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A SUMMARY OF RECENT WORK ON BEAM DYNAMICS AND COLLECTIVE PHENOMENA IN LEP VERSIONS 8 AND 9

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1. Introduction

This note gives a summary of recent work on beam dynamics and collective phenomena in LEP. This work was started in late 1980 and more or less completed in early 1980 before the reduction of the LEP circumference to about 27 km. It was undertaken with two particular objectives in mind, namely:

i) A review of the collective phenomena in LEP Phase 1, i.e. with only 1/6 of the RF system installed, operating in the neighbourhood of 50 GeV. This evaluation had not been done before.

ii) In the context of discussions about the choice of the RF frequency in LEP in the autumn of 1980 the question arose as to what frequency would be favoured for beam-dynamics reasons. This evaluation was carried out in parallel to the fresh look at the economic and engineering considerations in the choice between 350 and 500 MHz recently given by W. Schnell et al.

Since the techniques and data used in the two studies were the same, and since there was a large overlap in the results, we have decided to present all of them in one single note.

Chapter 2 summarizes our assumptions about the essential LEP lattice parameters for this study, the higher-mode losses, and the longitudinal and transverse impedances to be expected. The impedance estimates are based on more accurate calculations which have become available since the publication of the Pink Book1). They are higher than the previous values.

Chapter 3 contains our analysis of the phenomena associated with synchrotron oscillations and in particular with the synchrotron tune Q_s which increases with increasing RF frequency. There are several phenomena which become more serious when Q_s goes up.

Chapter 4 analyses collective phenomena. It contains our estimates for turbulent bunch lengthening and bunch widening, and for the coherent frequency shift of the dominant head-tail modes. Because of the higher impedances used, these effects are more serious than was estimated in the Pink Book.

Concerning the choice of the RF frequency we have arrived at the conclusion that the lower RF frequency, 350 MHz, is preferable, in all respects associated with the beam dynamics. This conclusion agrees with that reached by the economical and engineering study.

Compared to the estimates in the Pink Book the numbers for bunch lengthening and bunch widening have gone up slightly, and the numbers for the coherent tune shift have increased by a substantial factor. The reason for the large increase is the revised transverse impedance for the RF cavities.

The magnitude of the coherent tune shift depends on the fraction of RF cavities installed. In Phase 1 of LEP construction, with 272 m of RF cavities installed, the coherent tune shift remains below 0.5. In this case, the transverse impedance is dominated by the vacuum chamber. Vacuum chamber in this context denotes the whole LEP vacuum enclosure except the RF cavities, including transition pieces, pump ports, separator and other tanks, intersection chambers etc. The contribution of the smooth vacuum chamber to the impedances is actually quite small. The possibilities for changing the impedance by modifying the design of the vacuum chamber and/or the focusing in the arcs are quite limited. In addition, there is considerable uncertainty in the impedance scaling from PETRA which might be pessimistic.

With the full RF installation, 1629 m of cavities, the coherent tune shift approaches unity. In this case, the impedance is dominated by the RF cavities and can be computed quite accurately. Possibilities for reducing the coherent tune shift exist, such as increasing the size of the beam holes in the RF cavities, or increased focusing in the RF sections of the LEP lattice.

It is doubtful whether such large coherent tune shift can be handled in practice, but further work is needed to elucidate this problem.

2. Assumptions

In this chapter, we summarize our assumptions about the relevant parameters of the LEP lattice, about the higher-mode losses, and about the impedances presented to the beam by the LEP components.

2.1 Lattice parameters

We include two lattice configurations in this comparison which have essentially the same layout of magnetic elements.

The first configuration is that of the Pink Book[1) with a phase advance of 60° per lattice cell, while the second configuration has a phase advance of 90° and a smaller momentum compaction[2). The second lattice has a higher maximum energy and a smaller synchrotron tune Q_s in all stages of RF installation. These advantages are particularly noticeable at the higher energies. Therefore, it has been suggested that LEP should be operated with the 90° lattice. However, there are reasons to believe that a wide choice of tunes Q is desirable[3). This choice cannot be provided by a single configuration because of the strict rules for the phase advance per cell which must be followed in order to obtain an adequate correction of the chromatic effects. The relevant parameters of the two configurations can be found in refs. 1 and 2.

2.2 Higher-order mode losses

The higher-mode losses describe the energy losses of short intense bunches due to the excitation of electromagnetic fields in the RF cavities and similar objects of LEP; they are essential ingredients for the calculation of the RF voltage required, the RF power and Q_s.

