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ISOLATION OF THE NUCLEAR COMPRESSIBILITY WITH THE BALANCE ENERGY

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Previous theoretical studies of the disappearance of directed transverse flow showed a dual dependence on the equation of state (EOS) and the in-medium cross section ($\sigma_{\rm nn}$) for light systems. Also, the balance energy was shown experimentally to increase as a function of the impact parameter. However, Boltzmann-Uehling-Uhlenbeck model calculations show that the dependence on $\sigma_{\rm nn}$ weakens for heavy systems such as Au+Au, and data presented here show that the impact parameter dependence nearly vanishes for Au+Au. Therefore, the EOS parameter K can be isolated using the balance energy for the first time, and preliminary calculations show good agreement for a *soft* EOS. The reduction in $\sigma_{\rm nn}$ is then investigated using the experimental mass dependence of the balance energy.

The phenomenon of collective flow in heavy ion reactions has been used to study the properties of hot and compressed nuclear matter for a wide range of densities [1–3]. Of particular interest is the nuclear equation of state (EOS) [4,5], which is relevant to astrophysical events and objects such as the big bang, supernovae explosions, and neutron stars [6,7]. The nuclear equation of state is the description of the thermodynamic state of nuclear matter as a function of the state variables density, temperature, pressure, and entropy. We have strong evidence that suggests that we may be able to observe two phase transitions in this phase diagram. Collective flow may be of relevance to this physics because of the postulated softest point. Recently, the E895 Collaboration measured elliptic flow in Au+Au collisions that suggests a softening of the EOS at $E_{\rm beam} \sim 4A$ GeV [8]. At lower energies, theoretical Thomas-Fermi calculations showed that the existence of radial flow coincides with a first-order liquid-gas phase transition [9]. Other recently proposed experimental quantities for studying the EOS include differential flow [10] and elliptic flow near the balance energy [11]. Historically, much more attention, however, has been focussed on the curvature of the binding energy as a function of density for small (≈ 0) temperature, the nuclear compressibility.

The disappearance of directed transverse (sideward) flow, termed the balance energy $E_{\rm bal}$, was suggested as a powerful probe of the EOS [12]. However, numerous model calculations have demonstrated that the balance energy, while sensitive to the nuclear compressibility K, was also sensitive to the in-medium cross section $\sigma_{\rm nn}$ [13,15,14,16,17], as well as the momentum dependence of the nuclear mean field [18–23].

Zheng *et al.* recently used an isospin-dependent BUU model for the ^{48}Ca+^{48}Ca system to show that the same balance energy is obtained with a stiff EOS and vacuum cross section as with a soft EOS and reduced $\sigma_{\rm nn}$ [11]. In fact, $E_{\rm bal}$ was shown to have a weak dependence on K for light systems [25]. Also, $E_{\rm bal}$ was shown to depend strongly on the impact parameter [24–26], further hindering study of the EOS. However, all of these studies were carried out for systems with total mass of $A \lesssim 200$. Recently the balance energy for Au+Au was measured directly [27], extending the system mass dependence of $E_{\rm bal}$ and providing motivation for the present work.

In this paper we show for the first time that the EOS parameter K can be isolated using the balance energy. We show that the impact parameter dependence of $E_{\rm bal}$ weakens as the system mass increases and nearly vanishes for a heavy system such as Au+Au. We employ Boltzmann-Uehling-Uhlenbeck (BUU) model calculations [15,16,21] to show that the dependence of $E_{\rm bal}$ on $\sigma_{\rm nn}$ weakens as well for heavy systems. These findings, together with a strong dependence on the compressibility, allow for the first time isolation of EOS properties with the balance energy, which is particularly beneficial because the balance energy is a relatively model-independent observable [14]. Finally, the extended system mass dependence of $E_{\rm bal}$ can then be used to examine the magnitude of the in-medium modification of the baryon-baryon cross section, σ_{nn}.

