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LEP Note 291

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ULTEC - A COMPUTER PROGRAM FOR A CHROMATICALLY CORRECTABLE LATTICE

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1. INTRODUCTION

ULTEC gives a chromatically correctable lattice which has arbitrarily chosen Q-values and almost 90° phase advances in a periodic unit. The lattice is characterized by suitable phase advances in a periodic unit and insertions. A set of sextupole strengths is also generated by ULTEC to correct natural chromaticity and chromatic error vectors.

In this report the chromatic error vector is defined as follows:

$$\overline{W}(So) = \tfrac{1}{2} \int_0^{So} \beta(s)\, K(s)\, e^{2i\mu(s) \pm i\frac{\pi}{2}}\, ds \qquad \begin{matrix}\text{upper sign horizontal} \\ \text{lower sign vertical}\end{matrix} \qquad (1)$$

k(s): the field gradient normalized by particle momentum

μ(s): linear phase advance

ß(s): betatron amplitude function

Physically this vector gives the linear variation of Twiss parameters with particle momentum.

TRACHRO is a computer program which calculates this vector from an existing lattice. This chromatic error vector must be compensated by the following correcting vector, which is produced by correction sextupoles, installed in the regular arc, where the dispersion function does not vanish.

$$\overline{T}(So) = \tfrac{1}{2} \int_0^{So} \beta(s)\, K'(s)\eta(s) e^{2i\mu(s) \mp i\frac{\pi}{2}}\, ds \qquad \begin{matrix}\text{upper sign horizontal} \\ \text{lower sign vertical}\end{matrix} \qquad (2)$$

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k'(s): strength of sextupole field normalized by particle momentum
$\eta(s)$: horizontal dispersion function

ULTEC calculates the correcting vector, using the thin lens approximation, and gives suitable phase advances in the insertions, which are more exactly defined by phase advances from the collision point to the first correcting sextupole.

In order to get good agreement between the calculated and the arbitrarily chosen Q-values, ULTEC changes the phase advances in a cell, calculates new Q's and repeats this operation until the desired accuracy is obtained.

The following assumptions are made:

(a) There is an original lattice, which is to be corrected. Chromatic error vectors of this lattice, calculated by TRACHRO, will be used in the following correction procedure.
(b) The chromatic error vectors should vanish at the centre of the regular arc.
(c) Four sextupole families should be used in each half arc. As there are two kinds of insertion in the LEP ring and resulting chromatic errors differ from one another, there should be a total of 8 sextupole families in the whole ring.

Among the input parameters for ULTEC, the order of strong-weak sextupoles and the values of modulus should be judiciously chosen with the utmost care to avoid large deviation of phase advances in a cell from 90° and large deviation of phase advances in the insertions from the original values.

The former requirement is necessary to maintain the condition for automatic cancellation of non-linear kicks by sextupoles in every 4 sextupoles.

The latter will help to match a linear lattice and will not change substantially the chromatic error vector from the original ones.

2. THEORY

The following equation is a contribution to the correcting vector from one correction unit which contains 4 sextupoles:

$$\overline{T}_{unit} = \tfrac{1}{2}\Big\{ \left| K'_{f1} \right| \ell sf \ \beta_f \eta_f \ e^{\mp \frac{\pi}{2} i}$$

$$+ \left| K'_{d1} \right| \ell sd \ \beta_d \eta_d \ e^{\pm \frac{\pi}{2} i + i\, 2\phi_{fd}}$$

$$+ \left| K'_{f2} \right| \ell sf \ \beta_f \eta_f \ e^{\mp \frac{\pi}{2} i + i\, 2\phi_c}$$

$$+ \left| K'_{d2} \right| \ell sd \ \beta_d \eta_d \ e^{\pm \frac{\pi}{2} i + i\, 2(\phi_c + \phi_{fd})} \Big\} \qquad (3)$$

$$= \left| \overline{T}_{unit} \right| \cdot e^{i\Omega} \qquad (4)$$

The upper and lower signs enter to the horizontal and vertical planes, respectively.

ϕ_c is the phase advance in a unit linear lattice and is assumed to be near to 90°.

