QCD SUM RULES FOR HEAVY FLAVORS [a]

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We give a short review of QCD sum rule results for B and D mesons and Λ_Q and Σ_Q baryons. We focus mainly on recent developments concerning semileptonic $B \to \pi$ and $D \to \pi$ transitions, pion couplings to heavy hadrons, decay constants and estimates of the b quark mass from a baryonic sum rule, and the extraction of the pion distribution amplitude from CLEO data.

1 Introduction

The accurate study of B meson decays is a main source of information for understanding CP violation and the physics of heavy quarks. In particular, experiments at B factories will allow measurements of B decay properties with good precision [1]. On the theoretical side, the method of QCD sum rules [2] remains one of the main tools in applying Quantum Chromodynamics to hadron physics. Since its birth in 1979, the sum rule method has become more and more advanced not only technically, but also conceptually. In this talk, we give a short review of QCD sum rule results for B and D mesons and Λ_Q and Σ_Q baryons. We focus mainly on recent developments concerning semileptonic $B \to \pi$ and $D \to \pi$ transitions [3], including a new approach [4], which will be discussed in detail, and new results on the f^0 form factor [5]. Furthermore we will address pion couplings to B and D mesons and to Λ_Q and Σ_Q baryons, meson decay constants and corresponding matrix element for baryons, an estimate of the b quark mass from a baryonic sum rule, and finally a recent extraction of the pion distribution amplitude from CLEO data [6].

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2 Pion distribution amplitude from CLEO data

We start with the pion distribution amplitudes, which serve as input in the QCD sum rule method and allow the calculation of heavy-to-light form factors (e.g., $f_{B\to\pi}^+$ and $f_{D\to\pi}^+$) and hadronic coupling constants (e.g., $g_{B^*B\pi}$ and $g_{D^*D\pi}$). Recently, the CLEO collaboration has measured the $\gamma\gamma^* \to \pi^0$ form factor. In this experiment [8] [b], one of the photons is nearly on-shell and the other one is highly off-shell, with a virtuality in the range $1.5\,\text{GeV}^2 - 9.2\,\text{GeV}^2$. The possibility of extracting the twist-2 pion distribution amplitude from the CLEO data has been studied in the papers [6,7]. There, the light-cone sum rule (LCSR) method has been used to calculate the relevant form factor and to compare the calculation with the measurement of $\gamma\gamma^* \to \pi^0$.

In order to sketch the basic idea we begin with the correlator of two vector currents $j_\mu = (\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d)$:

$$\int d^4x\, e^{-iq_1 x}\langle\pi^0(0)|T\{j_\mu(x)j_\nu(0)\}|0\rangle = i\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta F^{\pi\gamma^*\gamma^*}(s_1, s_2), \qquad (1)$$

where q_1, q_2 are the momenta of the photons, and $s_1 = q_1^2$, $s_2 = q_2^2$ are the virtualities. In the CLEO data, one of the virtualities is small, i.e. $s_2 \to 0$. Since a straightforward OPE calculation is impossible, we have to use analyticity and duality arguments. One can write the form factor as a dispersion relation in s_2:

$$F^{\pi\gamma^*\gamma^*}(s_1, s_2) = \frac{\sqrt{2}\, f_\rho\, F^{\rho\pi}(s_1)}{m_\rho^2 - s_2} + \int_{s_0}^\infty ds\, \frac{\rho^h(s_1, s)}{s - s_2}. \qquad (2)$$

For the physical ground states ρ and ω we take $m_\rho \simeq m_\omega$; $\frac{1}{3}\langle\pi^0(p)|j_\mu|\omega(q_2)\rangle \simeq \langle\pi^0(p)|j_\mu|\rho^0(q_2)\rangle = \frac{1}{m_\rho}\,\epsilon_{\mu\nu\alpha\beta}\,e^\nu\, q_1^\alpha\, q_2^\beta\, F^{\rho\pi}(s_1)$; $3\langle\omega|j_\nu|0\rangle \simeq \langle\rho^0|j_\nu|0\rangle = \frac{f_\rho}{\sqrt{2}}\, m_\rho\, e_\nu^*$, e_ν being the polarization vector of the ρ meson and f_ρ being the decay constant. The spectral density of the higher energy states $\rho^h(s_1, s)$ is derived from the expression for $F_{QCD}^{\pi\gamma^*\gamma^*}(s_1, s)$ calculated in QCD, assuming semi-local quark-hadron duality for $s > s_0$. Equating the dispersion relation (2) with the QCD expression at large s_2, and performing a Borel transformation in s_2, one gets the LCSR:

$$\sqrt{2}\, f_\rho\, F^{\rho\pi}(s_1) = \frac{1}{\pi}\int_0^{s_0} ds\, \text{Im}\, F_{QCD}^{\pi\gamma^*\gamma^*}(s_1, s)\, e^{\frac{m_\rho^2 - s}{M^2}}, \qquad (3)$$

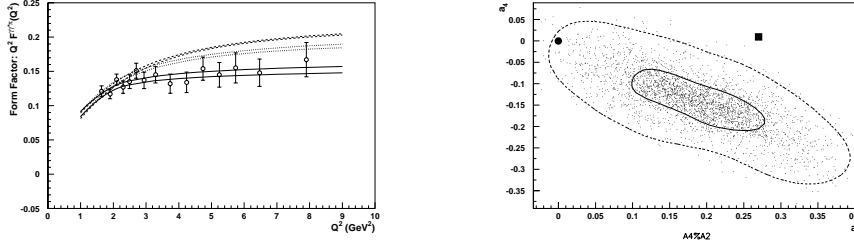[b]There also exist older results from the CELLO collaboration [9].

Figure 1: **Left:** The form factor $Q^2 F^{\gamma\gamma^*\pi}(Q^2)$ with different distribution amplitudes: Braun-Filyanov (dashed lines), Chernyak-Zhitnitsky (dotted lines) and φ_π extracted from CLEO data. **Right:** Ranges of the coefficients a_2 and a_4 suggested by CLEO data in comparison to the Chernyak-Zhitnitsky model (square) and the asymptotic distribution amplitude (circle).

where M is the Borel parameter. Substituting (3) into (2) and taking $s_2 \to 0$ one finally obtains

$$F^{\pi\gamma\gamma^*}(s_1) = \frac{1}{\pi\, m_\rho^2} \int_0^{s_0} ds\, \mathrm{Im}\, F_{QCD}^{\pi\gamma^*\gamma^*}(s_1, s)\, \mathrm{e}^{\frac{m_\rho^2 - s}{M^2}} + \frac{1}{\pi} \int_{s_0}^{\infty} \frac{ds}{s}\, \mathrm{Im}\, F_{QCD}^{\pi\gamma^*\gamma^*}(s_1, s).$$

(4)

This expression is the basic sum rule used for the numerical analysis. The calculation of the spectral density of the twist-2 operator including the $\mathcal{O}(\alpha_S)$ radiative correction gives[6]

$$\frac{1}{\pi}\, \mathrm{Im}_{s_2} F^{\pi\gamma^*\gamma^*}(s_1, s_2) = \frac{2\sqrt{2} f_\pi s_2 s_1}{(s_2 - s_1)^3} \Big(1 + \frac{\alpha_s(\mu) C_F}{12\pi} \cdot$$

(5)

$$\cdot \Big(-15 + \pi^2 - 3\log^2(-\frac{s_2}{s_1}) \Big) + a_2(\mu)\, A_2(s_2, s_1) + a_4(\mu)\, A_4(s_2, s_1) \Big).$$

As usual, the distribution amplitude of twist 2 is expanded in Gegenbauer polynomials, keeping only the first three terms: $\varphi_\pi = 6\, u(1-u) \left(1 + a_2 C_2^{3/2} + a_4 C_4^{3/2} \right)$. The coefficients $A_{2,4}$ in (5) are too complicated to be given here. They can be found in[6].

