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THE GMO SUMRULE AND THE πNN COUPLING CONSTANT

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Abstract

Abstract. The isovector GMO sumrule for forward πN scattering is critically evaluated using the precise π^-p and π^-d scattering lengths obtained recently from pionic atom measurements. The charged πNN coupling constant is then deduced with careful analysis of systematic and statistical sources of uncertainties. This direct determination gives $g_c^2(GMO)/4\pi = 14.17 \pm 0.09$ (statistic) ± 0.17 (systematic) or $f_c^2/4\pi = 0.0783(11)$. This value is half-way between that of indirect methods (phase-shift analyses) and the direct evaluation from from backward np differential scattering cross sections (extrapolation to pion pole). From the π^-p and π^-d scattering lengths our analysis leads also to accurate values for $\frac{1}{2}(a_{\pi^-p} + a_{\pi^-n})$ and $\frac{1}{2}(a_{\pi^-p} - a_{\pi^-n})$.

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1 Introduction

In approaching the present discussions concerning the πNN coupling constant in the present workshop it is most important not to confuse the issues. We are *NOT* arguing about a small versus a large coupling constant. We are discussing the *METHODOLOGY* that should be used to obtain the result. This means in particular the following:

-data should be falsifiable

-the analysis should be based on well recognized principles and be transparent and readily reproducible

-assumptions and consequences should be clearly spelled out

-the error analysis should be very detailed and critical with both statistical

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and systematical uncertainties well identified.
For a fundamental physical constant it is necessary, but not sufficient, that the data have sufficient sensitivity. Determination is an absolute statement and it should be clearly falsifiable and improvable.
In this perspective the Goldberger-Miyazawa-Oehme (GMO) sum-rule [1] is of special interest. The relation itself only involves very basic assumptions of physics, since it is a forward dispersion relation for πN scattering. Its validity assumes that scattering amplitudes are analytical functions and that they obey crossing symmetry. If anything is wrong with that it is a very important finding of physics and it is not likely. Usually it is stated as a relation for the isovector amplitude, but on close examination one sees that isospin may be used approximately for convenience to some level of precision, but it is not assumed. Isospin symmetry is an assumption which will break down not far beyond the level of present discussions. It is nice not to have to rely on it in principle. Similarly, the assumption must be made that electromagnetic corrections can be controlled, but we will see later that this assumption is not a strong one in practice.
Not going into details of little interest for to-day's discussion the sum-rule has the following form:

$$g_c^2/4\pi = -4.50J^- + 103.3(\frac{a_{\pi^-p} - a_{\pi^+p}}{2}). \qquad (1)$$

Here J^- is given in mb by the weighted integral of the difference between the $\pi^{\pm}$p total cross sections:

$$J^- = \frac{1}{4\pi^2}\int_0^\infty \frac{(\sigma^T_{\pi^-p} - \sigma^T_{\pi^+p})}{\sqrt{k^2+m_\pi^2}}dk \qquad (2)$$

and $a_{\pi^\pm p}$ are the scattering lengths in units m_π^{-1}. The combination in which they occur is the isovector one a^- assuming that isospin holds.

The beauty of this relation is that all of the ingredients are physical observables; there is no need in principle for theoretical transformations of these observables, extrapolations or interpolations of any kind. The situation is basically quite clean. The main point to be controlled theoretically is the electromagnetic corrections; experimentally the main crux in the past has been that the scattering lengths, which contribute 2/3 of the relation, have not been possible to determine experimentally to sufficient precision. As a consequence the previous applications of this relation have been no more than consistency checks, or, alternatively, as used by the VPI group, a constraint imposed on their analysis of the πN scattering data [2]. We report here on the first precision evaluation of this relation from actual data by B. Loiseau,

A. W. Thomas and myself; this investigation is now close to be completed and is reported briefly in PANIC99 in near final form [3].

Our first step is to write the GMO relation (1) in a simple, robust form in such a way that the dominant experimental contributions stand out clearly and with well identified separate uncertainties. In particular, it is important to exactly impose the condition that $a_{\pi^-p} = 0.0883(8)m_\pi^{-1}$ [4] is accurately known from the energy shift in pionic hydrogen:

$$g_c^2/4\pi = -4.50J^- + 103.3a_{\pi^-p} - 103.3(\frac{a_{\pi^-p} + a_{\pi^+p}}{2}). \tag{3}$$