The RF system and the vacuum chamber contribute to the higher-order mode losses. The BCI program[4) was used to compute the contribution of the RF cavities at 353 and 500 MHz. The cavities at both frequencies have a beam hole diameter of 10 cm and are optimized for maximum shunt impedance of the fundamental mode.

The higher-mode losses for the LEP vacuum chamber were obtained by scaling from measurements in PETRA. Their variation with bunch length was obtained from the resonator model used previously5). The resulting total higher-mode losses are shown in Figs. 1 and 2 as a function of the bunch length.

2.3 Impedances

The impedances describe the collective phenomena arising from the interaction of the beams in LEP with their surroundings.

2.3.1 Longitudinal broad-band impedance

The combined higher-mode impedance of the vacuum chamber and the RF cavities, at 353 and 500 MHz, was modelled by a single broad band resonator with a resonant frequency of f_r = 1.3 GHz and a quality factor Q = 1. For the vacuum chamber alone the shunt impedance of this resonator, which was chosen so that $|Z_L/n|$ (impedance at low frequencies divided by the mode number) has about the same value as that measured for the similar PETRA chamber. For the RF cavities this impedance was adjusted so that the loss parameter k_{pm} for a bunch length σ_s = 5 cm was the same as calculated with the program BCI. The resulting impedances are listed in Table 1.

2.3.2 Transverse broad-band impedance

The transverse broad-band impedance was estimated from the longitudinal resonator fit using the relation

$$Z_T(\pi) \simeq \frac{2c}{b^2} \frac{Z_L(\omega)}{\omega}$$

where the effective chamber radius is b = 0.04 m for the vacuum chamber and b = 0.055 m for the RF cavities. The latter number is 10% larger than the beam port of the RF cavities and has been estimated from an average value obtained for many computed transverse modes6). The resulting transverse shunt impedances $R_{st} = Z_t(\omega_r)$ are listed in Table 1 and have been used to calculated the transverse single-bunch effect. Later a better resonator fit was made directly to the transverse modes in the RF cavities giving the following values:

f_{rf}	(MHz)	353	500
fr	(GHz)	1.56	1.68
Q		1.26	1.03
R_{st} (full RF alone)	(MΩ/m)	14.1	17.3

Since these numbers are not very different from those listed in Table 1, the instability calculations were not repeated.

Table 1. Broad band impedance estimate for LEP

Resonator fit with f_r = 1.3 GHz, Q = 1.0

	k_{pm} V/pC (σ_s=5cm)	$R_{s\ell}$ MΩ	$\|Z_\ell/n\|$	R_{st} MΩ/m
Vacuum chamber	68.7	0.145	1.09	6.84
1/6 RF (353 MHz)	53.7	0.113	0.85	2.74
1/1 RF (353 MHz)	322.2	0.680	5.12	16.43
1/6 RF (500 MHz)	43.2	0.091	0.69	2.21
1/1 RF (500 MHz)	258.9	0.546	4.11	13.24
Total LEP Phase 1				
(353 MHz f-RF)	122.4	0.258	1.94	9.58
(500 MHz f-RF)	111.9	0.236	1.78	9.05
Total LEP Stage 1				
(353 MHz f-RF)	390.9	0.825	6.21	23.27
(500 MHz f-RF)	327.7	0.691	5.20	20.08

3. Phenomena associated with the synchrotron tune Q_s

3.1 Synchrotron tune Q_s vs energy and RF frequency

The minimum synchrotron tunes Q_s and several other longitudinal parameters as a function of energy, for the two lattices and the two RF frequencies under study, were obtained, using a self-consistent computer program[7)], by imposing a lower limit of 24 hours on the quantum lifetime. The figures apply to the accelerating mode of LEP in which the current required for e^+e^- collisions at the maximum energy is accelerated from injection energy upwards. Since there are many tables, we give a directory in Table 2.

Table 2. Directory of longitudinal parameter tables

Table No.	LEP Version	RF	MHz	Q_s
3	8	1/6	353	min
4	9	1/6	353	min
5	8	1	353	min
6	9	1	353	min
7	8	1	500	min
8	9	1	500	min
9	8	1/6	353	const
10	9	1/6	353	const
11	8	1	500	const
12	9	1	500	const
13	8	1	353	const
14	9	1	353	const

The notation in the tables is as follows:

V	Peak RF voltage
$_s$	stable RF phase angle
$_s$	rms bunch length
$_e/E$	rms bunch width
E/E	half height of RF bucket
Q_s	synchrotron tune
U_{vc}	higher-order mode losses into vacuum chamber
U_{rf}	higher-order mode losses into RF cavities

It may be necessary to keep Q_s constant while accelerating the beam from injection energy upwards in LEP, e.g. to avoid crossing synchro-betatron resonances. The resulting longitudinal parameters are also shown in the tables.

