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SIMPLE Z_o^* MODEL

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A B S T R A C T

A model for isoscalar KN scattering near the K*N threshold is constructed, and solved analytically in the approximation of retaining only pion exchange terms. Basic features of phenomenological phase shifts are reproduced, including a wide $J^P = \frac{1}{2}^+$ exotic resonance, $Z_o^*(1800)$.

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Possibly KN scattering contains exotic resonances Z^* near $\sqrt{s} \approx 1.7$ to 2.0 GeV [1),2)], and dynamical links with strong inelastic channels ($K\pi N$, K^*N, $K\Delta$,...) are conjectured [3)]. Here we are concerned with the $I = 0$ processes for which we construct a crude but interesting multichannel dynamical model. In the version presented here the model is simple enough to be analytically tractable, but contains what seems to be the essential dynamics. Indeed it predicts isoscalar S, P and D wave KN scattering amplitudes in the Z_0^* region in reasonable agreement with the favoured solution of a recent phase shift analysis [4)]. In particular, we find a wide exotic resonance in the $J^P = \frac{1}{2}^+$ (P wave) state. The important ingredient is a large amplitude for K^* production by π exchange, and there is one parameter which, if fixed by SU_6, gives a resonance mass of about 1800 MeV.

The $I = 0$ KN system is chosen for relative simplicity - the dominant $K\pi N$ inelastic channel becomes strong about $\sqrt{s} \approx 1.8$ GeV where it is almost all (~90%) taken up by K^*N [5)]. Thus to a good approximation the situation in the region of interest (1.7-2.0 GeV approximately) is represented by a two-channel (KN, K^*N) problem where the relatively narrow K^* can reasonably be treated as stable. For $I = 1$ (where there is less K^* production [5)]) the $K\Delta$ threshold is an additional complication and the wider Δ is less realistically taken as stable.

MODEL

The model we propose is a two-channel (KN, K^*N) K matrix representation valid near the K^*N threshold at 1.83 GeV. Phase shifts for S, P and D wave scattering are constructed from estimates of the K matrix elements made from phenomenological Regge pole exchange amplitudes. This follows an idea first pursued by Lovelace [6)] in a calculation of phase shifts from a Veneziano model for the coupled ($\pi\pi$, $\bar{K}K$) system. The Lovelace prescription identifies partial wave projections of the B_4 amplitudes as K matrix elements. The effects of unitarity (especially low energy interchannel coupling) are thus treated as perturbations on a dual Regge pole representation.

Further work on a dual link between Reggeon amplitudes and low energy K matrix elements has found considerable phenomenological success in πN scattering [7)]. It corresponds at higher energies to dressing exchanged bare poles in a phenomenologically and theoretically respectable way with Regge cut corrections [7),8)]. Diffractive scattering (or "background", or Pomeron $\mathbb{P}$ exchange) is neglected at low energy. Presumably such effects are built at higher energies by multiparticle intermediate states in the unitarity sum [9)]. The $\pi\pi$ calculation is especially successful for the non-resonant $(I=2)$ channel, where the full B_4 structure is in fact irrelevant and simpler estimates of the Regge exchanges give equally good phase shifts [10)]. Likewise, in the present exotic channels the lack of a satisfactory narrow width dual amplitude is no handicap - phenomenological Regge pole exchanges should serve. However, for the (KN, K*N) system two new features are present : (i) large inelasticity in the exotic channel, and (ii) strong π exchange amplitudes. Indeed these turn out to be the crucial features of the problem.

REGGE MODELS

Estimates of Regge pole exchange amplitudes are needed for the three reactions connected through unitarity : $KN \to KN$, $KN \to K^*N$, $K^*N \to K^*N$. For the first two, there are data to fit - the third is accessible only through higher symmetries. Note that we propose to extrapolate the Regge models down to the threshold region. Duality is the guiding principle indicating that this gives (up to unitarity corrections) the correct average low-energy phase. Since we treat exotic channels without (at this level) resonance structure, the "average" is arguably almost local. Certainly, down to very low energies, our $KN \to KN$ and $KN \to K^*N$ models give excellent fits to the rather unchanging data.

