THE IMPORTANCE OF STORED ENERGY IN THE LEP HIGH INTENSITY LINAC

K. Hübner

The first part of the linac has to provide an intense, short pulse of electrons for the positron production. The number of positrons generated is proportional to the energy*) of the electron pulse impinging on the converter target ; the proportionality factor is the conversion efficiency. If the pulse length is much shorter than the filling time of the linac the final pulse energy depends mainly on the electromagnetic energy stored in the linac at the moment the electron pulse passes. Thus the stored energy is the principal parameter of the high intensity linac. This note demonstrates how the required stored energy can be minimized by the proper choice of the charge in the pulse for a given linac and conversion characteristics. A numerical example is given where we compare the stored energy required for the LEP linac with the stored energy in existing linacs.

1. Positron production and stored energy

First consider the positron production in a target at the end of a given linac producing a beam of a few hundred MeV. The number of positrons generated per pulse is

$$N^+ = \eta \cdot n_s \cdot W \qquad (1)$$

where η - conversion efficiency, n_s - number of section and W - energy gain of the whole pulse in one section of the linac.

Both terms η and W depend on the charge per pulse q ; the pulse length is constant in our case. If more charge is injected from the gun, W will increase because more particles get accelerated but, due to beam loading, the increase will note be proportional to q. The increasing relative energy spread δ will widen the spot on the target due to chromatic aberrations of the focussing system. The growth of the transverse emittance with current will also increase the spot. Both of these effects contribute to the deterioration of η. The question arises what is the optimum charge for our given linac.

Consider first the energy gain W. It is defined by :

$$W = q \cdot \bar{U}/e \qquad (2)$$

with $\bar{U}$ the energy gain per particle in one section averaged over all particles in the pulse. The latter consists of a train of bunches. Define per section :

*) Whenever the term energy refers to the energy of a pulse or a bunch in this paper, the total kinetic energy of this ensemble of particles is meant.

U_o - energy gain from the fundamental mode experienced by a test particle preceding the pulse.

U_1 - same as above but for a particle following the pulse.

a - beam loading parameter (fundamental mode) $U_1 = U_o - aq$

bq - energy lost by the pulse to parasitic higher modes.

We agree to measure the W's in J and the U's in eV.

we get

$$\bar{U} = \frac{U_o + U_1}{2} - bq$$

$$\bar{U} = U_o - (\frac{a}{2} + b)q \tag{3}$$

and from (2)

$$W = \frac{1}{e}\left[U_o\, q - (\frac{a}{2} + b)q^2\right] \tag{4}$$

Monte-Carlo calculations give the conversion efficiency η as a function of δ and of the beam transverse emittance ε. They show that the conversion efficiency is a strong function of the relative energy spread δ because it is difficult to focus the beam properly above a certain δ. The spread δ is linked to q by

$$\delta = \frac{U_o - U_1}{\bar{U}} = \frac{aq}{U_o - (\frac{a}{2} + b)q} \tag{5}$$

In a precise calculation one would have to add to (5) the energy spread brought about by the non-zero phase extent of the bunches. Since this second contribution can be made much smaller than the first one in most practical cases, we can neglect it.

Knowing the dependence of ε on q, the conversion efficiency can be expressed as a function of a single variable, either δ or q, by virtue of (5). We choose δ as independent variable because it is easy to see what happens to η if δ gets large. We get

$$\eta = \eta(\delta) \tag{6}$$

Replacing q in (4) by δ yields

$$W = \left(\frac{U_o^2}{2ea}\right)\frac{8\delta}{\left[2 + \delta + 2(b/a)\delta\right]^2} \tag{7}$$

Inspecting standard formulae[1,5] we can identify the first term in (7) as the energy stored W_s in the accelerating mode of the section.

For a SW section

$$W_s = \frac{PQ}{\omega} \tag{8a}$$

where P - input RF power, Q - qualify factor, $\omega/2\pi$ - operating frequency. For a TW section

$$W_s = \frac{PQ}{\omega}\left(1 - e^{-2\alpha\ell}\right) \tag{8b}$$

with α - attenuation constant in nepers per unit length, ℓ - length of one section. The second term represents the power loss to the external load.

Rewrite (7)

$$W = W_s \cdot f(\delta) \tag{9a}$$

$$f(\delta) = \frac{8\delta}{\left[2 + \delta + 2(b/a)\delta\right]^2} \tag{9b}$$

Since

$$(b/a)\delta \ll 1$$

holds in practical cases, as shown in the appendix, we put b = 0. The function $f(\delta)$ is plotted in Fig.1 where it can be seen that it exhibits a maximum at $\delta = 2$. At this point U_1 vanishes as the beam loading is so strong that no stored energy is left for the last particle. It is clear that this maximum cannot be exploited because the focussing of the beam requires $\delta \ll 1$.

