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Z-SCALING AND JET PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES

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1 Introduction

The production of very large transverse momentum hadron jets in hadron-hadron collision at high energies at SpS at CERN observed by the UA2 [1] and UA1 [2] collaborations confirming the hints of jet production in the experiment of the AFS collaboration [3] at ISR is one of the outstanding discoveries confirming the theory of strong interaction - quantum chromodynamics (QCD). The prediction that the hard scattering of hadron constituents should results in the production of two hadronic jets having the same large transverse momentum as scattered partons has been direct evidence at the SpS collider at CERN when a dominant two-jet structure is observed in the events depositing a very large transverse energy. Systematic investigations of jet production in $\bar{p}p$ collisions are performed at the Tevatron at Fermilab by the CDF [4, 5] and D0 [6, 7, 8, 9] collaborations. A high colliding energy of hadrons guarantees that the QCD can be used to describe the interaction of hadron constituents. Main goals of the measurements of hadronic jet production are to study the energy and angular dependencies of jet cross section, the structure and content of jets and their fragmentation properties in the energy domain where a hard process can be separated clearly from soft hadronic interactions. Parton fragmentation into hadrons might be one of the least understanding features of QCD [10]. Even though a primary scattering process is described in terms of perturbative QCD, the hadronization chain contains very low $p_\perp$ hadrons respective to the parent parton. Therefore, the whole process is clearly a non-perturbative phenomenon involving final state interactions which have to conserve color and baryon numbers.

A search for general properties of jet production in hadron-hadron collisions is of great interest, especially in connection with commissioning such large accelerators of nuclei and protons as RHIC at Brookhaven and LHC at CERN. The main physics goals of the investigations on these colliders are to search for quark-gluon plasma - the hot and superdense phase of the nuclear matter, Higg's boson and particles of a new generation predicted by supersymmetry theories. The main features of jet production observed in hadron-hadron interactions allow us to extract nuclear effects and to study the influence of nuclear matter on jet formation in pA and AA collisions. The features established at the ISR and RHIC energies enable us to estimate background over the LHC energy range. Jets as direct photons and dilepton pairs are traditionally considered to be one of the good probes for studying the interaction between quarks and gluons in the surrounding nuclear matter and searching for new particles (quarks, leptons etc.)

A comparison of jets produced in hadron-hadron, hadron-nucleus and nucleus-nucleus interactions both in the central and non-central rapidity region allows us to understand in detail the physics phenomena underlying secondary jet production and, in particular, the hadronization process.

In this paper, we use the method proposed in [11] for the description of jet production in pp and $\bar{p}p$ collisions at high energies. In the framework of the method, such experimental observables as inclusive cross section and multiplicity particle density are used to construct scaling function $\psi(z)$ and variable z. The scaling, known as z-scaling, reveals interesting properties. Two of them are the independence of the scaling function, $\psi(z)$, of colliding energy and the angle of produced objects (hadron, photon). A general concept of the scaling is based on such fundamental principles as self-similarity, locality, fractality and scale-relativity [12, 13].

A new presentation, z-presentation, of the experimental data was used for the analysis of hadron and direct photon production in pp and pA collisions [11, 13, 14, 15, 16]. The scaling function was shown to be independent of center-of-mass energy $\sqrt{s}$ and the angle of produced particle θ over a wide kinematic range. The symmetry property of $\psi(z)$ under scale transformation was used to study the A-dependence of particle production [17]. The scaling function of direct photon production was found to reveal the power behaviour of $\psi(z)$ [14, 15, 16]. The properties of the scaling are assumed to reflect the fundamental properties of particle structure, interaction and production. The scaling function describes the probability to form the produced particle with formation length z. The existence of the scaling itself means that the hadronization mechanism of particle production reveals such fundamental properties as self-similarity, locality, fractality and scale-relativity.

The inclusive jet and dijet production in $\bar{p}p$ and pp collisions at high energies is studied in this paper. The available experimental data on the cross section obtained at ISR, SpS and Tevatron are used to construct scaling function $\psi(z)$. The dependence of $\psi(z)$ on the energy $\sqrt{s}$ and angle θ of the produced jet are studied. The power behaviour of the scaling function, $\psi(z) \simeq z^{-\alpha}$, is found. The properties of z-scaling for the jets produced in pp and $\bar{p}p$ collisions are used to predict jet yields at the RHIC, Tevatron and LHC energies.

The paper is organized as follows. A general concept of z-scaling and the method of constructing the scaling function for the $p + p \to jet + X$ process is described in Section 2. New results on the energy and angular dependence of scaling function $\psi(z)$ for pp and $\bar{p}p$ collisions based on the analysis of the experimental data, discussion of the obtained results, physical interpretation of the scaling function and variable z are presented in Section 3. Conclusions are summarized in Section 4.

2 General concept of z-scaling

In this section, we would like to remember the basic ideas of z-scaling [11, 13] dealing with the investigation of the inclusive process

$$P_1 + P_2 \to q + X. \tag{1}$$

The momenta and masses of the colliding and inclusive particles are denoted by P_1, P_2, q and M_1, M_2, m_1, respectively. In accordance with Stavinsky's ideas [18], the gross features of the inclusive particle distributions for reaction (1) at high energies can be described in terms of the corresponding kinematic characteristics of the exclusive subprocess written in the symbolic form

$$(x_1 M_1) + (x_2 M_2) \to m_1 + (x_1 M_1 + x_2 M_2 + m_2). \tag{2}$$

The parameter m_2 is introduced in connection with internal conservation laws (for isospin, baryon number, and strangeness). The x_1 and x_2 are the scale-invariant fractions of the incoming four-momenta P_1 and P_2 of colliding objects. The energy of the parton subprocess defined as

$$\hat{s}_x^{1/2} = \sqrt{(x_1 P_1 + x_2 P_2)^2} \tag{3}$$

represents the center-of-mass energy of the constituents involved in the collision. In accordance with a space-time picture of hadron interactions at the parton level, the cross section for the inclusive particle production is governed by a minimum energy of colliding partons

$$d\sigma/dt \sim 1/\hat{s}^2_{min}(x_1, x_2). \tag{4}$$

The corresponding energy $\hat{s}^{1/2}_{min}$ is fixed as a minimum of Eq. (3) which is necessary for production of the secondary particle with mass m_1 and four-momentum q. Below, we present a scheme from which a more general structure of the variables x_1 and x_2 follows. Moreover, we show that such the dynamic characteristic of process (1) as particle multiplicity density plays an important role in the framework of the developed scheme. We would like to emphasize two main points of this approach. The first one is a fractal character of the parton content of the involved composite structures. The second one is based on the self-similarity of the mechanism underlying the particle production at the level of elementary constituent interactions. Both points will be discussed in the other sections.

2.1 Momentum fractions x_1 and x_2

The elementary parton-parton collision is considered as a binary subprocess which is satisfied the condition

$$(x_1P_1 + x_2P_2 - q)^2 = (x_1M_1 + x_2M_2 + m_2)^2. \tag{5}$$

The equation expresses the 4-momentum conservation law for an elementary subprocess. Relationship between x_1 and x_2 is written in the form

$$x_1x_2 - x_1\lambda_2 - x_2\lambda_1 = \lambda_0, \tag{6}$$

where

$$\lambda_1 = \frac{(P_2q) + M_2m_2}{(P_1P_2) - M_1M_2}, \qquad \lambda_2 = \frac{(P_1q) + M_1m_2}{(P_1P_2) - M_1M_2}, \qquad \lambda_0 = \frac{0.5(m_2^2 - m_1^2)}{(P_1P_2) - M_1M_2}. \tag{7}$$

Considering process (2) as a parton-parton collision, we introduce the quantity Ω which connects kinematic and dynamic characteristics of the interaction. It is taken in the form

$$\Omega(x_1, x_2) = m(1 - x_1)^{\delta_1}(1 - x_2)^{\delta_2}, \tag{8}$$

where m is the mass constant and δ_1 and δ_2 are the factors relating the fractal structure of colliding objects [13]. We define the fractions x_1 and x_2 to maximize the value of $\Omega(x_1, x_2)$, simultaneously fulfilling condition (6)

$$d\Omega(x_1, x_2)/dx_1|_{x_2 = x_2(x_1)} = 0. \tag{9}$$

A prominent expression for $x_{1,2} = x_{1,2}(\lambda_0, \lambda_1, \lambda_2, \delta_1, \delta_2)$ was obtained in [13]. A physics interpretation of the quantity Ω and the factors δ_1 and δ_2 is given in Sec.4.