The balance energy arises from the canceling effects of the attractive part of the nuclear mean field, dominant at $E_{beam} \sim 10$ MeV/nucleon, and the repulsive nature of nucleon-nucleon scattering, which dominates at $E_{beam} \gtrsim 150$ MeV/nucleon. Coulomb repulsion plays an increasing role in the collision dynamics as system size increases [17,24], which explains the measured deviation from the anticipated value of $\tau = 1/3$ in the $E_{bal} \propto A^{-\tau}$ dependence [27]. Experimentally we observe that $\tau \sim 0.45$. Therefore, the Coulomb interaction certainly needs to be included when comparing experimental balance energies to model predictions for heavy systems such as Au+Au.

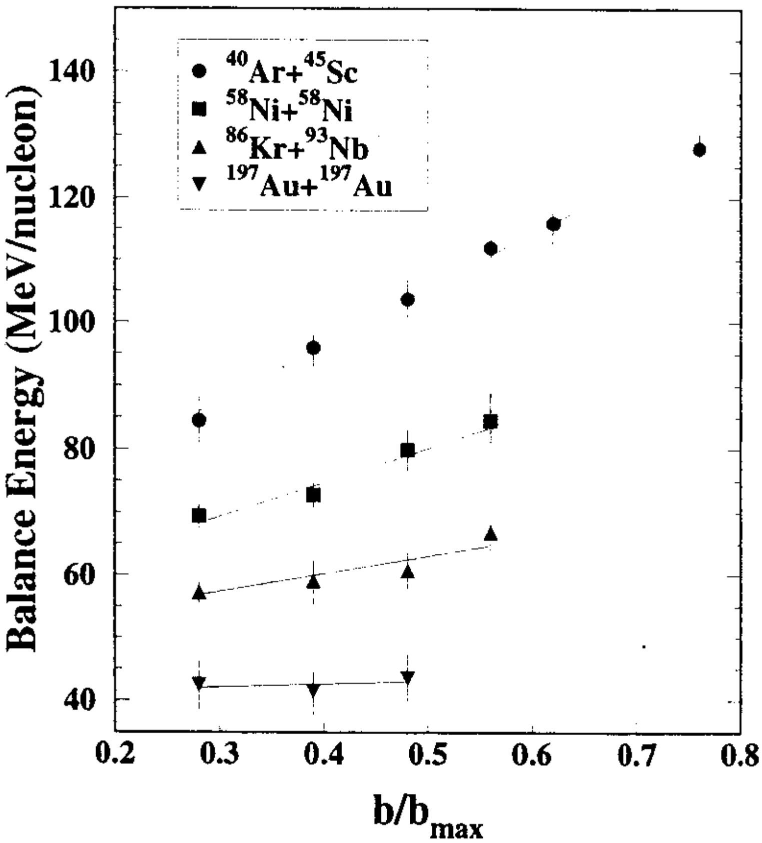

FIG. 1. Balance energy as a function of the reduced impact parameter for four systems. Data are taken with the 4π Array, the linear fits are intended to guide the eye.

Previously, the balance energy was observed to increase linearly as a function of impact parameter b for light systems [26]. This dependence was attributed to the need for a larger incident energy to overcome effects of the mean field as the participant zone gets smaller (with decreasing b). Figure 1 shows the balance energy as a function of the reduced impact parameter b/b_{max} (where b_{max} is the maximum estimated impact parameter) for the four systems Ar+Sc, Ni+Ni, Kr+Nb, and Au+Au. Data were recorded at the National Superconducting Cyclotron Laboratory with the 4π Array [29] in a consistent configuration which included a 45-element High Rate Array in the forward direction. Details of the experimental setup can be found in Refs. [26,27,30,31]. Not all impact parameter bins are shown due to detector acceptance effects at low incident energies and for less-central collisions. The lines represent linear fits to the data, included to guide the eye. As the system mass increases E_{bal} exhibits a weaker dependence on b/b_{max}, and for Au+Au the dependence nearly vanishes. The weakening could be due to the increasing role of the Coulomb interaction on the projectile's trajectory as b/b_{max} increases, counteracting the attractive mean field. Therefore, b can be regarded as a model-independent parameter when comparing E_{bal} to model calculations for Au+Au.