Combining (1) and (9) gives :

$$N^+ = \eta(\delta) \cdot f(\delta) \cdot n_s \cdot W_s \tag{10}$$

where $n_s W_s$ is the energy stored in the whole linac. It is the only linac parameter relevant for the positron production in this approximation. The conversion efficiency depends on the design of the converter and of the focussing system preceding it. It depends also on the emittance but the latter is mainly determined by the gun, prebuncher and buncher. Hence, η does not depend on the parameters of the linac proper.

Figure 2 shows qualitatively the individual terms. The conversion efficency is assumed to drop with increasing δ above a certain threshold in agreement with preliminary calculations[2]. The positron production reaches a maximum at δ_{opt}. Equation (5) gives the corresponding charge q_{opt} which is the parameter we were looking for. Thus we have determined the charge producing a maximum of positrons for a given linac.

Equation (10) provides also some rough guidance for the design of a new linac. One could propose the following procedure : work out $\eta(\delta)$; maximize $\eta(\delta) \cdot f(\delta)$ which yields δ_{opt}. Given the required N^+ the total stored energy $n_s \cdot W_s$ is defined. The two factors, n_s and W_s, should then be determined by a cost optimization. Having chosen the parameters of the sections the optimum charge can be calculated from (5).

Although this procedure must be complemented by consideration of cost, reliability and availability of components, it provides some insight and a useful guideline.

2. Example

We apply this procedure to the LEP high intensity linac. The required number of N^+ is given. The conversion efficiency η is not yet known over a large range of δ. First results from computations with our parameters show that η changes little up to 10 % but $\delta \approx 30$ % is unacceptable[2]. For the sake of our example we approximate $\eta(\delta)$ in the range $\delta \leq 10$ % by a constant equal to the value η_p given in the pink book ; for $\delta \geq 10$ % η is assumed to decrease like $1/\delta$. The required stored energy per section becomes according to (10) with b = 0

$$W_s = \frac{N^+}{\eta_p} \cdot \frac{1}{n_s} \frac{(2+\delta)^2}{8\delta} \qquad \text{for} \quad \delta \leq 10\ \% \qquad (11a)$$

$$W_s = \frac{N^+}{\eta_p} \cdot \frac{\delta}{0,1} \cdot \frac{1}{n_s} \frac{(2+\delta)^2}{8\delta} \qquad \text{for} \quad \delta \geq 10\ \% \qquad (11b)$$

Inspection of these equations shows that the W_s is minimum at $\delta = 10$ %.

The numerical value of N^+/η_p may be taken from the pink book ; it is equal to the nominal energy in the pulse at the target according to (1). We get

$$N^+/\eta_p = 6 \text{ J}$$

We assume 6 sections as in the pink book counting the buncher as a section and get from (11) with $\delta = 0,1$

$$W_s = 6 \cdot \frac{1}{6} \cdot 5,5 = 5,5 \text{ J} .$$

It is admitted that this result is sensitive to our choice of $\eta(\delta)$. Had we chosen $\delta = 15$ % as the point where η starts to drop, a lower $W_s = 3,9$ J would have been obtained. It shows only that the precise knowledge of $\eta(\delta)$ is imperative for the design of the linac.

Next we look at existing linacs to see whether W_s = 5,5 J is excessive or not. Table I gives the relevant parameters for S-band linacs used for positron production. We included SLED II, the LEP buncher (CGRMeV) and the two best performing examples considered recently by LAL. They are labelled LEP. The Q of the SW structures was degraded by 10 % to take into account the losses in the circulator protecting the klystron.

Table I, Parameters of S-band linac sections

Linac	f GHz	ℓ m	Q	P MW	$(1-e^{-2\alpha\ell})$	$\frac{W_s}{P}\omega$	$\frac{W_s}{\ell}$ J/m	W_s J
SLAC[3] (1970)	2,856	3,05	13000	6	0,68	8840	0,97	3,0
ADONE[3]	2,856	5,04	13300	12	0,81	10700	1,4	7,2
LEP buncher[4]	3	2,2	12600	13	1(SW)	12600	4,0	8,7
LEP[5]	3	2,5	12600	13,5	1(SW)	12600	3,6	9,0
DESY II[6]	2,998	5,2	14000	20	0,63	8850	1,8	9,4
LEP[7]	3	4,0	11800	27	0,70	8250	2,5	11,8
ORSAY[3]	2,998	6,0	11000	24	0,85	9350	2,0	11,9
SLAC(SLED II)[3,8]	2,856	3,05	13000	SLED	0,68	8840	4,3	13,2*

* This number will be doubled in some sections by doubling the klystrons[8].