Equation (6) satisfies the 4-momentum conservation law in the whole phase space. The variables $x_{1,2}$ are equal to unity along the phase space boundary and cover the full phase space accessible at any energy. The restriction $\lambda_1+\lambda_2+\lambda_0 \le 1$ can be obtained from the condition $x_i \le 1$. The inequality corresponds to the threshold condition

$$(M_1+M_2+m_2)^2+E^2-m_1^2 \le (\sqrt{s}-E)^2. \tag{10}$$

It kinematically restricts a maximum energy E of the inclusive particle m_1 in the c.m.s. of reaction (1).

2.2 Scaling function $\psi(z)$ and scaling variable z

In accordance with the self-similarity principle, we search for the solution depending on a single scaling variable z in the form

$$\psi(z) \equiv \frac{1}{<N>\sigma_{inel}}\frac{d\sigma}{dz}. \tag{11}$$

Here, σ_{inel} is the inelastic cross section and $<N>$ the average charged particle multiplicity. The function $\psi(z)$ has to be dependent on the scaling variable z. All the quantities in (11) refer to nucleon-nucleon interactions. The function $\psi(z)$, expressed via the invariant differential cross section for the production of inclusive particle m_1 is introduced as follows (see Ref.[13])

$$\psi(z) = -\frac{\pi s}{\rho(s,\eta)\sigma_{inel}}J^{-1}E\frac{d\sigma}{dq^3}, \tag{12}$$

where, the factor J is given by $J=(\partial y/\partial\lambda_1)(\partial z/\partial\lambda_2)-(\partial y/\partial\lambda_2)(\partial z/\partial\lambda_1)$. Here, s is the center-of-mass energy squared of the corresponding NN-system. Expression (12) relates the inclusive cross section $Ed^3\sigma/dq^3$ and the average charged particle multiplicity density $\rho(s,\eta)=d<N>/d\eta$ to the scaling function $\psi(z)$. As usual, the combination $y=0.5\ln(\lambda_2/\lambda_1)$ is approximated to (pseudo)rapidity η at high energies.

We choose z, in accordance with the ansatz suggested in Ref.[13], as a physically meaningful variable which could reflect self-similarity (scale invariance) as a general pattern of hadron production

$$z=\frac{\sqrt{\hat{s}_\perp}}{\Omega\cdot\rho(s)}, \tag{13}$$

where $\hat{s}_\perp^{1/2}$ is the transverse kinetic energy of subprocess (2) defined by the expression $\hat{s}_\perp^{1/2}=\hat{s}_\lambda^{1/2}+\hat{s}_\chi^{1/2}-m_1-(M_1x_1+M_2x_2+m_2)$; Ω is the fractal measure given by Eq.(8) and $\rho(s)=dN/d\eta|_{\eta=0}$ is the average multiplicity density of charged particles produced in the central region of the corresponding nucleon-nucleon interaction.

The transverse energy consists of two parts

$$\hat{s}_\lambda^{1/2}=\sqrt{(\lambda_1P_1+\lambda_2P_2)^2}, \qquad \hat{s}_\chi^{1/2}=\sqrt{(\chi_1P_1+\chi_2P_2)^2}, \tag{14}$$

which represent the transverse energy of the inclusive particle and its recoil, respectively. We would like to note that the form of z, as defined by Eq. (13), determines its variation range. The boundaries of the range are 0 and ∞. These values are scale independent and kinematically accessible at any energy.

2.3 Fractality and scale-relativity

Let us focus our attention on some symmetry properties of z-scaling construction. As noted above, Eq.(6) describes the 4-momentum conservation law for an elementary subprocess. The equation is a covariant one under the scale transformation

$$\lambda_{1,2} \to \rho_{1,2} \cdot \lambda_{1,2}, \qquad x_{1,2} \to \rho_{1,2} \cdot x_{1,2}, \qquad \lambda_0 \to \rho_1 \cdot \rho_2 \cdot \lambda_0. \tag{15}$$

The scale parameters, $\rho_{1,2}$, are chosen in accordance with the type of collisions. This is reasonable for the description of the pp and pA interaction to use $\rho_1 = 1, \rho_2 = 1$ and $\rho_1 = 1, \rho_2 = A_1$, respectively. Here, A_1 is the corresponding atomic number. The scale transformation allows us to consider the collision of complex objects in terms of a suitable subprocess of interacting elementary constituents. The choice of the elementary subprocess suitable to the problem is important for the development of a microscopic scenario of the collision.

The quantity Ω connects kinematic and dynamic characteristics of the interaction. The factors δ_1 and δ_2 are related to the fractal structure of colliding objects. Fractality as a general principle in particle physics means that internal structure of particles and their interactions reveal self-similarity at any scale. The fractal structure itself of colliding objects is defined by the structure of interacting constituents which is not an elementary one either. In this scheme, hadron-hadron and hadron-nucleus collisions are considered as an interaction of two fractals. The measure of the interaction is written as

$$\Omega = V^{\delta}, \tag{16}$$

where δ is the coefficient (fractal dimension) describing the fractal structure of the elementary collision. The factor V is a part of the full phase-space of fractions $\{x_1, x_2\}$ corresponding to such parton-parton collisions in which the inclusive particle can be produced. The fractal property of the collision reveals itself so that only the part of all multiscatterings corresponding to the phase space V^{δ} produces the inclusive particle.

The full phase-space is determined by the condition $\{0 \le x_1, x_2 \le 1.\}$ Equation (6) represents a kinematic restriction on binary parton-parton collisions in which the inclusive particle with momentum q can be produced. The volume V of the full phase-space, accessible for the production of the inclusive particle given by the values of kinematic variables P_1, P_2, and q, is approximately written as $V = (1-x_1)(1-x_2)$.

In the case of collisions of symmetric objects, the approximation for the measure Ω is written as follows

$$\Omega = [(1-x_1)(1-x_2)]^{\delta} = [(1-\bar{x}_1)(1-\bar{x}_2)]^{\bar{\delta}}. \tag{17}$$

The equation shows the correlation between the fraction x_i and fractal dimension δ. The measure is the invariant under simultaneity of the scale transformation of Lorentz invariants x_i and multiplicative transformation of δ. Invariance under the transformation $\{x_i, \delta\} \to \{\bar{x}_i, \bar{\delta}\}$ describes the relativity of any scales in the problem. This means that the resolution of one fractal structure with respect to the other one is a relative one. The parameter δ describes the accessible resolution for both structures or an absolute scale frame for the problem. Relativity of the scale frame to use for an other physical conditions in the problem relates to the applicability of the relativity principle to the laws of scale [12, 13]. The principle states that the laws of Nature are identical in all scale-inertial systems. In the considered perspective, the principle of scale relativity states that the Einstein-Lorentz composition law of velocities is applied to the systems of reference whatever their state of scale [12].

3 Jet production and z-scaling

In this section we study the properties of z-scaling for jets produced in $\bar{p}p$ and pp collisions. General formulas (12) and (13) are used to calculate the scaling function $\psi(z)$ and the variable z. The quantities $\rho(s,\eta)$ and σ_{inel} are the average multiplicity density and the inelastic cross section of jets production, respectively.

3.1 Jet definition

Two highly collimated collections of hadrons having an approximately equal transverse momentum are observed during the hard scattering interaction of colliding hadrons. These collimated beams of particles are called jets. Jets are experimentally defined as the amount of energy deposited in the cone of radius $R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}$ in the space (η,ϕ), where $\Delta\eta = \eta - \eta_0$ and $\Delta\phi = \phi - \phi_0$ specify the extent of the cone ΔR in pseudorapidity and azimuth. The coordinates of the jet axis are denoted by η_0 and ϕ_0. The pseudorapidity η is determined via the center mass angle θ by the formula $\eta = -ln(tg(\theta/2)))$. The jets are usually described using such variables as transverse momentum $(q_\perp)$, azimuth angle (ϕ), rapidity (y) and mass. In the case of the jet with zero mass, we have $\eta = y$ and $q_\perp = E_\perp$, where $E_\perp$ is the transverse energy of the jet. Different algorithms for jet finding are developed for the jet analysis of experimental data. The UA1 [2] and CDF [19] styles are most popular. The fixed cone algorithm is based on the assumption that the particles belonging to the jet are contained in the cone of radius R. The choice of R is usually made to ensure good reconstruction and energy resolution of jets.

3.2 Angular independence of $\psi(z)$

Now, let us study the angular dependence of the scaling function $\psi(z)$ of jet production in $\bar{p}p$ collisions. For analysis, we use two sets of cross section data [20] and [6] obtained by the UA2 and D0 collaborations, respectively. A strong dependence of the cross section on the angle of the produced jet was experimentally found.

We verify the hypothesis of angular scaling for data z-presentation for jet production in $\bar{p}p$ collisions using the available experimental data obtained at SpS and Tevatron.

Figures 1(a) and 2(a) show the dependence of the cross section of the $\bar{p} + p \rightarrow jet + X$ process on transverse momentum $q_\perp$ at $\sqrt{s} = 630$ [20] and 1800 GeV [6] for different rapidity intervals, $0.0 < |\eta| < 2.0$ and $0.0 < |\eta| < 3.0$, respectively.

