LEP Note 514
23 July 1984

Synchro-betatron Resonances due to Transverse Wakefields in Cavities

B. Zotter

Abstract

Synchro-betatron resonances due to various causes can be severe limitations of beam-current in large electron storage rings. In particular, they can be excited by transverse wakefields in cavities by off-axis particles. LEP will have a large number of RF cavities and we estimate the limitations by comparison with existing machines.

1. Introduction

This note is the summary of the second part of the presentation to the LEP Machine Advisory Committee on 7 and 8 March 1984 of work on synchro-betatron resonances. The first part by E. Keil on SBRs excited by dispersion in the cavities has appeared as LEP Note 505 (June 1984).

Synchro-betatron resonances ("SBRs") can be excited by bunched beams passing off-axis through cavities. This effect has been described by R. Sundelin[1)] and is therefore often called "Sundelin effect". An analytic treatment of linear resonances has been given by A. Piwinski[2)], and the strength of the effect in LEP was first estimated by B. Montague[3)]. The original conclusion was that LEP was likely to be more sensitive than PETRA. Fortunately, the parameters of LEP have been changed considerably since then, mainly in order to reduce the transverse mode-coupling instability, and thereby the situation has improved also for SBRs.

However, a new type of coherent SBRs has in the meantime been found[4)] which is excited even when the beams are perfectly centred in a cavity. These resonances - as all other SBRs - can be avoided essentially by the choice of betatron and synchrotron tunes and by a careful control of them during operation, in particular during injection and acceleration.

2. Sundelin Effect

Maxwell's equations show that transverse fields with a longitudinal gradient must be accompanied by longitudinal fields with a transverse gradient. The linear theory is valid at the first synchrotron sideband or "satellite". Then the transverse kick experienced by a particle traversing an excited cavity proportional to the longitudinal position in the bunch $\delta z' = A \cdot s$ while the energy change is proportional to the transverse position

$$\frac{\delta E}{E} = A \cdot z \ .$$

The proportionally constant A is the same in both cases. For a transverse resonance at frequency ω_r with impedance $R_\perp$ and quality factor Q one obtains

$$A = \frac{\omega_r^2 z_{co}}{f_0 c} \frac{R_\perp I_0}{QE/e} e^{-\frac{1}{2}\left(\frac{\omega_r \sigma}{c}\right)^2} \ .$$

It is thus proportional to the beam-current I_0 and to the transverse impedance over the quality factor $R_\perp/Q$ as well as to the transverse (closed-orbit) displacement z_{co}. For short bunches ($\omega_r\sigma/c \ll 1$) the exponential factor is nearly unity. The growth rate of the transverse displacement is then given by

$$\frac{dz}{dt} = \frac{A}{2} f_0 \langle\beta\rangle\sigma \ .$$

The bunch is stable if dz/dt is smaller than the transverse damping due to synchrotron radiation

$$\frac{dz}{dt} = \frac{z}{\tau_z} \ .$$

As radiation damping increases with amplitude, we can also define an "equilibrium amplitude" which becomes approximately

$$\frac{z_{eq}}{z_{co}} = \tau_z \cdot \frac{\langle\beta\rangle \omega_r R_\perp I_0}{2Q_\perp E/e} \frac{\omega_r \sigma}{c} .$$

In Table 1 we compare the equilibrium amplitudes of PETRA and LEP, both for the parameters used by B. Montague[3]) and for the present ones.

Table 1

Year	PETRA 1981	PETRA 1983	LEP 1981	LEP 1983
E (GeV)	7	7	20	20
I_b	4	8	2	0.75
β (m)	25	15	60	40
$R_\perp$ (Ω/m)	188	200	188	200
fr (GHz)	0.56	0.8*	0.56	0.56
τ_z (s)	0.1	0.1	0.3	0.3
σ (mm)	10	15	50	20
z_{co} (mm)	2	1	2	1
z_{eq} (mm)	0.11 [3])	0.13	0.72 [3])	0.04

* The lowest transverse resonance of a PETRA cavity has been scaled from the LEP cavity value (0.56 GHz) in the ratio of the RF frequencies (500/350)

These results are valid for a single resonance. For RF systems with a large number of cavity cells, each one having a large number of resonances, the situation will be much worse. However, as PETRA has now considerably more cells (60 × 5 + 56 × 7 + 152 = 842) than LEP, phase I (128 × 5 = 640) the comparison becomes only more advantageous for LEP.