We then combined the twist-2 contribution at NLO with the higher twist contributions up to twist 4, calculated in[7], and analyzed the LCSR for the form factor of the process $\gamma\gamma^* \to \pi^0$ numerically. Details of the analysis are given in[6]. The coefficients a_2 and a_4 of the twist-2 distribution amplitude can be determined by comparing the sum rule (4) with CLEO data[8]. We find that

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the deviation of the pion distribution amplitude from the asymptotic form is small. More definitely, putting $a_4 = 0$, we get[6]

$$a_2(\mu) = 0.12 \pm 0.03 \quad \text{at} \quad \mu = 2.4 \quad \text{GeV}. \tag{6}$$

This result agrees well with a recent analysis of the electromagnetic pion form factor [13]. Fig. 1 shows the form factor $Q^2 F^{\gamma\gamma^*\pi}(Q^2)$ calculated with different distribution amplitudes: Braun-Filyanov[11] (dashed lines), Chernyak-Zhitnitsky[12] (dotted lines) and (6). In principle, one can also extract the coefficient a_4. Unfortunately, the present data is not good enough to fix the values of a_2 and a_4 simultaneously. The ranges of a_2 and a_4 favored by CLEO data are shown in Fig. 1. Obviously, these are in qualitative agreement with (6) and also with the results derived in [14,15,16,13], where it has also been claimed that the pion distribution amplitude is very close to the asymptotic form.

3 Coupling constants $g_{B^*B\pi}$ and $g_{D^*D\pi}$

The hadronic $B^*B\pi$ coupling is defined by the on-shell matrix element

$$\langle \bar{B}^{*0}(p)\, \pi^-(q) \mid B^-(p+q) \rangle = -g_{B^*B\pi}(q \cdot \epsilon)\;, \tag{7}$$

where the meson four-momenta are given in brackets and ϵ_μ is the polarization vector of the B^*. An analogous definition holds for the $D^*D\pi$ coupling. These couplings play an important role in B and D physics. For example, they determine the magnitude of the weak $B \to \pi$ and $D \to \pi$ form factors at zero pion recoil. Moreover, the coupling constant $g_{D^*D\pi}$ is directly related to the decay width of $D^* \to D\pi$. The decay $B^* \to B\pi$ is kinematically forbidden. Theoretically, the $B^*B\pi$ and $D^*D\pi$ couplings have been studied using a variety of methods[c]. Among these, QCD light-cone sum rules (LCSR) have proved particularly powerful. The LCSR calculations of $g_{B^*B\pi}$ and $g_{D^*D\pi}$ including perturbative QCD effects in LO and NLO were reported in [17,18]. The final LCSR reads

$$f_B f_{B^*} g_{B^*B\pi} = \frac{m_b^2 f_\pi}{m_B^2 m_{B^*}} e^{\frac{m_B^2 + m_{B^*}^2}{2M^2}} \left[M^2 \left(e^{-\frac{m_b^2}{M^2}} - e^{-\frac{s_0^B}{M^2}} \right) \varphi_\pi(1/2, \mu) \right.$$

$$\left. + \frac{\alpha_s C_F}{4\pi} \int\limits_{2m_b^2}^{2s_0^B} f\left(\frac{s}{m_b^2} - 2 \right) e^{-\frac{s}{2M^2}} ds + F^{(3,4)}(M^2, m_b^2, s_0^B, \mu) \right] \tag{8}$$

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[c]An overview is given, e.g., in Tab. 1 of ref. [17].

LCSR	$f_B f_{B^*} g_{B^* B\pi}$	0.80 ± 0.23 GeV^2
2SR	f_B	180 ± 30 MeV
$\frac{\text{LCSR}}{\text{2SR}}$	$f_{B^*} g_{B^* B\pi}$	4.44 ± 0.97 GeV
2SR	f_{B^*}	195 ± 35 MeV
$\frac{\text{LCSR}}{\text{2SR}}$	$g_{B^* B\pi}$	22 ± 7

LCSR	$f_D f_{D^*} g_{D^* D\pi}$	0.54 ± 0.15 GeV^2
2SR	f_D	190 ± 20 MeV
$\frac{\text{LCSR}}{\text{2SR}}$	$f_{D^*} g_{D^* D\pi}$	2.84 ± 0.55 GeV
2SR	f_{D^*}	270 ± 35 MeV
$\frac{\text{LCSR}}{\text{2SR}}$	$g_{D^* D\pi}$	10.5 ± 3

