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Ground–γ band coupling in a collective scheme with a broken SU(3) symmetry

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Abstract

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Abstract

The link between the ground (g) and the γ- bands of even deformed nuclei is studied via the collective vector–boson model with a broken SU(3) symmetry. The g–γ bandmixing interaction as well as the limiting cases, in which the SU(3) symmetry is reduced completely, are estimated in terms of the energy splitting between these two bands. It is shown, that the systematical behavior of the g–γ splitting, observed in rotational regions, supports our analyses.

In a recent work we have started a systematical study of the link between the ground (g) and the γ- collective bands in even–even deformed nuclei [1]. The approach is based on the collective Vector–Boson Model (VBM) with a broken SU(3) symmetry [2, 3, 4]. In VBM the two bands (g and γ) belong to one split SU(3) multiplet, appearing in the $SU(3) \supset O(3)$ group reduction and labeled by a given irreducible representation (irrep) (λ, μ) of SU(3). The corresponding basis states are constructed with the use of two vector–boson creation operators $\boldsymbol{\xi}^+$, $\boldsymbol{\eta}^+$ and are denoted as [5]

$$\left| \begin{array}{c} (\lambda,\mu) \\ \alpha, L, M \end{array} \right\rangle . \tag{1}$$

The quantum number α distinguishes the various O(3) irreps, (L, M), appearing in a given SU(3) irrep (λ, μ) and labels the different bands of the multiplet. The SU(3)–symmetry breaking Hamiltonian is constructed by using three basic O(3) scalars, which belong to the enveloping algebra of SU(3) [4]:

$$V = g_1 L^2 + g_2 L \cdot Q \cdot L + g_3 A^+ A \ . \tag{2}$$

Here g_1, g_2 and g_3 are free parameters; L and Q are the angular momentum and quadrupole operators respectively; and $A^+ = \boldsymbol{\xi}^{+2} \boldsymbol{\eta}^{+2} - (\boldsymbol{\xi}^+ \cdot \boldsymbol{\eta}^+)^2$.

Using the VBM formalism, it has been shown, that for a given nucleus the physically significant features of SU(3)-symmetry should be studied in certain regions of (λ, μ) irreps instead of a single fixed irrep [1]. On this basis we have investigated various nuclei of rare earth region and actinides for which the model descriptions of the g- and γ-band energy levels and the concomitant B(E2) transition ratios have been evaluated [in the form of root mean square fits] in SU(3) irreps within the range $10 \leq \lambda \leq 160$ and $2 \leq \mu \leq 8$. It has been found, that the basic SU(3) properties of deformed nuclei depend on the SU(3) splitting. The latter is characterized by the ratio [1]

$$\Delta E_2 = (E_2^\gamma - E_2^g)/E_2^g \ , \tag{3}$$

where E_2^g and E_2^γ are the energy levels with angular momentum $L = 2$, which belong to the g- and the γ- band respectively. In the rare earth region this ratio varies within the limits $7 \leq \Delta E_2 \leq 18$, while in the actinides one observes values in the range $13 \leq \Delta E_2 \leq 25$. See figures 1 and 2.

In the nuclei with small band splitting ratios $\Delta E_2 \sim 8 - 10$ ($^{164-168}$Er, ^{164}Dy and ^{168}Yb) we have established clearly outlined regions of "favored" SU(3) multiplets (with relatively small λ-values $\lambda = 14 - 20$ and $\mu = 2, 4, 6$), where the descriptions of the energy levels are obtained essentially better than in the other irreps. Further with the increase of the splitting energy, as in the case of the nucleus ^{178}Hf (with $\Delta E_2 = 11.6$), the favored multiplets are shifted gradually to larger λ-values ($\lambda \sim 40$). In the nuclei where large band splitting is observed, $\Delta E_2 \sim 14 - 22$ (^{172}Yb, ^{176}Hf, ^{238}U), the present theoretical scheme provides almost equally good model descriptions in all (λ, μ)-multiplets with $\lambda > 60 - -80$ up to $\lambda = 160$ and $\mu = 6$ without presence of any upper limit for the quantum number λ. (See figures 1–8 of ref. [1])