Figures 1(b) and 2(b) demonstrate z-presentation of the same data. The obtained results show that the function $\psi(z)$ is independent of the angle θ over a wide rapidity range at the SpS and Tevatron energies.

The D0 and CDF collaborations have carried out the measurements [8, 5] of the angular dependence of the inclusive cross section of dijet production at $\sqrt{s} = 1800\ GeV$. In the first experiment [8], one jet was registered over a range of $0.0 < |\eta_1| < 1.0$ and the rapidity of the second jet changed over a range of $0.0 < |\eta_2| < 4.0$. In the CDF experiment [5], jets were registered over ranges of $0.1 < |\eta_1| < 0.7$ and $0.1 < |\eta_2| < 3.0$.

Figures 3(a) and 4(a) present the dependence of the cross section of the $\bar{p}+p \to 2jet + X$ process on transverse momentum at energy $\sqrt{s} = 1800\ GeV$ for different rapidity intervals. The obtained results show a strong angular dependence of the cross section for dijet production in $\bar{p}p$ collisions.

Figures 3(b) and 4(b) demonstrate $\psi(z)$ as a function of z for the same data. We see that the scaling function, $\psi(z)$, is practically independent of angular θ over a wide rapidity ranges. An indication of the angular dependence of ψ was found for large η.

3.3 Energy independence of $\psi(z)$

The hypothesis of energy scaling predicts that the scale invariant jet cross section $q_\perp^4 Ed^3\sigma/dq^3$ will be independent of center-of-mass energy $\sqrt{s}$ when plotted as a function of the variable $x_\perp = 2q_\perp/\sqrt{s}$. The UA1[21], UA2[22], CDF[4] and D0[7] collaborations performed the measurements of the inclusive cross section to study the scaling behaviour of jet production at different energy $\sqrt{s}$, and the dependence of jet spectra on colliding energy was found.

We verify the hypothesis of energy scaling for data z-presentation for jet production in pp and $\bar{p}p$ collisions using the available experimental data obtained at ISR, SpS and Tevatron.

Figure 5(a) shows the dependence of the cross section of the $p+\bar{p} \to jet+X$ process on jet momentum $q_\perp$ at $\sqrt{s} = 200, 500$ and $900\ GeV$ and a produced angle θ of 90^0. The experimental data on the cross section obtained by the UA1 collaborations [21] are used. We would like to note that the data cover the range of relatively small jet transverse momenta ($q_\perp < 35\ GeV/c$). The data demonstrate a strong dependence of the cross section on colliding energy $\sqrt{s}$.

Figure 5(b) shows z-presentation of the same data. Taking into account the experimental errors, we can conclude that the scaling function $\psi(z)$ of jet production in $\bar{p}p$ collisions demonstrates the energy independence.

A similar result is obtained for jet production in $\bar{p}-p$ collisions at the other energies, too.

Figures 6(a)-8(a) show the dependence of the cross section on momentum $q_\perp$ for jet production in $\bar{p}p$ collisions at SpS and Tevatron energies $\sqrt{s} = 546, 630$ and $1800\ GeV$ and a produced angle θ of 90^0. The experimental data on the cross section obtained by the CDF[4], D0[7], UA1[21] and UA2[22] collaborations are used. The z-presentations corresponding to the same data are shown in Figures 6(b)-8(b).

Figure 9 shows $q_\perp$- and z-presentations of the data for jet production in pp collisions at $\sqrt{s} = 38.8, 45$ and $63\ GeV$. The data were obtained by the AFS [3] and E557[23] collaborations at ISR and Tevatron, respectively.

Thus, based on the obtained results, we conclude that the energy scaling, an independence of the function $\psi(z)$ of jet production on colliding energy, is observed both in pp and $\bar{p}p$ collisions.

3.4 Jet multiplicity densitiy

The use of the average jet multiplicity density $\rho(s,\eta)$, is important for z-scaling construction.

Figure 10 shows the average multiplicity density $\rho(s)/\rho_0$ of the jet produced in $\bar{p}p$ and pp collisions in the central rapidity range as a function of NN center-of-mass energy $\sqrt{s}$ found in our analysis. Here, ρ_0 is the density at $\sqrt{s} = 1800\ GeV$ and $\eta \simeq 0$. The results of a special investigation of the energy dependence of the cross section performed by CDF [4] show the growth of the cross section with colliding energy. Therefore, the jet multiplicity density should increase with $\sqrt{s}$, too. The values of density found from the analysis of the UA1 data [21] are higher than the other ones. The bars shown in Figure 10 at $\sqrt{s} = 200, 500$ and $900\ GeV$ correspond to 10% errors. From our point of view, the discrepancy of the UA1 data [21] and the other one is connected with the problem of absolute cross section normalization at given energy $\sqrt{s}$. The total cross section falls very rapidly as the jet transverse energy increases. Thus, a large contribution to the cross section comes from low energy jets which are very difficult to measure. In conclusion it should be emphasized that high accuracy measurements of absolute cross section normalization and the jet multiplicity density are very important to verify the energy independence of the scaling function.

3.5 Power law of $\psi(z)$ and fractality

Here, we discuss a new feature of data z-presentation of jet production. This is the power law of the scaling function, $\psi(z) \sim z^{-\alpha}$.

From Figures 1(b)-4(b) and 6(b)-9(b) one can see that all data sets demonstrate a linear dependence of $\psi(z)$ on z on the log-log scale. The results of our analysis of the coefficient α are presented in Table 1.

The parameter α is the slope parameter of z-dependence of $\psi(z)$ on the log-log scale. The values of α_{pp} and $\alpha_{\bar{p}p}$ are found to be different ones for pp and $\bar{p}p$ collisions. It is interesting to note that the value of α is constant with a high accuracy and independent of rapidity intervals and energy $\sqrt{s}$ for separate data sets. But there is difference between the values corresponding to different data sets, too.

The values of parameter α found for jet and dijet production in $\bar{p}p$ and jet production in pp collisions differ from one author considerably. It allows us to study the features of these different processes in the same approach.

The mean values of α for jet and dijet production in $\bar{p}p$ were found to be 5.26 ± 0.12 and 4.98 ± 0.16, respectively. The mean value of α found from the pp data [3, 23] is equal to 5.95 ± 0.21. Single jets are mainly produced in a softer environment than dijets. This fact may be the chief cause of different values of the slope parameters $\alpha_{\bar{p}p}^{jet}$ and $\alpha_{\bar{p}p}^{2jet}$.

Table 1. Fractality dimension α of the scaling function, $\psi(z) \sim z^{-\alpha}$, of jet production in $\bar{p}p$ and pp collisions

Process	$\sqrt{s}$, GeV	$\alpha \pm \Delta\alpha$	Ref.
$\bar{p}+p \to Jet + X$	546,630,1800	5.20 ± 0.05	[4]
$\bar{p}+p \to Jet + X$	630	5.29 ± 0.01	[7]
$\bar{p}+p \to Jet + X$	630	5.51 ± 0.02	[20]
$\bar{p}+p \to Jet + X$	1800	5.16 ± 0.07	[7]
$\bar{p}+p \to Jet + X$	1800	5.15 ± 0.03	[6]
$\bar{p}+p \to 2Jet + X$	1800	4.82 ± 0.02	[5]
$\bar{p}+p \to 2Jet + X$	1800	5.14 ± 0.06	[8]
$p+p \to Jet + X$	38.8	6.16 ± 0.08	[23]
$p+p \to Jet + X$	45,63	5.74 ± 0.04	[3]

Taking into account the accuracy of the available experimental data, we conclude that the behaviour of $\psi(z)$ for the jets produced in pp and $\bar{p}p$ collisions reveals a power dependence with a high accuracy. The value of the slope parameter $\alpha_{\bar{p}p}$ is independent of colliding energy $\sqrt{s}$ and the angle of produced jet over a wide range of high transverse momentum $q_\perp$. The conclusion is partially supported by the pp data, too.

We would like to emphasize that the power behaviour is a new feature of scaling function found for jet production *. From our point of view, this power law, $\psi(z) \sim z^{-\alpha}$, means that the jet formation reveals a fractal behaviour. Therefore, the quantity α will be named the fractal dimension of jet formation.

4 Discussion

The results obtained in this paper support in general the properties of the z-scaling for jet production in pp and $\bar{p}p$ collisions at a high energy. Based on the available experimental data, we established an energy and angular independence of $\psi(z)$ over a wide range of $\sqrt{s}$ and θ. Moreover, z-dependence of $\psi(z)$ reveals a new property, the power behaviour, $\psi(z) \sim z^{-\alpha}$. The values of the slope parameter for jet production in pp and $\bar{p}p$ collisions are found to be different ones so that $\alpha_{pp} > \alpha_{\bar{p}p}$. It should be also noted that the found value of $\alpha_{\bar{p}p}$ for jet and dijet production at $\sqrt{s} = 1800$ GeV differs from one another. It might be due to different mechanisms of jet formation in the events having various particle multiplicity.

