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THE POSSIBILITY OF MEASURING ANGLE γ USING $B^0 \to K^+\pi^-$ AND $\bar{B}^0 \to K^-\pi^+$ DECAYS AT HERA-B: Preprint ITEP 3-99/

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1 Introduction

One of the main goals of HERA-B is to measure angles β (through $B \to J/\psi K_S^0$ decay mode) and $\beta + \gamma$ (through $B \to \pi^+\pi^-$ mode using the high-p_T trigger) in the the unitarity triangle. Recently, there has been particular interest in the $B \to K^\pm\pi^\mp$ decays, which could provide information on the angle γ [1, 2]. This paper discusses the capability of HERA-B to measure these decays.

2 Method

The principal idea of extracting the angle γ from $B^0 \to K^+\pi^-$ and $\bar{B}^0 \to K^-\pi^+$ decays is the following [1, 2]. The two diagrams which contribute mainly to these decays are the penguin (Fig. 1) and the tree diagram (Fig. 2). Total amplitudes for $B \to K^\pm\pi^\mp$ decays can be written as

$$A_{K^+\pi^-} = -e^{i\delta_P}|P| - e^{i\gamma+i\delta_T}|T|, \qquad A_{K^-\pi^+} = -e^{i\delta_P}|P| - e^{-i\gamma+i\delta_T}|T|. \qquad (1)$$

The minus sign is due to the definition of meson states. Here P(T) is the amplitude for the penguin(tree) diagram, $\delta_P(\delta_T)$ is the strong phase and γ is the CP violating weak phase arising from a $V_{ub}V_{us}^*$ factor in the tree amplitude. The penguin contribution can be extracted

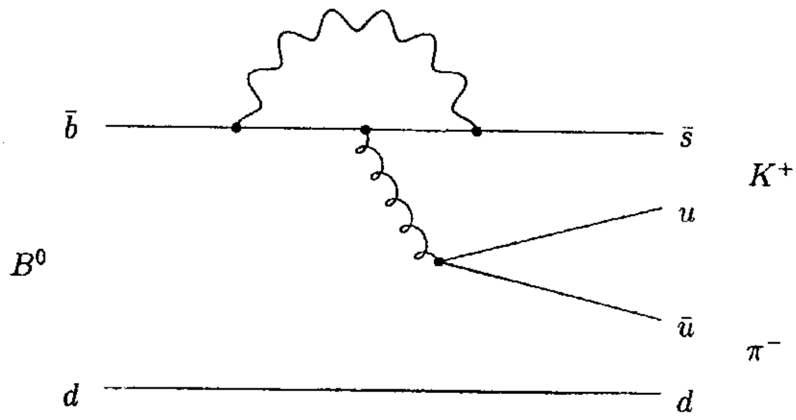

Figure 1: Penguin diagram for $B^0 \to K^+\pi^-$ decay.

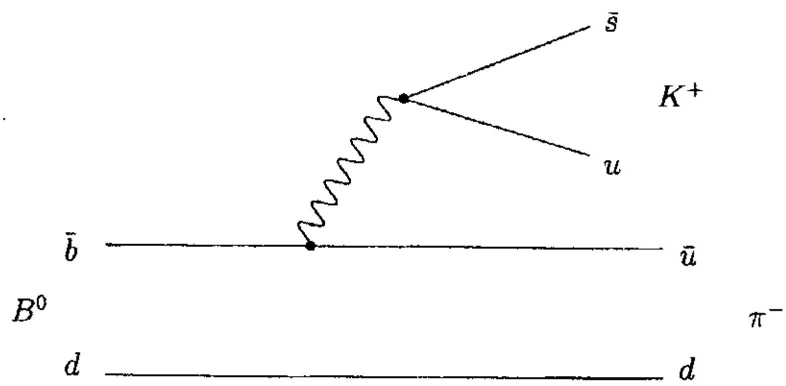

Figure 2: Tree diagram for $B^0 \to K^+\pi^-$ decay.

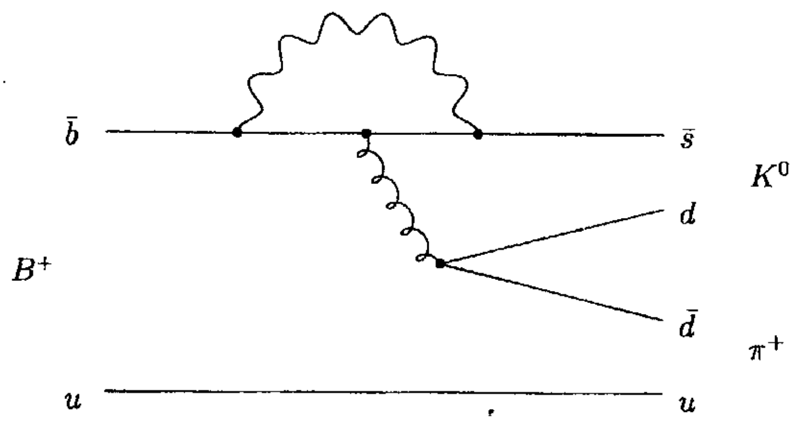

Figure 3: Penguin diagram for $B^+ \to K^0\pi^+$ decay.

using $B^+ \to K^0\pi^+$ ($B^- \to \bar{K^0}\pi^-$) decay modes where there is no corresponding tree diagram (Fig. 3). Isospin SU(2) symmetry of strong interactions implies:

$$A_{K^0\pi^+} = A_{\bar{K^0}\pi^-} = e^{i\delta_P}|P|. \qquad (2)$$

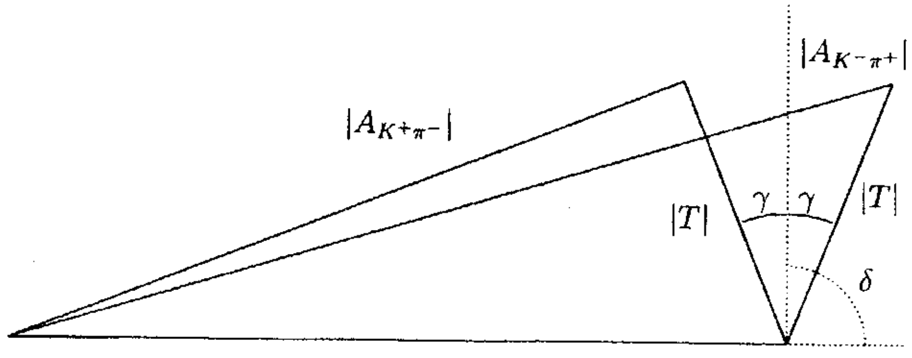

Figure 4: Triangles.