It can be seen from the last column that all of these linacs except SLAC would qualify if we needed no safety factor. If we add a reasonable safety factor, say $\gtrsim 2$, the choice is much reduced. Only the last three remain, if we insist on having only 6 sections.

The third last column gives the stored energy normalized to remove differences in input power and frequency.

$$\frac{W_s}{P}\omega = Q \cdot (1 - e^{-2\alpha\ell})$$

The two SW structures are the winners even with the degraded Q.

The last but one column displays stored energy per unit length. SLAC (SLED II) providing the most compact design is closely followed by the two SW examples.

Unfortunately, the most interesting column is missing, namely stored energy per unit cost.

The main conclusion we can draw from this exercise is that the nominal performance can be reached with only 6 sections using existing technology and design. However, this estimate provides only a rough guidance and is not sufficiently precise to assess with certainty the safety factor available. More precise calculations have to take into account the energy spread resulting from the phase spread, more refined computations of the conversion efficiency, etc.

The optimum number of sections must be reviewed once we have a better idea of the cost, keeping in mind that stored energy per section can be traded against the number of sections.

K. Hübner

Acknowledgements

I am grateful to D. Warner for many stimulating discussions and advice. I profited also from remarks by P. Brunet, R. Chehab, F. Dupont and L. Mélard (LAL).

References

1) P. Brunet, LAL-note PI 80-30 (1980)

2) R. Chehab, LAL-note in preparation.

3) G.A. Loew, R.B. Neal in Linear Acc. (P.M. Lapostolle and A.L. Septier Editors) 1970 p.108

4) Specifications CERN/LAL/CGR MeV (1979)

5) P. Brunet, LAL-note PI 80-20 (1980)

6) A. Febel and G. Stange, DESY Interner Bericht S 1-72/3 (1972)
N.C. Pering et al., IEEE Trans. Nucl. Sci. NS-18 (1971)579

7) L. Mélard, LAL-note PI 80-37 (1980)

8) Draft of SLAC single pass collider project, appendix A (1980)

9) R.F. Koontz, G.A. Loew, R.H. Miller, P.B. Wilson, IEEE Trans. Nucl. Sci. NS-24 (1977) 1493.

APPENDIX

The higher mode loss parameter b is given by

$$b = k'_{pm} \cdot \ell/n_m$$

k'_{pm} - energy loss experienced by a bunch of unit charge per unit length

ℓ - length of section

$n_m = \Delta t \cdot f$ - number of bunches in pulse of length Δt.

The beam loading parameter a in terms of linac parameters[1]

$$a = e \frac{r \cdot \omega}{2Q} \ell$$

r - shunt impedance per unit length

We get

$$b/a = \left(\frac{k'_{pm}}{e}\right) \frac{1}{\Delta t \cdot \pi f^2} \left(\frac{Q}{r}\right)$$

Assume that our section has the same parameters as SLAC

$k'_{pm} = 90 \cdot 10^{12}$ eV/C m [9] $Q = 13000$ [2]

$r = 56$ MΩ/m [2] $f = 3.10^9$ Hz [2]

With a pulse length of $\Delta t = 14$ ns the ratio becomes

$$b/a = 1/20$$

Introducting (r/Q) of other linacs does not change this ratio appreciably which always stays smaller than 0,1.

Hence, $(b/a)\delta \ll 1$ as claimed before.