In this section, we qualitatively discuss the obtained results in the framework of a fractal picture of constituent interaction. The idea of fractality of constituents (hadrons, nuclei etc.) and their interactions is its substantial element.

We would like to emphasize that z-scaling was observed in the production of particles with high $q_\perp$ at high energies. This means that the scaling function describes the fragmentation process of point-like produced partons into observable hadrons. The jet is usually considered as a group of collimated particles moving in one direction. The jet is a result of transformation of one or some partons into observable

* The power law of the scaling function was found for direct photon production in $\bar{p}p$ and pp collisions [14, 16].

hadrons. Thus, the process of jet formation is the process of formation of collimated hadrons considered as a jet.

The fractal character of initial states regards the parton compositeness of hadrons and nuclei and reveals itself with a larger resolution at high energies and a high transverse momentum. Led by these principles, the variable z is constructed according to Eq.(13). The fractal objects are usually characterized by a power law dependence of their fractal measures [12]. The fractal measure, considered in our case, is given by all possible configurations of elementary interactions that lead to the production of jets. Taking the following form

$$\Omega(x_1, x_2) \sim [(1-x_1)(1-x_2)]^{\delta}$$

it is described by a power law dependence in the space of fractions $\{x_1, x_2\}$. The measure reflects the number of constituent configurations in the colliding objects involved in jet production. The measure is characterized by the fractal dimension δ. Note that the fractal dimensions can be different for various colliding objects*.

In the framework of the fractal picture, the number of initial configurations is maximized according to Eq.(9), and the variable z describes the energy of an elementary constituent collision per initial configuration and per produced particle.

The existence of z-scaling itself is the confirmation of self-similarity of hadron interaction at the constituent level. The property reflects the simplest property of the corresponding equation, scale invariance. According to the principle of scale relativity, basic equations should be covariant under scale transformation. This means that the objects or phenomena described by the equations should demonstrate the properties of fractals.

Now, we would like to discuss the physical interpretation of the function ψ. As one can see from Figures 1(b)-4(b) and 6(b)-9(b), the z-dependence of ψ demonstrates a linear behaviour on the log-log scale over a wide z range. The range corresponds to a high transverse momentum range. The violation of the behaviour is observed at a low transverse momentum of jet (see Fig. 5). According to the general idea that the function ψ describes the hadronization mechanism, we interpret a different behaviour of the function as a transition from the soft to hard regime of jet formation.

The results presented in Figures 1(b)-9(b) give direct evidence of asymptotic regime of jet formation with increasing transverse momentum. The slope parameter α is independent of $\sqrt{s}$, η and $q_\perp$. It means that jet formation as a fractal process is characterized by the power law with the fractal dimension α.

We assume that the change of the fractal dimension of jet formation is a signature of new physics phenomena (quark compositeness, a new type of interaction, phase transition etc.). In the framework of the proposed scenario, the mechanism of jet formation is considered as a process of construction of a complex fractal (hadron,jet etc.) from elementary fractal blocks (interacting constituents). The size and structure of the blocks depend on the colliding energy and transverse momentum of the produced object. The value of z is proportional to the number of steps needed to

* As shown in Ref. [13] the fractal dimension of nucleus δ_A is related to the nucleon fractal dimension δ_N for different types of produced hadrons $(\pi^\pm, K^\pm, \bar{p})$ by the following simple form $\delta_A = A \cdot \delta_N$.

construct a complex object (fractal) from the elementary blocks. It can be considered as a relative formation length. We assume that inhomogeneous domains (for example, DCC [25]), a new type of interaction and quark compositeness might be the reason of z-scaling violation. The change of fractal dimension α of the jet formation process is a quantitative characteristic of violation.

We use the properties of the scaling function $\psi(z)$ for pp and $\bar{p}p$ collisions to predict jet spectra at the RHIC, Tevatron and LHC energies.

Figure 11 demonstrates the $Ed^3\sigma/dq^3$ dependence on the transverse momentum of the produced jet $q_\perp$ in $\bar{p}p$ and pp collisions in the central rapidity range at different colliding energy $\sqrt{s}$. The average jet multiplicity densities used during the calculation are presented in Table 2.

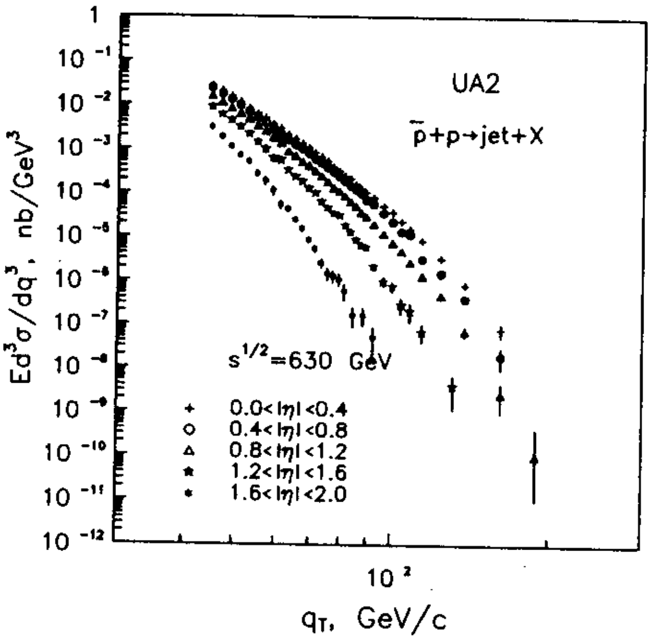
Table 2. The average multiplicity densities $\rho(s)/\rho_0$ of jet production in pp collisions

$\sqrt{s}$, GeV	63	$2. \cdot 10^2$	$5 \cdot 10^2$	$1.8 \cdot 10^3$	$7 \cdot 10^3$	$14 \cdot 10^3$
$\rho(s)/\rho_0$	0.35	0.5	0.67	1.0	1.5	1.85

5 Conclusions

The inclusive jet production in $\bar{p}p$ and pp collisions at high energies was considered. The function $\psi(z)$ describing the new scaling, z-scaling, was constructed and used to study the energy and angular dependence of data z-presentation. A general concept of z-scaling based on fundamental principles: self-similarity, locality, scale-relativity and fractality, was developed for jet production in hadron-hadron collisions. The experimental data on the inclusive cross sections for jet and dijet production over a high transverse momentum range, colliding energy and rapidity range obtained at ISR, SpS and Tevatron were used for the analysis. The function $\psi(z)$ was expressed via the invariant inclusive cross section $Ed^3\sigma/dq^3$ and normalized to the multiplicity density of the jet produced in $\bar{p}p$ collisions. The energy and angular independence of the scaling function $\psi(z)$ was found. The function $\psi(z)$ was interpreted as the quantity being proportional to the probability to form the jet with formation length z. The scaling function $\psi(z)$ of jet and dijet production demonstrates the power behaviour, $\psi(z) \sim z^{-\alpha}$ at high z. The mean values of the slope parameter (the fractal dimension of jet formation), α, for jet and dijet production in $\bar{p}p$ were found to be 5.26 ± 0.12 and 4.98 ± 0.11, respectively. The mean value of α found from the pp data was equal to 5.95 ± 0.21. The value of slope parameter $\alpha_{\bar{p}p}$ is independent of colliding energy $\sqrt{s}$ and the angle of the produced jet over a wide range of transverse momentum $q_\perp$. The obtained results confirm general properties of the jet production mechanism such as self-similarity, locality, scale-relativity and fractality revealing themselves in $\bar{p}p$ and pp collisions.