ϕ_{fd} is the phase advance from a focusing to a defocusing sextupole; it is not always equal to half of ϕ_c because sextupoles are not placed symmetrically in a cell.

β and η denote the amplitude and the dispersion functions, respectively; their suffices f and d denote that the parameters are measured at the focusing and defocusing sextupoles.

K' is the strength of the sextupole normalized by the momentum of circulating particles.

The polarity of the sextupole is expressed by f and d, for focusing and defocusing respectively.

Figures 1 and 2 show the order in which the sextupoles are placed, starting from the collision point.

ℓ_{sf} and ℓ_{sd} are the lengths of the focusing and defocusing sextupoles.

The explicit forms of $|\overline{T}\ \text{unit}|$ and Ω are incorporated in the program.

There are N_{sx} correction units in each half arc. They make up the following vector

$$\overline{T}_{total} = e^{2i\phi_I} \cdot \overline{T}_{unit} \cdot \sum_{k=o}^{N_{sx}-1} e^{4i\phi_c \cdot k} \qquad (5)$$

ϕ_I is the linear phase advance from the collision point to the first sextupole. Performing the summation gives

$$\overline{T}_{total} = e^{2i\phi_I} \cdot \overline{T}_{unit} \cdot e^{2i(Nsx-1)\phi_c} \cdot g \qquad (6)$$

where

$$g = \begin{cases} \dfrac{\sin 2Nsx\ \phi_c}{\sin 2\phi_c} & \text{for } \phi_c \neq \frac{\pi}{2} \\ N_{sx} \left\{ - (-1)^{Nsx\ -1} \right\} & \text{for } \phi_c = \frac{\pi}{2} \end{cases} \qquad (7)$$

Finally, the vector can be expressed in terms of polar co-ordinates

$$\overline{T}_{total} = \left| \overline{T}_{unit} \right| \cdot |g| \cdot e^{2i\ \phi_I + (Nsx-1)\ \phi_c\ + i\Omega + \frac{\pi}{2}(1 - \frac{g}{|g|})} \qquad (8)$$

TRACHRO gives the chromatic error vector in polar form:

$$\overline{W} = |\overline{W}|\ e^{iw} \qquad (9)$$

The correction criterion is

$$\overline{W} + \overline{T}_{total} = 0 \qquad (10)$$

Using equations (8) and (9), this criterion can be expressed by two equations for absolute values and arguments,

$$\left|\overline{T}_{unit}\right| \cdot \left|g\right| = \left|\overline{W}\right| \qquad (11)$$

$$\phi_I = \frac{w}{2} - (Nsx-1)\ \phi_c - \tfrac{1}{2}\Omega + \frac{\pi}{4}\left(1 + \frac{g}{|g|}\right) + m\ \pi \qquad (12)$$

where m is an arbitrarily chosen integer value.

ULTEC solves the equations (11) for both the horizontal and vertical planes simultaneously by means of the NEWTON-RAPHSON method.

The unknown parameters are:

$$\left|K'_{f1}\right| - \left|K'_{f2}\right| \text{ and}$$

$$\left|K'_{d1}\right| - \left|K'_{d2}\right|.$$

$\left|K'_{f1}\right| + \left|K'_{f2}\right|$ and $\left|K'_{d1}\right| + \left|K'_{d2}\right|$ are given as input parameters for correcting the natural chromaticity.

There should be four solutions corresponding to the four possible arrangements of strong-weak sextupoles. However, this cannot be guaranteed when ϕ_c is far from 90°.

ULTEC chooses one solution out of the four according to the arrangement of the sextupoles, assigned by the input parameters.

After a set of strengths has been given, the calculation proceeds straight forwardly to equations (12) and gives ϕ_I including modulus m provided by input parameters.

