RESUMMED COEFFICIENT FUNCTION FOR THE SHAPE FUNCTION

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Abstract

We present a leading evaluation of the resummed coefficient function for the shape function. It is also shown that the coefficient function is short-distance-dominated. Our results allow relating the shape function computed on the lattice to the physical QCD distributions.

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1 Introduction

In this note we present a leading evaluation of the coefficient function of the shape function [?]-[?]. The latter is also called structure function of the heavy flavours. The coefficient function allows relating the shape function — computed with a non-perturbative technique such as lattice QCD — to distributions in semi-inclusive heavyflavour decays. We consider in particular the processes

$$B \to X_s + \gamma$$
 (1)

and

$$B \to X_u + l + \nu \tag{2}$$

in the hard limit

$$Q \gg \Lambda.$$
 (3)

The quantity Q is the hard scale of these time-like processes: $Q \equiv 2\mathcal{E}$, where \mathcal{E} is the hadronic energy of the state X and Λ is the QCD scale¹. For a *B*-meson at rest, v = (1; 0, 0, 0), and the jet X flying along the minus direction (-z axis), the shape function is defined as

$$\varphi(k_{+}) \equiv \langle B(v) | h_{v}^{\dagger} \delta(k_{+} - iD_{+}) h_{v} | B(v) \rangle.$$
(4)

This represents the probability that the b-quark in the B-meson has momenta

$$p_b = m_B v + k' \tag{5}$$

with any transverse and minus components and with given plus component

$$k'_{+} = k_{+}.$$
 (6)

The static field $h_{v}(x)$ is related to the Dirac field of the beauty quark b(x) by

$$b(x) = e^{-im_B v \cdot x} h_v(x) + O(1m_B).$$
(7)

With the shape function, the decays (1) and (2) are related to their respective quark-level processes

$$b \to \hat{X}_s + \gamma$$
 (8)

and

$$b \to \hat{X}_u + l + \nu, \tag{9}$$

¹For the rare decay (1), one can actually set $Q = m_B$.

where the *b*-quark has the momentum (5) with the distribution (4). It can be shown that the invariant mass m of the state \hat{X} is related to the virtuality of the *b*-quark by

$$k_{+} = -m^2 Q. \tag{10}$$

The shape function is a non-perturbative distribution — analogous to parton distribution functions — and it describes the slice of the semi-inclusive region² in which

$$m^2 \sim \Lambda Q,$$

i.e.

$$k_+ \mid \sim \Lambda, \quad \text{or} \quad z \sim 1 - \Lambda Q,$$
 (11)

where

$$z \equiv 1 - m^2 Q^2. \tag{12}$$

Because of ultraviolet divergences affecting its matrix elements, the shape function has a dependence on the ultraviolet cut-off or renormalization point μ :

$$\varphi\left(k_{+}\right) = \varphi\left(k_{+};\mu\right),\tag{13}$$

and is related to a physical QCD distribution by means of a coefficient function by (cf. eq. (??))

$$\varphi(k_{+};Q) = \int dk'_{+} C\left(k_{+} - k'_{+};Q,\mu\right) \varphi\left(k'_{+};\mu\right).$$
(14)

The QCD distribution does not depend on μ :

$$dd\mu\varphi\left(k_{+};Q\right) = 0,\tag{15}$$

while the shape function does not depend on Q. As a consequence, the coefficient function depends on both Q and μ .

The shape function can be computed with a non-perturbative technique or extracted from experimental data. If it is computed inside a field-theory model — such as lattice QCD — its expression will exhibit a μ -dependence that cancels against that of the coefficient function. If it is instead computed inside a phenomenological model — such as a quark model — the situation is less transparent. The μ -independence is not "automatic" and one has to specify the value of μ appropriate for the model. Some care is needed also in extracting the shape function from the experimental data, in order to avoid double counting of perturbative corrections. A factorization scheme must be defined and the coefficient functions for the various processes all have to be computed in the same scheme³. In particular, if a branching MonteCarlo is used for the analysis, the perturbative corrections generated by the program must be subtracted.

The coefficient function is obtained by evaluating in leading approximation both φ and φ and inserting their expression in eq. (14). Since the coefficient function is expected to be a short-distance quantity, we compute the QCD distribution and the shape function in PT for an on-shell *b*-quark (k' = 0). This expectation will be verified *a posteriori*.

 $^{^{2}}$ This region is also called threshold region, large-x region, radiation-inhibited region and Sudakov region.

³The situation is analogous to usual hard processes, where various factorization schemes for the parton distribution functions are defined: DIS, $\overline{\text{MS}}$, etc.

2 The QCD distribution

The (perturbative) long-distance effects occurring in (1) and (2) can be factorized in the function

$$\mathbf{f}(z) = \delta (1 - z - 0) - A_1 \alpha_S (\log [1 - z] 1 - z)_+, \qquad (16)$$

where

$$A_1 = C_F \pi \tag{17}$$

and $C_F = (N_c^2 - 1) / (2N_c) = 4/3$. The plus-distribution is defined as usual as

$$P(z)_{+} \equiv P(z) - \delta (1 - z - 0) \int_{0}^{1} dy P(y).$$
(18)

The integrated or cumulative distribution is defined as

$$\mathbf{F}(z) \equiv \int_{z}^{1} dz' \, \mathbf{f}(z') \,. \tag{19}$$

Inserting expression (16) in this, one obtains the well-known double logarithm:

$$\mathbf{F}(z) = 1 - A_1 \alpha_S 2 \log^2 (1 - z).$$
(20)

The cumulative distribution satisfies the normalization condition $\mathbf{F}(0) = 1$. Multiple soft-gluon emission exponentiates the one-loop distribution, so that

$$\mathbf{F}(z) = \exp\left[-A_1 \alpha_S 2 \log^2 \left(1-z\right)\right]. \tag{21}$$

For further improvement, it is convenient to write the function $\mathbf{f}(z)$ in an "unintegrated" form, as

$$\mathbf{f}(z) = \delta(1-z) + A_1 \alpha_S \int_0^1 d\epsilon \epsilon \int_0^1 dtt \left[\delta(1-z-\epsilon t) - \delta(1-z)\right], \tag{22}$$

where we have defined the unitary energy and angular variables

$$\epsilon \equiv EQ$$
 and $t \equiv 1 - \cos \theta 2.$ (23)

The quantity E is two times the energy of the soft gluon, $E = 2E_g$, and θ is the emission angle. Leading logarithmic corrections are included replacing the bare coupling with the running coupling evaluated at the gluon transverse momentum squared [?]:

$$\alpha_S \to \alpha_S \left(l_\perp^2 \right),\tag{24}$$