LEP note 200

17.12.1979

Optimum Linac Energies

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The energy of the electron linac and of the positron linac can be determined by a cost optimization of the pre-injector. In this note, this procedure is reviewed and the sensitivity of cost to the choice of the linac energies is discussed. The result is applied to the LEP-8 pre-injector.

The energy dependent part of the cost C is to a good approximation a linear function of the linac energies (E_e - energy of electron linac, E_p - energy of positron linac and ACR)

$$C = k_e E_e + k_p E_p \tag{1}$$

The second term includes not only the cost of the linac but also the cost of ACR and the cost of the beam transfer between ACR and the injector synchrotron (ISY).

The linac energies E_e and E_p are not independent, they are linked by the average positron current the pre-injector has to provide during a LEP fill. This current is given by the desired e^+ filling rate of LEP and the cumulative transfer efficiency downstream of the point where this current is measured. We choose this point downstream of the positron linac, after the energy analysing slits. The resolved positron current $\bar{I}_p$ can be expressed in terms of pre-injector parameters

$$\bar{I}_p = \hat{Q}_e \cdot \frac{E_e}{E_c} \cdot \eta_p \cdot \frac{b}{\tau} \tag{2}$$

$\hat{Q}_e$ - charge in the electron pulse hitting the converter per linac pulse

E_e/E_c - conversion efficiency $e^- \rightarrow e^+$

η_p - transmission efficiency through positron linac

b - number of bunches in ACR

τ - damping time in ACR

b/τ - injection rate in ACR $\equiv$ linac repetition rate

The damping time can be expressed in terms of the ACR energy $E_A = E_p$ and other ACR parameters. If the energy of ACR is varied, the magnetic fields must change proportional to energy. The bending radius ρ_A remains constant. However, since the dipole field B_A is already rather high (1,6 T), it may be necessary to increase somewhat the length of the magnets if the energy is increased. In the limit, the field B_A may stay even constant. We treat the two extrema: ρ_A - constant, $B_A \sim E_p$; $\rho_A \sim E_p$, B_A - constant. The circumference of ACR is constant; it is given by the response times of the kickers and the number of bunches.

1. Constant bending radius in ACR

Under this assumption $\tau \sim E_p^{-3}$. Equation (2) can be written

$$\bar{I}_p = K_\rho \, E_e E_p^{\,3} \tag{3}$$

where K_ρ is a constant given by parameters independent of energy. Equation (1) becomes

$$C = k_e \frac{\bar{I}_p}{K_\rho} E_p^{-3} + k_p E_p \tag{4}$$

The energy E_p is chosen arbitrarily as independent variable. This function $C(E_p)$, sketched in Fig. 1, exhibits a minimum which occurs at

$$E_{po} = \left(3 \, \frac{k_e}{k_p} \, \frac{\bar{I}_p}{K_\rho} \right)^{1/4} \tag{5a}$$

From (3) we get the corresponding optimum energy of the electron linac

$$E_{eo} = \left(3 \, \frac{k_e}{k_p} \right)^{-3/4} \left(\frac{\bar{I}_p}{K_\rho} \right)^{1/4} \tag{5b}$$

and

$$E_{po}/E_{eo} = 3 \, \frac{k_e}{k_p} \tag{5c}$$

It is useful to calculate by how much the cost increases if the energy E_p is off the optimum E_{po}. The ratio of cost to minimum cost turns out to be

$$C/C_o = \left[3 + \left(\frac{E_{po}}{E_p} \right)^4 \right] \cdot \frac{1}{4} \cdot \left(\frac{E_p}{E_{po}} \right) \tag{6}$$

The function is plotted in Fig. 2. It can be seen that the cost is not very sensitive to the choice of energy.

2. Constant dipole field in ACR

In this case $\tau \sim E_p^{-2}$. Equation (2) becomes

$$\bar{I}_p = K_B \, E_e E_p^{\,2} \tag{7}$$

Eliminating E_e in (1) yields

$$C = k_e \, \frac{\bar{I}_p}{K_B} \, E_p^{-2} + k_p \, E_p \tag{8}$$

The positron energy minimizing the cost C is

$$E_{po} = \left(2 \, \frac{k_e}{k_p} \, \frac{\bar{I}_p}{K_B}\right)^{1/3} \tag{9a}$$

From (7)

$$E_{eo} = \left(2 \, \frac{k_e}{k_p}\right)^{-2/3} \left(\frac{\bar{I}_p}{K_B}\right)^{1/3} \tag{9b}$$

and

$$E_{po}/E_{eo} = 2 \, \frac{k_e}{k_p} \tag{9c}$$

The ratio of cost C to optimum cost C_o as a function of positron energy E_p is

$$C/C_o = \left[2 + \left(\frac{E_{po}}{E_p}\right)^3\right] \cdot \frac{1}{3} \cdot \left(\frac{E_p}{E_{po}}\right) \tag{10}$$

This function is plotted in Fig. 2. As expected, also this variation of cost with energy is weak.