Figure 2: **Left table:** Sum rule predictions for B and B^* mesons. **Right table:** Sum rule predictions for D and D^* mesons.

with

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\begin{aligned}
f(x) =\ & \frac{\pi^2}{4} + 3\ln\left(\frac{x}{2}\right)\ln\left(1+\frac{x}{2}\right) - \frac{3(3x^3 + 22x^2 + 40x + 24)}{2(2+x)^3}\ln\left(\frac{x}{2}\right) \\
& + 6\mathrm{Li}_2\left(-\frac{x}{2}\right) - 3\mathrm{Li}_2(-x) - 3\mathrm{Li}_2(-x-1) - 3\ln(1+x)\ln(2+x) \\
& - \frac{3(3x^2 + 20x + 20)}{4(2+x)^2} + \frac{6x(1+x)\ln(1+x)}{(2+x)^3} \ .
\end{aligned}
\tag{9}
$$

In (8), we have added the contributions $F^{(3,4)}$ from the pion distribution amplitudes of twist 3 and 4, which can be found in [17]. For the b-quark mass and the corresponding continuum threshold we use $m_b = 4.7 \pm 0.1$ GeV and $s_0^B = 35 \mp 2$ GeV^2 respectively. The running coupling constant is taken in the two-loop approximation with $N_f = 4$ and $\Lambda_{\overline{MS}}^{(4)} = 315$ MeV corresponding to $\alpha_s(m_Z) = 0.118$ [20]. In the charm case, the corresponding parameters are given by $m_c = 1.3 \pm 0.1$ GeV, $s_0^D = 6$ GeV^2, and $\Lambda_{\overline{MS}}^{(3)} = 380$ MeV [d]. Finally, for the pion distribution amplitude $\varphi_\pi(u,\mu)$ at $u = 0.5$ and $\mu = 2.4$ GeV we have $\varphi_\pi(1/2,\mu) = 1.23$ [6]. The decay constants and resulting coupling constants are summarized in the two tables in Fig. 2. We will make use of them in the next section. Here we just mention that from $g_{D^* D\pi}$ given in Table 2 one obtains $\Gamma(D^{*+} \to D^0 \pi^+) = 23 \pm 13$ keV. The current experimental limit

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[d]The meson masses are (in GeV) $m_B = 5.279, m_{B^*} = 5.325, m_D = 1.87, m_{D^*} = 2.01$, and $f_\pi = 132$ MeV.

[20] $\Gamma(D^{*+} \to D^0\pi^+) < 89$ keV is still too high to challenge the theoretical prediction.

4   The scalar form factor f^0

In general, the hadronic matrix element of the $B \to \pi$ transition is determined by two independent form factors, f^+ and f^-:

$$\langle \pi(q)|\bar{u}\gamma_\mu b|B(p+q)\rangle = 2f^+(p^2)q_\mu + \left(f^+(p^2) + f^-(p^2)\right)p_\mu, \qquad (10)$$

where $(p+q)$ and q denote the initial and final state four-momenta and $\bar{u}\gamma_\mu b$ is the relevant weak current. The form factor f^0 is usually defined through the matrix element

$$p^\mu \langle \pi(q)|\bar{u}\gamma_\mu b|B(p+q)\rangle = f^0(p^2)(m_B^2 - m_\pi^2), \qquad (11)$$

yielding together with (10) $f^0 = f^+ + \frac{p^2}{m_B^2 - m_\pi^2}f^-$. In order to determine f^0 from sum rules it is advantageous to consider f^+ and $f^+ + f^-$. The sum rule for f^+ has been analysed in [3,19]. The sum rule for the sum of form factors is given by

$$f^+ + f^- = -\frac{m_b f_\pi}{\pi\, m_B^2\, f_B} \int\limits_{m_b^2}^{s_0} ds \int\limits_0^1 du\, \exp\left(-\frac{s - m_B^2}{M^2}\right)\varphi_\pi(u)\,\mathrm{Im}\,\tilde{T}_{QCD}, \quad (12)$$