There is consensus in the literature about the approximate contribution from the dispersion integral J^-, although its uncertainty has only been discussed by R. Koch [5]. Its exact value is not so crucial, since it only contributes 30% to the relation. We make later a detailed discussion of its uncertainties. As a first numerical orientation we use momentarily the value $J^- = -1.077(47)$ mb [5]. This gives the following relation to be improved later:

$$g_c^2 = 4.85(22) + 9.12(8) - 103.3(\frac{a_{\pi^-p} + a_{\pi^+p}}{2}) = 13.97(23) - 103.3(\frac{a_{\pi^-p} + a_{\pi^+p}}{2}). \tag{4}$$

Note that this is not our final result and our best estimate of these terms is given in eq. (11) below. The key message from this way of writing the relation is that all the action is in the last term $1/2(a_{\pi^-p} + a_{\pi^+p})$, which, assuming isospin symmetry, is the isospin symmetric amplitude a^+: if this quantity is positive the coupling constant is smaller than 14, if it is negative it is larger. It is well known that we are dealing with a small quantity. If we use the old Koch-Pietarinen value [6] for $\frac{1}{2}(a_{\pi^-p} + a_{\pi^+p}) = a^+ = -83(38) \cdot 10^{-4}$ we would find $g_c^2/4\pi = 14.83(45)$, while the SM95 solution [7] with $a^+ = 20 \cdot 10^{-4}$ would lead to $g^2/4\pi = 13.75$. A small value for the coupling constant of the order of 13.6 requires $a^+ = 35 \cdot 10^{-4}$ or, as an alternative, a substantially smaller value for the cross section integral J^-. It is thus most important to obtain an accurate number for the small isoscalar amplitude. The main problem is that it is normally deduced from the cancellation to a few % between two much larger numbers and thus requires extreme precision in the extraction procedure. There are two main roads to attack this problem experimentally.

- The first one is to use that the charge exchange amplitude $\pi^-p \to \pi^0 n$ can be determined well from the 1s width of the π^-p atom [8]. Assuming isospin symmetry this gives directly the isovector term a^- in the GMO relation without any reference to the a_{π^-p} scattering length. At the present time however, the precision corresponds to $g_c^2/4\pi$ between 13.8 and 14.6. This is not now restrictive enough, so it is premature to use this road. Improvements will follow in coming years.

-The second road is to let nature do most of the work using the $\pi^- d$ scattering length $a_{\pi^- d} = (-0.0261(5) + i0.0063(7))m_\pi^{-1}$ [9], which is accurately determined from the pionic deuterium 1s energy level. To leading order this quantity is just the coherent sum of the π^-N scattering lengths from the neutron and the proton, which, assuming charge symmetry, is exactly the term required in our 'robust' relation above. The strong cancellation between the two terms is then achieved from the very beginning by the physics and only a summary description of the deuteron gives high absolute precision. In order to match the present precision using the first road, i. e., using the width, we only need a theoretical precision in the description of the deuteron scattering length to about 30%. Since the πd scattering length is a text book case of multiple scattering and 3-body theory, we will easily do better below.

This last road is the one we have followed. We now sketch how we have proceeded in order to achieve a quantitative control of theoretical uncertainties.

2 πN scattering lengths using the deuteron

As an orientation, consider first the problem in multiple scattering theory for the case of s-wave pions scattering from point-like nucleons. In the static (fixed scattering centers) approximation the leading structure and scale of the pion-deuteron scattering length is set by the coherent single scattering term S and the dominant s-wave double scattering term D proportional to the inverse deuteron radius $< 1/r >$ [10]. One has,

$$a_{\pi^- d}^{static} = S + D \ldots, \tag{5}$$

with

$$S = \frac{(1 + m_\pi/M)}{(1 + m_\pi/M_d)}(a_{\pi^- p} + a_{\pi^- n}), \tag{6}$$

where M and M_d are the nucleon and deuteron masses respectively. The two contributions to S cancel to nearly 1% precision. The double scattering term D plays therefore a disproportionately large role and it is the main correction:

$$D = 2\,\frac{(1 + m_\pi/M)^2}{(1 + m_\pi/M_d)}\,[(\frac{a_{\pi^- p} + a_{\pi^- n}}{2})^2 - 2(\frac{a_{\pi^- p} - a_{\pi^- n}}{2})^2] < 1/r > . \tag{7}$$

Its value using the final scattering lengths is $D = -0.0256\ m_\pi^{-1}$ to be compared to the experimental $a_{\pi^- d} = -0.0261\ m_\pi^{-1}$. This emphasises its dominant role, even if the agreement with observation is fortuitously good. The message is clear however: the theoretical description is fundamentally very good and this is supported by 3-body Faddeev calculations as well.