The total tune, Q, of a ring is given by:

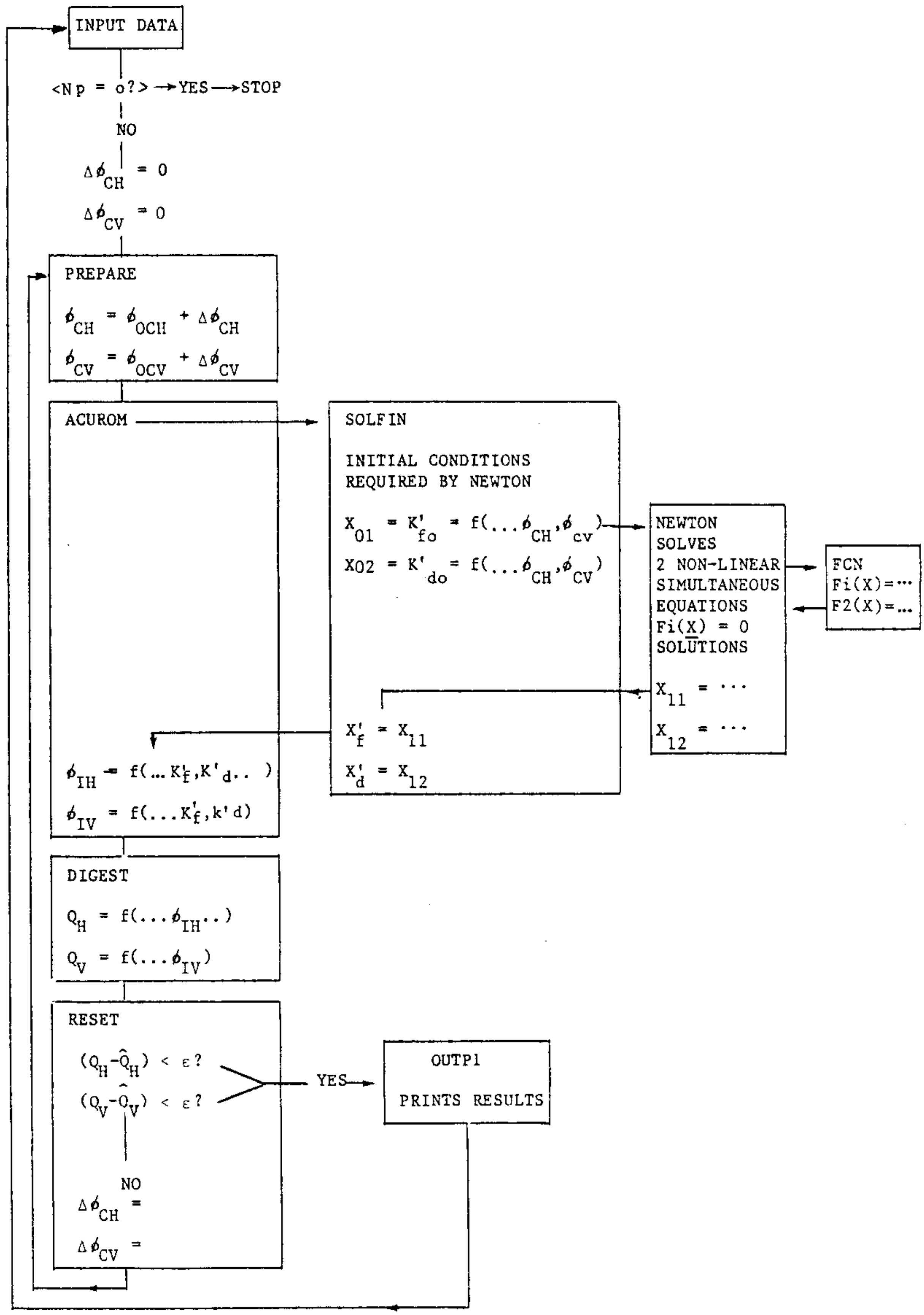
$$Q = 2\left\{\phi_{IS} + N_p \cdot \phi_c + \phi_{IL}\right\} \cdot N_{sp} \qquad (13)$$

ϕ_{IS} and ϕ_{IL} correspond to the phase advances in the different kinds of insertions, which are located on both sides of an arc.

Np and N_{sp} are the number of cells in each arc and the number of superperiods respectively.

The total tune Q is compared to the desired Q-value. If the agreement is not good enough then the whole calculation is repeated time and time again, each time with modified phase advances, until a required accuracy is obtained.

