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Broken SU(3) symmetry and ground–γ band coupling in deformed even–even nuclei

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Abstract

The link between the ground (g) and the γ- bands of even deformed nuclei is studied via the collective vector–boson model with a broken SU(3) symmetry. The g–γ band-mixing interaction as well as the limiting cases, in which the SU(3) symmetry is reduced completely, are estimated in terms of the energy splitting between these two bands. It is pointed out, that the systematic behaviour of the g–γ splitting, observed in rotational regions, supports our analyses.

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Abstract

The link between the ground (g) and the γ- bands of even deformed nuclei is studied via the collective vector–boson model with a broken SU(3) symmetry. The g-γ band-mixing interaction as well as the limiting cases, in which the SU(3) symmetry is reduced completely, are estimated in terms of the energy splitting between these two bands. It is pointed out, that the systematic behaviour of the g–γ splitting, observed in rotational regions, supports our analyses.

In a recent work we started a systematic study of the link between the ground (g) and the γ- collective bands in even–even deformed nuclei [1]. The approach is based on the collective Vector–Boson Model (VBM) with a broken SU(3) symmetry [2]. In VBM the two bands (g and γ) belong to one split SU(3) multiplet, appearing in the $SU(3) \supset O(3)$ group reduction and labelled by a given irreducible representation (irrep) (λ, μ) of SU(3). The corresponding basis states are constructed with the use of two vector–boson creation operators $\boldsymbol{\xi}^+$, $\boldsymbol{\eta}^+$ and are denoted as [2]

$$\left| \begin{array}{c} (\lambda,\mu) \\ \alpha, L, M \end{array} \right\rangle . \qquad (1)$$

The quantum number α distinguishes the various O(3) irreps, (L, M), appearing in a given SU(3) irrep (λ, μ) and labels the different bands of the multiplet. The SU(3)–symmetry breaking Hamiltonian is constructed by using three basic O(3) scalars, which belong to the enveloping algebra of SU(3) [2]:

$$V = g_1 L^2 + g_2 L \cdot Q \cdot L + g_3 A^+ A . \qquad (2)$$

Here g_1, g_2 and g_3 are free parameters; L and Q are the angular momentum and quadrupole operators respectively; and $A^+ = \boldsymbol{\xi}^{+2}\boldsymbol{\eta}^{+2} - (\boldsymbol{\xi}^+ \cdot \boldsymbol{\eta}^+)^2$.

Using the VBM formalism, we have shown that, for a given nucleus, the physically significant features of SU(3)–symmetry should be studied in certain regions of (λ, μ) irreps instead of a single fixed irrep [1]. On this basis we have investigated various nuclei of rare earth region and actinides for which the model descriptions of the g- and γ–bands have been evaluated in a wide range of SU(3) irreps. It has been found that the basic SU(3) properties of deformed nuclei depend on the SU(3) splitting. The latter is characterized by the ratio [1]

$$\Delta E_2 = (E_2^{\gamma} - E_2^{g})/E_2^{g} \ , \tag{3}$$

where E_2^g and E_2^γ are the energy levels with angular momentum $L = 2$, which belong to the g- and the γ- band respectively. In the rare earth region, this ratio varies within the limits $7 \leq \Delta E_2 \leq 18$, while in the actinides one observes values in the range $13 \leq \Delta E_2 \leq 25$.

In the nuclei with small band-splitting ratios, $\Delta E_2 \sim 8 - 10$ (for example ^{168}Er), we have established clearly outlined regions of "favoured" SU(3) multiplets (with relatively small λ-values $\lambda = 14 - 20$ and $\mu = 2, 4, 6$), where the model descriptions are essentially better than in the other irreps. With the increase of the splitting energy, $\Delta E_2 \sim 12$ (as in ^{178}Hf), the favoured multiplets are shifted to larger λ-values ($\lambda \sim 40$). For the nuclei with large band-splitting, $\Delta E_2 \sim 14 - 22$ (^{172}Yb), we have obtained almost equally good descriptions in all (λ, μ)-multiplets with $\lambda > 60 - 80$ without the presence of any upper limit for the quantum number λ.