Table 3. Longitudinal parameters for LEP Version 8, 1/6 RF, 353 MHz

Current: 4.96 mA

Energy GeV	V_{rf} GV	ϕ_s °	σ_s cm	σ_e/E $\times 10^{-3}$	$(\Delta E/E)$ bucket $\times 10^{-2}$	Q_s	U_{vc} GeV	U_{rf} GeV
51.8	0.401	142.8	5.0	2.72	1.056	0.078	0.0086	0.00679
50.0	0.364	143.9	5.16	2.73	1.054	0.076	0.00806	0.00648
40.0	0.207	150.5	6.12	2.79	1.043	0.065	0.00523	0.00470
30.0	0.114	158.0	7.33	2.90	1.048	0.056	0.00314	0.00306
22.0	0.068	164.1	8.60	3.09	1.062	0.051	0.00194	0.00194
18.0	0.052	166.8	9.38	3.26	1.073	0.050	0.00148	0.00146

Table 4. Longitudinal parameters for LEP Version 9, 1/6 RF, 353 MHz

Current: 4.96 mA

Energy GeV	V_{rf} GV	ϕ_s °	σ_s cm	σ_e/E $\times 10^{-3}$	$(\Delta E/E)$ bucket $\times 10^{-2}$	Q_s	U_{vc} GeV	U_{rf} GeV
51.8	0.360	138.1	5.33	3.72	1.23	0.050	0.0074	0.0061
50.0	0.325	139.1	5.49	3.73	1.23	0.049	0.0069	0.0058
40.0	0.179	145.8	6.54	3.78	1.22	0.042	0.0043	0.0041
30.0	0.094	153.8	7.91	3.91	1.23	0.036	0.0025	0.0025
22.0	0.054	160.7	9.35	4.15	1.25	0.032	0.0015	0.0015
18.0	0.040	164.1	10.24	4.37	1.27	0.031	0.0011	0.0010

Table 5. Longitudinal parameters for LEP Version 8, 1 RF, 353 MHz

Current: 8.37 mA

Energy	V_{rf}	ϕ_s	σ_s	σ_e/E	$(\Delta E/E)$	Q_s	U_{vc}	U_{rf}
GeV	GV	°	cm	$x10^{-3}$	$x10^{-2}$		GeV	GeV
88.8	2.255	133.8	5.19	4.75	1.46	0.131	0.0134	0.065
80.0	1.680	136.7	5.65	4.79	1.46	0.121	0.0108	0.056
70.0	1.141	140.8	6.32	4.81	1.45	0.109	0.0081	0.044
60.0	0.746	145.6	7.14	4.84	1.44	0.097	0.00574	0.033
51.8	0.517	150.3	7.94	4.90	1.44	0.088	0.00419	0.025
50.0	0.474	151.4	8.14	4.92	1.44	0.086	0.00388	0.0232
40.0	0.290	157.9	9.41	5.06	1.45	0.077	0.00248	0.0146
30.0	0.170	164.5	11.08	5.34	1.45	0.069	0.00148	0.00755
22.0	0.101	169.4	13.1	5.70	1.41	0.062	0.00087	0.0029
18.0	0.073	171.6	14.55	5.96	1.37	0.058	0.00062	0.0011

Table 6. Longitudinal parameters for LEP Version 9, 1 RF, 353 MHz

Current: 8.37 mA

Energy	V_{rf}	ϕ_s	σ_s	σ_e/E	$(\Delta E/E)$ bucket	Q_s	U_{vc}	U_{rf}
GeV	GV	°	cm	$x10^{-3}$	$x10^{-2}$		GeV	GeV
88.8	2.044	127.7	5.61	6.43	1.58	0.082	0.011	0.057
80.0	1.496	130.3	6.13	6.45	1.57	0.076	0.0088	0.047
70.0	0.988	133.9	6.85	6.45	1.54	0.0675	0.0064	0.036
60.0	0.621	138.4	7.85	6.46	1.52	0.059	0.0043	0.026
51.8	0.411	142.9	8.81	6.51	1.51	0.053	0.0030	0.018
50.0	0.372	143.9	9.06	6.52	1.50	0.052	0.0028	0.017
40.0	0.207	150.4	10.74	6.63	1.46	0.044	0.0016	0.0087
30.0	0.108	158.2	13.02	6.90	1.45	0.038	0.00088	0.0031
22.0	0.059	164.7	15.7	7.30	1.41	0.033	0.00049	0.00031
18.0	0.041	167.7	17.6	7.6	1.38	0.031	0.00034	0.0