Inside multiple scattering theory the state of the art description of the various correction terms has been the recent theoretical investigation by Baru and Kudryatsev (B-K) [11]. We take their work as the departure for a critical and quantitative assessment of the theoretical uncertainties. Their discussion of such effects is only partial and we have in several respects found it necessary to make substantial modifications and additions to their results. A comparison of typical contributions in the two cases is summarized in Table 1 in which a), b), etc. refer to the following points of discussion.

Table 1: Typical contributions to $a_{\pi d}$ in units $10^{-4} m_\pi^{-1}$.

Contributions	Present work	B-K [11]
D	-256(7)	-252
Fermi motion a)	61(7)	50
dispersion correction b)	-56(14)	not included
isospin violation f)	3.5	3.5
$(\pi^- p, \gamma n)$ double scattering	-2	not considered
form factor effect d)	17(9)	29(7)
higher order terms	4(1)	6
S-P interference c)	small	-44
non-static effects e)	11(6)	10
P-wave double scattering f)	-3	-3
virtual pion scattering f)	-7(2)	not considered
$a_{\pi d}$(experimental) [9]	-261(5)	

The principal corrections are as follows.

a) The Fermi motion of the nucleons: the nucleons have a momentum distribution and impinge on the pion at rest. Consequently the pions have a momentum in the πN CM system and this leads to p-wave πN scattering contributions. The single scattering term from the πN p-wave scattering produces a physically well understood attractive contribution which can be reliably evaluated to leading order from the expectation value $< p^2 >$ of the nucleon momenta in the deuteron and the spin-isospin averaged P-wave threshold scattering amplitude c_0=0.209 m_π^{-3}. This term would normally be quite small, but here it is the single largest correction. The uncertainty comes from the D-state percentage in the deuteron, since for each percentage point, the D state is 10 times more effective than the S state in its contribution. The well known difference in the D-state probability (P_D=5.7% vs. 4.3%) for the Machleidt1 [15] vs. the Paris [16] wave functions sets the uncertainty. Very likely this can be improved in the future.

b) A repulsive -20% contribution not described by multiple scattering is produced by the dispersive term from the absorption reaction π^-d$\rightarrow$nn, which has been repeatedly determined using 3-body Faddeev approaches [12, 13, 14]. It has typically a theoretical uncertainty of 20% of its numerical value, which gives $-0.0056(14)m_\pi^{-1}$ [14]. The dispersive contribution is a theoretically calculated correction and the cited studies are about 20 years old, when the experimental precision was far less than it is to-day. The uncertainties reflect the neglect of higher order terms and model dependence of the approach. Although there is no reason to question these careful studies, a modern reinvestigation of this term is highly desirable, which may reduce the quoted uncertainty. It is the second largest correction and the single largest source of uncertainty. This correction was not included in B-K.

c) B-K advocate a -15% correction from 'sp' interference, in which one scattering is s wave and the other one p wave due to 'Galilean invariant' off-mass-shell contributions. This last term generates a S-wave interaction even for a single nucleon. The procedure is model dependent. On analysing the contribution we find that it is generated nearly entirely by the spin averaged isovector p-wave πN scattering volume c_1. It is described very well by the Born term, which has no ambiguity in its off-mass-shell structure. The contribution vanishes exactly for the Born term and there is then no correction for this effect. There is a small part of the amplitude not described by the Born term which might behave differently. The remaining term will then be quite small and there is no reason why it should obey the B-K prescription. We have thus suppressed this contribution entirely. This is our most important conceptual difference from the results of B-K.

d) Corrections for non-pointlike interactions and form factors. The simplest approximation to the double scattering scattering term D assumes that the πN scattering is pointlike. This is appropriate if the two scatterers are well separated as is the case for the bulk of the contributions in the case of the deuteron as a consequence of its loose binding. It is not necessary to describe this effect accurately, but the sign and magnitude must be controlled. The non-local effects enter mainly via the dominant isovector πN s wave interaction, which is well known to be closely linked to ρ exchange. This effect represents only a correction of about -10% and no major uncertainty is introduced even with a liberal variation of parameters.

e) Non-static effects. These produce only a rather small correction of about 4%. The nature of the leading non-static corrections and the reasons why the static expression (fixed scattering centers) still remains an excellent approximation are well understood. In a multiple scattering description there are systematic cancellations between such contributions from single and double scattering. This phenomenon was first demonstrated in the present context for an analytically soluble model by Fäldt [17]. It has been numerically investigated by B-K [11] using a Hulthén wave function and a separable amplitude with a dipole form factor with a cut-off parameter $3m_\pi$. Following B-K we have adopted a value 0.0010(6), where the liberal uncertainty reflects a lack of independent verification.