The results give us some arguments to assume that the z-scaling of jet production in pp and $\bar{p}p$ collisions should be observed at an energy beyond ISR and Tevatron ones. Using the properties of z-scaling, the dependence of the cross section of the



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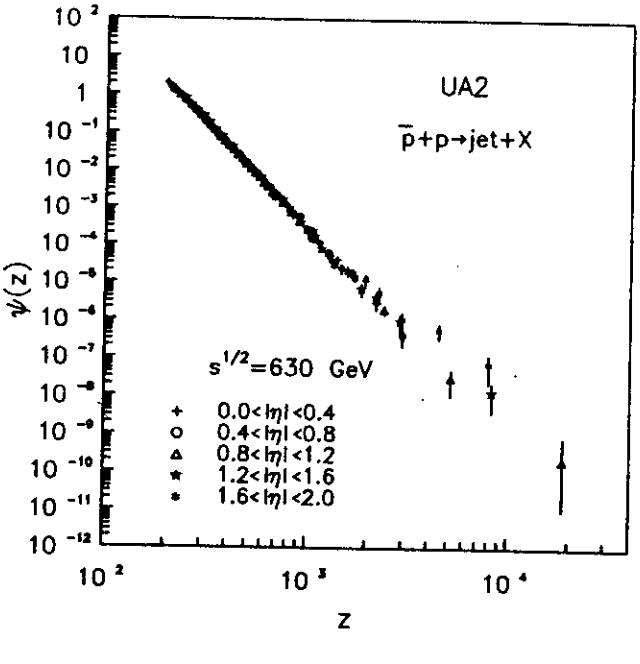

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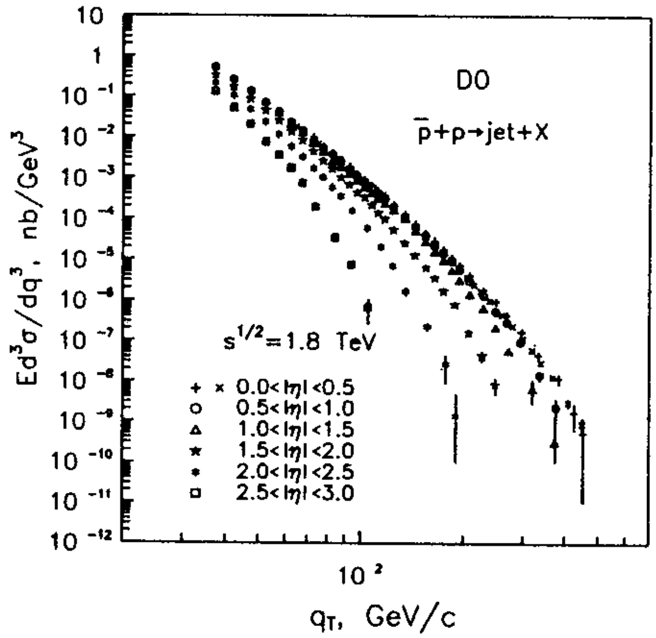
Figure 1. (a) The dependence of the inclusive cross section of jet production in $\bar{p}p$ collisions at $\sqrt{s} = 630\ GeV/c$ and different rapidity intervals on transverse momentum $q_\perp$. The experimental data on the cross section obtained by the UA2 collaboration are taken from Ref. [20]. (b) The corresponding scaling function $\psi(z)$.



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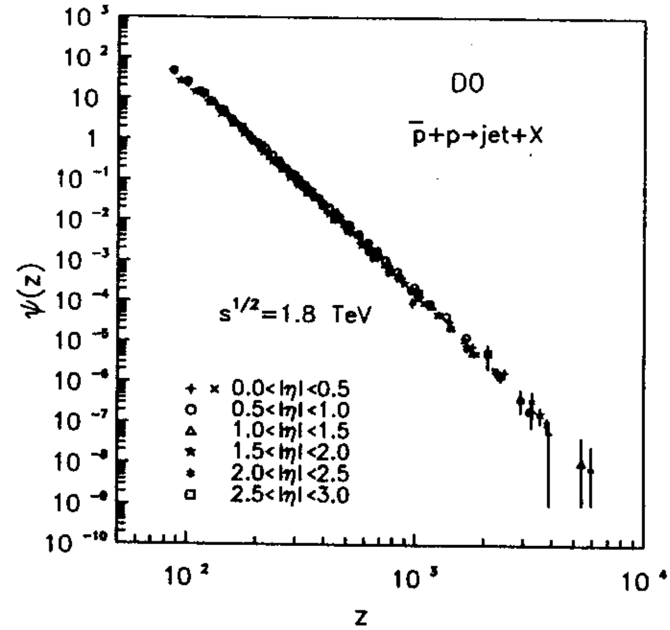

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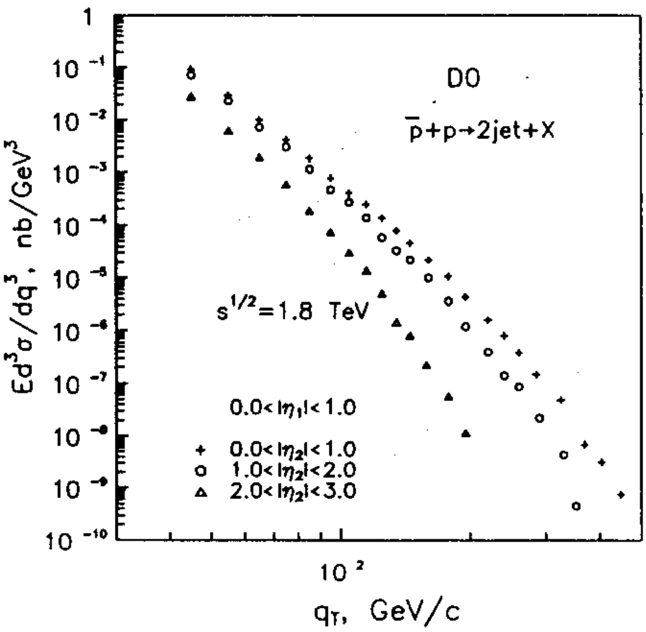
Figure 2. (a) The dependence of the inclusive cross section of jet production in $\bar{p}p$ collisions at $\sqrt{s} = 1800$ GeV/c and different rapidity intervals on transverse momentum $q_\perp$. The experimental data on the cross section obtained by the D0 collaboration are taken from Ref. [6]. (b) The corresponding scaling function $\psi(z)$.



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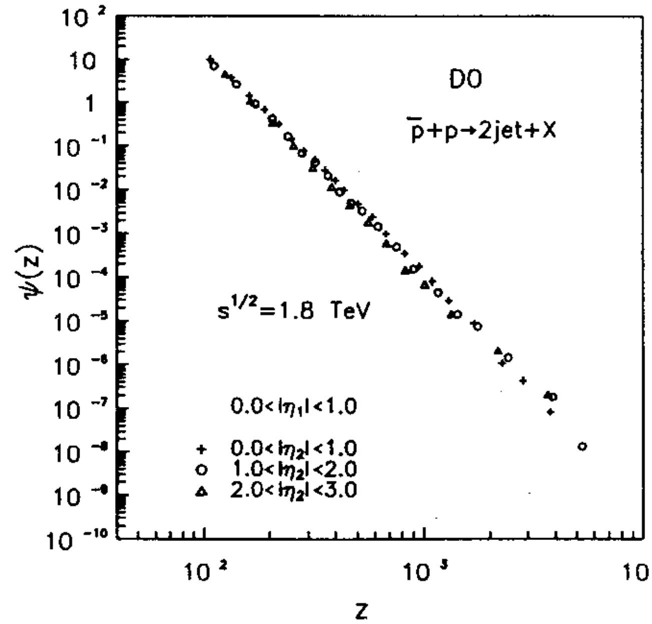

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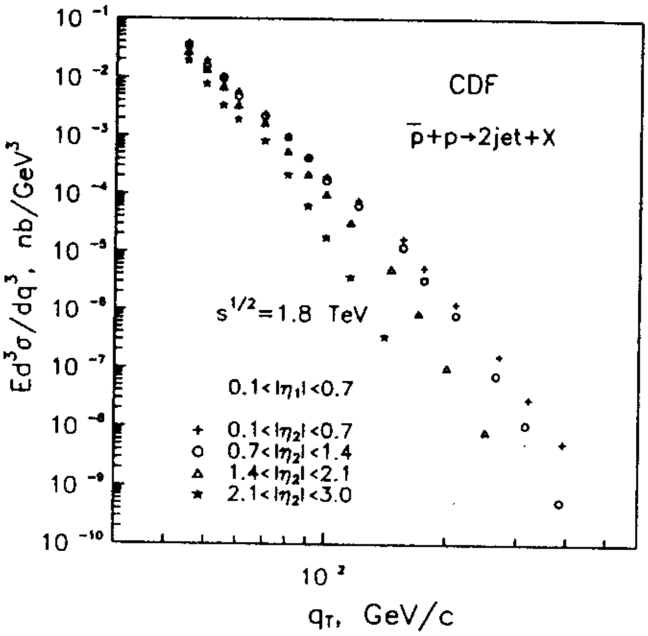
Figure 3. (a) The dependence of the inclusive cross section of dijet production in $\bar{p}p$ collisions at $\sqrt{s} = 1800$ GeV/c and different rapidity intervals on transverse momentum $q_\perp$. The experimental data on the cross section obtained by the D0 collaboration are taken from Ref. [8]. (b) The corresponding scaling function $\psi(z)$.