where M again denotes the Borel mass. The expression of the hard amplitude $\tilde{T}_{QCD}(p^2, s, u, \mu)$ in LO and NLO can be found in [28] and in [5], respectively. The leading twist-2 contribution to the imaginary part of the hard amplitude is given by [5]

$$\frac{1}{\pi}\mathrm{Im}\tilde{T}_{QCD}(s_1, s_2, u, \mu) = \left(\frac{C_F \alpha_s(\mu)}{2\pi}\right)\Theta(s_2 - m_b^2)\frac{m_b}{s_2 - s_1}$$

$$\left\{\Theta(u - u_0)\left[-\frac{(1-u)(u-u_0)(s_2-s_1)^2}{2u\rho^2} - \frac{1}{u(1-u)}\left(\frac{m^2}{\rho} - 1\right)\right]\right. \qquad (13)$$

$$\left. + \delta(u - u_0)\frac{1}{2u}\left[\frac{(s_1 - m_b^2)^2}{s_1^2}\ln\left(1 - \frac{s_1}{m_b^2}\right) + \frac{m_b^2}{s_1} - 1\right] - \frac{1}{1-u}\left(1 - \frac{m_b^2}{s_2}\right)\right\}$$

with $u_0 = \frac{m_b^2 - s_1}{s_2 - s_1}$. In Fig. 3, the form factor $f^0_{B\pi}$ is plotted together with the UKQCD lattice results [21]. We see that the radiative contributions improve the agreement between the lattice and the LCSR calculations. Also shown in Fig. 3 are the LCSR results for the form factor $f^0_{D\pi}$. We note that $f^0_{D\pi}(0) = 0.66$.
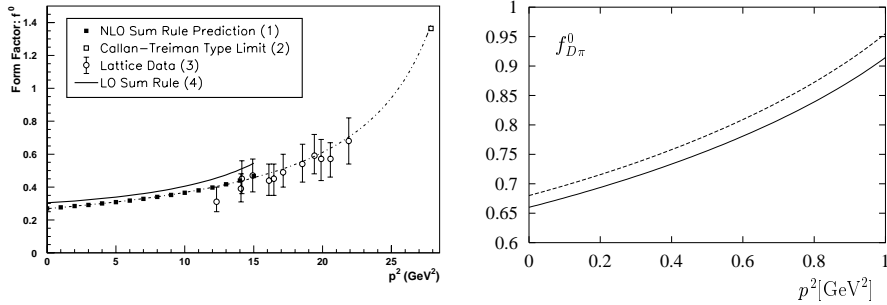
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Figure 3: **Left:** NLO LCSR prediction for $f_{B\pi}^0$ (solid squares) and fit to the later and the PCAC constraint shown by the empty square (dashed line). Also shown are the LO LCSR result (solid line) and the UKQCD lattice data (empty circles). **Right:** Light-cone sum rule results for the form factor $f_{D\pi}^0$ in NLO (solid line) and LO (dashed line).

5 New method of calculating f^+

In this section we review a new method suggested in [4] for calculating heavy-to-light form factors. The method is based on first principles. It is an extension of LCSR, but it has a much wider range of applicability, including the intermediate momentum region, where most of the lattice results are located, and even the region near zero recoil. The main idea is to use the operator product expansion with a combination of double and single dispersion relations. The resulting new sum rule has a term which corresponds to the ground-state, as well as contributions which account for all possible physical intermediate states. We start from the usual correlation function

$$F_\mu(p,q) = i \int dx e^{ip\cdot x} \langle \pi(q)|T\{\bar{u}(x)\gamma_\mu b(x), m_b\bar{b}(0)i\gamma_5 d(0)\}|0\rangle$$
$$= F(p^2,(p+q)^2)q_\mu + \tilde{F}(p^2,(p+q)^2)p_\mu, \tag{14}$$

focusing on the invariant amplitude $F(p^2,(p+q)^2)$. In the following, we use the definitions

$$\sigma(p^2,s_2) = \frac{1}{\pi} \operatorname{Im}_{s_2} F(p^2,s_2), \quad \rho(s_1,s_2) = \frac{1}{\pi^2} \operatorname{Im}_{s_1} \operatorname{Im}_{s_2} F(s_1,s_2). \tag{15}$$