f) Corrections for virtual pion scattering, isospin violation, p-wave double scattering and higher order terms are all small and controllable corrections. The isospin violation in the πN interaction appears in the double scattering term and can be controlled by the explicit evaluation of the effect of the $\pi^\pm - \pi^0$ mass difference; there are well understood systematic cancellations between single and double scattering contributions of the same nature as the non-static effects. An additional check in which the mass difference is implicit is the chiral approach [18] and both approaches give a small positive contribution of about 1%. In view of the small effect and the fact that the isospin violation beyond the mass difference is not presently established experimentally for the πN amplitudes we discuss it only as a measure of uncertainty in our estimate of systematic errors. At present it has has a near negligible influence on the conclusions. It can be improved in the future when needed.

Based on this, we obtain well controlled values for the symmetric and antisymmetric combinations of scattering lengths $\frac{1}{2}(a_{\pi^-p} \pm a_{\pi^-n}) \simeq a^\pm$ deduced from the data. Preliminary, though nearly final, values are

$$\frac{1}{2}(a_{\pi^-p} + a_{\pi^-n}) = (-17 \pm 3(\text{statistic}) \pm 9\ (\text{systematic}))\ 10^{-4} m_\pi^{-1}$$

and

$$\frac{1}{2}(a_{\pi^-p} - a_{\pi^-n}) = (900 \pm 12)\ 10^{-4} m_\pi^{-1}.$$

These results are in excellent agreement with the central values deduced from the pionic hydrogen shift and width by the experimental PSI group, which concludes [8] $a^+ = -22(43) \cdot 10^{-4} m_\pi^{-1}$; $a^- = 905(42) \cdot 10^{-4} m_\pi^{-1}$. Our values represent, however, a substantial improvement in accuracy as seen in Figure 1.

By this evaluation we have achieved quantitative control of the dominant contribution to the GMO relation from the scattering lengths to about 1% or better in $g_c^2/4\pi$.

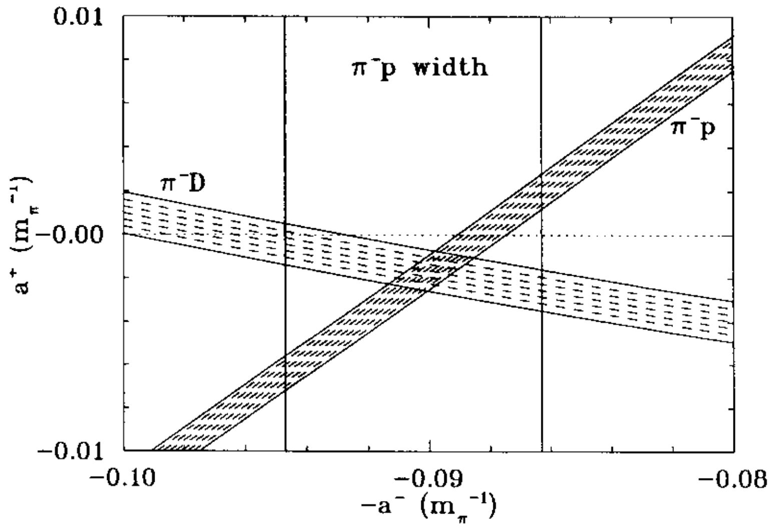

Figure 1: Our graphical determination of the πN scattering lengths

3 Cross section integral J^-

The cross section integral represents only one third of the GMO relation. This means that 3% uncertainty in the integral gives only 1% uncertainty in the coupling constant. Since total cross sections tend to be inherently accurate, the evaluation is easily performed with accuracy, but for the high energy region. There exists a vast amount of accurate measurements up to very high energies beyond 240 GeV/c. The possibility of systematic effects in the difference must be considered, particularly since Coulomb corrections have opposite sign for $\pi^{\pm}$p. The only previous evaluation with a detailed discussion and clearly stated sources of errors known to us is an unpublished study from 1985 by Koch which gives $J^- = -1.077(47)$ mb [5]. Later evaluations given in Table 2 find values within this band of errors, but the uncertainties are not stated and analyzed.

