussian to the data in the limit of a Gaussian correlation function. The power of this method lies in possibility of quantifying the deviation from Gaussian behavior in a systematic way via higher order moments. The only way that currently exists is to judge the $\chi^2$ of the fit. However as statistical precision increases, the $\chi^2$ value becomes a weaker indicator because it is very sensitive to the size of the errors. In order to quantify the deviation from Gaussian behavior, the 4$^{th}$ order moment of $q_i$ (i.e. $< q_i^4 >$) can be calculated as specified in equation 6:

$$< q_i^4 >= \frac{\int d^3\mathbf{q}\; q_i^4\, [C(q,K)-1]}{\int d^3\mathbf{q}\, [C(q,K)-1]} \tag{6}$$

and a quantity $\mathrm{G}_i$ may be defined:

$$\mathrm{G}_i = \frac{< q_i^4 >}{3 < q_i^2 >< q_i^2 >} - 1 \tag{7}$$

which vanishes in the case of a purely Gaussian correlation function. This quantity is able to take on positive or negative values depending on the specific shape of the function—negative if it has a flat profile and positive if it peaked sharply as illustrated in figure 1. This particular form of G is chosen because it removes the dependence on the width of the correlation function.

Although this approach does not make an a priori assumption on the shape of the distribution in order to extract the width, the same cannot be said about

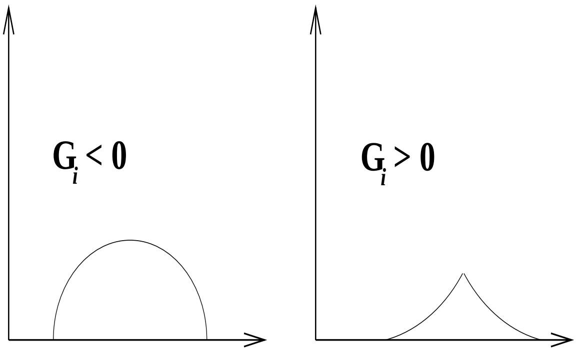


**Figure 1**: The sign of G is sensitive to the profile of the correlation function whether it is flatter (left) or more sharply peaked (right) than a Gaussian.

the extraction of the correlation strength $\lambda$. One must assume a value of the integral of the correlation function which is very much like assuming a functional profile that is done when fitting. If a Gaussian profile is chosen, $\lambda$ is given by:

$$\lambda = \frac{\sqrt{\mathrm{detD}}}{\pi^{\frac{3}{2}}} \int d^3\mathbf{q}\,[C(q,K) - 1] \tag{8}$$

Although this method is not as elegant in the case of the extraction of the width, comparisons between the value of $\lambda$ extracted from the fitted value and integrated value carries additional information on the shape of the correlation function.

# 3 Methodology

The best way to illustrate the details of the new method is to directly compare the parameters extracted by a Gaussian fitting procedure and the integral approach on the same one dimensional correlation function. This is shown in figure 2. The Gaussian fit method relies on a conventional $\chi^2$ minimization while the integral, as given by equation 5 is calculated bin-by-bin with increasing $Q$ starting at $Q$=0. A bin will contribute to the integral if the error is not larger than its contribution to the zeroth order moment; that is, if $[C(q,K) - 1] > \delta C(q,K)$). This is referred to as the "cross-over" point and is indicated by an arrow in figure 2. When this condition is no longer satisfied (i.e. at $Q_{Tout}$=160 MeV/c), there is no further contribution to the integral. The two sets of parameters in figure 2 refer to those extracted from the fit method (left) and the integral method (right).

The integral method gives a radius that is smaller than that extracted by fitting the correlation function to a Gaussian. This discrepancy has its origins in the non-Gaussian behavior of the correlation function. The Gaussian fit is sensitive to a very localized region of the correlation function—between 25-

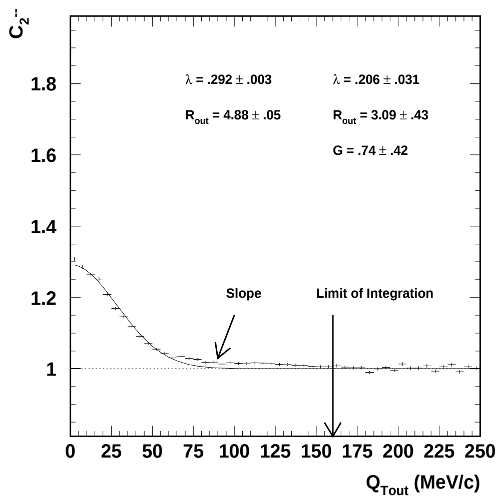


**Figure 2**: The "$Q_{Tout}$" component in the Pratt-Bertsch parameterization deduced from a correlation function constructed using $\pi^-$ pairs from 80k events. The curve is the best Gaussian fit to the data. It is evident that the shape of the correlation function differs significantly from that of a Gaussian profile. The parameters shown are those extracted from the fit (left) and integral (right) methods.