Therefore combining (1) and (2) one gets:

$$e^{-i\delta_P}A_{K^+\pi^-} + |A_{K^0\pi^+}| + e^{i\gamma+i\delta}|T| = 0, \qquad e^{-i\delta_P}A_{K^-\pi^+} + |A_{K^0\pi^+}| + e^{-i\gamma+i\delta}|T| = 0, \quad (3)$$

where δ is equal to $\delta_T - \delta_P$. These equations can be represented in the complex plane as two triangles (Fig. 4). They have the common base ($|A_{K^0\pi^+}| = |A_{\bar{K^0}\pi^-}|$) and their right sides have the same length ($|T|$). $|A_{K^0\pi^+}|$, $|A_{K^+\pi^-}|$ and $|A_{K^-\pi^+}|$ can be determined experimentally from the corresponding $Br(B^+ \to K^0\pi^+)$, $Br(B \to K^\pm\pi^\mp)$, e.g. for $Br(B \to K^\pm\pi^\mp)$ decay: $Br = \frac{(2\pi)^4}{2M_B}\tau_{B^0}|A|^2\Phi = k|A|^2$, where Φ is the phase space factor, $k = 8.8 \cdot 10^9$ GeV^{-2}. The color-allowed tree amplitude $|T|$ can be estimated with reasonable accuracy from theory, e.g. BSW [3] model predicts: $|T| = a_1 \cdot \left(\frac{|V_{ub}|}{3.2 \cdot 10^{-3}}\right) \cdot 7.8 \cdot 10^{-9} = 7.9 \cdot 10^{-9}$ GeV [4]. The form factor used in the model can be checked using $B \to \pi l \nu$ semileptonic decay. Another way to estimate $|T|$ is to use $B^+ \to \pi^+\pi^0$ decay. If one assumes factorization, parameterizes the SU(3) breaking by the factor $\frac{f_K}{f_\pi}$ and neglects the color-suppressed diagram for this decay it is possible to relate $|T|$ and $B^+ \to \pi^+\pi^0$ decay amplitude: $|T| = \sin\theta_C \frac{f_K}{f_\pi}\sqrt{2}|A(B^\pm \to \pi^\pm\pi^0)|$, where θ_C is the Cabibbo angle. Given the four triangle lengths one can draw both triangles and calculate the 2γ angle between their right sides.

Note that this can be done only with eightfold ambiguity. Fourfold ambiguity arises due to the fact that the triangles can be flipped around the triangles base $|A_{K^0\pi^+}| = |P|$ without changing their side lengths. In terms of γ and δ any of these flippings can be expressed by the combination of the following two transformations:

- exchanging of γ, δ: $\gamma \leftrightarrow \delta$ and
- the substitutions $\delta \to -\delta$, $\gamma \to -\gamma$.

Another twofold ambiguity is due to the fact that the 2γ (or 2δ) angle can always be substituted by $2\gamma + n \cdot 360^\circ$ ($2\delta + n \cdot 360^\circ$) where n is any integer number. Therefore one should add to the two transformations above the third one:

- $\gamma \to \gamma + 180^\circ$.

Combining the second and the third transformations together one can write the first 4 possible solutions in the form: $\gamma = \gamma_0$, $-\gamma_0$, $180^\circ + \gamma_0$, $180^\circ - \gamma_0$ or $\pm\gamma_0 + n \cdot 180^\circ$. For any $\gamma_0 \neq m \cdot 90^\circ$ where m is any integer number they correspond to the 4 points on the circle with unit radius situated in 4 different quadrants. The second 4 possible solutions are obtained when γ_0 is exchanged with δ_0: $\gamma = \delta_0$, $-\delta_0$, $180^\circ + \delta_0$, $180^\circ - \delta_0$. If $\gamma_0, \delta_0 \neq m \cdot 90^\circ$ there are exactly 2 solutions in the range $0^\circ < \gamma < 90^\circ$.

The presently allowed range for γ is [5]

$$\gamma = (67^{+11}_{-12})^\circ \tag{4}$$

Therefore at least 6 out of 8 solutions (negative and greater than 90°) should be discarded.

The asymmetry between $B^0 \to K^+\pi^-$ and $\bar{B}^0 \to K^-\pi^+$ branching ratios can be expressed by the quantity

$$\mathcal{A}_{CP} = \frac{Br(B^0 \to K^+\pi^-) - Br(\bar{B}^0 \to K^-\pi^+)}{Br(B^0 \to K^+\pi^-) + Br(\bar{B}^0 \to K^-\pi^+)} = \frac{-2|P||T|\sin\delta\sin\gamma}{|P|^2 + |T|^2 + 2|P||T|\cos\delta\cos\gamma}, \tag{5}$$

where we used $|A_{K^\pm\pi^\mp}|^2 = |P|^2 + |T|^2 + 2|P||T|\cos(\delta \pm \gamma)$. CLEO has measured recently the average branching ratio $\mathcal{B} = \frac{1}{2}(Br(B^0 \to K^+\pi^-) + Br(\bar{B}^0 \to K^-\pi^+)) = (1.4 \pm 0.3 \pm 0.2) \cdot 10^{-5}$ [6]. Combining it with $|T| = 7.9 \cdot 10^{-9}$ GeV and neglecting the terms $O((|T|/|P|)^2)$ one gets:

$$\mathcal{A}_{CP} = -\frac{2|P||T|\sin\delta\sin\gamma}{\mathcal{B}/k} = -\frac{2|T|}{\sqrt{\mathcal{B}/k}}(1 - \frac{|T|}{\sqrt{\mathcal{B}/k}}\cos\delta\cos\gamma) \cdot \sin\delta\sin\gamma$$

$$= -0.40(1 - 0.20\cos\delta\cos\gamma)\cdot\sin\delta\sin\gamma. \qquad (6)$$

Thus the maximal value of $\mathcal{A}_{CP}$ is 40%. The term in the brackets should be larger than 0.80 while $\sin\delta$ is not known.

It should be clear that if the CP violating asymmetry $\mathcal{A}_{CP}$ is found to be zero, γ still can be extracted from the triangles in Fig. 4. In this case the triangles are identical. As it can be seen from (6) or directly from the Fig. 4 two possible solutions are $\gamma = 0°, 180°$ which mean the absence of the CP-violation. They contradict the presently allowed range for γ (4) and should be discarded. However if the triangles are not degenerate there is always one nontrivial solution in the range $0° < \gamma \leq 90°$. It corresponds to the case $\delta = 0°$ or $\delta = 180°$ when the triangles are on the opposite sides of the triangles base.

An interesting situation arises when $Br(B^\pm \to K\pi^\pm)$ is larger than $Br(B \to K^\pm\pi^\mp)$. In this case even upper limits on $Br(B \to K^\pm\pi^\mp)$ are of particular importance. As it can be seen from Fig. 5 they allow the following constraints to be put on γ: $(0 < \gamma < \tilde{\gamma}$ or $180° - \tilde{\gamma} < \gamma < 180°)$, where $\tilde{\gamma}$ is shown in Fig. 5. This can be done without knowing the amplitude of the tree diagram $|T|$. These constraints are complementary to the presently allowed range (4).