The above results have a reasonable interpretation in terms of the band-mixing interactions. Some preliminary estimates, provided for the $(\lambda, 2)$ multiplets, show that the increase in the quantum number λ is connected with the corresponding decrease in the g–γ band-mixing interaction [1]. It follows, that in the nuclei with small SU(3) splitting the two bands should be strongly mixed, while in the cases with a large splitting they should interact weakly. This suggests a possible transition from the g–γ band coupling scheme to another scheme in which the bands belong to separate SU(3) irreps.

The study of such a possibility requires more quantitative information about the meaning in which the SU(3) symmetry is reduced in (λ, μ)-plane. For this reason, we extend our investigations towards the (λ, μ) multiplets with $\mu \geq 2$, by estimating analytically the g–γ interaction in various limits of the model. Also, we expect that the SU(3) splitting will play an important role in these considerations. It is therefore worthwhile to derive an analytic relation between the energy splitting and the SU(3) quantum numbers λ and μ. In order to implement such an analysis we study the energy levels E_2^g and E_2^γ [in Eq. (3)] in terms of VBM.

For any (λ, μ) multiplet ($\mu \geq 2$), the levels E_2^γ (labelled by $\alpha_1 = \mu/2 - 1$) and E_2^g (labelled by $\alpha_2 = \mu/2$) are the only possible ones, appearing at angular momentum $L = 2$. Hence for $L = 2$, the Hamiltonian matrix is always two-dimensional and the corresponding eigenvalue equation has the form:

$$det \begin{pmatrix} V_{1,1} - \omega^{L=2} & V_{1,2} \\ V_{2,1} & V_{2,2} - \omega^{L=2} \end{pmatrix} = 0 \tag{4}$$

where $\omega^{L=2}$ are the eigenvalues and

$$V_{j,j'} \equiv \langle \alpha_j, 2 | V | \alpha_{j'}, 2 \rangle = \left\langle \begin{matrix} (\lambda, \mu) \\ \alpha_j, 2, 2 \end{matrix} \middle| V \middle| \begin{matrix} (\lambda, \mu) \\ \alpha_{j'}, 2, 2 \end{matrix} \right\rangle \ , \tag{5}$$

with $j, j' = 1, 2$, are the corresponding Hamiltonian matrix elements. We have derived these matrix elements in the form:

$$V_{1,1} \ = \ \langle (\frac{\mu}{2} - 1), 2 | V | (\frac{\mu}{2} - 1), 2 \rangle = 6g_1 + 6g_2(2\lambda + 2\mu + 3) + g_3 P(\lambda, \mu) \ , \tag{6}$$

$$V_{2,2} = \langle\frac{\mu}{2},2|V|\frac{\mu}{2},2\rangle = 6g_1 - 6g_2(2\lambda+2\mu+3) + g_3Q(\lambda,\mu) \ , \quad (7)$$

$$V_{1,2} = \langle(\frac{\mu}{2}-1),2|V|\frac{\mu}{2},2\rangle = 12g_2\mu - 2g_3\mu(\mu-2) \ , \quad (8)$$

$$V_{2,1} = \langle\frac{\mu}{2},2|V|(\frac{\mu}{2}-1),2\rangle = -12g_2\lambda + 2g_3\lambda(\lambda+2\mu+2) \ , \quad (9)$$

where

$$P(\lambda,\mu) = \lambda(\mu-2)(\mu+2)(\lambda+2\mu+2) + \mu(\mu-2)(\mu+1)(\mu+3) \ , \quad (10)$$

$$Q(\lambda,\mu) = \lambda\mu^2(\lambda+2\mu+2) + \mu(\mu-1)(\mu+1)(\mu+2) \ . \quad (11)$$

This allows us to study the g–γ band-mixing interaction at $L=2$ in (λ,μ)-plane. Since the basis states (1) are determined for $\mu \le \lambda$, the model scheme has two limits: (i) $\lambda \to \infty$, with μ finite, and (ii) $\lambda \to \infty$, $\mu \to \infty$, with $\mu \le \lambda$. In each of them we estimate the λ- and/or μ- dependence of the matrix elements (6)–(9).

