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Soumis à Physical Review C

GANIL P 01 08

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Scaling properties of the fragment size distributions are studied in a microcanonical multifragmentation model. A new method based on the global quality of the scaling function is presented. Scaling is not washed out by the long range Coulomb interaction nor by secondary decays for a wide range of source masses, densities and deposited energies. However, the influence of these factors on precise value of the critical exponents as well as the finite size corrections to scaling are shown to be important and to affect the possible determination of a specific universality class.

PACS number(s): 24.10.Pa; 25.70.Pq; 64.60.Fr

In the past decade, the simultaneous breaking of a excited system in many pieces, has been recognized as a new decay mode of atomic nuclei. This multifragmentation has been tentatively associated with a liquid-gas phase transition which is expected to occur in nuclear matter because of the resemblance of the nucleon-nucleon interaction with the van der Waals force. In particular, scaling properties which are expected in the vicinity of a critical point in the thermodynamical limit, have been observed on the sizes of fragments as a function of multiplicity [1] and of excitation energy [2] and critical exponents have been extracted in reasonable good agreement with the liquid-gas universality class. These observations would plead in favor of the classification of nuclear multifragmentation as a second order phase transition.

It has been recently discussed in the framework of the grand canonical Lattice Gas Model that a scaling behavior can be observed at supercritical densities along the so called Kersetz line [3] and for canonical finite size systems in the liquid-gas coexistence region as well [4]. However, the lattice-gas model does not contain the specific features of nuclear physics concerning a quantum system of interacting fermions with a short range nuclear interaction but also a long range Coulomb force. Moreover, very little is known in general on scaling properties in first and second order phase transitions in the microcanonical ensemble and for systems subject to long range non saturating forces as the Coulomb interaction [5]. The present paper aims to investigate these issues about criticality within a realistic microcanonical nuclear multifragmentation model [6].

The model describes the disassembly of a statistically equilibrated nuclear source $(A,\ Z,\ E,\ V)$ (the mass number, the atomic number, the excitation energy and the freeze-out volume respectively). The basic assumption of the model is the equal probability between all configurations $C\ :\ \{A_i,\ Z_i,\ \epsilon_i,\ \mathbf{r}_i,\ \mathbf{p}_i,\quad i=1,\ldots,N\}$ (the mass number, the atomic number, the excitation energy, the position and the momentum of each fragment i of the configuration C, composed of N fragments) which are subject to standard microcanonical constraints: $\sum_i A_i = A$, $\sum_i Z_i = Z$, $\sum_i \mathbf{p}_i = 0$, E_t - constant. Fragments are assumed to be spherical, are not allowed to overlap and are placed in a spherical container of volume V. The integration of the total number of states of the system over fragment momenta can be analytically performed leading to a specific statistical weight for each partition in a smaller configuration space from which the momentum observables have been projected out [6]. Using these weights, a Metropolis-type simulation is employed in order to generate a microcanonical ensemble of events. Since fragments resulting from the primary break-up are excited the model is completed with a second stage treating secondary particle emission leading to the asymptotic fragments measured in the detectors [6].

Let us now discuss the scaling properties of the nuclear disassembly. Standard renormalization group arguments [7] lead to the following scaling relation [8] which is expected to hold in the proximity of the critical point of an infinite systems:

$$N\left(A,p\right) = A^{-\tau} f(A^{\sigma}\left(p-p_c\right)) \tag{1}$$

where $N\left(A,p\right)$ is the multiplicity of a cluster of size A when the control parameter of the considered system is p. σ and τ are the critical exponents while f is a universal scaling function.

For finite size systems, little is known about the survival of this scaling property in the different statistical ensembles. In the lattice-gas model for constant volume transformations scaling has been shown to hold in the canonical ensemble for which the control parameter is the temperature $(p=T)$ [3,4]. One of the aims of this paper is to verify whether such a scaling exists in microcanonical finite systems for which the control parameter is the energy $(p=E)$.

In order to study the critical behavior, two control parameters are of special interest in eq. (1): i) the critical point, from which one can deduce $N\left(A,p_c\right) = A^{-\tau} f(0)$, ii) the maximum of the scaling function, for which one has $N\left(A,p_{\max}\left(A\right)\right) = A^{-\tau} f_{\max}$ and $p_{\max}\left(A\right) = x_{\max} A^{-\sigma} + p_c$, where $x_{\max}$ is the value of x $(= A^{\sigma}(p-p_c))$ for which

f is maximized. These relations can be used to extract the exponents τ and σ [4].