The above results have a reasonable interpretation in terms of the band-mixing interactions. Some preliminary estimates, provided for the $(\lambda, 2)$ multiplets, show that the increase in the quantum number λ is connected with the corresponding decrease in the the g–γ band-mixing interaction [1]. This suggests, that in the nuclei with small SU(3) splitting the two bands are strongly mixed, while in the cases with a large splitting they interact weakly. The systematical behavior of rotational spectra, observed in the rare earth nuclei and actinides supports our analyses. The data show, that the g–γ band-splitting, which for the nuclei near the ends of rotational regions is relatively small, increases essentially towards the midshell nuclei. See figures 1 and 2. In terms of our considerations the strong g–γ splitting, observed in the middle of given rotational region, corresponds to the weak mutual perturbation and therefore to the good rotational behavior of the both bands . It is therefore clear, that the splitting plays an important role in the study of the link between the g- and the γ- band.

Here we shall extend our investigations towards the (λ, μ) multiplets with $\mu \geq 2$, by estimating the g–γ interaction in various limits of the model. On the other hand it is important to obtain more quantitative information about the the meaning in which the SU(3) symmetry is reduced in (λ, μ)-plane. It is therefore worthwhile to derive an analytical relation between the energy splitting and the SU(3) quantum numbers λ and μ. For this purpose, we shall study the energy levels E_2^g and E_2^γ [which determine the SU(3) splitting ratio ΔE_2, Eq. (3)] in terms of VBM.

We remark, that for any (λ, μ) multiplet ($\mu \geq 2$), the energy levels E_2^g and E_2^γ are the only possible ones, appearing at angular momentum $L = 2$. They are labeled by the quantum number α as follows [See inequality (3) of ref. [1]]:

$$\begin{array}{ll} \alpha_1 = \mu/2 - 1 \ , & \text{for the } \gamma\text{-band state } E_2^\gamma ; \\ \alpha_2 = \mu/2 \ , & \text{for the } g\text{-band state } E_2^g . \end{array} \qquad (4)$$

Hence for $L = 2$ the Hamiltonian matrix is always two-dimensional and the corresponding eigenvalue equation [See Eq. (12) of ref. [1]] has the form:

$$det \begin{pmatrix} V_{1,1} - \omega^{L=2} & V_{1,2} \\ V_{2,1} & V_{2,2} - \omega^{L=2} \end{pmatrix} = 0 \qquad (5)$$

where $\omega^{L=2}$ are the eigenvalues and

$$V_{j,j'} \equiv \langle \alpha_j, 2|V|\alpha_{j'}, 2\rangle = \left\langle \begin{array}{c} (\lambda,\mu) \\ \alpha_j, 2, 2 \end{array} \middle| V \middle| \begin{array}{c} (\lambda,\mu) \\ \alpha_{j'}, 2, 2 \end{array} \right\rangle , \qquad (6)$$

with $j, j' = 1, 2$, are the matrix elements of the Hamiltonian (2) between the highest-weight basis states ($L = M = 2$) [See Eq. (1)].

Eq. (5) has two solutions:

$$\omega_\pm^{L=2} = \frac{1}{2}\left\{V_{1,1} + V_{2,2} \pm \sqrt{(V_{1,1}+V_{2,2})^2 - 4(V_{1,1}V_{2,2} - V_{1,2}V_{2,1})}\right\} . \qquad (7)$$

The energy levels E_2^g and E_2^γ are determined as:

$$E_2^g = \omega_-^{L=2} - \omega^{L=0} , \qquad (8)$$

$$E_2^\gamma = \omega_+^{L=2} - \omega^{L=0} , \qquad (9)$$

where $\omega^{L=0}$ is the zero-level eigenvalue

$$\omega^{L=0} = \langle \frac{\mu}{2}, 0|V|\frac{\mu}{2}, 0\rangle . \qquad (10)$$

By using the analytical form of the matrix elements of the operators $L \cdot Q \cdot L$ and A^+A [given in Table 1 of ref. [1]], we have calculated all necessary matrix elements:

$$V_{1,1} = \langle (\frac{\mu}{2} - 1), 2|V|(\frac{\mu}{2} - 1), 2\rangle = 6g_1 + 6g_2(2\lambda + 2\mu + 3) + g_3 P(\lambda,\mu) , \qquad (11)$$

$$V_{2,2} = \langle \frac{\mu}{2}, 2|V|\frac{\mu}{2}, 2\rangle = 6g_1 - 6g_2(2\lambda + 2\mu + 3) + g_3 Q(\lambda,\mu) , \qquad (12)$$