3. Application to LEP-8 pre-injector

From the linac energies in the Pink Book (E_e = 200 MeV, E_p = 600 MeV) the coefficients $\bar{I}_p/K_\rho$ and $\bar{I}_p/K_B$ can be calculated. Their numerical value is given in Table I. The relations (3) and (7) give then pairs of values E_p, E_e which yield the required average positron current. These pairs can be read from the two curves in Fig. 3 which cross at E_p = 600 MeV as this is our starting point.

In order to find the pair which minimizes the cost, the ratio k_e/k_p must be determined. We use the information [1] gathered for the Blue Book and take into account the up-date for the Pink Book. The resulting values k_e/k_p are shown in Table I. For ρ_A constant, ACR contributes to k_p about 10%, the beam transfer to ISY about 7%; if B_A is constant, the corresponding values are 14% and 6%. The main contribution to k_p comes from the positron linac.

Knowing k_e/k_p the optimum energies can be calculated from (5) and (9). The result is shown in Table I together with the ratio of actual to optimum positron energy. We learn that our choice for the Pink Book is optimum for ρ_A constant. It is a bit off optimum for B_A constant. However, consideration of the relative cost variation (Fig. 2) shows that the concurrent cost increase must be insignificant.

Table I - Optimum energies of the LEP pre-injector linacs

Constant in ACR	Bending radius	Bending field
$\bar{I}_p/K$	$4{,}3 \cdot 10^{-2}$ GeV^4	$7{,}2 \cdot 10^{-2}$ GeV^3
k_e/k_p	1,0	0,90
E_{po} GeV	0,60	0,50
E_{eo} GeV	0,20	0,28
E_p/E_{po}	1,0	1,2

Although the absolute value of k_e and k_p may not be known very well, it is believed that their ratio k_e/k_p has been estimated to a reasonable accuracy, the more so as k_e/k_p is mainly determined by the linac cost. Furthermore, since the cost is not very sensitive to the choice of the linac energies, we are confident that the set chosen by LAL, E_e = 200 MeV and E_p = 600 MeV, is reasonably close to the cost minimum. The choice was governed also by the consideration that both linacs should have an even number of sections, and that a high positron energy is beneficial for ACR and ISY.

1) M. Sommer (editor); "Projet d'un ensemble pre-injecteur pour LEP 70", report LAL-78/33 (1978).