85 MeV/c— and especially to the area immediately around the inflection point of the function (i.e. $Q$ ∼65 MeV/c). Because the Gaussian fitting procedure is sensitive to a small region of the correlation function, the radii deduced from this method are relatively independent of the number of bins that are used in the fit. Furthermore, the fitting method is usually insensitive to the very small $Q$ bins (i.e. $Q_{inv}$ <15 MeV/c), because the errors in this region are usually an order of magnitude larger than the points at larger relative momentum. Thus it is rather unimportant whether these points are included in the Gaussian fit. This insensitivity has not been viewed as serious because although the correlation strength is maximal in this region, it also carries the largest uncertainty as a result of the experimental difficulties in measuring closely separated tracks— either in momentum or configuration space, for both large acceptance and small acceptance experiments. In a large acceptance experiment like NA49, it is imperative that tracks not be split in reconstruction as this can mimic low $Q_{inv}$ pairs and overestimate the yield in this region. On the other hand a small acceptance experiment is adversely affected by the need to generate a large event sample to be statistically competitive. A thick target is usually employed to increase the trigger rate, but this degrades the momentum resolution which distorts the low $Q$ region of the spectrum. In contrast to the Gaussian fit method which is sensitive over a very localized region, the integral method is sensitive to a more extended region both at small and large relative $Q$. Small relative momentum bins make a large contribution because the correlation function has its largest values in this region. Large relative momentum bins make a sizable

contribution to the integral of equation 5 because they are weighted with $Q_i^2$ of the bin. In this respect the integral method is more democratic because it uses a wider range of the correlation function. However it demands that the errors be small and well understood because the parameters extracted are very sensitive to the position of the cross-over point. Using the criterion of the error on the bin being smaller than its contribution to the zeroth order moment implies that large $Q$ bins will be used when statistical error is small. The value of the second order moment will increase with the number of bins that contribute to the integral. Since the value of $<\mathrm{q}^2>$ is inversely proportional to the radius parameter, the radius will appear smaller, as observed.

The integral method extracts a smaller value for the correlation strength, $\lambda$ as well. This is expected because $\lambda$ is not independent of $R_i$. For example the magnitude of the second order moments determine the magnitude **D**. A large second order moment implies a small radius which translates into a smaller $\lambda$ than expected if the correlation function was Gaussian in profile. The non-zero value of G is also expected from a visual inspection. It is noteworthy that the error on the parameters extracted from the integral method are an order of magnitude larger than those extracted via the fitting method. This will become more important when multi-dimensional correlation functions are integrated.

## 3.1 Comparison between the two methods

The performance of the q-variance method will be further illustrated by comparing different Coulomb corrections. It is probably fair to say that the Coulomb correction is the least understood factor in the interpretation of correlation functions. Although corrections may differ based on their theoretical considerations most corrections are parameterized as a function of $Q_{inv}$. NA49 is no exception to this however an attempt is made to absorb the effect of the finite source effect into the correction and reducing calculational dependency by fitting an exponentially damped Gamov correction factor, with suitable boundary conditions, to an experimentally measured correlation function constructed from opposite charge sign pairs. The functional form of the correction is seen in equation 9:

$$C_2^{+-} = ((G(\eta) - 1)\exp(-Q_{inv}/Q_{eff}) + 1) \tag{9}$$

where $G(\eta)$, the Gamov factor$= \frac{\eta}{e^{\eta}-1}$, where $\eta = \frac{2\pi m}{q}$, where $m$ is the mass of the particle of interest and $q$ is the momentum difference of the pair, and $Q_{eff}$ is a free parameter that is fit to the data. This correction still introduces some asymmetry into the profile, however it is an improvement over the Gamov factor which is seen to have a very destructive effect on the correlation function beyond $Q_{inv}$ ~500 MeV/c as seen in figure 3. There is a strong singularity common in both the Gamov and NA49 correction factor at small relative momentum (i.e. $Q_{inv}$ = 0 MeV/c). However the strength of the pole decreases and the quality of the fit increases with the introduction of a second parameter in $\eta$:

$$\eta \rightarrow \eta' = \frac{2\pi\alpha m_\pi}{q - q_o} \tag{10}$$

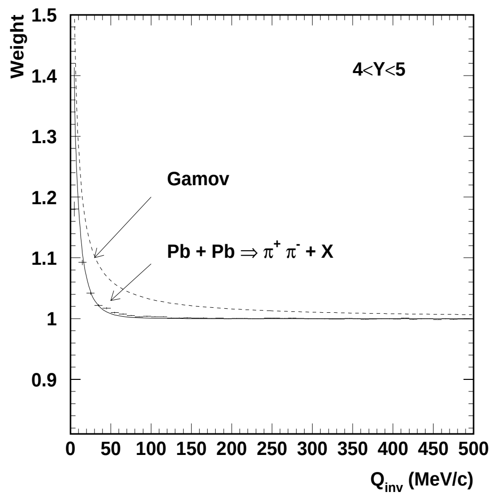


**Figure 3**: The weight assigned to particle pairs as a function of $Q_{inv}$ given by two different Coulomb corrections; the Gamov factor (dashed) and the NA49 fit to data (solid). Although both corrections introduce strong non-Gaussian components at small relative momentum, the Gamov factor also introduces a long tail that extends well out beyond 250 MeV/c.

Using NA49 data the parameterization of the Coulomb correction (Eqn. 9 and 10) is fitted to a correlation function constructed from $10^8$ "$h^+h^-$" pairs. This function is then used to correct a like-sign correlation function constructed from $10^7$ pairs ($\pi^-$) as a function of $Q_{inv}$. Although $Q_{inv}$ is not the preferred variable for extracting spatial and temporal dimensions of the source, it is useful to illustrate the differences between the corrections with the same data. This is illustrated in figure 4. The top panel shows the correlation function with no Coulomb correction applied. The middle panel shows the same data where the single parameter correction (i.e. eqn. 9) has been used and the bottom panel uses the additional parameter which shifts the pole in the Gamov factor (i.e. eqn. 10). The curves drawn in all three cases is the best Gaussian fit to the data. "**Fit**" specifies the parameters extracted from fitting a Gaussian curve to the data while "$<\mathbf{q}^2>$" specifies the parameters extracted via the moments calculated as defined in equation 6. It is immediately obvious that the Coulomb correction imposes a large non-Gaussian distortion on the correlation function. It is also somewhat unsettling that the width of the correlation function seems to increase (i.e. the radius decreases) when the Coulomb correction is applied. This is observed in both the fit and integral methods. This occurs because the Coulomb correction appears to introduce a slope in the correlation function which was illustrated in figure 3. Because the value of the correlation function differs from unity at large $Q_{inv}$ it contributes to the width of the function thus increasing its width. The radii are again systematically smaller in the case of the values extracted via the integration method because of the sensitivity of the

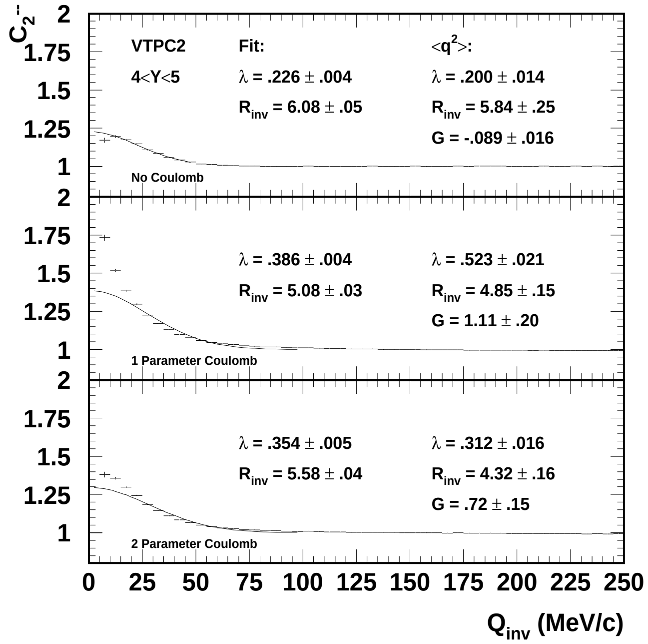


**Figure 4**: Correlation functions in the rapidity region 4<y<5 with no (top) Coulomb correlation, the one parameter (middle) Coulomb correction, and the two parameter (bottom) Coulomb correction. The curves are the best Gaussian fits to the data. Also shown are results utilizing a conventional $\chi^2$ minimization and the integral method. The parameter G describes the deviation from Gaussian behavior where a perfect Gaussian would have G=0.

integral method to the form of the correlation function at large $Q$.