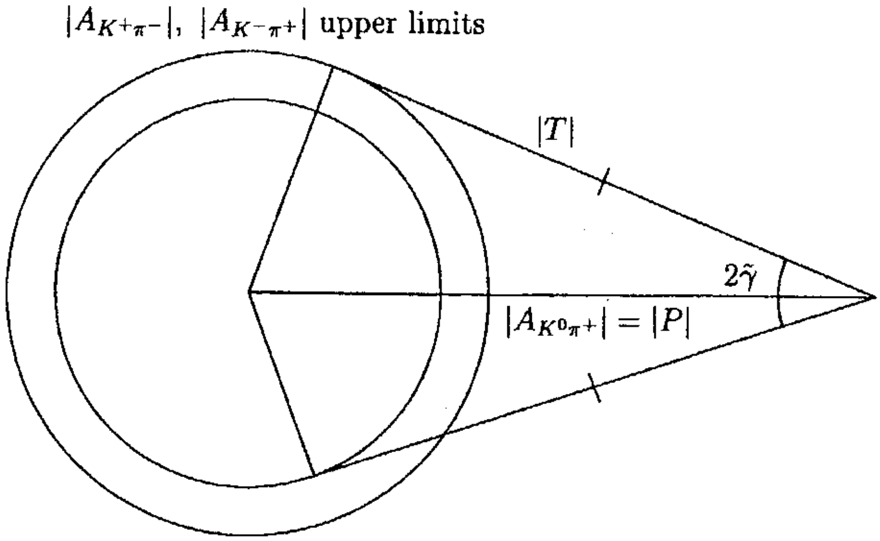

Figure 5: Triangles in the case when $Br(B^\pm \to K\pi^\pm) > Br(B \to K^\pm\pi^\mp)$.

In view of the recent CLEO results it was pointed out in the literature [2] that such constraints can be derived even in the case when only the average branching ratio $B = \frac{1}{2}(Br(B^0 \to K^+\pi^-) + Br(\bar{B}^0 \to K^-\pi^+))$ is known and the ratio $R = \frac{B}{Br(B^\pm \to K\pi^\pm)}$ is smaller than 1. The worst constraint with the largest $\tilde{\gamma}$ is obtained if one assumes that $Br(B^0 \to K^+\pi^-)$ and $Br(\bar{B}^0 \to K^-\pi^+)$ are equal. In this case two circles in Fig. 5 are identical and the maximum possible angle $\tilde{\gamma}$ is simply $\arcsin \frac{|A_{K^\pm\pi^\mp}|}{|P|} = \arcsin \sqrt{R}$. Taking the current CLEO values $\mathcal{B} = (1.4 \pm 0.3 \pm 0.2) \cdot 10^{-5}$ and $Br(B^\pm \to K\pi^\pm) = (1.4 \pm 0.5 \pm 0.2) \cdot 10^{-5}$ [6] and adding the errors in quadrature one gets $R = 1.00 \pm 0.46$.

One should note that the method of γ determination briefly discussed in this section can suffer from additional theoretical corrections caused by effects of the color-suppressed electroweak penguin terms in $B \to K^\pm\pi^\mp$ and $B^+ \to K^0\pi^+$ and effects of the annihilation amplitude in $B^+ \to K^0\pi^+$. Their discussion can be found elsewhere [7, 1, 2].

3 HERA-B capabilities

The decays $B \to K^\pm\pi^\mp$ can be selected by the HERA-B high-p_T trigger just as decays $B \to \pi^+\pi^-$ since they have almost the same kinematics. Thus no special redesign of HERA-B is required. But in contrast to the analysis of the $\pi^+\pi^-$ mode the measuring of $Br(B \to K^\pm\pi^\mp)$ requires neither tagging nor time dependent measurements.

Assuming the CLEO value $\mathcal{B} = (1.4 \pm 0.3 \pm 0.2) \cdot 10^{-5}$ the expected number of $B \to K^\pm\pi^\mp$ events passed the trigger level and vertex cut during one year of HERA-B operation is

$$I \cdot T \cdot \frac{\sigma_{b\bar{b}}}{\sigma_{inel}} \cdot (P_{\bar{b}\to B^0} + P_{b\to \bar{B}^0}) \cdot \mathcal{B} \cdot \mathrm{eff}_{trig} \cdot \mathrm{eff}_{\mathrm{VX}} =$$

$$40\ MHz \cdot 10^7\ s \cdot 10^{-6} \cdot 0.8 \cdot 1.4 \cdot 10^{-5} \cdot 0.28 \cdot 0.57 \approx 720. \tag{7}$$

Note that the B^0 and $\bar{B}^0$ production rates are expected to be slightly different: $\frac{P_{\bar{b}\to B^0} - P_{b\to \bar{B}^0}}{(P_{\bar{b}\to B^0} + P_{b\to \bar{B}^0})/2} \approx 2 - 3\%$ so that if $Br(B^0 \to K^+\pi^-) = Br(\bar{B}^0 \to K^-\pi^+)$ the expected numbers of events with $B^0 \to K^+\pi^-$ and $\bar{B}^0 \to K^-\pi^+$ decays are different too: approximately 360 and 350 respectively. This should be taken into account in asymmetry $\mathcal{A}_{CP}$ determination.

The efficiency to identify $K(\pi)$ in the $K\pi$ pair is found to be 44%. This value is obtained by convoluting the kaon momentum spectrum with the RICH kaon identification efficiency $\mathrm{eff}^{\mathrm{id}}_{\mathrm{K}}(\mathrm{p})$ from [8]. The pion misidentification probability is 5%. The pion identification efficiency is assumed to be the same as that of the kaon: $\mathrm{eff}^{\mathrm{id}}_{\pi}(\mathrm{p}) = \mathrm{eff}^{\mathrm{id}}_{\mathrm{K}}(\mathrm{p})$. The value of 44% corresponds approximately to the sharp momentum cut $p_{K(\pi)} < 50$ GeV/c. The probability to identify both K and π is 24%. In order to increase statistics we shall also consider, during analysis, partially reconstructed events selected in the following way. One particle (K or π) is identified, another (π or K respectively) is not and in addition $p_{\pi(K)} > 50$ GeV/c. The last momentum requirement reduces soft background. The efficiency for such events is 14% when K is identified and 14% when π is identified. Note that all three samples which have efficiencies of 24%, 14% and 14% are statistically independent.