The standard sum rule for the form factor $f^+(p^2)$ is obtained by writing a single dispersion relation for $F(p^2,(p+q)^2)$ in the $(p+q)^2$-channel, inserting

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the hadronic representation for $\sigma(p^2, s_2)$ and Borelizing in $(p+q)^2$:

$$\mathcal{B}_{(p+q)^2}F = \mathcal{B}_{(p+q)^2}\left(\frac{2m_B^2 f_B f^+(p^2)}{m_B^2 - (p+q)^2} + \int\limits_{s_2 > s_0} ds_2 \frac{\sigma^{hadr}(p^2, s_2)}{s_2 - (p+q)^2}\right). \quad (16)$$

Note that any subtraction terms which might appear vanish after Borelization. Similarly, the standard light-cone sum rule for the coupling $g_{B^* B\pi}$ is obtained from a double dispersion relation:

$$\mathcal{B}_{p^2}\mathcal{B}_{(p+q)^2}F = \mathcal{B}_{p^2}\mathcal{B}_{(p+q)^2}\left(\frac{m_B^2 m_{B^*} f_B f_{B^*} g_{B^* B\pi}}{(p^2 - m_{B^*}^2)((p+q)^2 - m_B^2)}\right.$$
$$\left. + \int\limits_{\Sigma} ds_1 ds_2 \frac{\rho^{hadr}(s_1, s_2)}{(s_1 - p^2)(s_2 - (p+q)^2)}\right), \quad (17)$$

where Σ denotes the integration region defined by $s_1 > s_0$, $s_2 > m_b^2$ and $s_1 > m_b^2$, $s_2 > s_0$.

In contrast to the above procedure we suggest to use a dispersion relation for $\sigma(p^2, s_2)/(p^2)^l$ in the p^2-channel (with l being an integer):

$$\sigma(p^2, s_2) = -\frac{1}{(l-1)!}(p^2)^l \frac{d^{l-1}}{ds_1^{l-1}}\frac{\sigma(s_1, s_2)}{s_1 - p^2}\bigg|_{s_1 = 0} + \int\limits_{s_1 > m_b^2} ds_1 \frac{(p^2)^l}{s_1^l}\frac{\rho(s_1, s_2)}{s_1 - p^2}, (18)$$

and to replace $\sigma(p^2, s_2)$ in (16) by the r.h.s of (18) [e]. Then, writing a double dispersion relation for $F(p^2, (p+q)^2)/(p^2)^l$ and comparing it with the previous result, we obtain the sum rule

$$f^+(p^2) = \frac{1}{2}\frac{(p^2)^l}{(m_{B^*}^2)^l}\frac{f_{B^*} g_{B^* B\pi}}{m_{B^*}\left(1 - \frac{p^2}{m_{B^*}^2}\right)} - \frac{1}{(l-1)!}(p^2)^l \frac{d^{l-1}}{ds_1^{l-1}}\frac{f^+(s_1)}{s_1 - p^2}\bigg|_{s_1 = 0}$$
$$+ \frac{1}{2m_B^2 f_B}\int\limits_{\Sigma'} ds_1 ds_2 \frac{(p^2)^l}{s_1^l}\frac{\rho(s_1, s_2)}{s_1 - p^2} e^{-\frac{s_2 - m_B^2}{M^2}}, \quad (19)$$

where the integration region Σ' is defined by $s_1 > s_0$ and $m_b^2 < s_2 < s_0$. This sum rule is valid in the whole kinematical range of p^2. As input we need the first $(l-1)$ terms of the Taylor expansion of $f^+(p^2)$ around $p^2 = 0$. These