The correlation strength or $\lambda$ has an interesting behavior. In the top and bottom panels of figure 4, the integral method extracts a smaller value for $\lambda$ than the fit method. This is expected because the radii parameters are also smaller. This is also the case for the middle panel, however the $G$ parameter is much larger than in the other two cases. It is also true that the difference in the radii parameters extracted from the two methods is smallest in this case. Even though the radii are nearly in agreement, $G$ indicates that the profile is far from the Gaussian expectation. If the moments are interpreted as moments of inertia, this can be recast into the statement that although the functions have the same moment of inertia, the distribution of the mass is very different. $G$ manages to quantify this deviation in a systematic way that can be compared without having to worry about how the statistical precision folds in if a conventional $\chi^2$ is used.

It is apparent that if the non Coulomb corrected correlation function is Gaussian, the application of a Coulomb correction will not conserve the Gaussian shape. This means that either:

- Gaussian assumption is manifestly wrong or,

- Coulomb correction must be rethought.

It is true that the Coulomb correction is not understood and this is evident in the number of different Coulomb corrections which have been proposed recently. Comparing the correlation function within figure 4, it is apparent that the Coulomb correction is a manifestly non-Gaussian in origin and does serve to enhance the non-Gaussian features.

### 3.2 Application to Multi-Dimensional Correlation Functions

Because the two parameter Coulomb correction imposed the smallest non-Gaussian perturbation, it was selected for the remainder of the analysis. In the following the effect of the integral method on the correlation parameters with the Pratt-Bertsch parameterization is investigated. In order to utilize the more powerful multi-dimensional parameterizations, it is necessary to project such correlation functions into 1 or 2 dimensions and then integrate because the statistical errors become so large that it is not possible to extract statistically significant parameters. Even with $10^9$ pairs distributed over three units of rapidity[1] the statistical power is not great enough to extract parameters with meaningful significance because of the $Q^2$ factor in the integral. This projection has the unfortunate side effect of diluting the correlation strength and artificially enhancing the width of the correlation function. Results from one dimensional projections are shown in figure 5 Although the errors are large, the radii extracted via the integral method are 20-30% smaller than those extracted via a Gaussian fit, independent of whether one looks at the radii as a function of rapidity, or the $k_T$ (pair transverse momentum) dependence within a single rapidity bin. Nonetheless, the large errors will preclude the use of this method for all but the highest statistic studies.

### 3.3 How do Moments change Physics

It is interesting that although the radii extracted via the integral method are systematically smaller by 20-30% than those extracted via a Gaussian fit, the qualitative behavior of the radii parameters as a function of rapidity (or $k_T$) do not change significantly; or at least the errors are large enough that one can not ascertain any differences. An illustration of this is shown in figure 6.

This difference emanates from the small deviations from unity of the correlation function at large relative momentum as has been explained previously. This decrease in the radii parameters is significant because if this is a real effect, and not simply a result of careless imposition of a non-Gaussian artifact such as a poor Coulomb correction, this method extracts radii that are on average 25% smaller than those extracted via a fitting algorithm. This is very significant because it reduces the volume of the source by nearly 50% and would influence the magnitude of the energy density by the same factor. This is important because

[1] the acceptance of NA49.