The Boltzmann-Uehling-Uhlenbeck (BUU) model has been successful in studying the flow of nuclear matter and energy. The BUU model treats the single-body phase space distribution as it evolves through time. In the present numerical implementation of the BUU model, the inter-nucleon potential is split among two mechanisms: a mean field for soft, low-momentum processes, and hard nucleon-nucleon scattering

$$\sigma_{\text{nn}} = \sigma_{\text{free}}(1 + \alpha\frac{\rho}{\rho_0}), \tag{1}$$

where σ_{free} is the cross section in the vacuum, and α is the the first-order coefficient of the Taylor-expansion of the in-medium cross section in terms of the density [16]. Previous studies had found a value between -0.2 and -0.3 for α, in good agreement with finite temperature G-matrix calculation results [33]. The mean field can be expressed solely in terms of the compressibility K by using the saturation binding energy and initializing all nucleons with the Fermi momentum. A value of $K = 200$ MeV is commonly used for a *soft* EOS, while $K = 380$ corresponds to a *stiff* EOS. Previously, the BUU model exhibited a dual dependence of E_{bal} on K and σ_{nn}. However, these calculations were generally carried out for light systems.

We present results of a systematic study of the balance energy using the BUU model for a wide range of system sizes, $63 < A < 394$. For each system size, several energies near the anticipated balance energy were chosen, and several combinations of (K, α) were selected: $K = 200$, 235, and 380 MeV and $\alpha = 0$,-0.1,-0.2, and -0.3. The values for K were chosen in accordance with parameter sets used in previous studies. Each set of parameters was calculated using four different random number seeds to minimize any effect from the choice of seed. For all systems, an impact parameter corresponding to $b/b_{max} = 0.28$ was used in order to compare to the most central bin of our experimental data (for which the mean $b/b_{max} \approx 0.28$). Momentum-dependent mean fields were not included in the present numerical implementation, because E_{bal} is affected very little by momentum dependence at low beam energies and in near-central collisions [32].

Figure 2 shows balance energies extracted from BUU calculations as a function of the system mass for four different cross sections, assuming a soft equation of state. Lines represent power law fits to the simulated values, as suggested by the experimental mass dependence of E_{bal} [27,28]. The error bars are associated with the linear fit of the flow excitation function. The balance energy clearly shows a strong dependence on α for light systems, in agreement with previous theoretical work. However, as the system size increases, the α dependence of E_{bal} nearly vanishes. For $A = 394$ (Au+Au), all of the extracted balance energies are well within error bars.

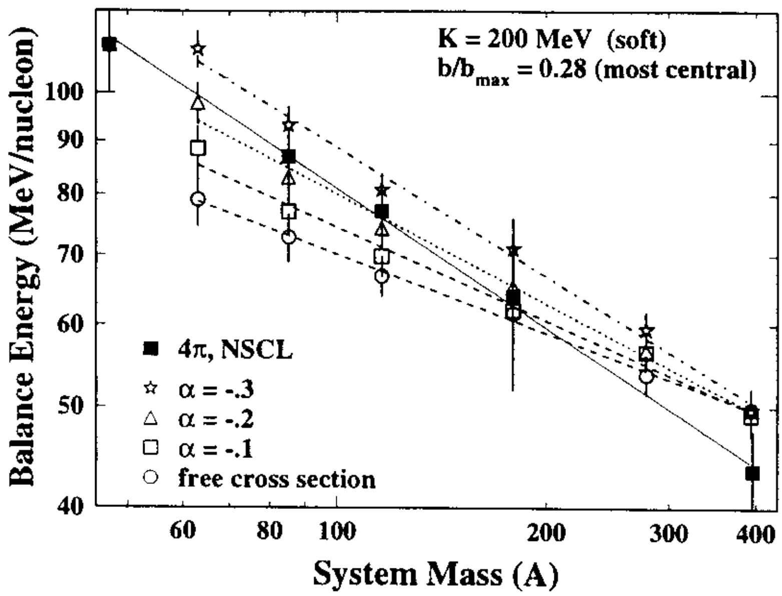

FIG. 2. BUU model calculations of the mass dependence of $E_{\rm bal}$ for values different reductions of the in-medium cross section $\sigma_{\rm nn}$. Experimental measurements of $E_{\rm bal}$ are shown as solid squares. The calculated balance energy for Au+Au depends very weakly on the value of α.