There are two sources of background: the combinatorial background and the reflections from other two-body $B_{(s)}$ decays. The main contribution to the combinatorial background like in the $B \to \pi^+\pi^-$ case comes from minimum bias events. Table 1 shows the number of minimum bias events (out of 32 millions of generated events) after the following trigger cuts: $p_T^{1,2} > 1.5$ GeV/c, $M_{1,2} > 4.5$ GeV/c^2, energy asymmetry cut $\frac{|E_1-E_2|}{E_1+E_2} < 0.5$ (first row) and after an additional B^0 mass window cut $|M_{1,2} - M_{B^0}| < 50$ MeV/c^2 (second row; to increase statistics the number of events here was taken as 1/4 of the number of events in the range $|M_{1,2} - M_{B^0}| < 200$ MeV/c^2). The detector resolution is expected to be 35 MeV/c^2 so that this window corresponds approximately to $\pm 1.5\sigma$. An unidentified particle is denoted in the table by an X, the $\pi^+\pi^-$ case is given for a comparison.

Mass cut, GeV/c^2	$K^+\pi^-$	$K^-\pi^+$	K^+X^-	K^-X^+	π^+X^-	π^-X^+	$\pi^+\pi^-$
$M_{1,2} > 4.5$	277	147	91	64	265	346	931
$\|M_{1,2} - M_{B^0}\| < 0.05$	15.8	5.5	3.8	3	11	14.8	41.5

Table 1: Background from minimum bias events.

The average background suppression factors for the samples with fully identified $K\pi$ pairs, with identified K and unidentified π, $p_\pi > 50$ GeV/c and vice versa are

$$\frac{1}{3.2 \cdot 10^7}(10.6;\ 3.4;\ 12.9) \approx (1;\ 0.3;\ 1.2) \cdot 3.3 \cdot 10^{-7}. \quad (8)$$

In addition, one should apply a secondary vertex cut and cuts using the presence of a second B in the event. The signal to background ratio in $M_{B^0} \pm 50$ MeV/c^2 mass window in case of full $K\pi$ identification is:

$$\frac{S}{B} = \frac{\sigma_{b\bar{b}} \cdot \frac{1}{2}(P_{\bar{b}\to B^0} + P_{b\to \bar{B}^0}) \cdot \mathcal{B} \cdot \mathrm{eff}^{\mathrm{S}}_{\mathrm{trig}} \mathrm{eff}^{\mathrm{S}}_{\pm 50} \mathrm{eff}^{\mathrm{S}}_{\mathrm{VX}} \mathrm{eff}^{\mathrm{S}}_{\mathrm{id}} \mathrm{eff}^{\mathrm{S}}_{\mathrm{2d\ B}}}{\sigma_{inel} \cdot 3.3 \cdot 10^{-7} \cdot \mathrm{eff}^{\mathrm{B}}_{\mathrm{VX}} \mathrm{eff}^{\mathrm{B}}_{\mathrm{2d\ B}}} =$$

$$= 10^{-6} \cdot \frac{0.4 \cdot 1.4 \cdot 10^{-5} \cdot 0.28 \cdot 0.85 \cdot 0.57 \cdot 0.24 \cdot \mathrm{eff}^{\mathrm{S}}_{\mathrm{2d\ B}}}{3.3 \cdot 10^{-7} \cdot \mathrm{eff}^{\mathrm{B}}_{\mathrm{VX}} \mathrm{eff}^{\mathrm{B}}_{\mathrm{2d\ B}}} = \frac{\mathrm{eff}^{\mathrm{S}}_{\mathrm{2d\ B}}}{1.8 \cdot 10^{6} \cdot \mathrm{eff}^{\mathrm{B}}_{\mathrm{VX}} \mathrm{eff}^{\mathrm{B}}_{\mathrm{2d\ B}}}, \quad (9)$$

where eff^S, eff^B are efficiencies for signal and background. $\mathrm{eff}^{\mathrm{B}}_{\mathrm{VX}}$ and $\mathrm{eff}^{\mathrm{B}}_{\mathrm{2d\ B}}$ are not known yet, but to keep S/B around unity they should provide a suppression factor of the order of $1/(1.8{\cdot}10^6)$. In the following we assume that the ratio between the backgrounds (8) remains the same after applying all the cuts.

In case of partial identification one should take into account reflections from other two-body $B_{(s)}$ decays. The kinematics of such reflections is discussed in the *Appendix*. If one identifies only K^+ there is a reflection from $B_s \to K^+K^-$ when K^- is misidentified as π^-. This produces a bump 60 MeV/c^2 higher than M_{B^0}. In case of π^- identification there is a reflection from $B \to \pi^+\pi^-$ which produces a bump 30 MeV/c^2 higher than M_{B^0}. Final state in the decay $B \to \pi^+\pi^-$ ($B_s \to K^+K^-$) is totally symmetric relative to the change $\pi^+ \leftrightarrow \pi^-$ ($K^+ \leftrightarrow K^-$). Therefore the reflections are of the same magnitude for $B^0 \to K^+\pi^-$ and $\bar{B}^0 \to K^-\pi^+$ and do not produce any shift in asymmetry $\mathcal{A}_{CP}$. They can be considered like the combinatorial background.

We assume $Br(B_s \to K^+K^-) = Br(B^0 \to K^+\pi^-)$ and $\sigma_{B^0_s} = 0.2 \cdot \sigma_{B^0}$. For $Br(B \to \pi^+\pi^-)$ the value of $0.84 \cdot 10^{-5}$ which is reported by CLEO as an upper limit at 90% CL [6] is taken as a conservative estimation. Then before applying any cuts the ratio of the number of events in the $B_s \to K^+K^-$ reflection and in the signal is $0.2{*}2 = 0.4$, while for the $B \to \pi^+\pi^-$ reflection the corresponding ratio is $0.84/1.4 * 2 = 1.2$. (The additional factor of 2 arises because both $B^0_s(B^0)$ and $\bar{B}^0_s(\bar{B}^0)$ contribute to the reflection). The number of events in the reflections within ± 50 MeV/c^2 around the M_{B^0} mass which passed the $p_T^{1,2} > 1.5$ GeV/c cuts, energy asymmetry cut $\frac{|E_1-E_2|}{E_1+E_2} < 0.5$ and the cuts on partial identification described above relative to the corresponding number of events in the signal is given in the last row of Table 2. Without smearing with $\sigma = 35$ MeV/c^2, the reflections are quite narrow. After applying all the cuts they are contained in about a 15 MeV/c^2 mass range. The $B_s \to K^+K^-$ reflection has only a small overlap with the region $|M - M_{B^0}| < 50$ MeV/c^2. Therefore the ratio of efficiencies for reflection and the signal is only 2% in this case (see Table 2). However, taking into account the detector resolution drastically changes the situation. Instead of 2% one gets 49% so that the ratio between the number of events in the reflection and in the signal becomes $0.4 \cdot 49\% = 20\%$.