Table 7. Longitudinal parameters for LEP Version 8, 1 RF, 500 MHz

Current: 8.37 mA

Energy GeV	V_{rf} GV	ϕ_s °	σ_s cm	σ_e/E $x10^{-3}$	$(\Delta E/E)$ bucket $x10^{-2}$	Q_s	U_{vc} GeV	U_{rf} GeV
88.8	2.474	137.9	3.97	4.78	1.46	0.172	0.0236	0.0837
80.0	1.882	141.3	4.36	4.93	1.48	0.161	0.0197	0.0714
70.0	1.316	145.7	4.89	5.04	1.49	0.147	0.0154	0.0577
60.0	0.894	150.5	5.51	5.12	1.49	0.133	0.0116	0.0455
51.8	0.644	154.9	6.09	5.22	1.50	0.122	0.0089	0.0369
50.0	0.597	155.9	6.23	5.25	1.50	0.120	0.0084	0.0351
40.0	0.375	160.9	7.20	5.39	1.47	0.107	0.0056	0.0251
30.0	0.230	165.7	8.41	5.70	1.45	0.097	0.0035	0.0169
22.0	0.144	168.9	9.77	6.12	1.41	0.089	0.0022	0.0108
18.0	0.107	170.2	10.76	6.41	1.37	0.085	0.0016	0.0079

Table 8. Longitudinal parameters for LEP Version 9, 1 RF, 500 MHz

Current: 8.37 mA

Energy GeV	V_{rf} GV	ϕ_s °	σ_s cm	σ_e/E $x10^{-3}$	$(\Delta E/E)$ bucket $x10^{-2}$	Q_s	U_{vc} GeV	U_{rf} GeV
88.8	2.189	131.47	4.38	6.58	1.58	0.108	0.0195	0.0707
80.0	1.618	134.17	4.82	6.68	1.57	0.100	0.0158	0.0592
70.0	1.086	138.0	5.44	6.74	1.55	0.089	0.0120	0.0467
60.0	0.699	142.4	6.20	6.77	1.52	0.079	0.0085	0.0355
51.8	0.476	146.8	6.93	6.84	1.51	0.071	0.0063	0.0276
50.0	0.434	147.7	7.12	6.85	1.50	0.069	0.0058	0.0259
40.0	0.254	153.6	8.36	6.98	1.47	0.060	0.0036	0.0171
30.0	0.143	160.1	9.95	7.31	1.45	0.053	0.0021	0.0102
22.0	0.083	164.9	11.78	7.78	1.41	0.047	0.0012	0.0057
18.0	0.060	167.0	13.1	8.13	1.38	0.045	0.0009	0.0038

Table 9. Longitudinal parameters for LEP Version 8, 1/6 RF, 353 MHz

Current: 8.37 mA

Energy GeV	V_{rf} GV	ϕ_s °	σ_s cm	σ_e/E $x10^{-3}$	$(\Delta E/E)$ bucket $x10^{-2}$	τ_q h	U_{vc} GeV	U_{rf} GeV
51.8	0.401	142.8	5.00	2.72	1.056	24.0	0.00865	0.00680
50.0	0.378	145.3	5.10	2.76	1.11	342	0.00845	0.00669
40.0	0.273	157.4	5.44	2.97	1.39	∞	0.00707	0.00591
30.0	0.199	166.4	5.97	3.26	1.62	∞	0.0056	0.0049
22.0	0.146	171.0	6.60	3.60	1.75	∞	0.0043	0.0040
18.0	0.121	172.5	7.03	3.84	1.80	∞	0.0036	0.0034

Table 10. Longitudinal parameters for LEP Version 9, 1/6 RF, 353 MHz

Current: 8.37 mA

Energy GeV	V_{rf} GV	ϕ_s °	σ_s cm	σ_e/E $x10^{-3}$	$(\Delta E/E)$ bucket $x10^{-2}$	τ_q h	U_{vc} GeV	U_{rf} GeV
51.8	0.360	138.1	5.33	3.72	1.23	24.0	0.0074	0.0061
50.0	0.334	140.34	5.41	3.76	1.29	324.9	0.0072	0.0060
40.0	0.233	153.70	5.81	4.04	1.67	∞	0.0060	0.0052
30.0	0.166	164.15	6.37	4.43	2.01	∞	0.0047	0.0043
22.0	0.121	169.68	7.03	4.89	2.20	∞	0.0035	0.0034
18.0	0.100	171.54	7.49	5.21	2.27	∞	0.0030	0.0029