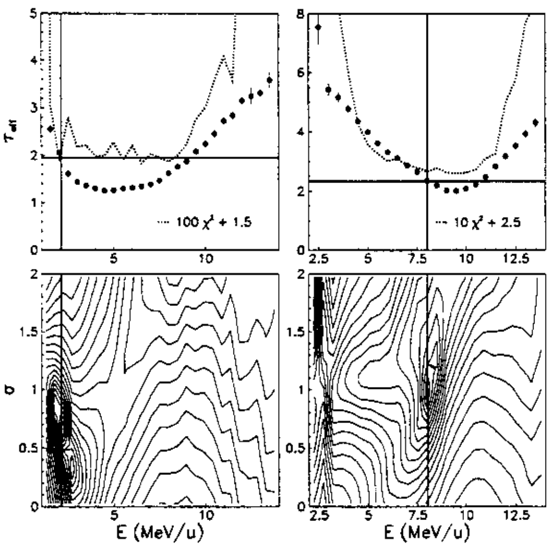

FIG. 1. Upper part: exponent of a power law fit of the size distribution as a function of the excitation energy (symbols) and χ^2 of the fit (dotted line); horizontal line: exponent of a power law fit of the maximum fragment production yield of a given size. Lower part: global quality of the scaling surface ϵ_r. The calculations on the left (right) panels correspond to the model with (without) the Coulomb interaction.

Relation (i) has been widely used for defining a critical point and extracting the critical exponent τ from both experimental data [1] and model predictions [9]. A standard technique is to calculate $\tau_{eff}(p)$ from a power law fit of the fragment size distribution at a given value of the control parameter p. In the first studies, it has often been claimed on heuristic grounds that the minimum of τ_{eff} as a function of p should correspond to the critical point [10,9]. It is well known now [11] and easy to demonstrate from equation (1) that this statement is not correct and it has been proposed to rather look at the minimum value of the χ^2 of the power law fit [12]. However, due to finite size effects, only a small mass window is expected to scale because the mass should be much larger then the elementary building block of the clusters to not be affected by the details of the interaction and small enough compared to the total mass to not feel the finite size restrictions. To assure that τ_{eff} is meaningful near the critical point, we choose to fix the mass interval for the fit at the energy where the power law region is the widest.

In Fig. 1 $\tau_{eff}(E)$ is represented for a system with $A = 200$, $Z = 82$ and a freeze-out recipient of volume $V = 3\ V_0$ (where V_0 is the volume of the system at normal nuclear matter density) in which the centers of the spherical fragments are positioned. The fit is performed at the asymptotic stage (i.e. after secondary particle emission) in both the standard version of the model and in the case in which the Coulomb interaction has been switched off. It can be noticed that the χ^2 of the power law fit has a rather flat minimum spread over a wide range of excitation energies. Both one and two parameter fits, for which the power law normalization is related to τ or is free to vary, have been tested leading to similar results. One can conclude that the minimum χ^2 method gives rather uncertain results concerning both τ and the critical excitation energy E_c.

In order to identify in a less ambiguous way the critical region, we can test the quality of the whole scaling (1). For this evaluation E_c and σ have been varied independently while τ is taken as the τ_{eff} estimated from the power law fit at an energy $E = E_c$ (upper part of figure 1). The estimator of the quality of scaling is defined as the average (subject to x) relative dispersion (subject to A) of the scaling functions $f_A(x)$, corresponding to the same mass interval $A_{\min}, \dots, A_{\max}$ as that used for determining τ_{eff}, over the interval $0 < x < A_{min}^{\sigma}\Delta E$ with ΔE=8 MeV/u (here $x = A^{\sigma}(E - E_c)$ and $f(x) = N(A, E)A^{\tau}$).

The result is presented in the bottom part of figure 1. We observe a deep minimum of ϵ_r which clearly indicates the location of the critical (E_c, σ) region. This minimum is sharper in the energy direction defining in a rather precise way the domain of acceptable E_c which in turn defines the possible τ using the relation $\tau = \tau_{eff}(E_c)$. A comparable quality of the scaling is obtained with and without the inclusion of the Coulomb interaction. This indicates that in the nuclear case, the presence of a long range non saturating force does not cause important deviations from the expected critical behavior. This might be due to the fact that the Coulomb force represents only a relatively small correction to the energy and thus to the relative weight of the various partitions.