$$V_{1,2} = \langle (\frac{\mu}{2} - 1), 2|V|\frac{\mu}{2}, 2\rangle = 12g_2\mu - 2g_3\mu(\mu - 2) , \qquad (13)$$

$$V_{2,1} = \langle \frac{\mu}{2}, 2|V|(\frac{\mu}{2} - 1), 2\rangle = -12g_2\lambda + 2g_3\lambda(\lambda + 2\mu + 2) , \qquad (14)$$

where

$$P(\lambda,\mu) = \lambda(\mu-2)(\mu+2)(\lambda+2\mu+2) + \mu(\mu-2)(\mu+1)(\mu+3) , \qquad (15)$$

$$Q(\lambda,\mu) = \lambda\mu^2(\lambda+2\mu+2) + \mu(\mu-1)(\mu+1)(\mu+2) . \qquad (16)$$

For the the zero-level matrix element we have:

$$\langle \frac{\mu}{2}, 0|V|\frac{\mu}{2}, 0\rangle = g_3\mu^2(\lambda+\mu+1)^2 . \quad (17)$$

Now we are able to study the g–γ band-mixing interaction at $L = 2$ in (λ, μ) plane. Since the basis states (1) are determined for $\mu \leq \lambda$, there are two possible limiting cases: *i)* $\lambda \to \infty$, with μ finite, and *ii)* $\lambda \to \infty$, $\mu \to \infty$, with $\mu \leq \lambda$. In both cases we estimate the λ- and/or μ- dependence of the matrix elements (11)–(14).

In the limiting case *i)* the matrix elements are determined by the corresponding highest degrees of λ, so that the Hamiltonian matrix $(V_{i,j})$ [Eq. 6] obtains the following asymptotical form:

$$(V)_{\lambda\to\infty} = \begin{pmatrix} \lambda^2 & * \\ \lambda^2 & \lambda^2 \end{pmatrix} , \quad (18)$$

where the upper offdiagonal element (denoted by $*$) does not depend on λ. Then one can easily deduce, that the relative contribution of the offdiagonal (band-mixing) terms in the Hamiltonian matrix decreases with the increase of λ as $\lambda^2/\lambda^4 = 1/\lambda^2$. An exception occurs in the particular case $\mu = 2$ in which the $V_{1,1}$ is proportional to λ instead of λ^2 [See Eqs. (11) and (15)]. In this case the offdiagonal contribution decreases as $1/\lambda$. So, we find that for all (λ, μ)- multiplets with $\mu \geq 2$ ($\mu \leq \lambda$), the increase in the quantum number λ is connected with a corresponding decrease in the g–γ band-mixing interaction within the framework of the SU(3) symmetry.

Consider the case *ii)* $\lambda \to \infty$, $\mu \to \infty$, ($\mu \leq \lambda$). Since the difference $\lambda - \mu$ is always finite, it is enough to take $\mu = \lambda$. Then the asymptotical form of the matrix $(V_{i,j})$ is:

$$(V)_{\substack{\lambda\to\infty \\ \mu\to\infty}} = \begin{pmatrix} \lambda^4 & \lambda^2 \\ \lambda^2 & \lambda^4 \end{pmatrix} . \quad (19)$$

Here we find that the relative contributions of the band-mixing interaction decreases as $\lambda^4/\lambda^8 = 1/l^4$, i.e., more rapidly in comparison to the previous case.

Generally, our analysis shows that in the both limits, *i)* and *ii)*, the g–γ mixing decreases asymptotically to zero. In such a way the SU(3) symmetry disappears completely, and the two bands do not belong anymore to the same SU(3) multiplet. Furthermore, our estimates give a physical insight into the so called group contraction process in which the SU(3) algebra reduces to the algebra of the triaxial rotor group $T_5 \wedge SO(3)$ [6]. The contraction limit appears when the eigenvalues of the second order Casimir operator of SU(3), $C_2(SU(3))$, go to infinity. Since $\langle C_2\rangle$ is a quadratic function of both λ and μ, it is clear that considered above cases exactly reproduce this limit.