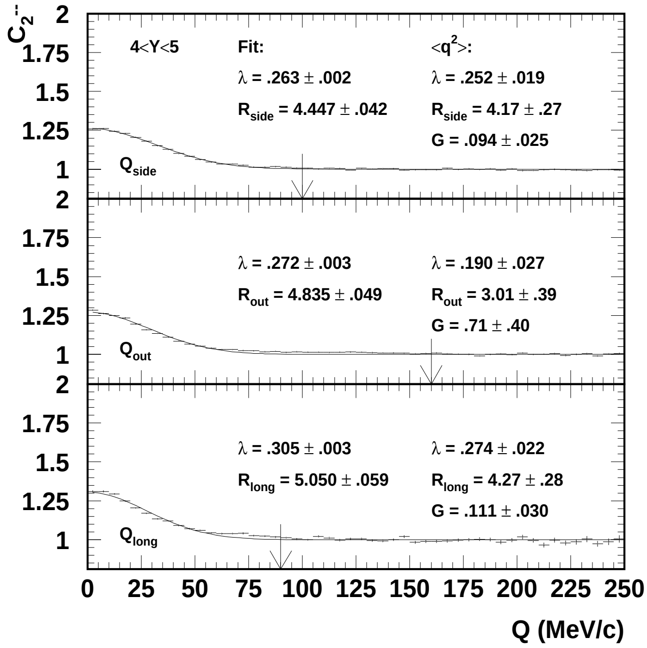


**Figure 5**: One dimensional projection of the three dimensional correlation functions as well as the extracted radii parameters from the fit (left) and integral (right) methods. Shown are the three momentum components in the rapidity region from 4<y<5 in VTPC2.

the energy regime that is being explored at CERN-SPS is believed to be near the threshold that deconfinement occurs. In this case a factor this large in the uncertainty in the energy density is very significant.

# 4 Future Studies

This method of extracting the width of the correlation function allows a systematic and definite method for comparing fixed and specific intervals of the correlation function, and this has a direct application in Event-by-Event studies of correlation functions.

Although it is possible to construct using signal derived in one event with an ensemble background, it is also possible to construct the background from a single event using the "+−" pairs to measure the uncorrelated background as well as the Coulomb effect via:

$$C_2^{EBE} = \frac{Y(\pi^-\pi^-) + Y(\pi^+\pi^+)}{Y(\pi^+\pi^-)} \tag{11}$$

Figure 7 shows the correlation function constructed from two single events from the NA49 data set. Even at CERN-SPS energies the statistical power is not

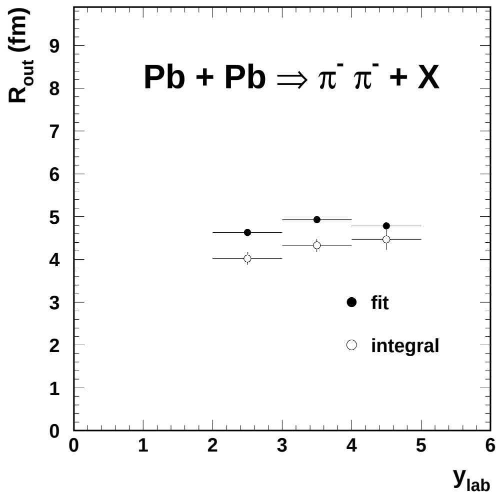


**Figure 6**: The differences between the radii extracted via the integral and fit methods.

strong and it appears that the construction of multi-dimensional correlation functions will not be possible. Nonetheless, it is possible to extract radii parameters from such functions.

Since the errors are large, the integral method will be used because the region of sensitivity can be easily adjusted with the integration limits. A truncation at the cross-over point, as was done in the ensemble analysis, would result in the use of only 2-3 bins which would be insufficient to characterize the correlation function. Instead, a fixed interval was defined to be integrated—the first 100 MeV/c. The reason the interval 0-100 MeV/c was chosen was that it contains the complete region where the enhancement is expected and extends slightly beyond. Thus, a small bit of shape information is also included such that fluctuations should be detectable at both the size and shape level. As mentioned previously the interval that is integrated—0-100 MeV/c in our case—determines the general size. That is, the fewer the number of bins used in the integration, the larger the size.

With such values it is possible to correlate the radius parameter, $R_{inv}$ with the event multiplicity while varying the severity of the cuts imposed on the data. This is shown in figure 8. This not only shows that the radius parameter extracted is a very stable parameter with event multiplicity but also the magnitude of the number of tracks that are rejected with the cuts. Although it may be suggested that there is a very slight increase with multiplicity when no cuts are made, this effect disappears with the introduction of the cuts. The error shown in figure 8 is only statistical. In this data set a 4% interaction trigger was used and although it may be argued that the trigger does not allow a large enough variation in centrality, there will be difficulties in constructing correlation functions in events with smaller multiplicities. Still there is no identification