Possible reflections from two-body Λ_b decays are expected to be negligible. Firstly they

decay	$B_s \to K^+K^-$	$B \to \pi^+\pi^-$
identification	K^+	π^-
shift from M_{B^0}, MeV/c^2	+60	+30
$2\sigma Br(\text{reflection})/\sigma Br(\text{signal})$	0.4	1.2
cuts	efficiency(reflection) / efficiency(signal)	
$\lvert M - M_{B^0}\rvert < 50$ MeV/c^2	0.37	0.52
$\lvert M - M_{B^0}\rvert < 50$ MeV/c^2, $p_T^{1,2} > 1.5$ GeV/c, $\frac{\lvert E_1-E_2\rvert}{E_1+E_2} < 0.5$, partial identification	0.02	1.00
after smearing with $\sigma = 35$ MeV/c^2	0.49	0.83
N(reflection) / N(signal)	20%	100%

Table 2: Reflections from two-body $B_{(s)}$ decays. The last row shows the number of events in the reflections found in the $|M - M_{B^0}| < 50$ MeV/c^2 window of the $K\pi$ mass spectra relative to the number of signal events.

are suppressed by the smallness of the Λ_b production rate $\sigma(B)/\sigma(\Lambda_b) \approx 10 - 20$. Secondly the reflections are far away from the B^0 mass. Even the decay $\Lambda_b \to pK^-$ in case of a misidentification of a p as a π^+ produces a reflection which is contained in the region 5.49 - 5.55 GeV/c^2 (assuming $M_{\Lambda_b} = 5.64$ GeV/c^2). This region is about 5σ away from the B^0 range $|M - 5.28| < 0.05$ GeV/c^2.

The relative statistical error in $Br(B^0 \to K^+\pi^-)$ and in $Br(\bar{B}^0 \to K^-\pi^+)$ can now be written as:

$$\frac{\delta Br}{Br} = \frac{\delta(N^{tot} - R - B)}{N^{tot} - R - B}$$

for each of the samples of full identification, K identification and π identification. Here N^{tot} is the total number of observed candidates of the decay $B \to K^\pm\pi^\mp$ in the corresponding sample. It includes the background contributions from the reflections (R) and from the minimum bias events (B). In the following we neglect the errors in the estimations of R and B. R can be determined using samples of fully reconstructed $B \to \pi^+\pi^-$ and $B_s \to K^+K^-$ decays. B can be estimated from sidebands on the mass spectra of $K^\pm\pi^\mp$ pairs or by varying the cuts on the $K^\pm\pi^\mp$ vertex or cuts on the second B in the event. Let us denote the number of signal events $N^{tot} - R - B$ by S:

$$\frac{\delta Br}{Br} = \frac{\delta(N^{tot} - R - B)}{S} = \frac{\delta N^{tot}}{S} = \frac{\sqrt{N^{tot}}}{S} = \frac{\sqrt{1 + \frac{R}{S} + \frac{B}{S}}}{\sqrt{S}}.$$

To estimate S in the following we simply assume $Br(B^0 \to K^+\pi^-) = Br(\bar{B}^0 \to K^-\pi^+) = \mathcal{B} = 1.4 \cdot 10^{-5}$ and neglect the difference between production rates of B^0 and $\bar{B}^0$. Thus for samples of full identification, K identification and π identification the expected value of S in one year of HERA-B operation is equal to $\frac{1}{2} \cdot (0.24, 0.14, 0.14) \cdot 720 = \frac{1}{2} \cdot (170, 100, 100)$ respectively (see (7)). The minimum bias backgrounds for three samples are assumed to follow the relation (8):

$$(B/S)_{K\pi} : (B/S)_K : (B/S)_\pi = \frac{1}{0.24} : \frac{0.3}{0.14} : \frac{1.2}{0.14}.$$

The ratios R/S are taken from Table 2. The resulting value of $\frac{\delta Br}{Br}$ in one year of HERA-B operation is shown by dotted lines in Fig. 6 for three separate samples for different minimum bias background to signal ratios $(B/S)_{K\pi}$ in case of full identification.

After averaging over all samples the statistical error is:

$$< \frac{\delta Br}{Br} > = \sqrt{\frac{1}{\frac{1}{\left(\frac{\delta Br}{Br}\right)^2_{K\pi}} + \frac{1}{\left(\frac{\delta Br}{Br}\right)^2_{K}} + \frac{1}{\left(\frac{\delta Br}{Br}\right)^2_{\pi}}}} = \frac{1}{\sqrt{720/2}} \times$$

$$\left(\frac{0.24}{1 + (B/S)_{K\pi}} + \frac{0.14}{1 + 0.20 + (B/S)_{K\pi}\left(\frac{0.24}{0.14} \cdot 0.3\right)} + \frac{0.14}{1 + 1.00 + (B/S)_{K\pi}\left(\frac{0.24}{0.14} \cdot 1.2\right)} \right)^{-1/2}.$$

It is shown in Fig. 6 by the solid line and is also given in Table 3.

$(B/S)_{K\pi}$	0	0.5	1	2	3	5	7	10	15	20	50
$< \delta Br/Br >$, %	8.1	9.6	10.9	13.0	14.8	17.8	20.3	23.6	28.3	32.3	49.9
$< \delta A^0_{CP} >$, %	5.7	6.8	7.7	9.2	10.4	12.6	14.4	16.7	20.0	22.8	35.3

Table 3: Relative statistical error in $Br(B \to K^\pm\pi^\mp)$ and absolute statistical error in $\mathcal{A}_{CP}$ in one year of HERA-B operation for different minimum bias background to signal ratios $(B/S)_{K\pi}$ in case of full identification.

The contribution to the *absolute* statistical error δBr from the minimum bias background is shown in Fig. 7. It is obtained by first assuming that the fluctuation of the minimum bias background is the only source of the statistical error. For each of the samples one gets:

$$\delta Br^{(B)} = \frac{\sqrt{B}}{n_B \cdot \mathrm{eff}^S_{tot}},$$

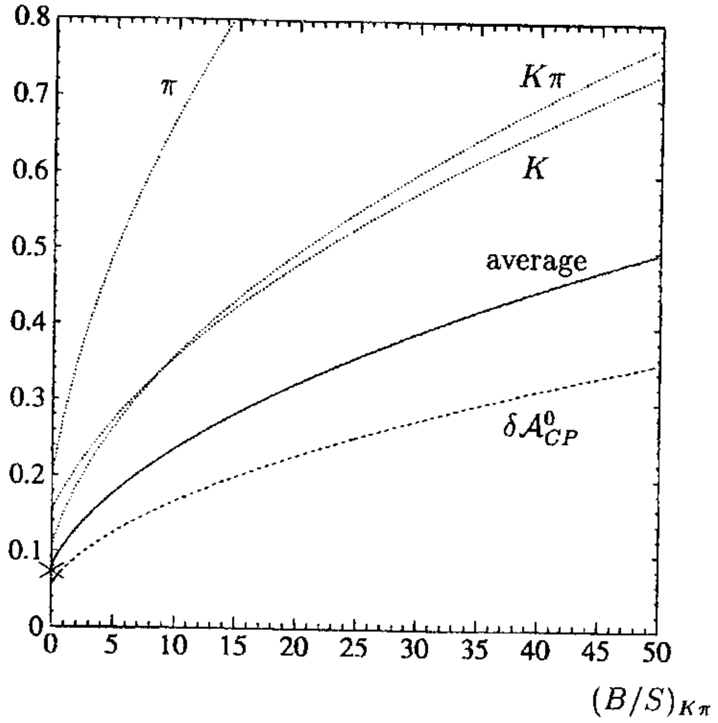

Figure 6: The relative statistical errors in $Br(B \to K^{\pm}\pi^{\mp})$ in one year of HERA-B operation for the separate samples with full, K and π identifications (dotted lines) and after averaging over all three samples (solid line). The cross on the vertical axis shows, for a comparison, the error $\delta Br/Br$ in the ideal case when there are no background contributions from the reflections and minimum bias events. The dashed line shows the corresponding absolute statistical error in the asymmetry $\delta\mathcal{A}^0_{CP} = \frac{1}{\sqrt{2}}\delta Br/Br$.

where n_B is the total number of B^0 (or $\bar{B}^0$) mesons, eff^S_{tot} is the total efficiency for the signal. Then averaging over all three samples and using the values from (7) and (9) yields

$$< \delta Br^{(B)} >= \left(\sqrt{\frac{0.24^2}{1} + \frac{0.14^2}{0.3} + \frac{0.14^2}{1.2}}\right)^{-1} \frac{\sqrt{40MHz \cdot 10^7 s \cdot 3.3 \cdot 10^{-7}}}{40MHz \cdot 10^7 s \cdot 10^{-6} \cdot 0.4 \cdot 0.28 \cdot 0.57} \times$$

$$\sqrt{\frac{\mathrm{eff}^B_{VX}\mathrm{eff}^B_{2d\ B}}{(\mathrm{eff}^S_{2d\ B})^2}} = 1.21 \cdot 10^{-6}\sqrt{\frac{\mathrm{eff}^B_{VX}\mathrm{eff}^B_{2d\ B}}{(\mathrm{eff}^S_{2d\ B})^2} \cdot 10^6},$$

where the efficiencies eff^B, eff^S are defined as in (9). Note that $< \delta Br^{(B)} >$ does not depend on the assumed value of $Br(B \to K^{\pm}\pi^{\mp})$.

The absolute statistical error in asymmetry $\mathcal{A}_{CP}$ can be estimated in a similar way. In the following the variables related to $B^0 \to K^+\pi^-$ ($\bar{B}^0 \to K^-\pi^+$) decays are marked by $+-$

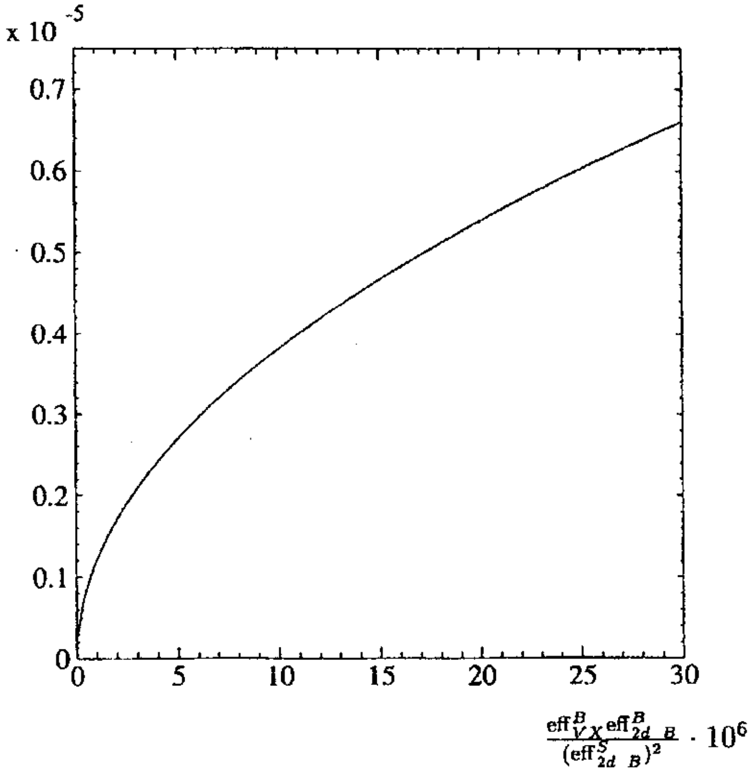

Figure 7: The contribution to the absolute statistical error in $Br(B \to K^{\pm}\pi^{\mp})$ from minimum bias background in one year of HERA-B operation versus the background suppression factor $\frac{eff^B_{VX} eff^B_{2d\_B}}{(eff^S_{2d\_B})^2}$ explained in the text.

$(-+)$ subscript. The asymmetry is determined by the equation

$$\mathcal{A}_{CP} = \frac{N^{tot}_{+-} - N^{tot}_{-+}}{(N^{tot}_{+-} - R - B) + (N^{tot}_{-+} - R - B)}.$$

Here it is assumed that the background contributions $(R+B)$ are the same for $B^0 \to K^+\pi^-$ and $\bar{B}^0 \to K^-\pi^+$ decays so that they cancel in the enumerator. For simplicity we also neglect the difference between B^0, $\bar{B}^0$ production rates. The absolute statistical error of $\mathcal{A}_{CP}$ is

$$\delta\mathcal{A}_{CP} = \sqrt{\left[\left(\frac{\partial\mathcal{A}_{CP}}{\partial N^{tot}_{+-}}\right)\delta N^{tot}_{+-}\right]^2 + \left[\left(\frac{\partial\mathcal{A}_{CP}}{\partial N^{tot}_{-+}}\right)\delta N^{tot}_{-+}\right]^2}.$$

R and B are assumed again to be known precisely, $\delta N^{tot}_{+-,-+} = \sqrt{N^{tot}_{+-,-+}}$. The calculation gives:

$$\delta\mathcal{A}_{CP} = \frac{\sqrt{(1-\mathcal{A}^2_{CP}) + (\frac{R}{S}+\frac{B}{S})(1+\mathcal{A}^2_{CP})}}{\sqrt{S_{+-}+S_{-+}}}.$$

Here $S_{+-}+S_{-+}$ is the total number of signal events $N^{tot}_{+-}+N^{tot}_{-+}-2(R+B)$ which is determined by $\mathcal{B} = \frac{1}{2}(Br(B^0 \to K^+\pi^-) + Br(\bar{B}^0 \to K^-\pi^+))$ and does not depend on asymmetry. The errors $\delta\mathcal{A}_{CP}$ calculated at the point $\mathcal{A}_{CP} = 0$:

$$\delta\mathcal{A}_{CP}|_{\mathcal{A}_{CP}=0} = \delta\mathcal{A}^0_{CP} = \frac{\sqrt{1+\frac{R}{S}+\frac{B}{S}}}{\sqrt{S_{+-}+S_{-+}}} = \frac{1}{\sqrt{2}}\frac{\delta Br}{Br}$$

are given in Table 3 and are shown in Fig. 6 by a dashed line.