Let us now turn to the SU(3) splitting, which will be studied in terms of the ratio ΔE_2 [Eq. (3)]. After introducing the matrix elements (11)–(14) and (17) into Eqs. (7), (8), (9), we obtain the following expressions for the energy levels E_2^g and E_2^γ:

$$E_2^g = 6g_1 - 2Fg_3 - 2\sqrt{Ag_2^2 + Bg_3^2 - Cg_2g_3} , \quad (20)$$

$$E_2^\gamma = 6g_1 - 2Fg_3 + 2\sqrt{Ag_2^2 + Bg_3^2 - Cg_2g_3} , \quad (21)$$

where

$$\begin{aligned}
A &= A(\lambda,\mu) = 9[(2\lambda+2\mu+3)^2 - 4\lambda\mu] \; ; && (22)\\
B &= B(\lambda,\mu) = [\lambda(\lambda+2\mu+2)+\mu(\mu+1)]^2 - \\
&- \lambda\mu(\lambda+2\mu+2)(\mu-2) \; ; && (23)\\
C &= C(\lambda,\mu) = 6(2\lambda+2\mu+3)[\lambda(\lambda+2\mu+2)+\mu(\mu+1)] - \\
&- 6\lambda\mu(\lambda+3\mu) \; ; && (24)\\
F &= F(\lambda,\mu) = \lambda(\lambda+2\mu+2)+2\mu(\mu+1) \; . && (25)
\end{aligned}$$

Hence the energy splitting ratio ΔE_2 obtains the following analytical form:

$$\Delta E_2 = \frac{2}{(3g_1 - Fg_3)/\sqrt{Ag_2^2 + Bg_3^2 - Cg_2g_3} - 1} \; . \qquad (26)$$

The above relation allows one to study the g–γ band-splitting in (λ,μ) plane as well as to estimate it in the two limiting cases.

So, in the limit $\lambda \to \infty$, with μ finite, the functions (22)-(25) behave like:

$$A_{\lambda\to\infty} = 36\lambda^2; \quad B_{\lambda\to\infty} = \lambda^4; \quad C_{\lambda\to\infty} = 12\lambda^3; \quad F_{\lambda\to\infty} = \lambda^2 \; .$$

After applying the above asymptotical quantities in Eq. (26), we find:

$$\lim_{\lambda\to\infty} \Delta E_2 = \frac{2}{-g_3/|g_3| - 1} \; . \qquad (27)$$

We remark that the application of VBM in rare earth nuclei and actinides requires $g_3 < 0$, which gives $\lim_{\lambda\to\infty} \Delta E_2 = \infty$. Therefore in this case the g- and the γ- band are completely split, i.e., they should not be considered anymore in the same energy scale.

On figure 3 the λ–dependence of the theoretically derived splitting ratio ΔE_2 [Eq. (26)] is illustrated numerically. The three curves are obtained for $\mu = 2$ and correspond to the three sets of parameters in the nuclei ^{168}Er, ^{178}Hf and ^{170}Yb [given in Table 2 of ref. [1]] . The ΔE_2 values, which correspond to the best model descriptions are denoted by the open circles.

In the limit $\lambda = \mu \to \infty$, one has:

$$A_{\lambda=\mu\to\infty} = 108\lambda^2; \quad B_{\lambda=\mu\to\infty} = 13\lambda^4; \quad C_{\lambda=\mu\to\infty} = 72\lambda^3; \quad F_{\lambda=\mu\to\infty} = 5\lambda^2 \; .$$

Then the SU(3) splitting ratio goes to:

$$\lim_{\lambda=\mu\to\infty} \Delta E_2 = \frac{2}{-(5/\sqrt{13})g_3/|g_3| - 1} \; . \qquad (28)$$

For $g_3 < 0$ we obtain $\lim_{\lambda=\mu\to\infty} \Delta E_2 = 2/(5/\sqrt{13} - 1) = 5.1$. This is an interesting finding. In the case the band-mixing interaction vanishes, while the energy splitting between the two bands remains finite. The latter is even relatively smaller compared to the splitting ratios, observed in rotational nuclei. The above result is illustrated numerically on figure 4, where the μ-dependence of ΔE_2 is shown for a typical set of model parameters ($g_1 = 10\mathrm{KeV}$, $g_2 = -0.1\mathrm{KeV}$, $g_3 = -0.1\mathrm{KeV}$).

We note, that excepting the sign of g_3, in considered above cases the obtained limits do not depend on the model parameters. (It is assumed that g_1, g_2 and g_3 are finite).

The further interpretation of the presented results will be a subject of forthcoming discussions.