22.8.80

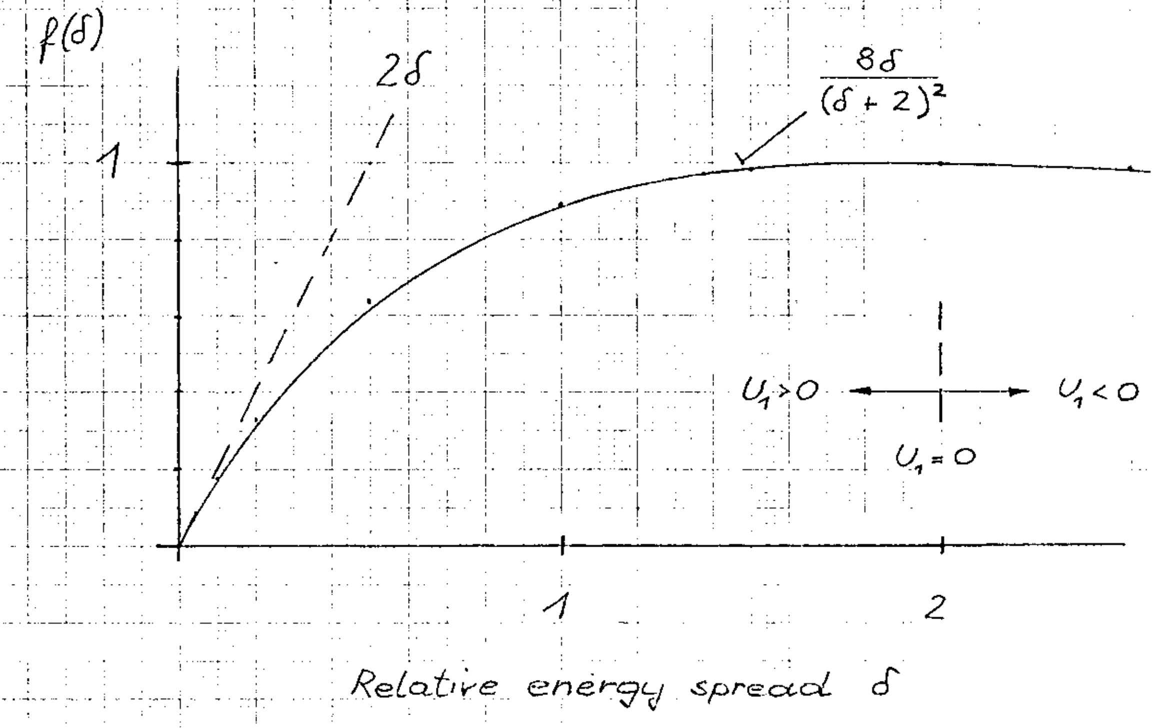

Fig. 1

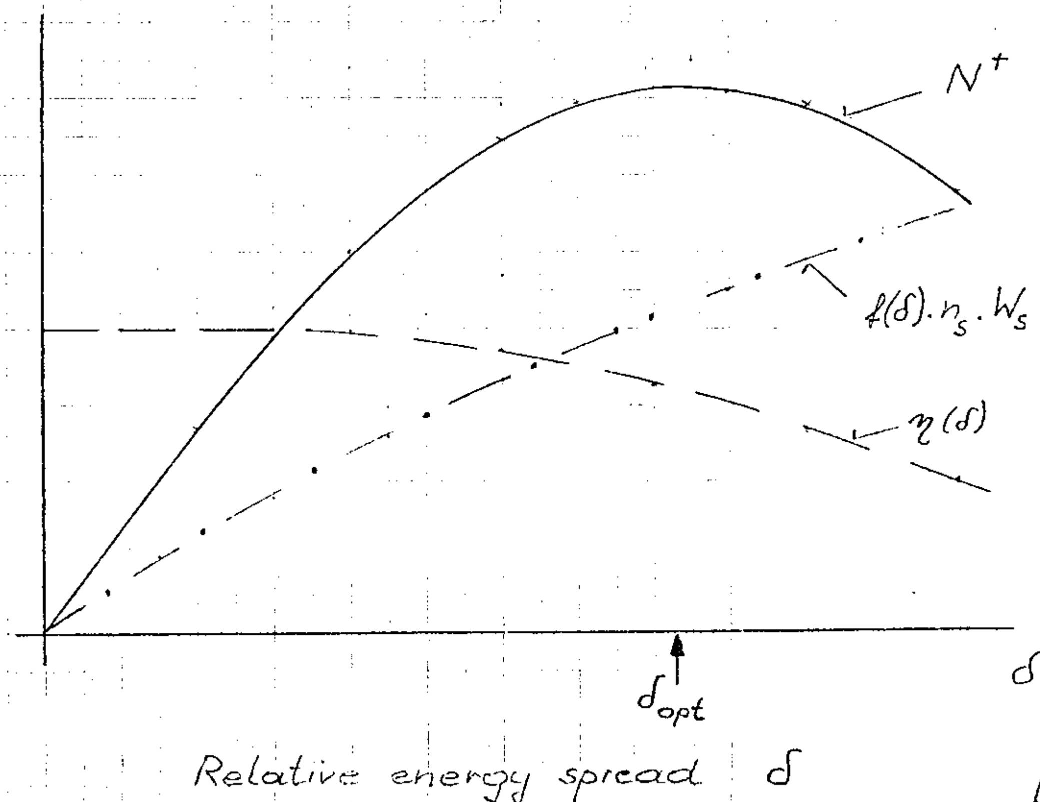

Fig. 2