Table 2: Values of J^- from the literature

Source	J^- mb
Koch 1985 [5]	-1.077(47)
Workman et al. 1992; K-H [19]	-1.056
Workman et al. 1992; VPI [19]	-1.072
Arndt et al. 1995 [20]	-1.05
Gibbs-Kaufman 1998 [21]	-1.051
Present work	-1.083(31)

In view of the importance of obtaining a clear picture of the origin of present uncertainties we have reexamined this problem in spite of the approximate consensus. The uncertainty we find in the table of ±0.031 is a comfortable one at present in the sense that increasing it to ±0.040 has only a small influence on the uncertainty of the coupling constant. We limit the discussion to the critical features. The typical shape of the integrand is seen in Figure 2.

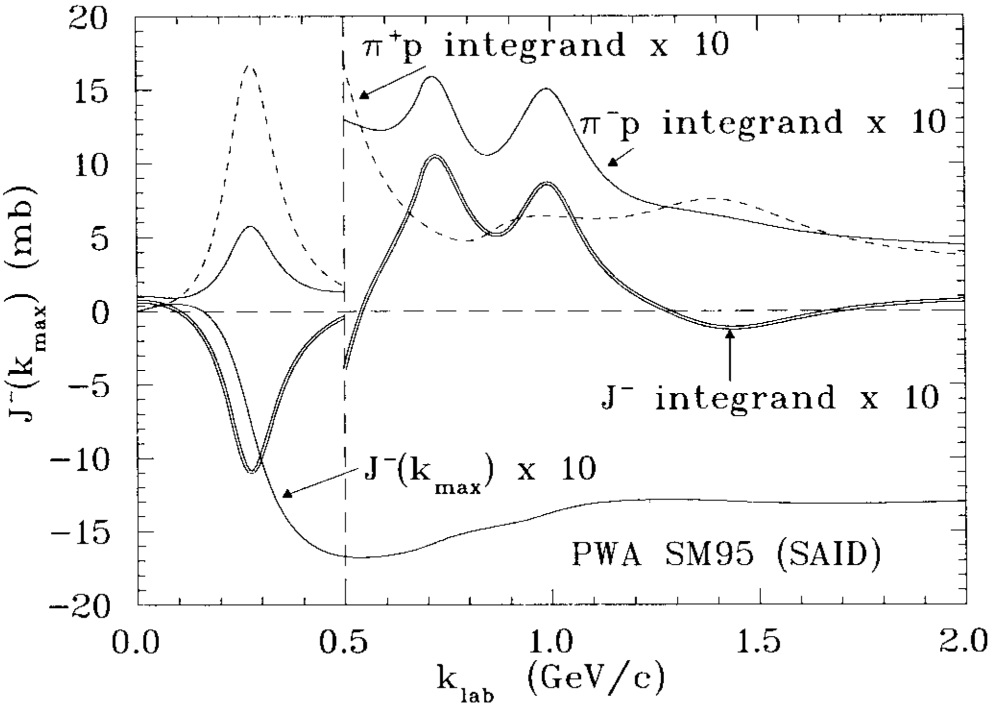

Figure 2: The separate integrands for $\pi^{\pm}$p as well as for their difference as function of k_{lab} as well as the cumulative value of the integral $J^{-}(k_{lab})$ from the region $0 < k < k_{lab}$. The integrands are in units of mb GeV/c

As one might expect the main contributions come from the region of the Δ resonance and just above. It would be false however to believe that this is the region that produces the uncertainty of the integral. There are no strong cancellations in the difference between the total $\pi^{\pm}$p cross sections in that region and the cross sections have been very carefully analyzed. Systematic uncertainties of 2-3% or more are very unlikely indeed; if they occur, they would certainly influence other determinations of the coupling constant importantly. We have first evaluated the purely hadronic cross sections up to 2 GeV/c based on the VPI phase-shift solution [7]. In this approach Coulomb corrections and penetration factors have been taken into account