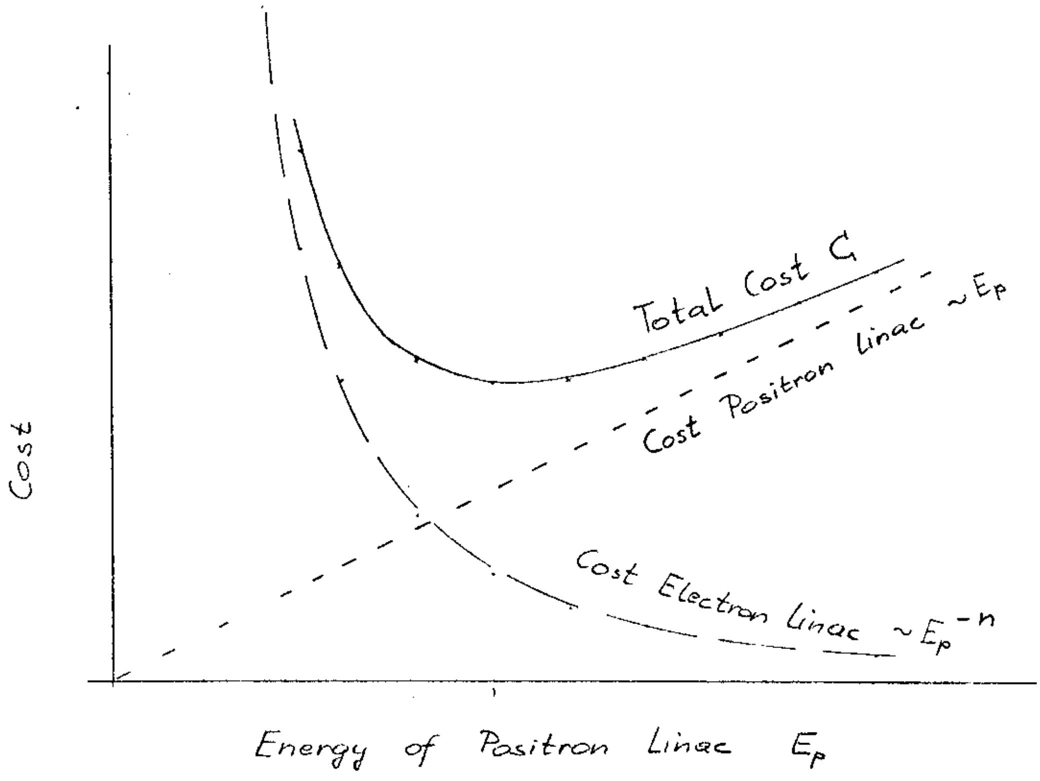

Fig. 1

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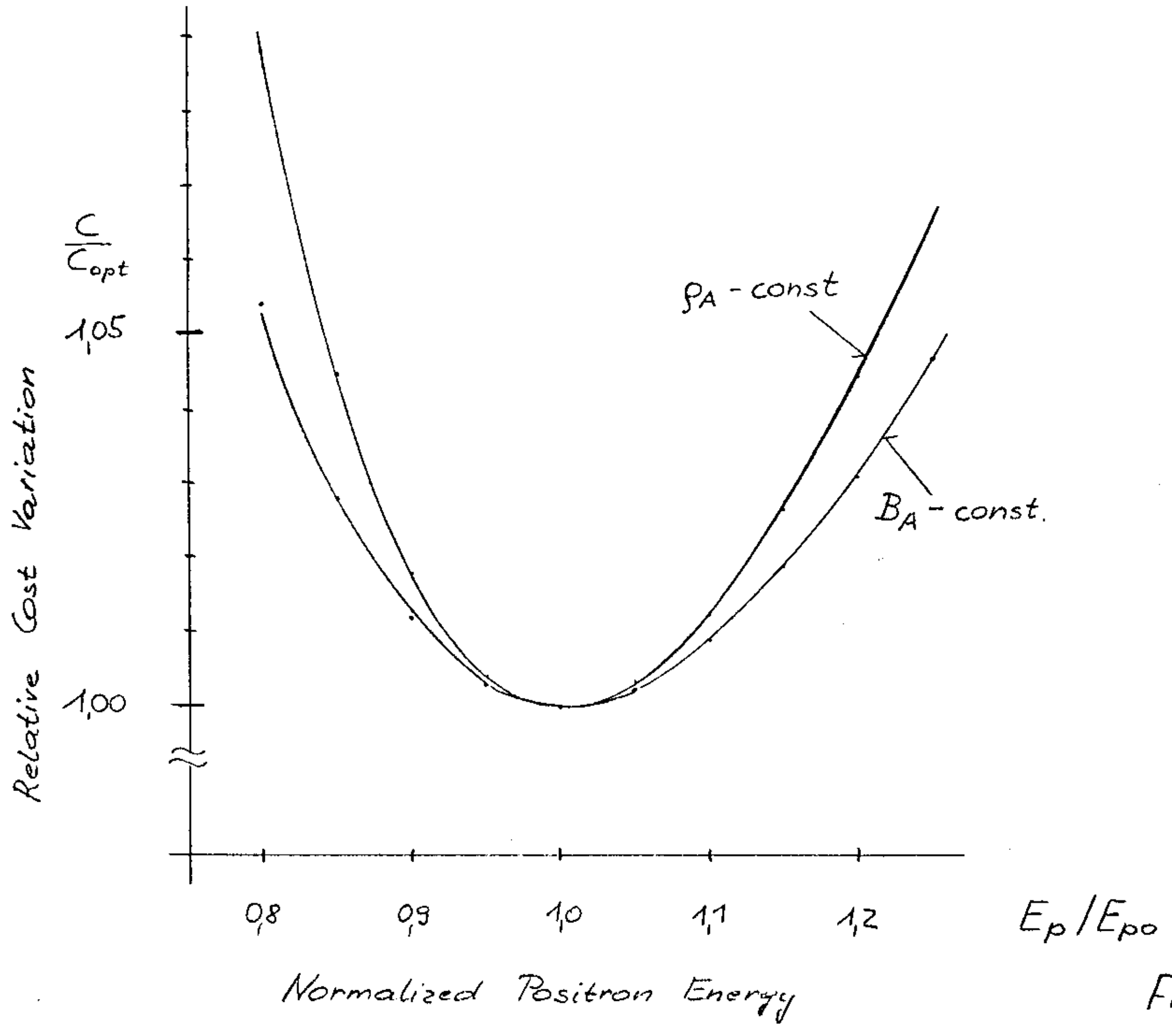