3. FLOW DIAGRAM FOR PROGRAM ULTEC



4. AN EXAMPLE OF A 7600 RUN

```
NAME.
ACCOUNT (name, group, account number)
CERNLIB.
FIND, OLDPL, ULTEC, CY = 1, ID = IS490HANN.
UPDATE (F,L = 0)
FTN (I = COMPILE)
LGO.
        END-OF-RECORD
        END-OF-RECORD
$ DATA   AWLH = 0.28545, AWLV = -0.2585, AWSH = -0.3094, AWSV = -0.26381
KHP = 0.19779825, KVP = 0.253459, LC = 79.0, LDDL = 1.46, LDDS = 1.46,
LFFL = 1.28, LFFS = -1.28, LSD = 0.76,LSF = 0.4, MODLH = 1.5,
MODLV = 2.0, MODSH = 1.5, MODSV = 1.5, NP = 37, NSX = 8,
PHI = 0.0197666, QHB = 102.3158, QVB = 106.5443, SOL = 1221, SOS = 2112,
UOH = 0.24558, UOV = 0.251, WHL = 14.24, WLV = 44.79, WSH = 24.15,
WSV = 41.16 $

$DATA  NP = 0 $

END-OF-FILE
```

5. (a) <u>INPUT</u>

VARIABLES IN NAMELIST $DATA

NAME FORTRAN	NAME SYMBOLIC	OF TYPE VARIABLE	UNITS	MEANING
				Argument of chromatic error vector* for:
AWLH	arg $(\overline{WH})_L$	REAL	rad/2π	The HORIZONTAL plane near a LONG insertion
AWLV	arg $(\overline{WV})_L$	"	"	" VERTICAL " " " LONG "
AWSH	arg $(\overline{WH})_S$	"	"	" HORIZONTAL " " " SHORT "
AWSV	arg $(\overline{WV})_S$	"	"	" VERTICAL " " " Short "
KHP	$\overline{K}'_F$	"	m^{-3}	Average strength of SF-magnets $\frac{1}{2}(\lvert K'_{f1}\rvert + \lvert K'_{f2}\rvert)$
KVP	$\overline{K}'_D$	"	m^{-3}	Average strength of SD-magnets $\frac{1}{2}(\lvert K'_{d1}\rvert + \lvert K'_{d2}\rvert)$
LC	L_C	"	m	Length of one linear periodic unit
LDDL	ℓ_{DDL}	"	m	Distance between centre of SD magnet and neighbouring QD magnet near a LONG insertion
LDDS	ℓ_{DDS}	"	m	" " " " SD " " " QD " " " SHORT "
LFFL	ℓ_{FFL}	"	m	" " " " SF " " " QF " " " LONG "
LFFS	ℓ_{FFS}	"	m	" " " " SF " " " QF " " " SHORT "
				NB. The sign is positive when the S-magnet is downstream of the Q-magnet otherwise it is negative.
LSD	ℓ_{SD}	"	m	Length of D sextupole magnet
LSF	ℓ_{SF}	"	m	" " F " "
MODLH	–	"	0.5	Modulus for HORIZONTAL phase advance in a LONG insertion
MODLV	–	"	0.5	" " VERTICAL " " " " LONG "
MODSH	–	"	0.5	" " HORIZONTAL " " " " SHORT "
MODSV	–	"	0.5	" " VERTICAL " " " " SHORT "
NP	N_P	INTEGER	1	Number of linear periodic structures in an arc (N_P = 37 for LEP Version 10)
NSX	N_{SX}	"	1	Number of sextupole magnets in each family
PHI	ϕ	REAL	rad	Total bending angle in one periodic unit
QHB	$\hat{Q}_H$	"	–	Desired value of the HORIZONTAL oscillations
QVB	$\hat{Q}_V$	"	–	" " " " VERTICAL "
SOL	–	INTEGER	–	A four digit number representing the order of the sextupole stengths near a LONG insertion
SOS	–	"	–	" " " " " " " " " " " " SHORT "
				Convention: 2 stands for strong and 1 for 'weak'. For example SOS = 2211 means that, a short insertion, the first SF and SD sextupole strengths are strong and the second SF and SD sextupole strengths are weak.
UOH	ϕ_{OCH}	REAL	rad/2π	Initial value at the HORIZONTAL phase advance in a cell
UOV	ϕ_{OCV}	"	"	" " " " VERTICAL " " " " "
WLH	$\overline{WH}_L$	"	–	Absolute value of chromatic error vector for the HORIZONTAL plane near a LONG insertion
WLV	$\overline{WV}_L$	"	–	" " " " " " " " VERTICAL " " " LONG "
WSH	$\overline{WH}_S$	"	–	" " " " " " " " HORIZONTAL " " " SHORT "
WSV	$\overline{WV}_S$	"	–	" " " " " " " " VERTICAL " " " SHORT "