In conclusion, the HERA-B capabilities to measure $Br(B^0 \to K^+\pi^-)$ and $Br(\bar{B}^0 \to K^-\pi^+)$ have been studied. These quantities are needed to draw the triangles in Fig. 4 and calculate γ as a half of the angle between the right sides of the triangles. The high-p_T trigger system allows for the selection of the decays $B \to K^\pm\pi^\mp$ in parallel to the decays $B \to \pi^+\pi^-$. Neither tagging nor time dependent measurements are required. In order to increase statistics along with fully reconstructed events involving $K^\pm\pi^\mp$ pairs, the partially reconstructed events are also included in the analysis where one particle is identified while the other is not. Two main sources of the background are considered: background from minimum bias events and reflections from other two-body $B_{(s)}$ decays. The reflections from Λ_b two-body decays are expected to be negligible. The estimated statistical errors in $Br(B \to K^\pm\pi^\mp)$ and in asymmetry $\mathcal{A}_{CP}$ in one year of HERA-B operation are shown in Fig. 6 and are also given in Table 3 for different minimum bias background to signal ratios in the sample of fully reconstructed events. The contribution to the absolute statistical error in $Br(B \to K^\pm\pi^\mp)$ from minimum bias background is shown in Fig. 7.

Appendix. Kinematics of two-body decay reflections.

Let's consider the reflection from the decay of a particle with a mass M into two particles with masses m_1 and m_2: $\mathbf{M \to m_1 m_2}$. If the first daughter particle is misidentified as a particle with mass m'_1 this shifts the mass M into some new value M'. Let's consider the case of perfect detector resolution when the momenta of daughter particles p_1, p_2 are measured precisely. The energies E_1, E_2 are calculated from masses and momenta. In the units with the speed of light c=1 one gets:

$$M^2 = (\sqrt{p_1^2 + m_1^2} + \sqrt{p_2^2 + m_2^2})^2 - (\vec{p_1} + \vec{p_2})^2, \quad M'^2 = (\sqrt{p_1^2 + m'^2_1} + \sqrt{p_2^2 + m_2^2})^2 - (\vec{p_1} + \vec{p_2})^2,$$

$$M'^2 - M^2 = (\sqrt{p_1^2 + m'^2_1} + \sqrt{p_2^2 + m_2^2})^2 - (\sqrt{p_1^2 + m_1^2} + \sqrt{p_2^2 + m_2^2})^2 = (E'_1 + E_2)^2 - (E_1 + E_2)^2 =$$

$$(E'^2_1 - E_1^2) + 2E_2(E'_1 - E_1) = (m'^2_1 - m_1^2) + 2E_2(E'_1 - E_1).$$

Let's denote $m'^2_1 - m_1^2$ and $M'^2 - M^2$ by $\Delta(m_1^2)$ and $\Delta(M^2)$ respectively. Since $E'^2_1 - E_1^2 = \Delta(m_1^2)$: $E'_1 = \sqrt{E_1^2 + \Delta(m_1^2)}$ and:

$$\Delta(M^2) = \Delta(m_1^2) + 2E_2(\sqrt{E_1^2 + \Delta(m_1^2)} - E_1).$$

One can see that if $\Delta(m_1^2) > 0$: $\Delta(M^2) > \Delta(m_1^2)$. If $\Delta(m_1^2) < 0$: $\Delta(M^2) < \Delta(m_1^2) < 0$. In both cases: $|\Delta(M^2)| > |\Delta(m_1^2)|$.

Let's suppose now that $|\Delta(m_1^2)| \ll E_1^2$. For HERA-B this is a very good approximation, e.g. $m_K^2 - m_\pi^2 = 0.22$ GeV^2. (For $\Upsilon(4S)$ energies it is not so good, e.g. for π with $p = 0.8$ GeV: $E_\pi^2 = 0.66$ GeV^2).

$$\Delta(M^2) = \Delta(m_1^2) + 2E_1E_2\left(\frac{1}{2}\frac{\Delta(m_1^2)}{E_1^2} - \frac{1}{8}\left(\frac{\Delta(m_1^2)}{E_1^2}\right)^2 + \ldots\right) =$$

$$\Delta(m_1^2)\left(\left(1 + \frac{E_2}{E_1}\right) + \frac{E_2}{E_1}\left(-\frac{1}{4}\left(\frac{\Delta(m_1^2)}{E_1^2}\right) + \ldots\right)\right).$$

In the following we shall consider only the main term so that

$$\Delta(M^2) \approx \Delta(m_1^2)\left(1 + \frac{E_2}{E_1}\right) = \Delta(m_1^2)\frac{E}{E_1}.$$

In this linear approximation the normalized shift $\Delta(M^2)/\Delta(m_1^2)$ does not depend on the characteristic parameter of misidentification $\Delta(m_1^2) = m'^2_1 - m_1^2$ and is determined only by the spectrum of E/E_1. It is the same for $\pi \to K$, $K \to p$ or $\pi \to p$ misidentifications.

If $|\Delta(M^2)| \ll M^2$ one can write

$$\Delta M = M' - M = \sqrt{M^2 + \Delta(M^2)} - M = \frac{\Delta(M^2)}{2M}(1 - \frac{1}{4}\frac{\Delta(M^2)}{M^2} + \dots)$$

$$\approx \frac{\Delta(M^2)}{2M} \approx \frac{\Delta(m_1^2)}{2M} \cdot \frac{E}{E_1}. \qquad (10)$$

Now let's analyze the shape of E/E_1 spectrum. To determine it one needs to consider the decay **M** $\to$ $\mathbf{m_1 m_2}$ in the rest frame of **M** and then make a boost to the laboratory system. Let's denote by $\vec{\beta}$, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ the characteristics of the boost. Then the energy E_1 can be related to the corresponding energy and the momentum in the rest frame by the formulae

$$E_1 = \gamma(E_1^{cm} + \vec{\beta} \cdot \vec{p_1^{cm}}) = \gamma(E_1^{cm} + \beta p_1^{cm} \cos\theta),$$

where θ is the angle between the momentum $\vec{p_1^{cm}}$ in the rest frame and the direction of the boost. The total energy E in the laboratory system is given by $E = \gamma M$. Thus the ratio

$$\frac{E}{E_1} = \frac{M}{E_1^{cm} + \beta p_1^{cm} \cos\theta}. \qquad (11)$$

The quantities E_1^{cm} and p_1^{cm} are totally determined by the masses M, m_1 and m_2 and therefore should be considered as constants here. β and $\cos\theta$ are variables.