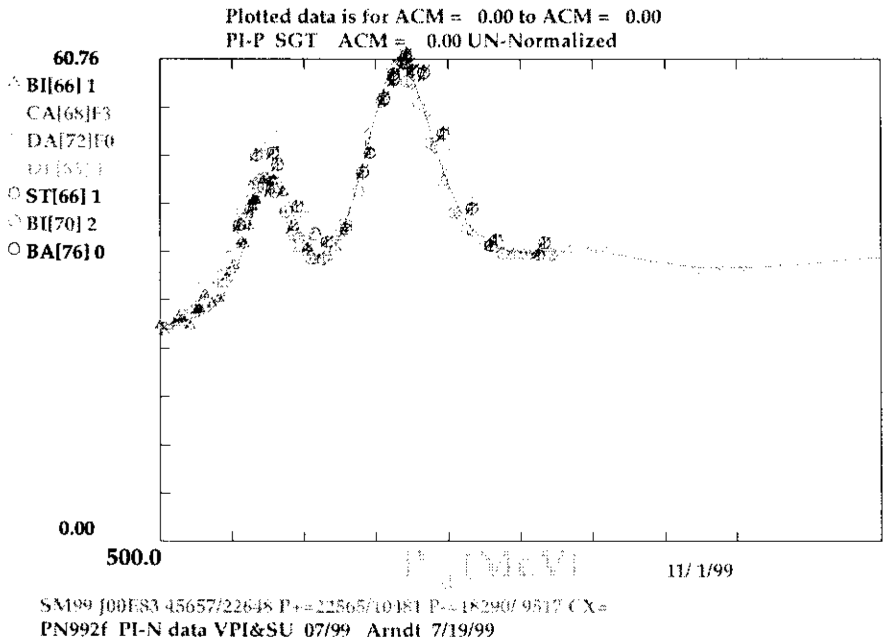

Figure 3: The π^-p total cross sections in the region $0.5 < p_{lab} < 2$ GeV/c according to R. Arndt including penetration factors compared to experimentally measured cross sections.

in the adjustment to experimental data even if the treatment may not be optimal. It also allows for some isospin breaking, since the Δ mass splitting is parametrised. In view of the low accuracy we aim for this should be adequate in principle. Bugg [22, 23] has emphasised that in the π^+p scattering the total cross sections are systematically reduced at all energies by the Coulomb repulsion between the particles and, conversely, in π^-p scattering, the total cross sections are systematically enhanced. One must correct for this effect, which gives a negative contribution to J^-; the coupling constant will be underestimated in the absence of such corrections. Below 500 MeV/c the correction is explicitly applied to the data analysis using the standard Tromborg procedure [24]. At higher energies a rough estimate of the magnitude is readily obtained using ray optics and assuming the total cross section produced by a black diffracting sphere of radius R (or a grey diffracting disk) with the Coulomb potential $V(R)$ evaluated at the surface [22]. This gives

$$\sigma^T(\pi^- p) - \sigma^T(\pi^+ p) \simeq A(\sigma^T(\pi^- p) + \sigma^T(\pi^+ p))\omega/k^2, \tag{8}$$

where $A = 3.7$ MeV for $R = 0.8$ fm. The correction factor is then about

1.5% at 500 MeV/c. The integrated contribution to the integral above a momentum k and assuming a constant total cross section is $2A/k^2\omega\sigma_T(R)$, which typically gives $1/k_0$(GeV/c)*0.007mb. A similar correction is included in the SM95 Coulomb corrections below 2 GeV/c. As an illustration the resulting fits for the solution SM99 of Arndt et al. is shown in Figs. 3 and 4 in the range $0.5 < p_{lab} < 2$ GeV/c. We have not made any correction for it at higher energies, which means that we systematically will underestimate the coupling constant somewhat. This effect has little bearing on the issue of a large vs. a small coupling constant.