* The vector is measured from the colliding point to the mid-point of the arc by the PROGRAM-TRACHRO described in LEP Note 165.

A data card $ DATA NP = 0 $ terminates the program.

5. (b) <u>ULTEC OUTPUT</u> 11/03/81

QH	= 102.31580	TOTAL TUNE IN HORIZONTAL DIRECTION
QV	= 106.54430	TOTAL TUNE IN VERTICAL DIRECTION
PH_{CH}	= .24542 RAD/2.PI	HORIZONTAL TUNE IN A NORMAL CELL
PH_{CV}	= .25325 RAD/2.PI	VERTICAL TUNE IN A NORMAL CELL
PH_{IHS} + MODHS	= 1.72100 RAD/2.PI	HORIZONTAL PHASE ADVANCE IN A SHORT INSERTION
PH_{IVS} + MODVS	= 1.83459 RAD/2.PI	VERTICAL PHASE ADVANCE IN A SHORT INSERTION
PH_{IHL} + MODHL	= 1.98788 RAD/2.PI	HORIZONTAL PHASE ADVANCE IN A LONG INSERTION
PH_{IVL} + MODVL	= 2.11316 RAD/2.PI	VERTICAL PHASE ADVANCE IN A LONG INSERTION
$\dot{K}F + \dot{K}FS$	= .24676 M^{-3}	MOMENTUM NORMALISED STRENGTH FOR THE FIRST SF IN SHORT SIDE
$\dot{K}F - \dot{K}FS$	= .14884 M^{-3}	MOMENTUM NORMALISED STRENGTH FOR THE SECOND SF IN SHORT SIDE
$\dot{K}D + \dot{K}DS$	= .15323 M^{-3}	MOMENTUM NORMALISED STRENGTH FOR THE FIRST SD IN SHORT SIDE
$\dot{K}D - \dot{K}DS$	= .35369 M^{-3}	MOMENTUM NORMALISED STRENGTH FOR THE SECOND SD IN SHORT SIDE
$\dot{K}F + \dot{K}FL$	= .17030 M^{-3}	MOMENTUM NORMALISED STRENGTH FOR THE FIRST SF IN LONG SIDE
$\dot{K}F - \dot{K}FL$	= .22530 M^{-3}	MOMENTUM NORMALISED STRENGTH FOR THE SECOND SF IN LONG SIDE
$\dot{K}D + \dot{K}DL$	= .35846 M^{-3}	MOMENTUM NORMALISED STRENGTH FOR THE FIRST SD IN LONG SIDE
$\dot{K}D - \dot{K}DL$	= .14845 M^{-3}	MOMENTUM NORMALISED STRENGTH FOR THE SECOND SD IN LONG SIDE

APPENDIX

Lattice parameters in the thin lens approximtion

In a cell of length L_c and bending angle ϕ, the lattice parameters used in ULTEC are calculated in the thin lens approximation.

If ϕ_{CH} and ϕ_{CV} are the horizontal and vertical phase advances in a cell, then the focal lengths of the focusing and defocusing quadrupoles, f_F and f_D are given by:

$$L_c/f_F = \tfrac{1}{2}\left\{\sqrt{B^2 + 4A} - B\right\} \qquad \text{A-1}$$

$$L_c/f_D = \tfrac{1}{2}\left\{\sqrt{B^2 + 4A} + B\right\} \qquad \text{A-2}$$

$$A = 4\left\{2 - \left(\cos\phi_{CH} + \cos\phi_{cv}\right)\right\} \qquad \text{A-3}$$