Table 11. Longitudinal parameters for LEP Version 8, 1 RF, 500 MHz

Current: 8.37 mA, $Q_s = 0.130$

Energy GeV	V_{rf} GV	ϕ_s °	σ_s cm	σ_e/E $x10^{-3}$	$(\Delta E/E)$ bucket $x10^{-2}$	τ_q h	U_{vc} GeV	U_{rf} GeV
88.8	2.474	137.9	3.97	4.78	1.47	24.0	0.0237	0.0837
80.0	2.039	144.4	4.15	5.00	1.68	∞	0.0218	0.0778
70.0	1.645	152.5	4.37	5.26	1.95	∞	0.0196	0.0711
60.0	1.343	159.8	4.62	5.57	2.21	∞	0.0174	0.0641
51.8	1.139	164.7	4.87	5.87	2.41	∞	0.0155	0.0581
50.0	1.096	165.6	4.93	5.94	2.45	∞	0.0151	0.0568
40.0	0.877	169.9	5.31	6.40	2.63	∞	0.0126	0.0490
30.0	0.666	172.5	5.84	7.03	2.75	∞	0.0100	0.0404
22.0	0.497	173.6	6.44	7.76	2.83	∞	0.0077	0.0326
18.0	0.412	173.9	6.86	8.27	2.86	∞	0.0064	0.0282

Table 12. Longitudinal parameters for LEP Version 9, 1 RF, 500 MHz

Current: 8.37 mA, $Q_s = 0.0824$

Energy GeV	V_{rf} GV	ϕ_s °	σ_s cm	σ_e/E $x10^{-3}$	$(\Delta E/E)$ bucket $x10^{-2}$	τ_q h	U_{vc} GeV	U_{rf} GeV
88.8	2.189	131.47	4.38	6.58	1.58	24.0	0.0195	0.0707
80.0	1.759	138.3	4.56	6.84	1.85	∞	0.0180	0.0660
70.0	1.377	147.2	4.78	7.18	2.23	∞	0.0162	0.0604
60.0	1.099	155.8	5.037	7.57	2.61	∞	0.0143	0.0545
51.8	0.921	161.7	5.29	7.95	2.89	∞	0.0130	0.0495
50.0	0.885	162.9	5.35	8.04	2.95	∞	0.0124	0.0483
40.0	0.702	168.1	5.76	8.65	3.22	∞	0.0103	0.0416
30.0	0.531	171.5	6.31	9.48	3.41	∞	0.0081	0.0341
22.0	0.396	173.0	6.9	10.46	3.52	∞	0.0062	0.0273
18.0	0.327	173.40	7.42	11.15	3.56	∞	0.0051	0.0233

Table 13. Longitudinal parameters for LEP Version 8, 1 RF, 353 MHz

Current: 8.37 mA, $Q_s = 0.172$

Energy (GeV)	V_{rf} (GV)	ϕ_s °	σ_s cm	σ_e/E $\times 10^{-3}$	$(\Delta E/E)$ bucket $\times 10^{-2}$	τ_q h	U_{vc} GeV	U_{rf} GeV
88.8	2.255	133.76	5.19	4.75	1.46	24.0	0.0134	0.0649
80.0	1.833	140.75	5.36	4.92	1.72	∞	0.0124	0.0613
70.0	1.454	149.6	5.61	5.14	2.05	∞	0.0111	0.0567
60.0	1.173	157.8	5.89	5.40	2.37	∞	0.0098	0.0515
51.8	0.988	163.4	6.17	5.66	2.61	∞	0.0086	0.0467
50.0	0.951	164.5	6.24	5.73	2.66	∞	0.0083	0.045
40.0	0.756	169.5	6.70	6.15	2.88	∞	0.0069	0.039
30.0	0.571	172.7	7.34	6.74	3.04	∞	0.0053	0.031
22.0	0.424	174.3	8.10	7.43	3.13	∞	0.0039	0.024
18.0	0.349	174.8	8.63	7.92	3.17	∞	0.0032	0.0194

Table 14. Longitudinal parameters for LEP Version 9, 1 RF, 353 MHz

Current: 8.37 mA, $Q_s = 0.108$

Energy (GeV)	V_{rf} (GV)	ϕ_s °	σ_s cm	σ_e/E $\times 10^{-3}$	$(\Delta E/E)$ bucket $\times 10^{-2}$	τ_q h	U_{vc} GeV	U_{rf} GeV
88.8	2.044	127.6	5.61	6.43	1.58	24.0	0.011	0.057
80.0	1.607	134.4	5.82	6.64	1.87	∞	0.010	0.053
70.0	1.230	143.8	6.08	6.93	2.30	∞	0.0090	0.048
60.0	0.962	153.2	6.38	7.28	2.75	∞	0.0079	0.043
51.8	0.797	160.0	6.69	7.63	3.10	∞	0.0069	0.039
50.0	0.764	161.3	6.76	7.71	3.16	∞	0.0067	0.038
40.0	0.601	167.6	7.26	8.28	3.51	∞	0.0055	0.032
30.0	0.451	171.6	7.95	9.07	3.75	∞	0.0042	0.025
22.0	0.333	173.75	8.78	10.01	3.89	∞	0.0031	0.018
18.0	0.274	174.51	9.36	10.68	3.94	∞	0.0025	0.015