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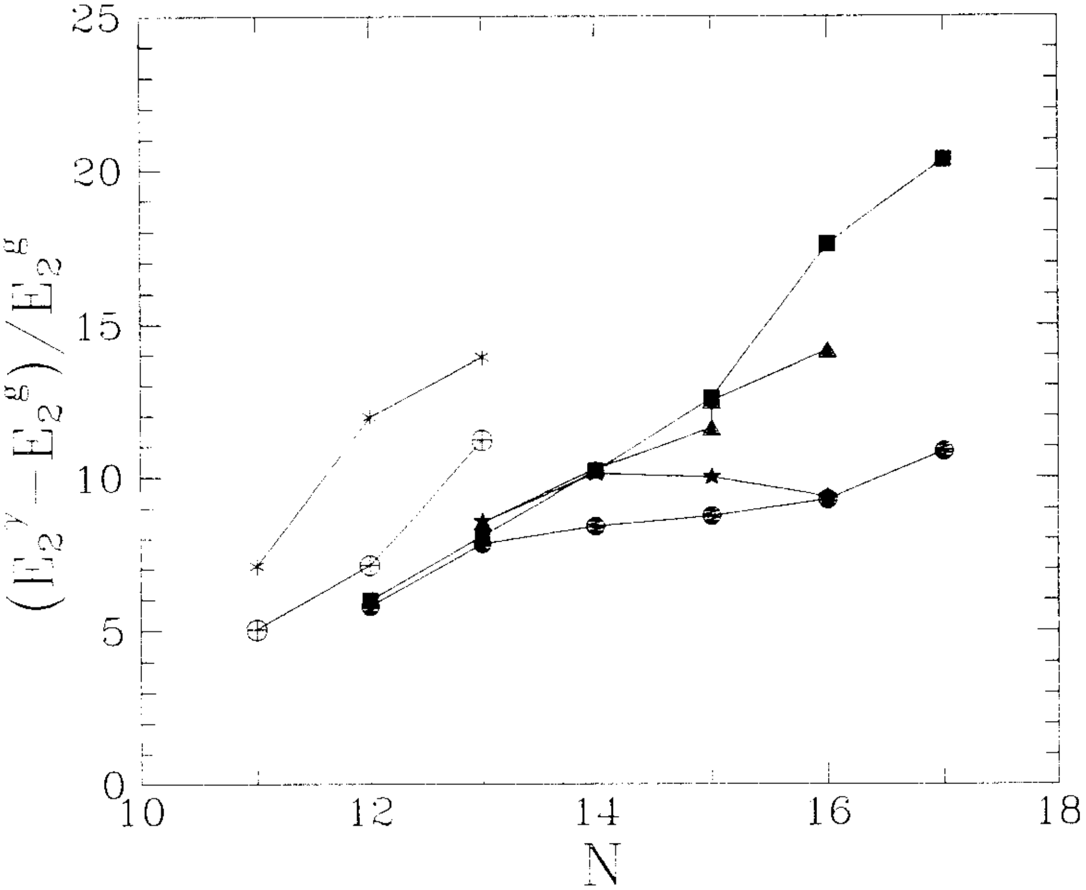