The gradual loss of the sensitivity of $E_{\rm bal}$ to $\sigma_{\rm nn}$ can be attributed to the change in the collision dynamics at lower beam energies. As A increases, the corresponding balance energy decreases, and hard scattering processes play a lesser role in the dynamics of the collision [24]. This is due to the Pauli Exclusion Principle, which forbids an increasing number of collisions as the number of nucleons present increases [15]. Pauli blocking also becomes more dominant as beam energy is decreased. Therefore, for heavy systems the balance energy is due mostly to combined effects of the attractive mean field and the repulsive Coulomb interaction. Without the Coulomb interaction included in the BUU calculations, $E_{\rm bal}$ for Au+Au is ~10-15 MeV/nucleon larger [27].

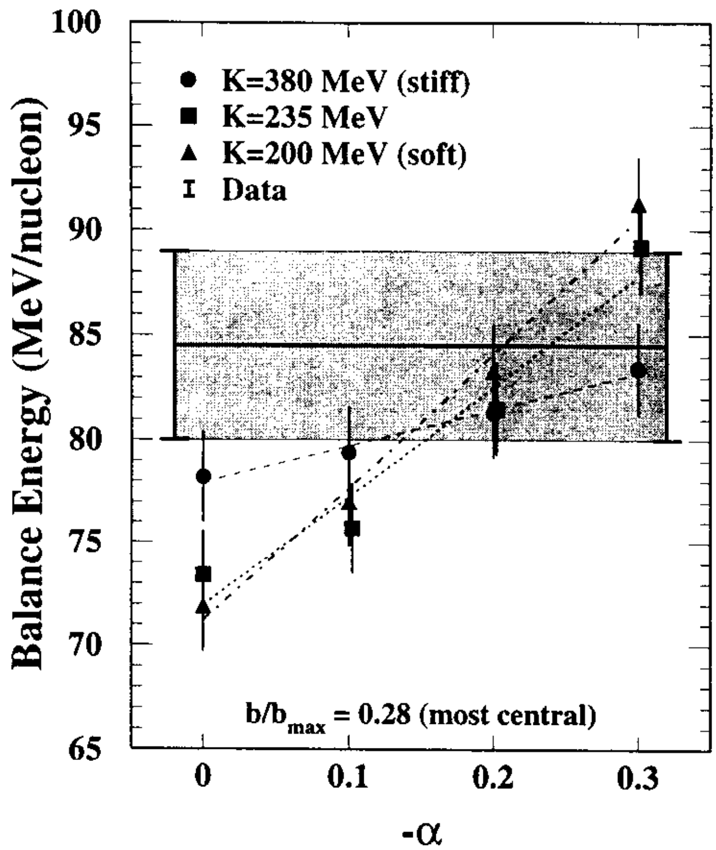

FIG. 3. BUU balance energies plotted as a function of the σ_{nn} reduction parameter $-\alpha$ for three different values of the compressibility K for Ar+Sc. The experimental measurement is represented by a flat line with error bars.

Because the balance energy for Au+Au is nearly independent of the reduction in in-medium cross sections and the impact parameter, BUU predictions can be compared directly to the experimental value of the balance energy to estimate the nuclear compressibility K. This lack of dependence on b and α for Au+Au differs from lighter systems that showed strong dependence on b and α, which made the isolation of K difficult. Figure 3 shows BUU balance energies for Ar+Sc ($A = 85$) as a function of the cross section reduction parameter $-\alpha$ for three different values of the nuclear compressibility: K=200, 235, and 380 MeV. Dashed lines are included only to guide the eye. The single experimental value is plotted as a horizontal line with error bars. Depending on the α selected, all three K's can agree within error bars of the experimental value for E_{bal}.