Naturally, it would be senseless to operate any machine on the first satellite, and higher satellites are considerably weaker. Nevertheless, a good tune-control system which keeps both the coherent and the incoherent betatron frequency well between satellites is important. Such a system is under development for LEP[6]). In particular, the tunes change during accumulation at injection and during acceleration, but also in operation due to beam-loss. The synchrotron frequency would change in particular during acceleration, but the RF voltage in LEP will be programmed such that it remains approximately constant.

One of the cures proposed for overcoming the threshold due to transverse mode-coupling was to operate with increased synchrotron tune[7]. In that case, operation at constant tune would become impractical and resonances may have to be crossed. This can be done with little or no beam loss if it is done fast enough. Resonances can be jumped, e.g. by changing the phases in part of the RF cavities as has been demonstrated in PETRA, where this was necessary during acceleration with the higher-harmonic system, as this was not powerful enough to keep the synchrotron frequency constant up to top energy.

As can be seen from Table 1, the equilibrium amplitude in LEP has become much lower than in PETRA, while it was more than 6 times larger in the earlier estimate. This indicates that it should be easier to avoid these resonances in LEP than in PETRA. In PEP, SBRs do not seem to have been a problem, which is explained partially at least by the fact that PEP can inject at a higher energy and thus does not need to accelerate.

3. Coherent Resonances

During computer simulation of transverse stability of single bunches in LEP it was found that the threshold for transverse mode-coupling was very low whenever the (non-integer part of the) betatron tune was a multiple of the synchrotron tune. This occurred even in the absence of longitudinal fields (in the simulation they can be switched off easily to keep the bunch length constant) - and hence it was clearly a different mechanism from the Sundelin effect which is based on a closed loop of transverse and longitudinal interactions. The small asymmetry of a finite number of randomly distributed particles in a centred beam of non-zero height is sufficient to initiate a slow growth of the transverse beam dimension. Usually the beam remained centred about the symmetry axis, fluctuating with slowly increasing amplitude due to the increasing asymmetries as the beam-height increased.

This behaviour was clearly different from the much faster blow-up of both beam-height and displacement usually found for tunes located well between the resonances but at higher currents, although the border line between the two phenomena is rather smeared out. There is also a third type of behaviour, a very rapid growth of both beam height and displacement at low currents, for betatron tunes between an integer and the first (positive) synchrotron sideband.

Two-particle models using both distributed[8] and localized transverse wake potentials[5] also show similar resonances, which were explained by the fact that the longitudinal emittance is much larger than the transverse ones and energy is flowing even if the loop is not closed. Recent analytical work with localized impedances[9] suggest a somewhat different explanation in terms of coupling of higher satellites (e.g. m = 4 or m = 5) to the m = 0 mode (coherent betatron oscillation). This can happen since the frequency of the satellites is reflected by the integer or half-integer resonances. This conclusion appears to be supported by the first results of a multiparticle model[10] (typically 10 or 20 particles), where again the coupling of higher satellites to the m = 0 mode appears. This mechanism, however, is not valid in the two-particle model since only two modes are present there. Nevertheless the agreement of all these models is remarkable. Experimental verification is more difficult, as there are a number of competing mechanisms which lead to resonant blow-up at the synchrotron satellites, in particular closed-orbit errors and/or dispersion in the RF cavities (or any other element with large transverse impedance). In PETRA, one attempts to minimize the effects of SBRs by compensation using closed-orbit bumps in the RF cavities and dispersion bumps all around the ring. In that machine, the major contribution to the transverse impedance appears not to be due to the RF cavities and hence careful orbit correction will be important everywhere.