FIG. 1

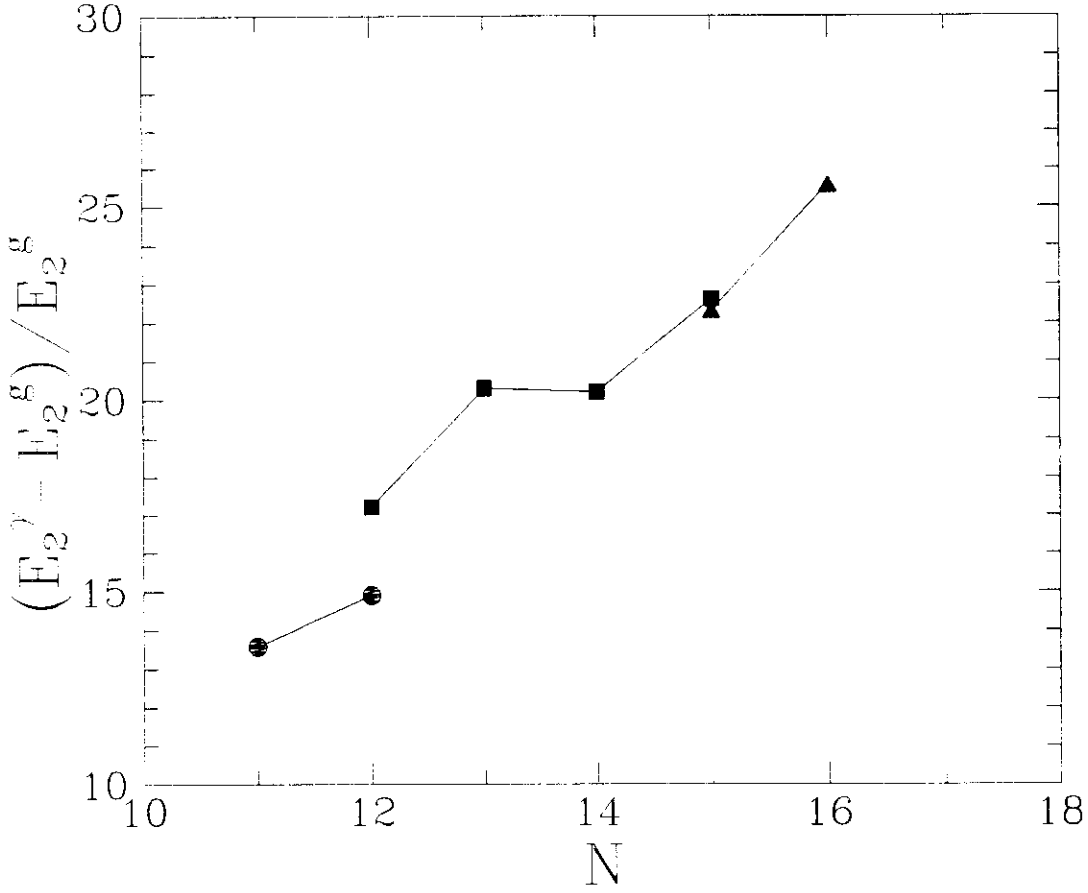
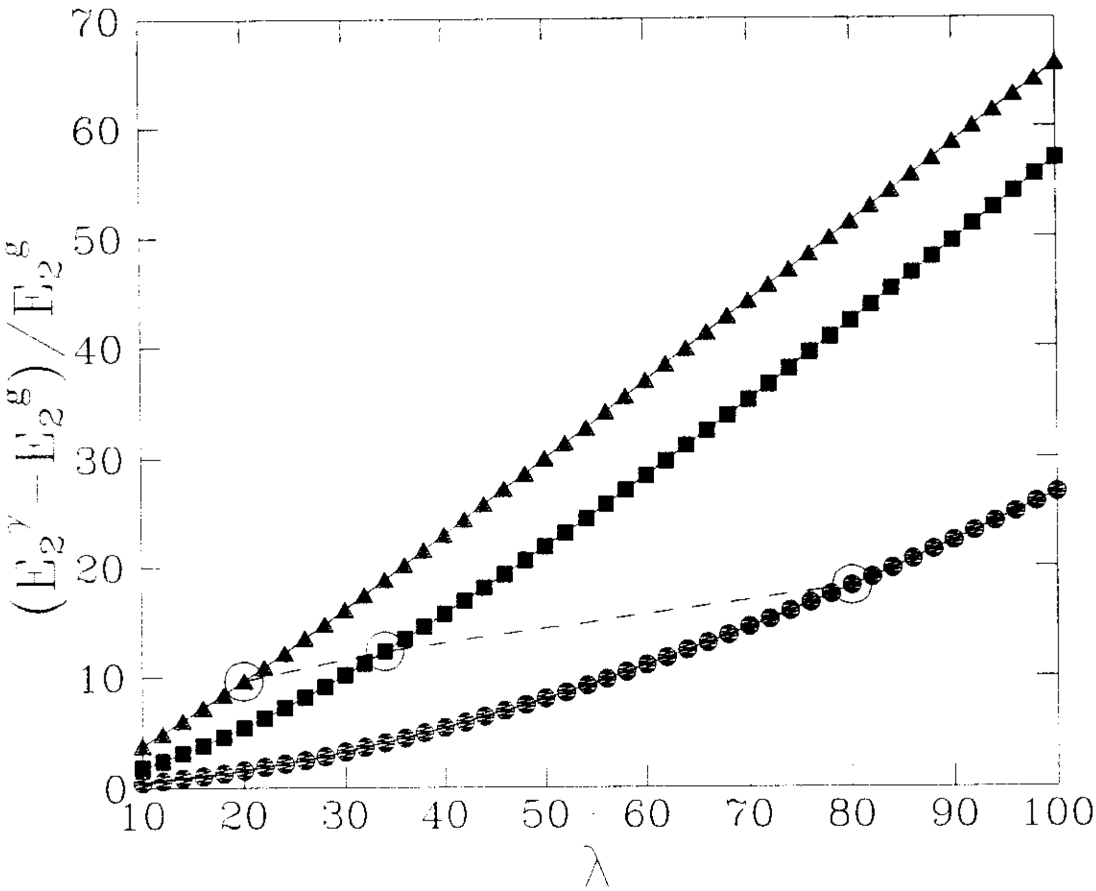