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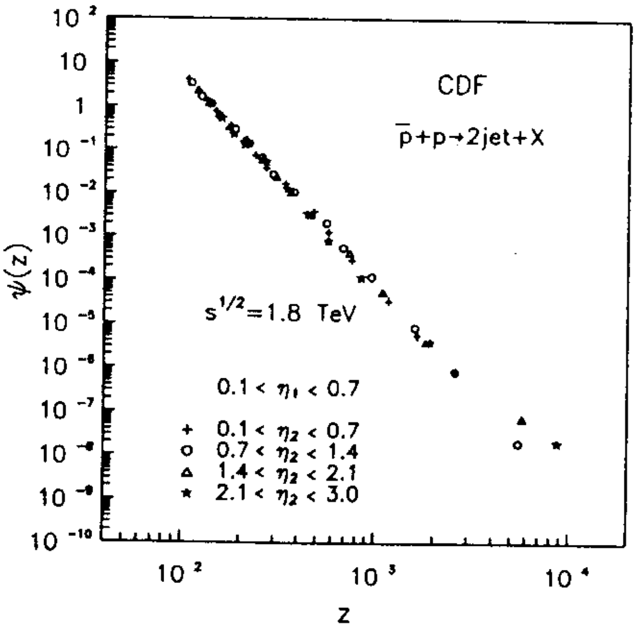

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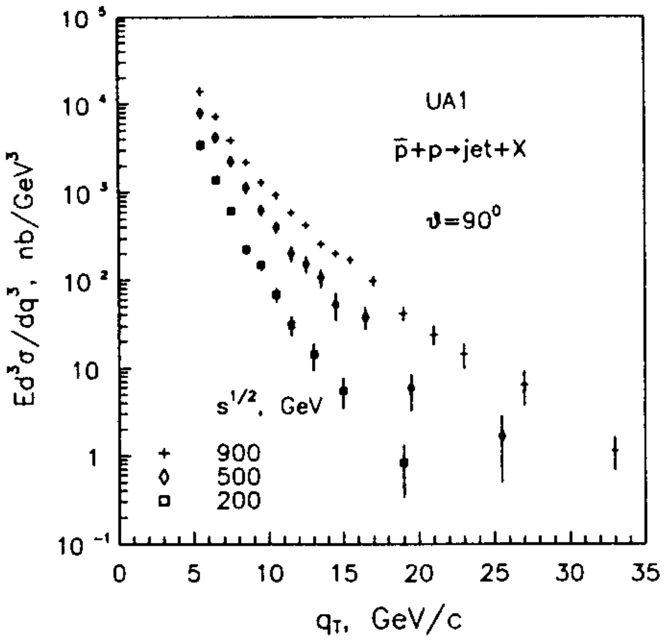
Figure 4. (a) The dependence of the inclusive cross section of dijet production in $\bar{p}p$ collisions at $\sqrt{s} = 1800$ GeV/c and different rapidity intervals on transverse momentum $q_\perp$. The experimental data on the cross section obtained by the CDF collaboration are taken from Ref. [5]. (b) The corresponding scaling function $\psi(z)$.



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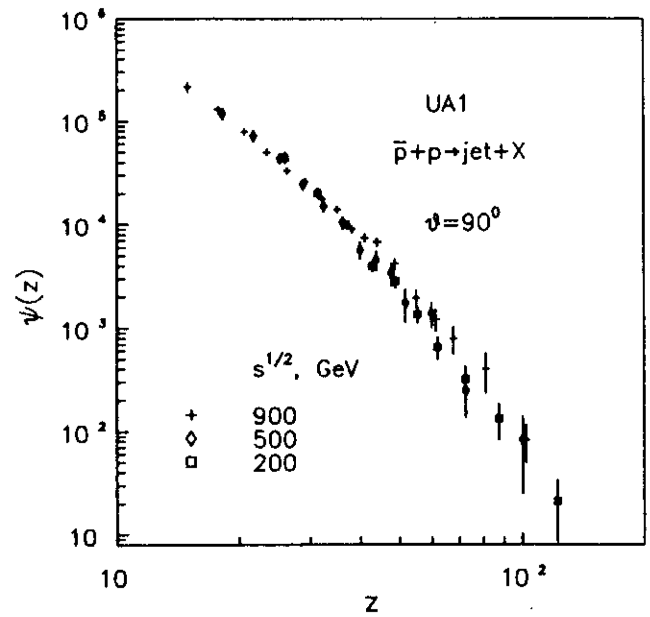

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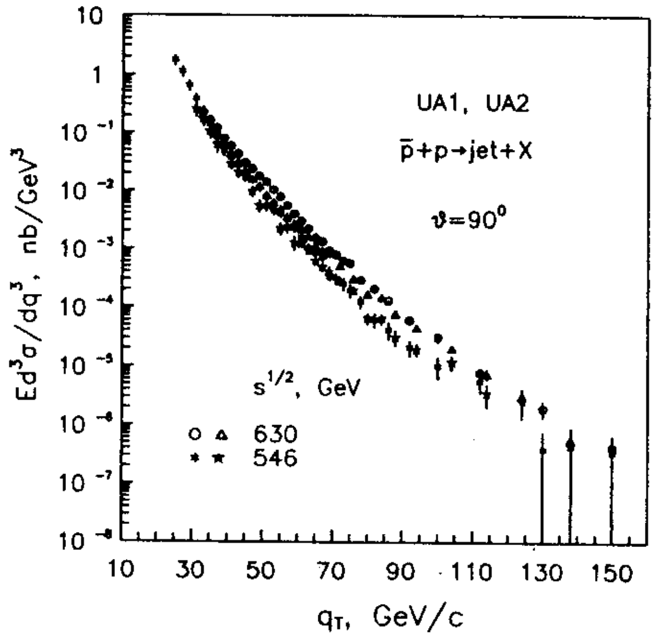
Figure 5. (a) The dependence of the inclusive cross section of jet production in $\bar{p}p$ collisions at $\sqrt{s} = 200, 500$ and 900 GeV/c and over a central rapidity range on transverse momentum $q_\perp$. The experimental data on the cross section obtained by the UA1 collaboration are taken from Ref. [21]. (b) The corresponding scaling function $\psi(z)$.



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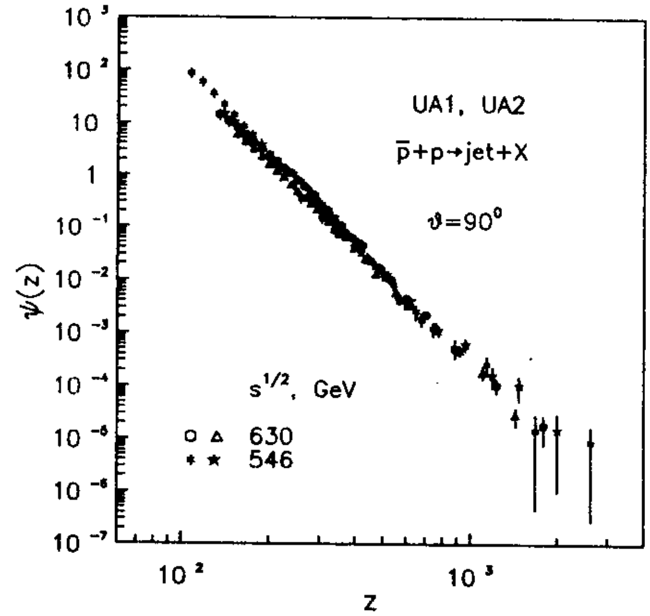

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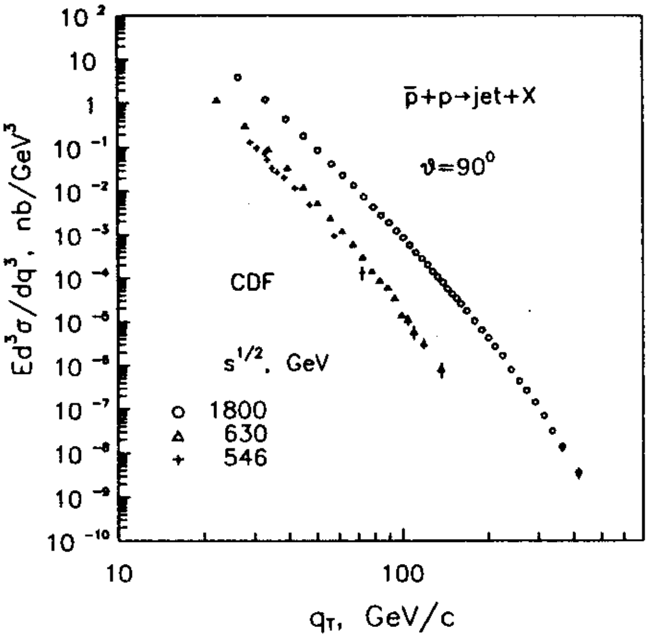
Figure 6. (a) The dependence of the inclusive cross section of jet production in $\bar{p}p$ collisions at $\sqrt{s} = 546$ and 630 GeV/c and central rapidity range on transverse momentum $q_\perp$. The experimental data on the cross section obtained by the UA1 and UA2 collaborations are taken from Refs. [21, 22]. (b) The corresponding scaling function $\psi(z)$.