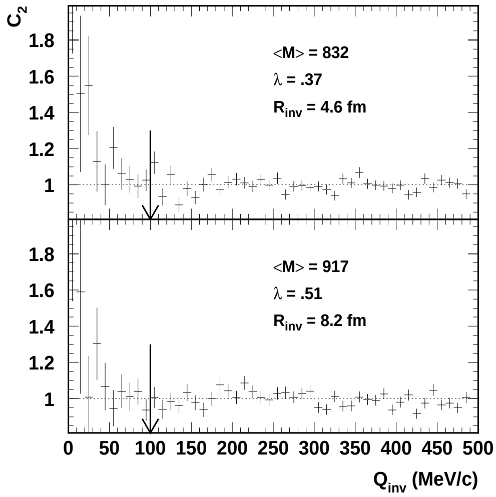


**Figure 7**: Typical correlation functions constructed from the tracks within the VTPCs of a single event with *no* cuts made. Shown are the radius parameter $R_{inv}$, the correlation strength $\lambda$, and the event multiplicity M. Although the statistics are marginal, the correct shape and reasonable radius parameter is found.

of transitional behavior in such analysis.

# 5 Conclusions

A new method which extracts radii parameters from correlation functions without the use of a conventional $\chi^2$ minimization fitting method. This new method is capable of quantifying the deviation from Gaussian behavior in a systematic way and has power to discriminate between different shapes. The radii parameters extracted from the integral method are systematically 20-30% smaller than those extracted via the conventional Gaussian fitting procedure as a result of correlation function differing from unity at large $Q$. This discrepancy is most noticeable in the direction parallel to the pairs transverse momentum or $k_T$. This implies that a non-Gaussian component in the correlation function will cause an over estimate in the magnitude of the spatial extension of the source which is important since it is believed that CERN-SPS energies are near threshold for deconfinement to occur. HBT radii that are 20-30% smaller would reduce the volume of the source by nearly a factor of 2! Although the radii are systematically smaller with this new method, application to multi-dimensional correlation functions shows the same qualitative behavior of the parameters as a function of rapidity remains. However the statistical power required to fully exploit the integral method is large and this method will be limited to those studies using only the largest ensembles.

An open question remains if this non-Gaussian behavior is a result of long-

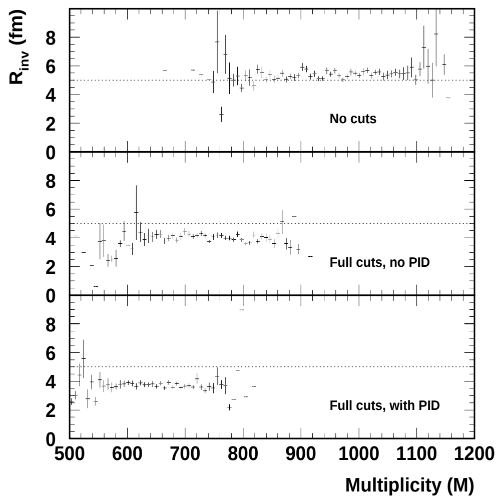


**Figure 8**: The correlation between $R_{inv}$ and event multiplicity for varying severity of cuts. The top panel imposes no cuts outside that of acceptance. The middle panel imposes the full set of cuts used in the ensemble analysis which includes track length and vertex position as well as two-track resolution, while the bottom panel illustrates all cuts in the middle panel *and* ionization consistent with that of a $\pi$. Although it is obvious there is a change in the radius parameter with the different cuts, there is no strong variation in source geometry, characterized by $R_{inv}$ with multiplicity. A line at 5 fm is drawn as a reference mark only.

lived resonance decays and if so, whether it is possible to utilize $G$ in order to estimate the resonance population. This will be heavily model dependent and require an increased understanding of the systematic errors in correlation functions especially that of the Coulomb correction. This does open up the possibility that it may be wiser to extract information from $+-$ or even neutral pairs with $K^o$ and $\Lambda$ being interesting candidates. The first look at event-by-event correlation functions seems to preclude the possibility of constructing any sort of multi-dimensional correlation function and it appears that a construction in $Q_{inv}$ does not identify any translational behavior over the restricted range of the data set. If any time is to be invested into this pursuit one must ask the question of whether there is any useful information that can be extracted from single dimensional functions ($Q_{inv}$).

# 6 Acknowledgements

This work is supported by a grant from the US Department of Energy (DOE) DE-FG03-88ER40424 A011 (FDP) and DE-FG02-91ER40609.

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