FIG. 2



70
60
50
40
30
20
10
0
10 20 30 40 50 60 70 80 90 100
λ
$(E_2^\gamma - E_2^g)/E_2^g$

FIG. 3

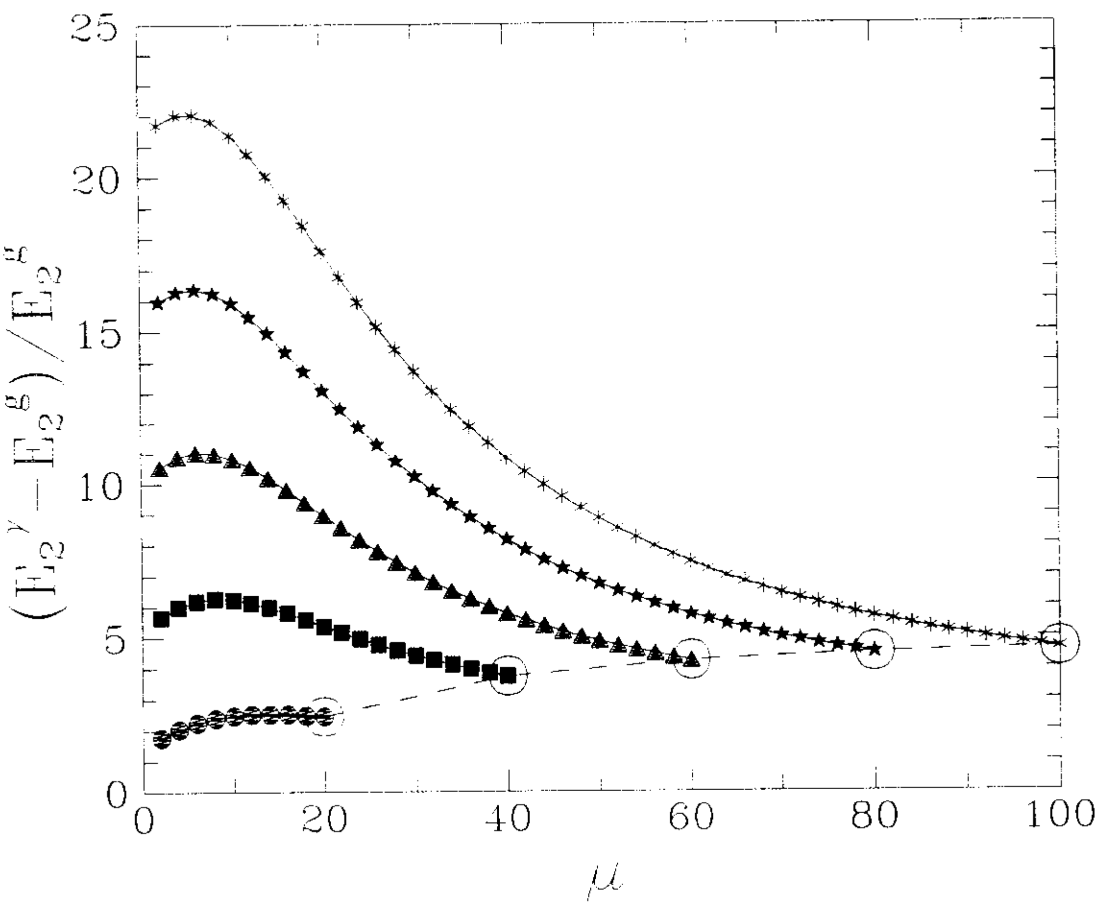


FIG. 4