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[e]By choosing l large enough the dispersion relation (18) will be convergent.
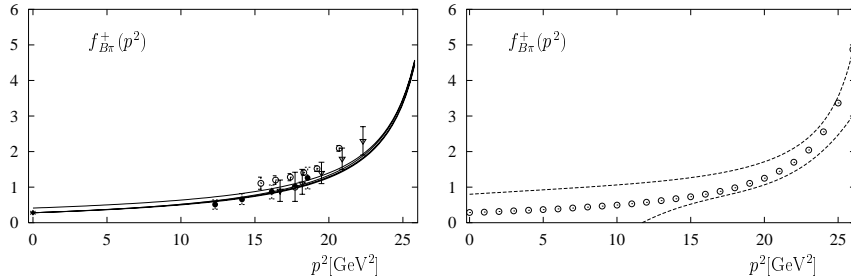
Figure 4: **Left:** The LCSR prediction for the $B \to \pi$ form factor at $l = 0, 1, 2, 3$ in comparison to lattice results. The lattice results come from FNAL (full circles), UKQCD (triangles), APE (full square), JLQCD (open circles), and ELC (semi-full circle). **Right:** The LCSR prediction on the form factor $f_{B\pi}^+$ (circles) in comparison to the constraint (dashed) derived by Boyd and Rothstein.

parameters can be obtained numerically from the standard sum rule for $f^+(p^2)$:

$$f^+(p^2) = \frac{1}{2m_B^2 f_B} \int_{m_b^2}^{s_0} \sigma^{QCD}(p^2, s_2) e^{-\frac{s_2 - m_B^2}{M^2}} \qquad (20)$$

following from (16). We further need the residue at the pole $p^2 = m_{B^*}^2$, which can be obtained from the sum rule (17), as discussed in the previous section (see (8)).

The case $l = 0$ has a very transparent physical meaning. The first term represents the contribution of the ground state resonance with mass m_{B^*}, while the second term corresponds to the contributions of all other physical states in this channel. As shown explicitly in[4] for twist 2, 3 and 4, the last term is of $O(\alpha_s)$ only. This provides an explanation of the empirical fact that the single pole model describes many form factors quite well. In addition, we are now able to quantify the deviation from the pole model, in a model-independent way, applying QCD and light-cone OPE. It should be noted that the parameter l plays a similar role as the Borel parameter M^2. There is a lower limit on l such that the dispersion relation (18) converges. Going to higher values of l will improve the convergence of the dispersion relations and will suppress higher resonances in the B^*-channel. But there is also an upper limit on l. The higher the value of l, the more derivatives of $f^+(p^2)$ at $p^2 = 0$ enter. At some point, one starts probing the region $p^2 > m_b^2 - 2\chi m_b$, where the standard sum rule (20) breaks down. Details on the numerical analysis of the new sum rule can be found in [4]. This analysis nicely supports the qualitative results

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obtained in [3]. Using the convenient parameterization[27]

$$f_{B\pi}^+(p^2) = \frac{f_{B\pi}^+(0)}{(1 - p^2/m_{B^*}^2)(1 - \alpha_{B\pi}p^2/m_{B^*}^2)} \ , \tag{21}$$

and $f_{B\pi}^+(0) = 0.28\pm0.05$[3,19], we get $\alpha_{B\pi} = 0.4\pm0.04$ in remarkable agreement with $\alpha_{B\pi} = 0.32\pm_{0.07}^{0.21}$ derived in [3]. Fig. 4 shows a comparison of (21) with recent lattice results [21,22,23,24,25]. The agreement within uncertainties is very satisfactory. Finally, the LCSR prediction also obeys the constraints derived from sum rules for the inclusive semileptonic decay width in the heavy quark limit [26]. This is also demonstrated in Fig. 4.

The above results on $f_{B\pi}^+$ can be used to calculate the width of the semileptonic decay $B \to \pi\bar{l}\nu_l$ with $l = e, \mu$. For the integrated width, one obtains[3]

$$\Gamma = \frac{G^2|V_{ub}|^2}{24\pi^3} \int dp^2(E_\pi^2 - m_\pi^2)^{3/2} \left[f_{B\pi}^+(p^2)\right]^2 = (7.3 \pm 2.5) \ |V_{ub}|^2 \ \text{ps}^{-1} \ . \tag{22}$$