3.2 Synchro-betatron resonances

Two driving mechanisms for synchro-betatron resonances are considered: dispersion in the RF cavities and transverse deflecting fields excited by the passage of the bunches.

3.2.1 Synchro-betatron resonances driven by dispersion

The growth rate $1/T_r$ for the synchro-betatron resonance

$$Q - Q_s = p\, N_{RF}$$

driven by dispersion in the RF cavities was calculated by Piwinski and Wrulich[8]. Here Q and Q_s are the betatron and synchrotron tunes, respectively, p is an integer and N_{RF} the number of equidistant RF stations. The growth rate can be written in the form:

$$\frac{1}{T_r} = N_{rf}\, f_o \left(1 - \cos \frac{2\pi Q_s}{N_{rf}}\right) \left(\frac{N_{rf} H_o}{\alpha C \sin \frac{2\pi Q_s}{N_{rf}}}\right)^{\frac{1}{2}}$$

Here, f_o is the revolution frequency, α the momentum compaction, C the circumference and H_o is the expression

$$H_o = \frac{1}{\beta} \{D^2 + (\beta D' - \tfrac{1}{2}\beta' D)^2\}$$

taken at an RF station. Here D is the dispersion and β is the betatron amplitude function. The prime denotes differentiation along the orbit. Note that H_o is an invariant in any section of magnetic lattice which does not include bending magnets.

If one imposes the condition that the vertical synchrotron damping time τ_y is smaller than T_r, i.e. if

$$\tau_y \leq T_r$$

one can obtain a tolerance for H_o:

$$H_o \leq \left(\frac{1}{N\,f_o\,\tau_y\,(1 - \cos 2\pi Q_s/N_{rf})}\right)^2 \left(\frac{\alpha\ C\ \sin 2\pi Q_s/N_{rf}}{N_{rf}}\right)$$

In a given machine, H_0 depends on the beam energy, because of the energy dependence of the damping time τ_y, and, to a lesser extent, of Q_s. For $Q_s << 1$ as will usually be the case, the dependence on N_{rf} is rather weak. It drops out altogether when the trigonometric functions are expanded.

The most stringent condition arises at injection energy when filling LEP. Precisely in order to avoid synchro-betatron resonances, the synchrotron tune Q_s should not be changed during energy ramping. It therefore has to be at its high-energy value already at injection energy. Sometimes it is more convenient to express these tolerances in terms of the vertical dispersion D_y:

$$D_y \approx (H\beta)^{\frac{1}{2}}$$

The relevant parameters and the tolerances on H_0 and D_y arrived at in this manner are shown in Table 15.

Table 15. Tolerances on H_0 and D_y in LEP at 22 GeV

Phase advance μ	$\pi/3$	$\pi/3$	$\pi/2$	$\pi/2$
Momentum compaction α	2.939×10^{-4}	2.939×10^{-4}	1.463×10^{-4}	1.463×10^{-4}
Vertical damping time τ_y/s	0.3433	0.3433	0.3044	0.3044
RF frequency f_{rf}/MHz	350	500	350	500
Synchrotron tune Q_s	0.1576	0.2044	0.1002	0.1283
Tolerance on H_0/µm	3.278	1.502	8.075	3.486
Max. vert. ß in RF section/m	84.7	84.7	78.2	78.2
Tolerance on vert. dispersion D_y/m	0.017	0.011	0.025	0.017

The tolerances on D_y must be compared to the expected values due to alignment errors which were calculated for LEP with phase advance $\pi/3$ to be $\langle D_y \rangle \approx 0.16$ m. Clearly the tolerances are much lower than the expected values and difficulties with synchro-betatron resonances are to be foreseen.

The theory used strictly applies only to the first satellite which can be avoided by the choice of the tunes. This is more difficult for the higher satellites

$$Q - nQ_s = pN_{RF}$$

where $n > 1$ is an integer. The relative strength of these satellites is known to decrease with n, the rate of decrease depending on the harmonic spectrum of the synchrotron oscillations due to potential-well distortions, or due to a higher-harmonic RF system if one is present.