4. Cures for SBRs

LEP has about 12 times more circumference than the two largest electron storage rings in existence (PEP and PETRA). The transverse impedance is essentially proportional to the circumference, since the number of high-impedance elements (such as bellows, flanges, transitions, pick-ups, coupling slots, etc.) grows linearly in first approximation. Therefore, in LEP particular care has to be taken to reduce the impedance of each single element to lower values than has been done before. Since the larger circumference is coupled to a higher energy, also those elements increase which are related to energy, in particular RF cavities and separator plates, but also injector tanks because of higher injection energy. Low-impedance design of each single element is thus important in order to reduce the total transverse impedance. In addition, the following measures were already implemented or could be taken in case of need:

1) Reduce betafunctions in the RF-cavities. By adjusting the phase-advance in the RF lattice[11], the average betafunctions were reduced from over 60 m to about 40 m. In case of need, the vertical beta-function could be reduced even more - at the expense of the horizontal one - by powering the quadrupoles in the RF region unevenly[12].

2) Avoid crossing of resonances during accumulation and aceleration by careful control of both the coherent and the incoherent betatron tune as well as control of the synchrotron frequency[6]. The working point is chosen between two higher satellites but below the half-integer resonance[13].

3) A careful closed-orbit correction, in particular in the RF region. Present computer estimates put the RMS value there at 0.5 to 0.6 mm, with a concurrent reduction of the vertical dispersion to about 45 mm RMS[14].

4) The possibility of producing closed-orbit and/or dispersion bumps all around the machine, and in particular in the RF region, by a sufficient number of pick-ups and correctors in both planes. In case of need, correctors could be added in the RF region near the quadrupoles QS9 and QS10, in which case the beam-stoppers would have to be displaced[15].

5) Increase the synchrotron frequency by using a higher Rf voltage. This has already been done in LEP in order to increase the mode-coupling threshold, but it is limited by the available RF voltage at top energy if the synchrotron tune is to be kept constant, i.e. if one wants to avoid resonance crossing. Fast jumps across resonances by phase-changes in the RF cavities have been tested in PETRA, but not used operationally[16].

6) Low momentum compaction α, which can be achieved by special optics. This method has been used successfully in PETRA when all the availabe space was filled with RF cavities[17].

7) Increase of synchrotron frequency spread: using higher-harmonic cavities permits to make the synchrotron frequency zero in the bunch-centre while it is increased at the edges. This method is quite

effective in damping SBRs, but it is usually limited to the lower energy range because of restrictions in space for cavities and in RF power. At higher energies, satellites may have to be crossed as discussed in Section 5. Space for higher-harmonic cavities has been reserved in the RF region[7)].

8) Increase transverse damping with special wiggler magnets[18)] in the dispersion-free region. This may be a more economic alternative than 7) but needs more study.

Finally, SBRs (like most instabilities) are inversely proportional to energy, for which reason PEP - which injects at operating energy - is better off than PETRA which has to accelerate from 7 GeV injection energy. In LEP, the injection energy cannot be raised much due to limitations of the transfer lines and injection equipment, but also due to restrictions on top energy in the SPS in order to avoid radiation damage.

References

1) R. Sundelin, Trans. IEEE NS 26 (1978) p. 3604.
2) A. Piwinski, Proceedings 11th Conf. Part. Acc. (1980) p. 638.
3) B.W. Montague, LEP Note 288 (1981).
4) D. Brandt and B. Zotter, Proceedings 12th Conf. Part. Acc. (1983).
5) D. Brandt and B. Zotter, LEP Report 84-02 (1984).
6) S. Myers, LEP Note 499 (1984).
7) LEP Design Report, in preparation.
8) J. Jowett, LEP Note 474 (1983).
9) F. Ruggiero, Report in preparation.
10) M. Bassetti, private communication.
11) B. Zotter, LEP Note 448 (1983).
12) A. Hutton, LEP Note 464 (1983).
13) J. Jowett, LEP Note 494 (1984).
14) G. Guignard, private communication.
15) 27th meeting of the Lattice Layout Working Group, June 1984.
16) Minutes of LEP Advisory Committee Meeting, March 1984.
17) A. Piwinski and J. Rossbach, DESY-M-84-02.
18) A. Hofmann and J. Jowett, CERN/ISR-TH/81-23.