In case (i), the matrix elements are determined by the corresponding highest degrees of λ. So, for $\mu > 2$ the Hamiltonian matrix $(V_{i,j})$ obtains the following asymptotic form:

$$(V)_{\lambda\to\infty} = \begin{pmatrix} \lambda^2 & * \\ \lambda^2 & \lambda^2 \end{pmatrix} \ , \quad (12)$$

where the upper off-diagonal element (denoted by $*$) does not depend on λ. Then the relative contribution of the off-diagonal (band-mixing) terms in the eigenvalue equation (4) decreases with the increase of λ as $\lambda^2/\lambda^4 = 1/\lambda^2$. For $\mu = 2$ the term $V_{1,1}$ is proportional to λ instead of λ^2 [See Eqs. (6) and (10)], so that in this particular case the off-diagonal contribution decreases as $1/\lambda$.

Consider case (ii), $\lambda \to \infty$, $\mu \to \infty$, $(\mu \le \lambda)$. Since the difference $\lambda - \mu$ is always finite, we can take $\mu = \lambda$. Then the asymptotic form of the matrix $(V_{i,j})$ is:

$$(V)_{\substack{\lambda\to\infty \\ \mu\to\infty}} = \begin{pmatrix} \lambda^4 & \lambda^2 \\ \lambda^2 & \lambda^4 \end{pmatrix} \ . \quad (13)$$

Here we find that the band-mixing interaction decreases as $\lambda^4/\lambda^8 = 1/\lambda^4$, i.e., more rapidly in comparison to the previous case.

Generally, our analysis shows that the increase in the quantum numbers λ and/or μ is connected with a corresponding decrease in the g–γ band-mixing interaction within the framework of the SU(3) symmetry. In both limits, (i) and (ii), the g–γ mixing decreases asymptotically to zero. In such a way the SU(3) symmetry disappears completely, and the two bands do not belong to the same SU(3) multiplet any more.

We further turn to the energy levels E_2^g and E_2^γ, which are determined as

$$E_2^g = \omega_-^{L=2} - \omega^{L=0} \ , \quad (14)$$

$$E_2^\gamma = \omega_+^{L=2} - \omega^{L=0} \ . \quad (15)$$

Here

$$\omega_\pm^{L=2} = \frac{1}{2}\left\{V_{1,1} + V_{2,2} \pm \sqrt{(V_{1,1}+V_{2,2})^2 - 4(V_{1,1}V_{2,2} - V_{1,2}V_{2,1})}\right\} \quad (16)$$

are the solutions of the eigenvalue equation (4), and $\omega^{L=0} = g_3\mu^2(\lambda+\mu+1)^2$ is the zero-level eigenvalue. After using Eqs. (6)–(9) we obtain the following analytic expressions for E_2^g and E_2^γ:

$$E_2^g = 6g_1 - 2Fg_3 - 2\sqrt{Ag_2^2 + Bg_3^2 - Cg_2g_3} \ , \quad (17)$$

$$E_2^\gamma = 6g_1 - 2Fg_3 + 2\sqrt{Ag_2^2 + Bg_3^2 - Cg_2g_3} \ , \quad (18)$$

where

$$A = A(\lambda,\mu) = 9[(2\lambda+2\mu+3)^2 - 4\lambda\mu] \ ; \quad (19)$$

$$B = B(\lambda,\mu) = [\lambda(\lambda+2\mu+2) + \mu(\mu+1)]^2 - \lambda\mu(\lambda+2\mu+2)(\mu-2) \ ; \quad (20)$$

$$C = C(\lambda,\mu) = 6(2\lambda+2\mu+3)[\lambda(\lambda+2\mu+2)+\mu(\mu+1)] - 6\lambda\mu(\lambda+3\mu) \ ; \quad (21)$$

$$F = F(\lambda,\mu) = \lambda(\lambda+2\mu+2) + 2\mu(\mu+1) \ . \quad (22)$$

Then the energy splitting ratio ΔE_2 takes the following analytic form:

$$\Delta E_2 = \frac{2}{(3g_1 - Fg_3)/\sqrt{Ag_2^2 + Bg_3^2 - Cg_2g_3} - 1} \ . \quad (23)$$

The above relation allows the study of the g–γ band-splitting in (λ,μ) plane as well as its estimation in the two limiting cases.