Because of the rapid decrease of the damping time with increasing energy, the tolerable dispersion in the RF cavities quickly increases, and the strength of synchro-betatron resonances becomes smaller. This is one of the reasons why the injection energy in LEP should not be too low.

3.2.2 Transverse mode excitation

Synchro-betatron resonances can be excited through deflecting fields induced in a cavity by a bunch passing with a transverse displacement from the mode axis[9,10]. A basic feature of such deflecting modes is a longitudinal gradient of deflecting force related to the transverse gradient of accelerating force by Maxwell's equations. An off-axis bunch as a whole excites the deflecting modes whilst individual particles in the bunch are subjected to the resulting coupling forces between longitudinal and transverse motion. A detailed discussion of this effect is given in LEP Note 288 and the main results are summarized below.

For the first-order synchro-betatron satellite $Q_{x,y} \pm Q_s$ = integer, the equilibrium betatron amplitude $\hat{y}$ between resonant growth and radiation damping, arising from a single cell of an RF cavity, is given by

$$y = -\frac{Ne^2 \tau_y f_o \beta s}{2p} \cdot y_o \cdot \frac{R_t''L}{2Q} \cdot \frac{\omega^2}{c^2} \exp\left(-\frac{\omega^2 \sigma_z^2}{2c^2}\right)$$

where: N = number of particles in a bunch,
τ_y = transverse damping time,
f_o = revolution frequency,
β = betatron function at the RF cavity,
s = longitudinal displacement of a particle from the bunch centre,
p = beam momentum
y_o = transverse beam displacement from the axis,
$R_t''L$ = transverse impedance (Ωm^{-1})
Q = quality factor of mode
$\omega/2\pi$ = mode frequency
σ_z = standard deviation of bunch length

This formula applies for the case of large machines in the single-pass regime, where the fields decay substantially between bunch passages.

We use the transverse impedance for the lowest mode of a single LEP RF cell, for which:

$$R_t''L/2Q = 188.3 \quad (\Omega m^{-1})$$
$$\omega/2\pi = 5.65\text{x}10^8 \quad (\text{Hz})$$

We further assume an orbit error of:

$$y_o = 2 \text{ mm}$$

and a longitudinal excursion s of a particle equal to the bunch standard deviation σ_z. The results are shown in Table 16 for four examples.

Table 16. Synchro-betatron resonances driven by transverse modes

		LEP Stage 1	LEP Phase I		PETRA
N		$1.33\mathrm{x}10^{12}$	$0.75\mathrm{x}10^{12}$	$0.75\mathrm{x}10^{12}$	$2\mathrm{x}10^{11}$
cp	(GeV)	22	22	18	7
$s = \sigma_z$	(mm)	50	50	50	10
τ_y	(s)	0.3	0.3	0.42	0.1
ß	(m)	60	60	60	25
f_o	(s^{-1})	10^4	10^4	10^4	$1.3\mathrm{x}10^5$
$\hat{y}$	(mm)	0.58	0.33	0.56	0.12

Even for a single cell, LEP appears to be in a worse situation than that of PETRA, which is known to suffer appreciably from synchro-betatron resonances$^{11)}$. LEP however will have many more RF cells and the effects are likely to be still more serious, though the present calculation does not tell us by what factor.

The present model is inadequate to deal with extended RF systems, non-linear (higher-order) satellites and higher-order transverse modes, for which computer programs will have to be developed.

4. Collective phenomena

Based on the broad band impedances listed in Table 1 the single-bunch collective effects, bunch lengthening and frequency shifts have been calculated using the program BBI (version December 1980).

4.1 Bunch lengthening

Turbulent and potential well bunch lengthening have been calculated using the methods described in LEP Note 168$^{5)}$. This lengthening is accompanied by a shift of the incoherent phase oscillation frequency while the frequency of the coherent dipole mode is hardly changed. Due to the effect of the potential well the bunch length suffers a larger relative increase than the energy spread. The results are listed in Tables 17 and 18 for LEP having 1/6 or the full 353 MHz RF system. Table 19 gives the results for LEP Stage 1 with Q_s kept constant while accelerating from 22 to 80 GeV. In Table 20 the different phase advances per cell and the different RF frequencies are compared. The first part of each table gives the nominal parameters ignoring collective effects. The second part gives the estimated final bunch length and energy spread, the incoherent Q_s and the Q_{s1} of the coherent dipole mode.