Fig. 2

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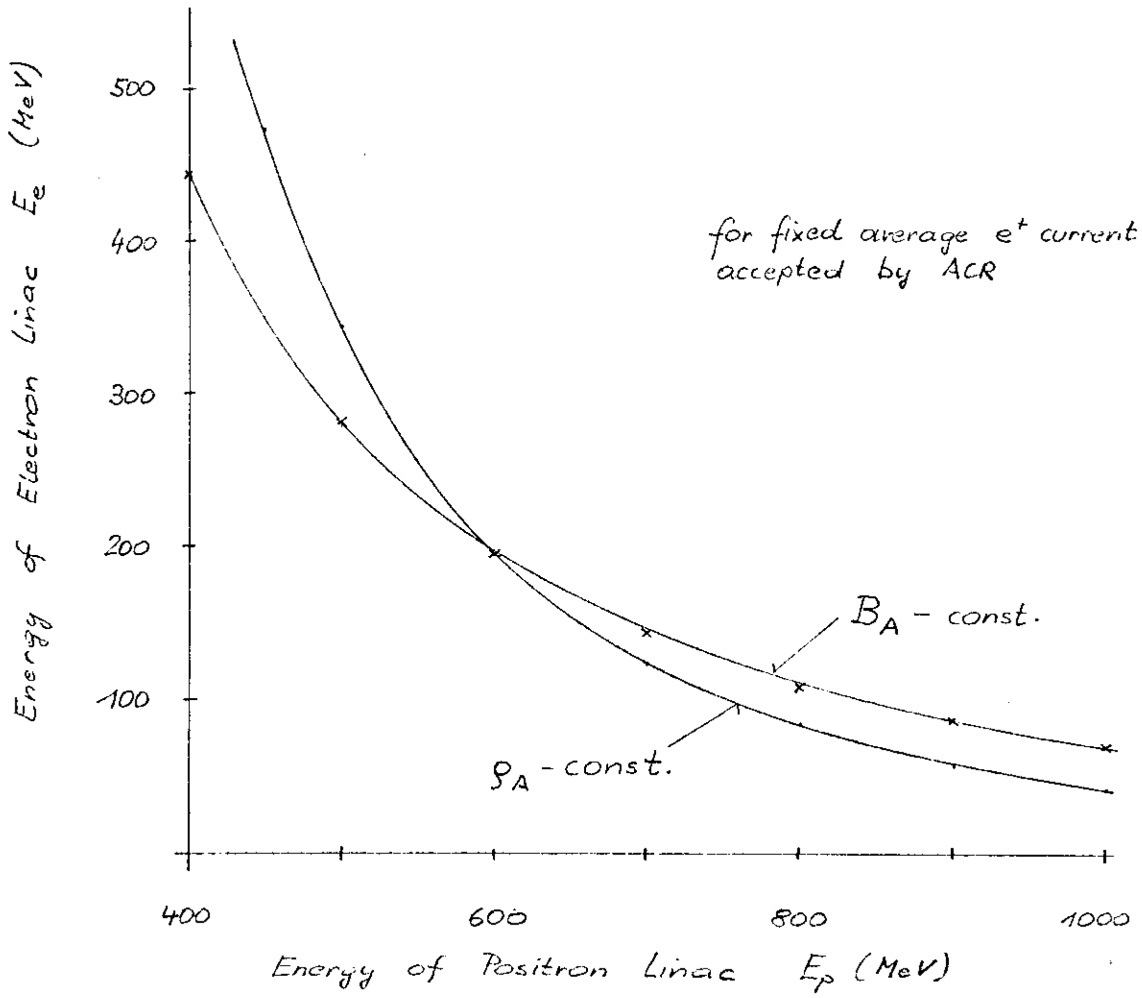

500
400
300
200
100
Energy of Electron Linac E_e (MeV)
for fixed average e^+ current accepted by ACR
B_A - const.
ρ_A - const.
400
600
800
1000
Energy of Positron Linac E_p (MeV)

Fig.3