Another method to evaluate the critical control parameter and the critical exponents based on the relation (ii) has recently been used. If scaling is fulfilled, the maximum fragment production yields $N(A, E_{\max}(A))$ should behave as a power law of the fragment mass with the exponent τ. As shown in the upper part of figure 2, this turns out to be the case in a large mass region leading to $\tau = 1.95$ when the Coulomb force is included and $\tau = 2.33$ when it is not. The maximum mass for which the power law is still valid, can be understood as a finite size cutoff while the minimum mass defines a maximum energy above the critical point up to which the scaling holds. σ and E_c can be defined using the quality test of scaling ϵ_r. The lower part of figure 2 shows the minimum of ϵ_r as a function of E. We observe a sharp minimum in both considered cases, with and without the Coulomb interaction. In the case in which the Coulomb interaction has been considered, a second minimum at high energy appears as well. However, this second minimum can be eliminated by considering the χ^2 of the fit

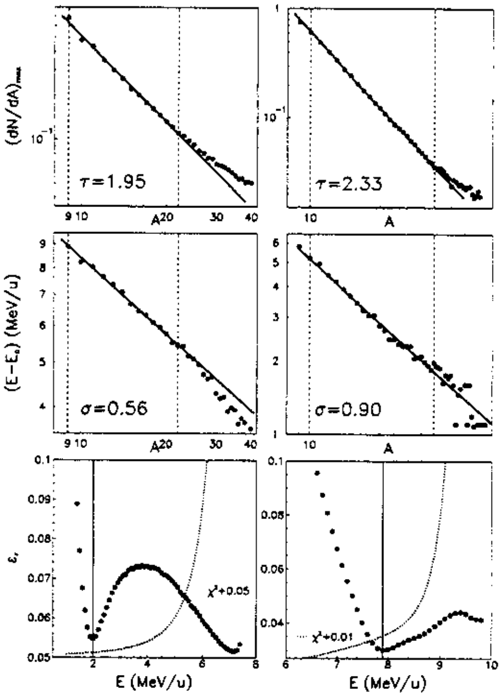

FIG. 2. Microcanonical multifragmentation model calculations with (left) and without (right) Coulomb. Upper panels: power law fit of the τ exponent from the maximum production yield as a function of the size. Lower panels: χ^2 of the power law fit of the σ exponent from the energy of the maximum production yield as a function of the possible critical energy (dotted line) and global quality of the scaling. Middle panels: power law fit of the σ exponent from the energy of the maximum production yield if the critical energy is taken at the minimum of ϵ_τ in the lower panel.

of $(E_{\max}(A) - E_c)$ with a power law of exponent A^σ. The middle part of figure 2 shows the quality of the σ fit when E_c and σ are taken as the (first) minimum of ϵ_τ. It is clear that the fit is excellent in the very same energy interval defined by the determination of the exponent τ.

If we compare the results of Fig. 1 with those of Fig. 2 we note a remarkable agreement between the two methods ((i) and (ii)) both for the critical exponents and the critical energy, making us confident on the precision of the present evaluation. Moreover, it appears that the intersection between the τ_{eff} curve (using (i)) and the τ deduced from the maximum production (ii) provides a good estimation for the critical energy E_c. This method has been already used for extracting critical information in the canonical liquid gas model framework [4].

The analysis described above has been repeated for nuclei of various sizes and for different volumes (see figure 3). We have obtained a comparable quality scaling for the various mass studied. The more surprising result is that scaling is also observed in a wide range of volumes, with a monotonic increase of the critical excitation energy with increasing density. The critical parameters associated with the asymptotic fragment mass distributions obtained with the inclusion of the Coulomb interaction are summarized in Table I.

TABLE I. Critical parameters for the asymptotic distributions of the statistical multifragmentation model

	$A = 50$, $V = 3V_0$	$A = 200$, $V = 3V_0$	$A = 200$, $V = 2V_0$	$A = 200$, $V = 4V_0$
E_c	3.10(1)	2.05(6)	3.0(1)	1.7(2)
τ	1.60(1)	1.95(3)	1.87(1)	1.98(3)
σ	0.83(3)	0.56(1)	0.71(2)	0.55(2)

The mass of the source has a non negligible influence on the critical quantities while the density only induces a small variation of the critical exponents. Important fluctuations are seen in the σ exponent, which however are due to the intrinsic imprecision in the determination of σ as can be seen from the flatness of the quality test in the σ direction (see lower panels of figure 1). These results are in qualitative agreement with the lattice gas results of ref. [4], where finite size effects were shown to induce criticality in the cluster observables along a line in the temperature/density plane passing close to the thermodynamical critical point and extending inside the coexistence region of the first order phase transition.

A final test for the accuracy of the critical analysis is to consider the behavior of the scaling function f. The scaled mass distributions are shown for a A=200 system in figure 3 at two different volumes. Coulomb interaction is included. An almost perfect scaling is observed in the considered energy range $2 < E < 22$ MeV/u for the primary mass partitions in a volume $V = 2V_0$. To our knowledge this is the first evidence of criticality in the cluster size distributions for a microcanonical finite system. This demonstrates that energy can be used as a control parameter and that the long range Coulomb interaction can be viewed as a minor correction as far as the critical behavior is concerned. It is also apparent from figure 3 that calculations at large volumes lead to a violation of the scaling which can be appreciated only for excitation energies higher than 12 MeV/u (grey symbols in figure 3). This tends to demonstrate that the apparent scaling observed at lower densities is not an effective critical line [4], but rather a spreading effect of the thermodynamical critical point into an effective critical region due to finite size; in particular the thermodynamical critical point should be located at smaller freeze-out volumes than those considered in this paper. However, if one restricts the analysis to the $2 < E < 12$ MeV/u energy interval (black symbols), all densities lead to a comparable scaling.