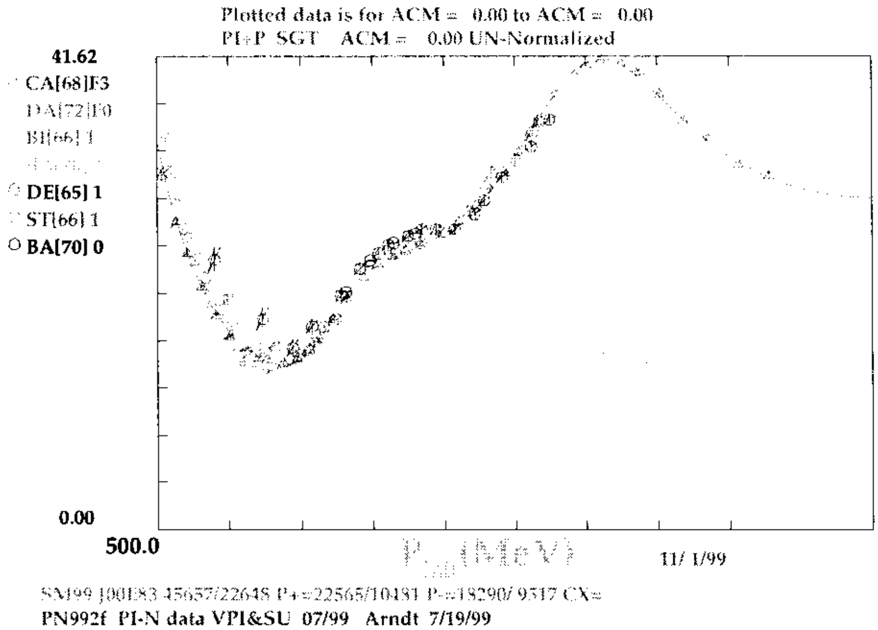

Figure 4: As in Fig. 3 but for the π^+p total cross sections

There are no explicit measurements of total cross sections below 160 MeV/c, but the accurately known scattering lengths give a very strong constraint assuming isospin symmetry. The s- and p-wave contributions give nearly cancelling terms from this region each representing only about 5% of the total J^- or less than 2% of the coupling constant. A small correction occurs, since isospin is broken by the 3.3 MeV lower threshold for the π^0n channel below the physical π^-p threshold. The additional phase space contributes to the integral. The corrections are easily evaluated and partly cancel. They are small and of the order of a few parts in 10^3 and less in the coupling constant. The effect appears nearly irrelevant in the present context.

Another isospin breaking effect is the Δ mass splitting, which may affect the deduced coupling constant also in determinations based on πN data. The magnitude of the effect is apparent from the textbook difference of the $\pi^{\pm}$d cross sections by Pedroni et al. [25]: the dominant Δ^{++} contribution is shifted to lower energies. To leading order the deuteron gives the sum of the cross sections on the neutron and the proton and thus the Pedroni data gives a direct measure of the splitting in a null experiment, i.e., it would give zero in the limit of isospin symmetry and no Coulomb effects. As compared to strict isospin symmetry and guided by these data the value of J^- becomes more *negative* by less than 1%, which *increases* the coupling constant by less than 0.3%. In SM95 the effective position of the Δ resonance is chosen as the average between the Δ^{++} mass and the Δ^0 one. The splitting is small and eliminated as an e.m. correction from the hadronic cross sections. In this case the effect in J^- is only 0.05% (private communication by R. Arndt) as a consequence of a 7 times smaller mass splitting. The GMO dispersion relation does not assume the validity of isospin, however, and it concerns the difference between cross sections for which the splitting effect is not eliminated by the choice of an effective resonant mass. Apart from the effect of the Coulomb interaction the splitting contribution is real. We consider the estimate above a liberal upper limit for the size of this small systematic effect.

It is interesting to quantify the Coulomb penetration effects more in detail. In the region 550 MeV/c to 1.2 GeV/c the correction to J^- is -0.029 mb (-2.7%) as compared to using direct experimental cross sections and from 1.2 to 2 GeV/c -0.027mb (-2.5%). Even if these corrections would be accurate only to 33%, the uncertainty from this source is only $\pm$0.019 mb in J^- or $\pm$0.6% in the coupling constant. Thus, the uncertainty of simply using hadronic cross sections from a good modern PWA description up to 2 GeV/c appears quite satisfactory[7].

In the region around 500 MeV/c there are long-standing problems with the experimental total cross section data. Those of Davidson et al. [29] are generally thought to have an incorrect energy calibration by about 10 MeV/c. Its 72 data points must either be recalibrated or eliminated from the analysis [26], [27]. Similarly, the SM95 solution driven by modern angular distributions is systematically lower than the data of Carter et al. [28] below 600 MeV/c, a region were the data for experimental reasons are less reliable than at higher energies. These points have been omitted in the PWA analysis [27] (see e.g. Figs. 3 and 4). This experimental uncertainty, which is much larger than the Coulomb penetrability effects, should be resolved. Under the circumstances we have preferred to use the SM95 PWA solution as the best guide.

The real uncertainty in J^- comes from the high energy region and is associated with the relatively slow convergence of the integral. The region 2 GeV $< k <$4.03 GeV/c gives a moderate contribution with a modest ± 0.007 mb systematical error when using the Particle Data Group 1998 tables [30]. The penetration factors would make this contribution more negative, but are neglected. At still higher energies an important effort to measure and analyse cross sections has been made, since the issue of the rate at which the $\pi^{\pm}$p cross sections become the same is important for the discussion of high energy asymptotics. Since these studies rely on the difference of the cross sections as here, we have accepted the PDG treatment of these data as far as the Coulomb penetration problem is concerned. Cross section data with considerable systematic accuracy exist from 4.03 GeV/c $< k <$370 GeV/c and are listed in the Particle Data Group tables [30, 31]. These have been carefully parameterized above 10 GeV/c by the Particle Data Group in terms of a Regge expansion in s. Errors are not cited for the region 10 to 240 GeV/c in the 1994 tables [31], but use of the 1998 Regge fit [30] from 240 GeV/c to ∞ gives a contribution of 0.018(3). The 1994 version also lists a fit from 4.03 GeV/c to 240 GeV/c corresponding to a contribution of 0.155 mb. The main contribution comes from the region well below 240 GeV/c. An evaluation using selected data gives a lower value 0.133$\pm$0.005 (statistic) $\pm$0.022 (systematic) with the error dominated by the large systematic uncertainty. The larger value has been used in several previous GMO evaluations [21, 32]. We prefer to use the lower value, in particular since this is consistent with the 1998 Data Table fit to the region above 240 GeV/c which gives a lower 0.018(3) mb than the 0.030 mb obtained with the Regge theory fit given in the 1994 Tables. We ascribe a 0.4% uncertainty from this contribution. It is noteworthy that a small, but non-negligible, contribution of +0.6% to the final coupling constant comes from the very high energy region, above 240 GeV/c.