$$B = \cos\phi_{CH} - \cos\phi_{CV} \qquad \text{A-4}$$

The Twiss parameters at the exit from the quadrupoles can be expressed as follows:

$$\beta_{FH} = \frac{L_c}{\sin\phi_{CH}}\left(1 + \frac{1}{4}\frac{L_c}{f_D}\right) \qquad \text{A-5}$$

$$\beta_{FV} = \frac{L_c}{\sin\phi_{CV}}\left(1 - \frac{1}{4}\frac{L_c}{f_D}\right) \qquad \text{A-6}$$

$$\alpha_{FH} = \frac{1}{2\sin\phi_{CH}}\;\frac{-L_c}{f_F}\left(1 + \frac{1}{4}\;\frac{L_c}{f_D}\right) \qquad \text{A-7}$$

$$\alpha_{FV} = \frac{1}{2\sin\phi_{CV}}\;\frac{L_c}{f_F}\left(1 - \frac{1}{4}\;\frac{L_c}{f_D}\right) \qquad \text{A-8}$$

$$\beta_{DH} = \frac{L_c}{\sin\phi_{CH}}\left(1 - \frac{1}{4}\frac{L_c}{f_F}\right) \qquad \text{A-9}$$

$$\beta_{DV} = \frac{L_c}{\sin\phi_{CV}}\left(1 + \frac{1}{4}\frac{L_c}{f_F}\right) \qquad \text{A-10}$$

$$\alpha_{DH} = \frac{1}{2\,\sin\phi_{CH}} \cdot \frac{L_c}{f_D} \left(1 - \frac{1}{4}\frac{L_c}{f_F} \right) \qquad \text{A-11}$$

$$\alpha_{DV} = \frac{1}{2\,\sin\phi_{CV}} \quad \frac{-L_c}{f_D} \left(1 + \frac{1}{4}\frac{L_c}{f_F} \right) \qquad \text{A-12}$$

The values at the entrance of the quadrupoles are given by changing the sign of α.

The dispersion function takes the following values in a focusing and defocusing quadrupole,

$$\eta_F = \frac{1}{1-\cos\phi_{CH}} \left(1 + \frac{1}{8}\frac{L_c}{f_D}\right) \frac{\phi L_c}{2} \qquad \text{A-13}$$

$$\eta_D = \frac{1}{1-\cos\phi_{CH}} \left(1 - \frac{1}{8}\frac{L_c}{f_F}\right) \frac{\phi L_c}{2} \qquad \text{A-14}$$

If a focusing sextupole is located at a distance ℓ_{FF} downstream from a focusing quadrupole, and a defocusing sextupole is located at a distance ℓ_{DD} downstream from a defocusing quadrupole, then the phase advances from the quadrupoles to their adjacent sextupoles are given by the following:

$$\phi_{FFH} = \arctan \left\{ \ell_{FF} \Big/ \left(\beta_{FH} - \alpha_{FH} \cdot \ell_{FF} \right) \right\} \qquad \text{A-15}$$

$$\phi_{FFV} = \quad '' \quad \left\{ \ell_{FF} \Big/ \left(\beta_{FV} - \alpha_{FV} \cdot \ell_{FF} \right) \right\} \qquad \text{A-16}$$

$$\phi_{DDH} = \quad '' \quad \left\{ \ell_{DD} \Big/ \left(\beta_{DH} - \alpha_{DH} \cdot \ell_{DD} \right) \right\} \qquad \text{A-17}$$

$$\phi_{DDV} = \quad '' \quad \left\{ \ell_{DD} \Big/ \left(\beta_{DV} - \alpha_{DV} \cdot \ell_{DD} \right) \right\} \qquad \text{A-18}$$

When the sextupole position is upstream, the phase advances are given by changing the sign.

The phase advances from a focusing to a defocusing sextupole are given by:

$$\phi_{FDH} = \tfrac{1}{2}\,\phi_{CH} - \left(\phi_{FFH} - \phi_{DDH} \right) \qquad \text{A-19}$$

$$\phi_{FDV} = \tfrac{1}{2}\,\phi_{CV} - \left(\phi_{FFV} - \phi_{DDV} \right) \qquad \text{A-20}$$