4.2 Shift of the betatron tune

The reactive part of the transverse impedance produces a shift of betatron frequency of the different head-tail modes. This effect depends on the value of the beta function at the impedance in question. This was taken into account by using an effective Q-value obtained by weighting the beta value in the RF section and the vacuum chamber with the corresponding impedance. This results in $Q_{eff} = 78.5$ for the 90° lattice and full RF system, $Q_{eff} = 71$ for the 90° lattice and 1/6 RF system, and $Q_{eff} = 70$ for the 60° lattice and full RF system. The shifts of the vertical Q values are listed in Tables 17 to 20 for the different conditions.

Table 17. Single-bunch collective effects for LEP Phase 1 (1/6 RF system)

90° lattice, acceleration mode (I = 4.96 mA) for f_{rf} = 353 MHz

E		50	40	30	22	18
V_{rf}	GV	0.329	0.187	0.106	0.068	0.054
Q_{so}		0.065	0.057	0.051	0.048	0.047
σ_{so}	cm	1.22	1.37	1.53	1.60	1.58
σ_{eo}/E	%	0.111	0.109	0.109	0.107	0.105
collective effects						
σ_s	cm	6.0	7.0	8.3	9.6	10.3
σ_e/E	%	0.40	0.41	0.43	0.47	0.49
$Q_s$inc		0.048	0.042	0.037	0.035	0.034
Q_{s1}coh		0.062	0.059	0.053	0.051	0.050
ΔQ_y		-0.21	-0.23	-0.27	-0.33	-0.37

Table 18. Single-bunch collective effects for LEP Stage 1 (Full RF system)
90° lattice, acceleration mode ($I = 8.37$ mA) for $F_{RF} = 353$ MHz

E		80	70	60	50	40	30	22
V_{rf}	GV	1.43	0.96	0.63	0.41	0.27	0.19	0.15
Q_{so}		0.093	0.85	0.077	0.077	0.065	0.063	0.064
σ_{so}	cm	0.87	0.94	1.04	1.04	1.21	1.24	1.19
σ_{eo}/E	%	0.114	0.113	0.111	0.111	0.109	0.109	0.107
collective effects								
σ_s	cm	7.1	7.8	8.8	9.9	11.2	12.6	13.8
σ_e/E	%	0.68	0.68	0.69	0.70	0.73	0.80	0.89
$Q_s$inc		0.069	0.062	0.055	0.050	0.047	0.045	0.046
Q_{s1}coh		0.093	0.087	0.080	0.074	0.070	0.068	0.070
ΔQ_y		-0.43	-0.46	-0.48	-0.52	-0.57	-0.68	-0.86

Table 19. Single-bunch collective effects for LEP Stage 1 (Full RF system)
90° lattice, acceleration mode ($I = 8.37$ mA) for $f_{rf} = 353$ MHz
with $Q_s$inc kept constant. ΔQ_{y0} and ΔQ_{y1} are the shifts for the two lowest head-tail modes.

E		80	70	60	50	40	30	22
V_{rf}	GV	1.60	1.22	0.952	0.765	0.603	0.454	0.336
Q_{so}		0.109	0.110	0.110	0.111	0.111	0.112	0.113
σ_s	cm	0.75	0.74	0.72	0.71	0.70	0.69	0.67
σ_{eo}/E	%	0.114	0.113	0.111	0.111	0.109	0.109	0.107
collective effects								
σ_s	cm	6.3	6.6	6.9	7.3	7.8	8.6	9.5
σ_e/E	%	0.72	0.75	0.79	0.84	0.90	0.98	1.09
$Q_s$inc		0.0813	0.0814	0.0814	0.0815	0.0815	0.0816	0.0817
Q_{s1}coh		0.106	0.108	0.110	0.112	0.114	0.117	0.119
ΔQ_{y0}		-0.470	-0.522	-0.587	-0.674	-0.796	-0.981	-1.225
ΔQ_{y1}		-0.161	-0.188	-0.222	-0.268	-0.332	-0.431	-0.561

Table 20 Single-bunch collective effects for LEP Stage 1 (Full RF system) at top energy for 353 and 500 MHz RF frequency, 60° and 90° lattice

RF frequency	MHz	353	353	500
Phase adv./cell		60°	90°	90°
E	GeV	86.11	88.99	88.99
V_{rf}	GV	1.95	1.94	2.06
Q_{so}		0.158	0.097	0.124
σ_{so}	cm	1.12	0.84	0.66
σ_{eo}/E	%	0.124	0.115	0.115
I	mA	9.15	8.4	8.4
collective effects				
σ_s	cm	6.2	6.6	5.3
σ_e/E	%	0.51	0.66	0.69
$Q_s$inc.		0.117	0.071	0.094
Q_{s1}coh		0.152	0.094	0.114
ΔQ_y		-0.52	-0.41	-0.41

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