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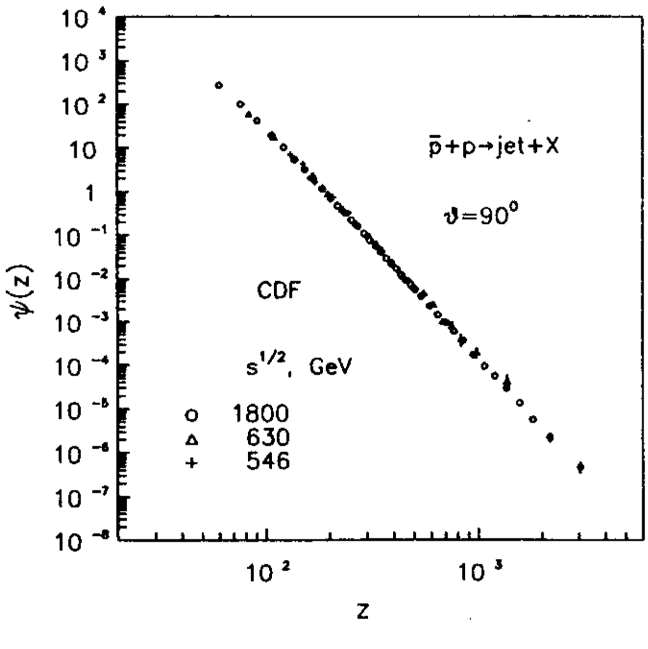

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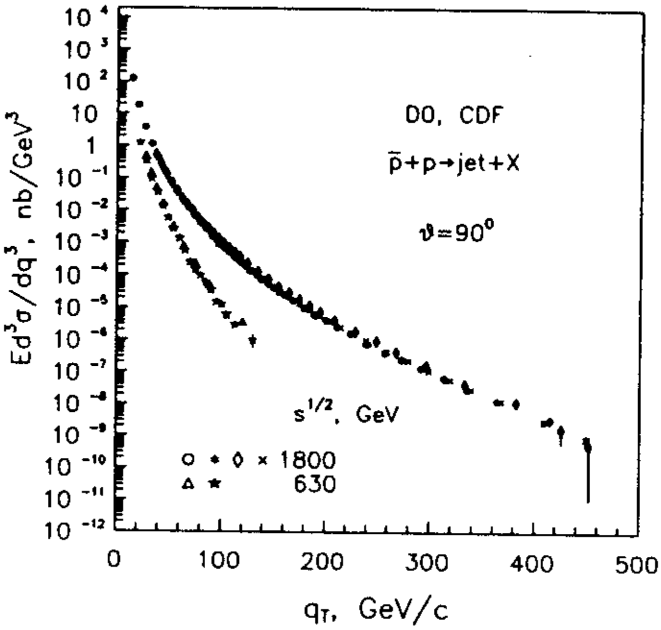
Figure 7. (a) The dependence of the inclusive cross section of jet production in $\bar{p}p$ collisions at $\sqrt{s} = 546, 630$ and $1800\ GeV/c$ and central rapidity range on transverse momentum $q_\perp$. The experimental data on the cross section obtained by the CDF collaboration are taken from Ref. [4]. (b) The corresponding scaling function $\psi(z)$.



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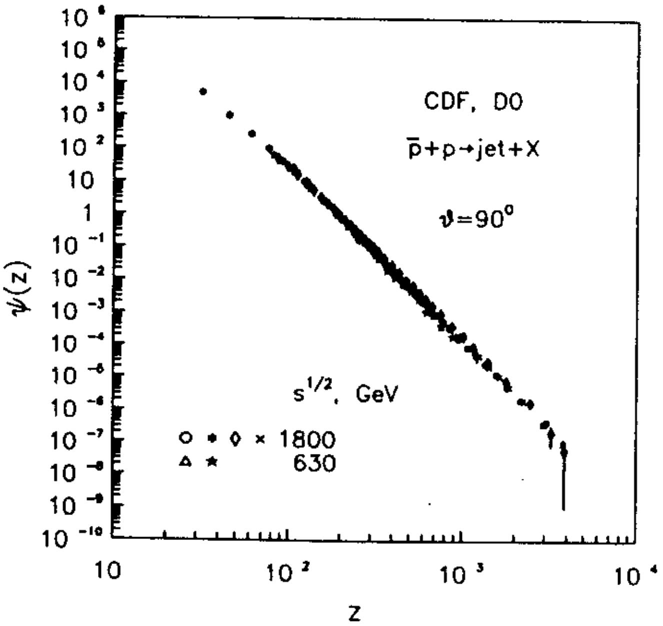

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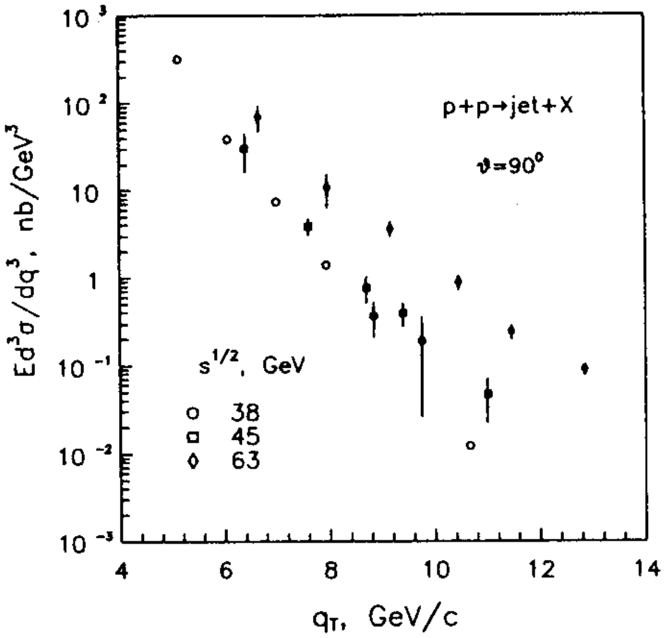
Figure 8. (a) The dependence of the inclusive cross section of jet production in $\bar{p}p$ collisions at $\sqrt{s} = 630$ and 1800 GeV/c and central rapidity range on transverse momentum $q_\perp$. The experimental data on the cross section obtained by the CDF and D0 collaborations are taken from Refs. [4, 7]. (b) The corresponding scaling function $\psi(z)$.



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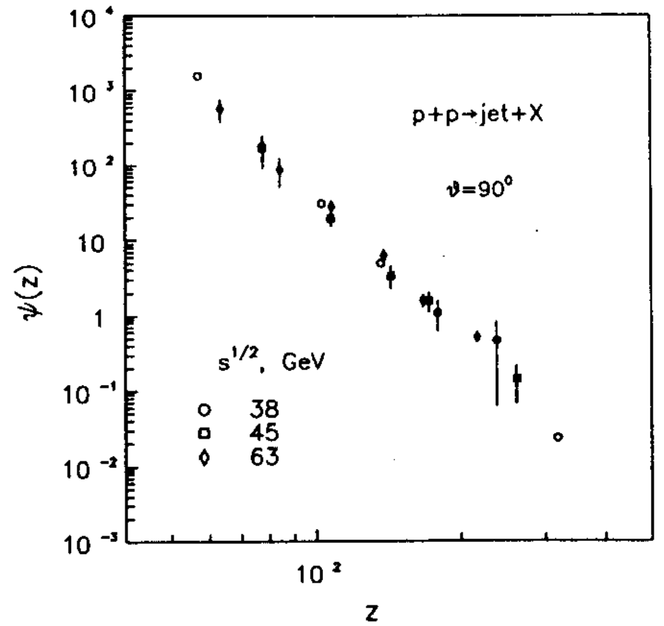

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Figure 9. (a) The dependence of the inclusive cross section of jet production in pp collisions at $\sqrt{s} = 38.8, 45$ and 63 GeV/c and central rapidity range on transverse momentum $q_{\perp}$. The experimental data on the cross section obtained by the AFS and E557 collaborations are taken from Refs. [3, 23]. (b) The corresponding scaling function $\psi(z)$.