It is also interesting to look at the distortion induced by secondary decay by comparing the scaling of the primary (left part of figure 3) and asymptotic fragments. One can see that the influence of the decay stage is small and does not modify the expected properties. This means that the experimental observation of finite size scaling in nuclear multifragmentation data [1,2] is compatible with low freeze out densities specific to the coexistence region of the first order phase transition and does not imply a second order phase transition.

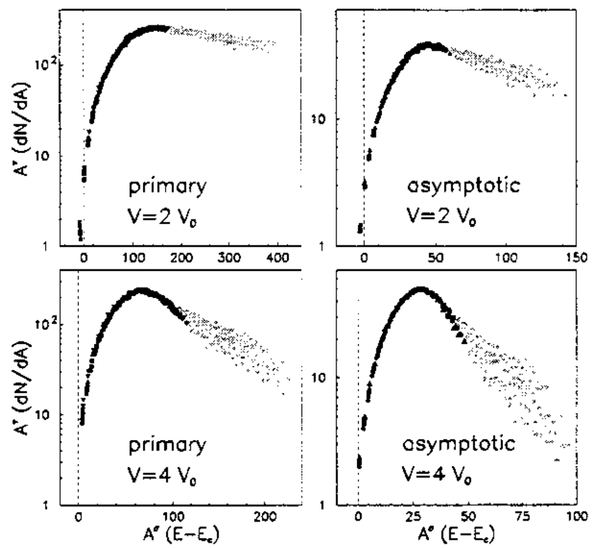

FIG. 3. Scaling function f from eq.(1) from primary (left side) and asymptotic (right side) size distributions at two different volumes with the inclusion of the Coulomb interaction. Black symbols: energies ranging from 2 to 12 MeV/u. Grey symbols: energies ranging from 12 to 22 MeV/u.

It is important however to stress that the measured values of the critical exponents which refer to the asymptotic fragment distributions, suffer of finite size effects and Coulomb distortions, and depend on the volume which needs not to be the thermodynamical critical volume. Therefore, it may be very difficult to relate the observed critical behavior to any specific universality class. As an example in our calculation for a mass A=200, the τ exponent calculated at $V = 2V_0$ from primary fragments is 2.26(2) but it becomes $\tau = 1.98(3)$ if asymptotic fragments are considered in a larger volume $V = 4V_0$. Similarly the σ exponent changes from $\sigma = 0.91(4)$ to $\sigma = 0.55(2)$. In the same way, even if the quality of scaling itself is not distorted by the Coulomb interaction, the critical quantities change considerably. If we consider for example primary fragments at $V = 2V_0$, not only the critical energy decreases from $E_c = 7.28(2)$ MeV/u to $E_c = 2.7(3)$ MeV/u taking into account the Coulomb interaction, but also the τ exponent changes from $\tau = 2.62(2)$ to $\tau = 2.26(2)$.

In conclusion, a clear scaling behavior has been identified for the first time in the fragment size distributions of a microcanonical multifragmentation model. This scaling can be observed even for source sizes as small as $A = 50$ and in a wide range of freeze-out volumes. A new method for identifying the critical region based on the evaluation of the relative dispersion of the scaling function has been proposed. Using this method the very same critical region has been identified if the τ exponent is obtained by fitting either the fragment size distribution at a given excitation energy ($\tau_{eff}(E)$), or the maximum production yield of fragments with size A. As a side result we show that the energy at which the $\tau_{eff}(E)$ extracted from the power law fit is equal to the one obtained from the maximum fragment production can provide a good estimation of the critical parameters E_c. This method is precise and easy to implement both in experimental and simulated data. Two important fundamental results should be stressed: scaling is not washed out by secondary decay of the primary excited fragments nor seriously affected by the long range non saturating Coulomb interaction. However, the critical parameters change with the size of the system, the freeze-out volume and the inclusion of the Coulomb interaction. An analysis of the quality of scaling when volume is varied shows that scaling improves notably when the volume is diminished towards the expected thermodynamical critical point. This improvement is not visible if one restricts to the analysis of experimentally accessible energies ($E < 12$ MeV/u). This means that the experimental observation even of a very high quality scaling does not allow to determine the thermodynamical critical point of nuclear matter and gives only an approximate estimation of the critical exponents.

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