So, in case (i), the functions (19)–(22) have the following asymptotic behaviour:

$$A_{\lambda\to\infty} = 36\lambda^2; \quad B_{\lambda\to\infty} = \lambda^4; \quad C_{\lambda\to\infty} = 12\lambda^3; \quad F_{\lambda\to\infty} = \lambda^2 \ .$$

After applying them in Eq. (23), we find the limit:

$$\lim_{\lambda\to\infty} \Delta E_2 = \frac{2}{-g_3/|g_3| - 1} \ . \quad (24)$$

We remark that the application of VBM in rare earth nuclei and actinides requires $g_3 < 0$ [1], which gives $\lim_{\lambda\to\infty} \Delta E_2 = \infty$. Therefore in this case the g- and the γ- band are completely split, i.e., they should not be considered in the same energy scale any more.

In the limiting case (ii) one has:

$$A_{\lambda=\mu\to\infty} = 108\lambda^2; \quad B_{\lambda=\mu\to\infty} = 13\lambda^4; \quad C_{\lambda=\mu\to\infty} = 72\lambda^3; \quad F_{\lambda=\mu\to\infty} = 5\lambda^2 \ .$$

Then the SU(3) splitting ratio goes to:

$$\lim_{\lambda=\mu\to\infty} \Delta E_2 = \frac{2}{-(5/\sqrt{13})g_3/|g_3| - 1} \ . \quad (25)$$

For $g_3 < 0$ we obtain $\lim_{\lambda=\mu\to\infty} \Delta E_2 = 2/(5/\sqrt{13} - 1) = 5.1$. Therefore in this case the band-mixing interaction vanishes, while the energy splitting between the two bands remains finite.

The limits considered allow some directions to be outlined in (λ, μ)–plane, which could be appropriate to a study of the transition between the different band coupling schemes. We note that in case (i) ($\lambda \to \infty$, with μ finite) a continuous decrease in the band-mixing interaction corresponds to a continuous increase in the g–γ band splitting. Therefore, this case is not able to reproduce a vanishing interband interaction at finite energy splitting. The limiting case (ii) ($\lambda = \mu \to \infty$) implies that the strong suppression of the band interaction as well as the transition between the different band coupling schemes could be realized at reasonable (finite) SU(3) splitting. Based on the above considerations, we deduce that the possibly interesting, physically meaningful, directions in the (λ, μ)–plane should be associated with a consistent increase in the quantum numbers λ and μ. Thus, any particular direction of interest could be easily estimated by using its intermediate behaviour between the two considered limiting cases.

The systematic behaviour of rotational spectra, observed in rare earth nuclei and actinides, supports the above theoretical analysis. The data show, that for the nuclei near the ends of rotational regions, the g–γ band-splitting is relatively small, while for the mid-shell nuclei it is essentially larger. In terms of our considerations, the strong g–γ splitting, observed in the middle of a given rotational region, corresponds to the weak mutual perturbation of the bands. This is consistent with the respective good rotational behaviour of the g-band, which in this case could belong to a separate SU(3) multiplet.

So, the theoretical results and experimental data suggest that the g–γ band coupling scheme is more appropriate near the ends of rotational regions while in the midshell regions another coupling scheme, such as the Interacting Boson Model (IBM) scheme [3] (with β–γ band coupling) could take a place. In this respect, the detailed comparison of VBM and IBM band-coupling mechanisms would be of great interest. The consistent study (within both models) of the limits in which the SU(3) symmetry is reduced completely could give important information about the rearrangement of rotational bands into different SU(3) irreps.

In conclusion, we have examined within the VBM formalism the possible ways in which the g–γ band coupling scheme could be reduced. The results obtained are consistent with the experimental behaviour of rotational spectra and suggest detailed comparison of the different classification schemes in deformed nuclei.

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