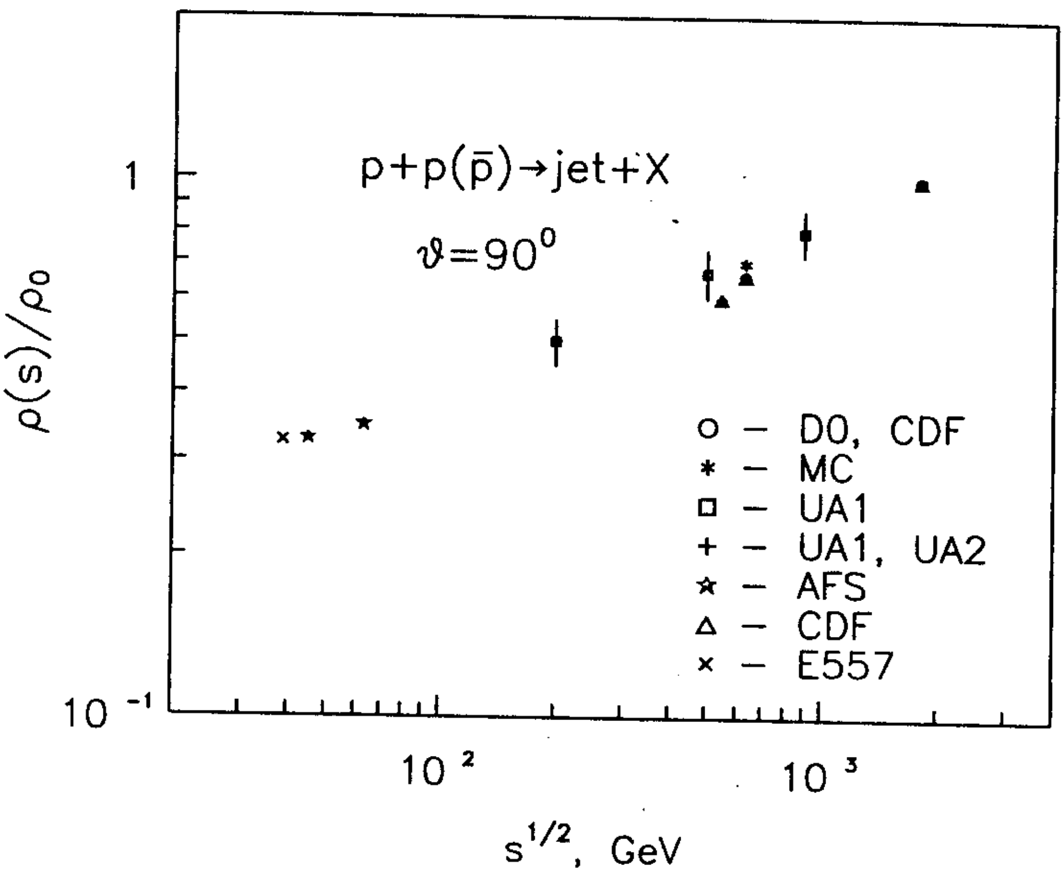
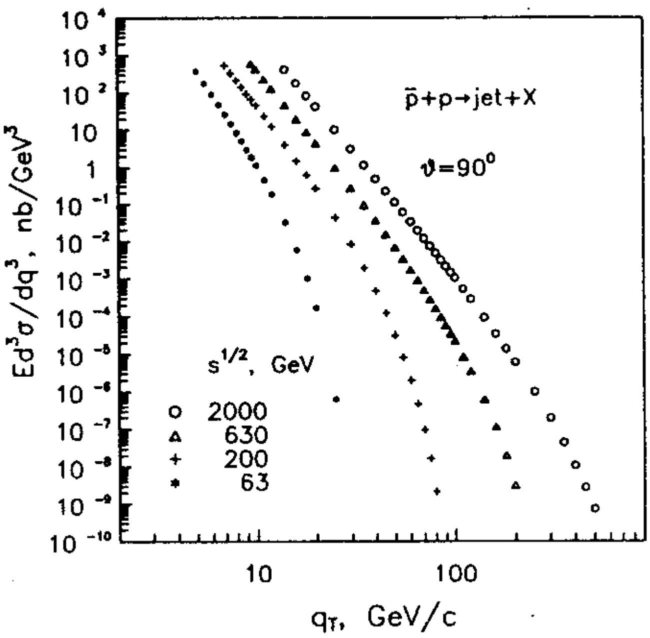


Figure 10. The average multiplicity density $\rho(s)/\rho_0$ of jets produced in $\bar{p}p$ and pp collisions over the central rapidity range as a function of colliding energy $\sqrt{s}$. Points are the results of our analysis on experimental data of the cross section taken from [3, 4, 7, 21, 22, 23, 24].



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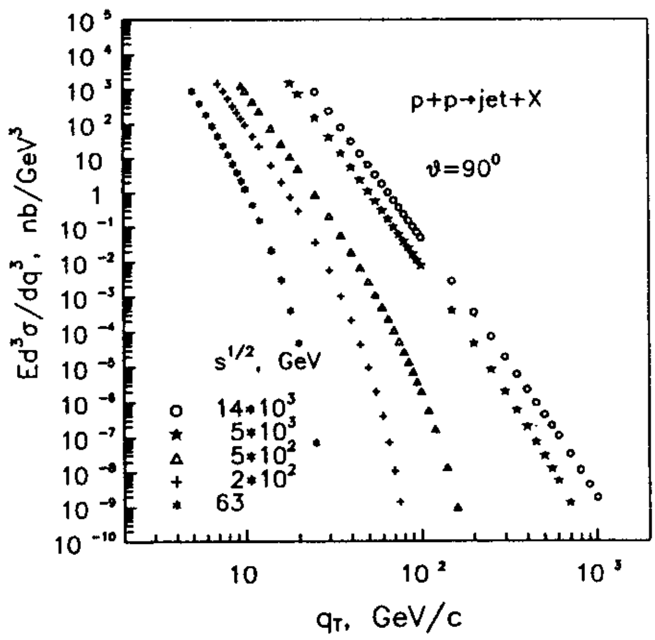

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Figure 11. The inclusive cross section of jet production in $\bar{p}p$ (a) and pp (b) collisions as a function of transverse momentum $q_\perp$ for different energies $\sqrt{s}$ at an angle of $\theta = 90^0$.

jet produced in pp collisions on transverse momentum over the central range at RHIC, Tevatron and LHC energies was predicted. The search for z-scaling violations of jet production in hadron-hadron, hadron-nucleus and nucleus-nucleus collisions, especially in the region of high transverse momenta, could be very interesting for our understanding such fundamental problems as hadronization and phase transition. The verification of the predictions and the study of z-scaling in pp, pA and AA collisions in future experiments were suggested to perform at RHIC (BNL), Tevatron (Fermilab), HERA (DESY) and LHC (CERN).

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Токарев М.В., Дедович Т.Г. E2-99-300
Z-скейлинг и рождение струй в адрон-адронных взаимодействиях при высоких энергиях

Изучается рождение струй в $\bar{p}p$- и pp-взаимодействиях в рамках концепции z-скейлинга. Для построения скейлинговой функции $\psi(z)$ используются экспериментальные данные по сечениям рождения струй, полученные коллаборациями UA1, UA2, CDF и D0. Скейлинговая функция ψ выражается через экспериментально измеряемые величины, инвариантное сечение $Ed^3\sigma / dq^3$ и плотность распределения струй $\rho(s,\eta)$. Установлены свойства z-представления, энергетическая и угловая независимости скейлинговой функции $\psi(z)$ и степенное поведение $\psi(z) \sim z^{-\alpha}$. Свойства z-скейлинга используются для предсказательных расчетов сечения рождения струй в $\bar{p}p$- и pp-взаимодействиях при энергиях RHIC и LHC. Полученные результаты представляют интерес для поиска новых явлений в адрон-адронных, в адрон-ядерных и ядро-ядерных взаимодействиях в экспериментах, планируемых на установках RHIC, LHC, HERA и Tevatron.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 1999

Tokarev M.V., Dedovich T.G. E2-99-300
Z-Scaling and Jet Production in Hadron-Hadron Collisions at High Energies

In the framework of the concept of z-scaling inclusive jet production in $\bar{p}p$- and pp-collisions at high energies is studied. The available experimental data on the cross section of jet production obtained by the UA1, UA2, CDF and D0 Collaborations are used for analysis. The scaling function $\psi(z)$ is expressed via inclusive cross section $Ed^3\sigma / dq^3$ and jet multiplicity density $\rho(s,\eta)$. The properties of z-scaling, the energy and angular independence of $\psi(z)$ and the power behaviour, $\psi(z) \sim z^{-\alpha}$, of jet and dijet production were found. Based on the properties of z-scaling, the dependence of the cross section of jets produced in $\bar{p}p$- and pp-collisions on transverse momentum $q_{\perp}$ over the central range is predicted. The obtained results can be of interest for future experiments planned at RHIC, LHC, HERA and Tevatron to search for new phenomena in hadron-hadron, hadron-nucleus and nucleus-nucleus collisions.

The investigation has been performed at the Laboratory of High Energies, JINR.

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