The upshot of this is that the uncertainty in J^- comes mostly from a region which has attracted strong theoretical interest in connection with the asymptotics of high energy cross sections. We have evaluated the different contributions with no other Coulomb and penetration corrections than those introduced by the experimental authors above 2 GeV or by the theoretical analysis below 2 GeV. We find, based on the SM95 and Arndt 12/98 analysis below 2 GeV/c [7], and on the Regge pole PDG94 and PDG98 extrapolation beyond 240 GeV/c, the values $J^- = (-1.075 \pm 0.008)(25)$ mb and $(-1.090 \pm 0.008)(25)$ mb respectively. Based on this we have adopted the mean value $J^- = (-1.083 \pm 0.008)(30)$ mb for the integrated cross section. Here we have added a systematic uncertainty of ± 0.017 from the region less than 2 GeV/c and note should be taken that such corrections above this region will make the integral systematically more negative. We note that the different evaluations of this quantity by different authors all stay within the errors quoted above.

4 Results

In conclusion, we have derived first new values for the πN scattering lengths from the π^-d one by carefully examining the statistical and systematic contributions. We have shown that previous approaches have been incomplete and in part contain spurious terms. We obtain,

$$a^+ \simeq \frac{a_{\pi^-p} + a_{\pi^-n}}{2} = (-17 \pm 3)(9)) \cdot 10^{-4} m_\pi^{-1} \tag{9}$$

and

$$a^- \simeq \frac{a_{\pi^-p} - a_{\pi^-n}}{2} = 900(12) \cdot 10^{-4} m_\pi^{-1}. \tag{10}$$

Our second conclusion concerns the charged πNN coupling constant which can be derived from the GMO forward dispersion relation using these new accurate values for the πN scattering lengths. According to eq. 3 using $J^- = (-1.083 \pm 0.008)(30)$ and charge symmetry:

$$g_c^2/4\pi = (4.87 \pm 0.04)(14) + (9.12 \pm 0.08) + (0.18 \pm 0.03)(9) = (14.17 \pm 0.09)(17). \tag{11}$$

The uncertainty no longer originates in the uncertainty in the scattering lengths, but it comes mainly from the weighted integral J^- of the difference between the charged pion total cross sections. The value we obtain for the coupling constant is intermediate between the low value deduced from the large data banks of NN and πN scattering data and the high value from np charge exchange cross sections [33]. It is fully compatible with the latter, differing statistically by only about one standard deviation. The uncertainty is mainly of systematical origin. Such errors do not have a Gaussian probability distribution, and in the present case it is a much narrower one. Consequently it is not easy to reconcile the present GMO value with a low value of about 13.6. The modification of the value of J^- required to accommodate a low value of about 13.6 is about 10%. Most of such a modification must come from the region above 2 GeV, which implies changes in the contributions from that region of the order of 50%. Such large changes appear unlikely to us.

We therefore conclude that the present evaluation of the GMO sumrule with quantitatively controlled uncertainties in the input values for the πN isoscalar scattering length as well as for the cross section integral J^- does not readily support the conclusion of the indirect determinations that the πNN coupling is in the range of 13.5.

Additional support for a relatively large coupling constant are the recent measurements by Raichle et al. of polarised np total cross sections [34]. From these the pion dominated ϵ_1 parameter can be determined. They find that it is systematically larger than the values in the old phase shift analysis PWA93 by the Nijmegen group [35]. If the discrepancy persists in other PWA's, it suggest as a possible partial explanation that the coupling constant is too small. In any case, it points to inconsistencies in the PWA analysis on which the argument for a low coupling constant is based, as do the Uppsala and Freiburg np elastic data as discussed elsewhere in this workshop.

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