Experimentally, combining the branching ratio $BR(B^0 \to \pi^-l^+\nu_l) = (1.8 \pm 0.6) \cdot 10^{-4}$ with the B^0 lifetime $\tau_{B^0} = 1.54 \pm 0.03$ ps [20], one gets $\Gamma(B^0 \to \pi^-l^+\nu_l) = (1.17\pm0.39)\cdot10^{-4}\,\text{ps}^{-1}$. From that and (22) one can then determine the quark mixing parameter $|V_{ub}|$. The result is

$$|V_{ub}| = (4.0 \pm 0.7 \pm 0.7) \cdot 10^{-3} \tag{23}$$

with the experimental error and theoretical uncertainty given in this order. For the $D \to \pi$ transition and using (21) analogously one obtains [3] $\alpha_{D\pi} = 0.01_{-0.07}^{+0.11}$ and $f_{D\pi}^+(0) = 0.65\pm0.11$, which nicely agrees with lattice estimates, for example, the world average [22] $f_{D\pi}^+(0) = 0.65 \pm 0.10$, or the most recent APE result [24], $f_{D\pi}^+(0) = 0.64 \pm 0.05_{-.07}^{+.00}$. For more details one should consult[3].

6 Heavy baryons

The study of heavy baryons such as Λ_b, Λ_c and Σ_b, Σ_c, is more complicated. Two and three point QCD sum rules have been investigated in[31,33,34,32,30], and have been applied to the heavy-to-light baryon transitions[41,42,40]. However, we are not aware of applications of LCSR to heavy-to-light baryon transitions. In the following we collect the results available at present (see also [30]). One has estimated the binding energies, $\bar{\Lambda} = M - m_Q$, of the ground state baryons and the residues of the baryonic currents, $\langle B|J_B|0\rangle = F_B u_B$, at NLO. The results are[31,32,33,34]

$$\bar{\Lambda}(\Lambda_Q) = 0.77 \pm 0.05\text{GeV} \quad \text{and} \quad |F_{\Lambda_Q}| = 0.027 \pm 0.001\text{GeV}^3, \tag{24}$$

$$\bar{\Lambda}(\Sigma_Q) = 0.94 \pm 0.05\text{GeV} \quad \text{and} \quad |F_{\Sigma_Q}| = 0.038 \pm 0.003\text{GeV}^3. \tag{25}$$

Using the experimental value for the mass of the Λ_b baryon, $m(\Lambda_b) = 5.642 \pm 0.05$ GeV [20] one finds for the pole mass of the b quark: $m_b = 4.88 \pm 0.1$, and for the related $\overline{MS}$ mass: $\bar{m}(\bar{m}) = 4.25 \pm 0.1$GeV. These values are in good agreement with the mass estimates in the meson case[37,38]. Coupling constants have been derived from sum rules in the external axial field [29] with the result: $g_{\Sigma^*\Sigma\pi} = 0.83 \pm 0.3$ and $g_{\Sigma^*\Lambda\pi} = 0.58 \pm 0.2$. The semileptonic transition $\Lambda_b \to \Lambda_c$ has been studied[35] using sum rule techniques. The matrix elements of this weak transition are determined by the Isgur-Wise function

$$\langle \Lambda_b | \bar{c}\Gamma b | \Lambda_c \rangle = \xi(w)\bar{u}_c \Gamma u_b \tag{26}$$

where $w = v_b \cdot v_c$ and Γ is the Dirac matrix. The baryonic Isgur-Wise function has been estimated by QCD sum rules in [35]. The slope of this function at $w = 1$ is found to be $\rho^2 = -1.15 \pm 0.2$ and the shape is fitted very well by $\xi(w) = \frac{2}{1+w} exp\left((2\rho^2 - 1)\frac{w-1}{w+1}\right)$. Taking into account $1/m$ corrections[39], one obtains the width $\Gamma(\Lambda_b \to \Lambda_c e\nu) = |\frac{V_{cb}}{0.04}|^2 \cdot 6 \cdot 10^{-14}$ GeV.

Conclusion: We have given a short review of selected topics concerning QCD sum rules for heavy hadrons. One main development during the past few years has been the NLO improvement. A second important development is the ongoing update of the pion wave functions [6]. Very recently a new sum rule for $B \to \pi$ has been suggested[4]. Among the remaining problems, we have mentioned the application of LCSR to baryons requiring the knowledge of baryonic distribution amplitudes.

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