The E/E_1 spectrum is the function $\frac{dN}{d(E/E_1)}$. It shows the number of events dN in the bin of the variable E/E_1: $(\frac{E}{E_1}, \frac{E}{E_1} + d(\frac{E}{E_1}))$. Let's consider the case when the energy E is fixed so that β is constant, $\cos\theta$ can vary. Then

$$\frac{dN}{d(E/E_1)} = -\frac{dN}{\frac{E}{E_1^2}dE_1} = -\frac{dN}{\left(\frac{E}{E_1}\right)^2 \frac{dE_1}{E}} =$$

$$-\frac{1}{(E/E_1)^2} \cdot \frac{dN}{d\cos\theta \cdot \frac{d(E_1/E)}{d\cos\theta}} = -\frac{1}{(E/E_1)^2} \cdot \frac{dN}{d\cos\theta} \cdot \frac{1}{\left(\frac{E_1^{cm} + \beta p_1^{cm}\cos\theta}{M}\right)'_{\cos\theta}} =$$

$$-\frac{1}{(E/E_1)^2} \cdot \frac{dN}{d\cos\theta} \cdot \frac{M}{\beta p_1^{cm}}.$$

To understand this formulae let's denote E/E_1 by x, $\frac{dN}{d\cos\theta}$ by some function $f(\cos\theta, \beta)$. Since $x = \frac{M}{E_1^{cm} + \beta p_1^{cm}\cos\theta}$: $f(\cos\theta, \beta) = f(\frac{M/x - E_1^{cm}}{\beta p_1^{cm}}, \beta)$ and

$$\frac{dN}{dx} = -\frac{1}{x^2} \cdot f(\frac{M/x - E_1^{cm}}{\beta p_1^{cm}}, \beta) \cdot \frac{M}{\beta p_1^{cm}}. \qquad (12)$$

To consider the general case when the energy E can vary one should convolute the above formulae with the β spectrum.

To go further let's make two assumptions. First let's consider only the simplest case when $f(\cos\theta, \beta) = const = A$. This means there is no correlation between the direction of $\overrightarrow{p_1^{cm}}$ in the rest frame and the direction of the boost to the laboratory system. The only quantity which can bring the information on the direction of the boost to the rest frame of **M** is the spin of **M**. Therefore the correlation under consideration does exist only if

1) there is a correlation between the momentum of **M** and its spin in the laboratory system

and

2) in the rest frame of **M** the direction of $\overrightarrow{p_1^{cm}}$ is correlated to **M**'s spin.

The correlation does not exist for example if **M** decays in S-wave isotropically. This is true for all two-body decays of $B_{(s)}$ mesons since they are spinless.

Our second assumption is that $\beta \approx 1$. If $p_{B^0} = 10\ (20)$ GeV: $\beta = 0.88\ (0.97)$, so for $B_{(s)}$ mesons at HERA-B this is good approximation.

With these assumptions equation (12) can be rewritten as

$$\frac{dN}{dx} = -\frac{A}{x^2} \cdot \frac{M}{p_1^{cm}}$$

and taking into account the relation $\Delta M = \frac{\Delta(m_1^2)}{2M} \cdot x$:

$$\frac{dN}{d(\Delta M)} = -\frac{A}{(\Delta M)^2} \cdot \frac{\Delta(m_1^2)}{2p_1^{cm}}. \tag{13}$$

The corresponding normalized spectrum for the case $\Delta(m_1^2) > 0$ is shown in Fig. 8. The minimal shift as can be seen from (10) and (11) is equal to $(\Delta M)_{\min} = \frac{\Delta(m_1^2)}{2M} \frac{M}{E_1^{cm} + \beta p_1^{cm}}$ $\approx \frac{\Delta(m_1^2)}{2M} \frac{M}{E_1^{cm} + p_1^{cm}}$. If $m_2 \ll M$ so that $p_1^{cm} = p_2^{cm} \approx E_2^{cm}$ and $E_1^{cm} + p_1^{cm} \approx E_1^{cm} + E_2^{cm} = M$ one gets:

$$(\Delta M)_{\min} = \frac{\Delta(m_1^2)}{2M}. \tag{14}$$

For the reflections from $B^0 \to \pi^+\pi^-$ and $B_s^0 \to K^+K^-$ to $B^0 \to K^+\pi^-$: $(\Delta M)_{\min} = 21$ MeV. In the region $(\Delta M) > (\Delta M)_{\min}$ the spectrum goes as $1/(\Delta M)^2$. Note that this behaviour does not depend on the energy spectrum of **M** (until $\beta \approx 1$).

It is interesting to determine how the reflection is affected by the energy asymmetry cut discussed in the text: $\frac{|E_1 - E_2|}{E_1 + E_2} < \alpha$, $0 < \alpha < 1$. For the high-p_T trigger $\alpha = 0.5$. This requirement is equivalent to: $\frac{2}{1+\alpha} < x = \frac{E}{E_1} < \frac{2}{1-\alpha}$, therefore

$$\frac{2}{1+\alpha}(\Delta M)_{\min} < \Delta M < \frac{2}{1-\alpha}(\Delta M)_{\min}.$$

Thus for $\alpha = 0.5$ the cut sets the following bounds for the reflection:

$$\frac{4}{3}(\Delta M)_{\rm min} < \Delta M < 4(\Delta M)_{\rm min}.$$

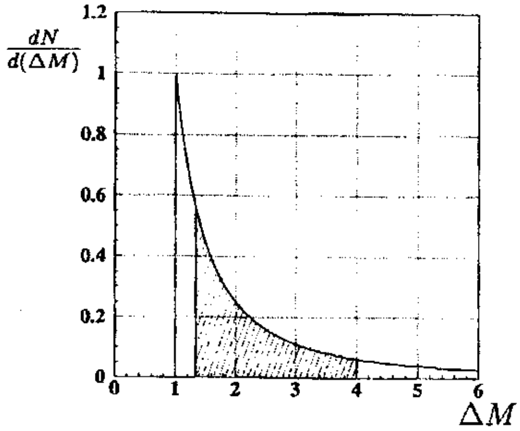

Figure 8: Reflection from two body decay. One unit on ΔM axis is equal to $\frac{\Delta(m_1^2)}{2M}$. The region selected by the energy asymmetry cut $\frac{|E_1-E_2|}{E_1+E_2} < 0.5$ is hatched.

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