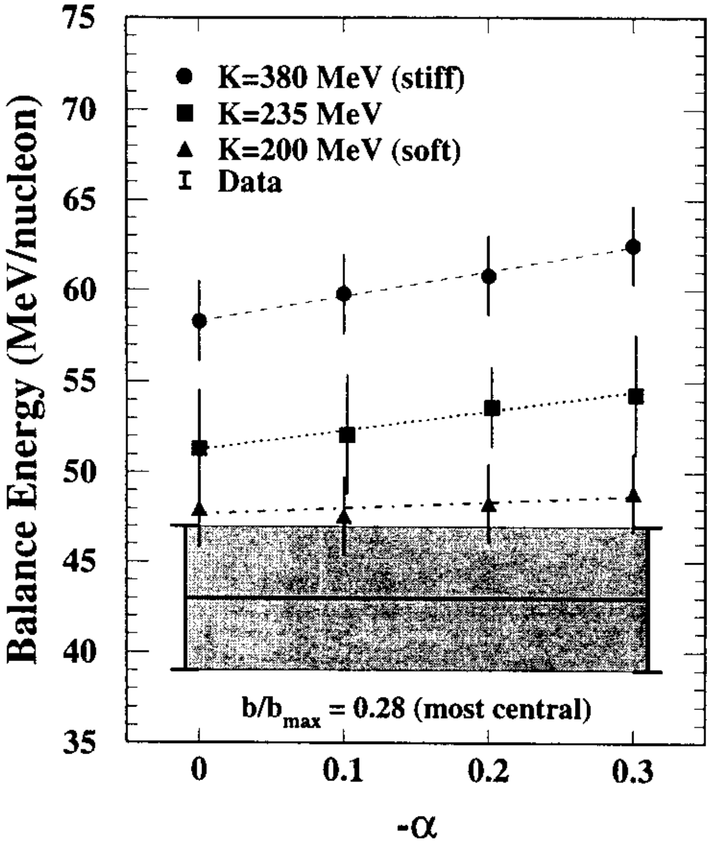

FIG. 4. BUU balance energies plotted as a function of the σ_{nn} reduction parameter $-\alpha$ for three different values of the compressibility K for Au+Au. The experimental measurement is represented by a flat line with error bars.

In Figure 4, BUU balance energies for Au+Au are plotted vs. $-\alpha$, and again the experimental value is represented by a horizontal line. Only K=200 MeV, which corresponds to a *soft* equation of state, falls within error bars of the experimental measurement. The approximate value of K is in good agreement with other measurement techniques. Studies of the isoscalar monopole resonance indicate K=200$\pm$20 MeV [34], while recent Thomas-Fermi model calculations pointed to K=234 MeV [35]. Also, Pan and Danielewicz estimated that

K lies between 165 and 220 MeV by studying the dependence of sideward flow on multiplicity [20].

Once a value for K in the BUU parameterization is established, the system mass dependence can be used to investigate the magnitude of σ_{nn}'s deviation from the vacuum cross section. Figure 2 shows the experimental data for the mass dependence of the balance energy (filled boxes) [27]. A reduction of α=-0.2 in the cross section agrees well with the data for light- and medium-sized systems, while α=-0.3 most closely reproduces the slope (power-law exponent) on the experimental mass dependence.

In conclusion, we have shown that the impact parameter dependence of the balance energy nearly vanishes for heavy systems such as Au+Au, which we attribute to the increased strength of the Coulomb repulsion counteracting the attractive mean field as b/b_{max} increases. We have also performed a systematic set of BUU calculations to show that the sensitivity of E_{bal} to the in-medium cross section weakens as the system size increases and nearly disappears for Au+Au. This effect is ascribed to the lesser role of hard scattering processes at lower beam energies due to Pauli blocking. These two findings make Au+Au a very promising system for extracting the nuclear compressibility K from the balance energy. Boltzmann-Uehling-Uhlenbeck calculations for Au+Au with K=200 MeV, corresponding to a *soft* equation of state, produce balance energies which lie within error bars of the recently measured value. The present findings warrant further theoretical and experimental study of the balance energy for very heavy systems, as a more thorough study has the potential to determine the compressibility in a relatively model-independent way. In addition, the experimental mass dependence can employed to estimate the cross section reduction parameter. Calculations presented in this paper estimate α to be -0.2 to -0.3.

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