We summarize only pertinent details of the Regge fits. Further details are given elsewhere, in a wider context [11)]. As we shall show, the only important feature in the $I=0$ channel is π exchange in $KN \to K^*N$, $K^*N \to K^*N$. Natural parity exchanges are relatively negligible.

We start with $KN \to K^*N$. The data [12)] in the usual Regge region (about 4 GeV/c and above) show $K^{\pm}p \to K^{*\pm}p$ to the mainly natural parity exchange and the charge-exchange reactions $K^-p \to \bar{K}^{*0}n$ and $K^+n \to K^{*0}p$ to be dominated by unnatural parity exchange. A good fit to available cross-section and density matrix data [12)] is possible with a very simple Regge pole exchange model with the strongly EXD quartet ρ, f^0, ω, A_2 (collectively M) plus the π. The M and π exchange couplings are chosen according to a prescription for Reggeizing elementary exchanges described by simple Lagrangians, which has considerable success elsewhere in applications to resonance production [11]. In K* production elementary M exchange is treated as vector exchange V. The simplest Lagrangians for the couplings VVP, VPP, PNN [$P \equiv$ pseudoscalar (π, K), $N \equiv$ nucleon] are unique : respectively of the form $\phi\varepsilon(p_1 p_2 V_1 V_2)$, $\phi \overleftrightarrow{\partial}_\mu \phi V^\mu$, $\bar{\psi}\gamma_5\psi\phi$. Data demand the VNN coupling to be of the form $\bar{\psi}\partial_\mu \psi V^\mu$ [11]. To Reggeize, Feynman propagators are replaced by (signature factor) $\times\, (s/s_o)^{\alpha(t)}$ and coupling constants by residues with ghost-killing factors as appropriate. With this prescription surviving amplitudes for $KN \to K^*N$ are $M_{\pm 1}$ and π_o, where subscripts refer to t channel helicities, there is no t channel nucleon helicity flip, and the amplitudes $M_o = \pi_{\pm 1} \equiv 0$ *). For the KN $I = 0$ channel we have specifically

$$M_{\pm 1} = \gamma\left[\varphi(s,t)\right]^{1/2} \Gamma(1-\alpha_M) \left(s/s_o\right)^{\alpha_M - 1} , \qquad (1)$$

$$\pi_o = \tfrac{1}{2}\beta\left[1 + e^{-i\pi\alpha_\pi}\right] \left(s/s_o\right)^{\alpha_\pi} . \qquad (2)$$

The kinematical factors in the Kibble function φ appear automatically, and the residues β, γ are taken as constant with t. Trajectories are $\alpha_\pi(t) = (t - m_\pi^2)\alpha'$ and $\alpha_M(t) = 1 + (t - m_\omega^2)\alpha'$ with α' fixed at 1.0 $(\mathrm{GeV}/c)^{-2}$.

Figure 1 illustrates the fit to data, where the natural and satisfactory value $s_o = 1.0$ $(\mathrm{GeV})^2$ is fixed by the t dependence of differential cross-sections **), and the ratio $\beta/\gamma = 1.34$ is fixed by the ratio $\rho_{oo}/\rho_{1,-1}$ of t channel K* density matrix elements. Note that Reggeization of π exchange is vital in fitting ρ_{oo} at smaller $|t|$ - an exchange term proportional to $(m_\pi^2 - t)^{-1}$ gives much too narrow a forward peak. Over-all normalization is somewhat problematical [13] - normalizing to data at 4.6 (2.11 to 2.72) GeV/c gives $\beta = 60$ (71) GeV^{-3}, [using $d\sigma/dt = (1/64\,\pi\, p^2 s)\tfrac{1}{2}\Sigma|T|^2$]. The splendid simplicity of the model allows all three parameters to be determined analytically.

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*) The unconventional M exchange flip/non-flip ratios are in fact irrelevant, as shown in later discussion.

**) A small difference in slopes of $d\sigma/dt(K^- p \to \bar{K}^{*o} n)$ and $d\sigma/dt(K^+ n \to K^{*o} p)$ is not described by the bare pole exchange model.

The lower-lying π trajectory has a larger contribution relative to M exchange at lower energies. Indeed the lower energy $d\sigma/dt$ data (Fig. 1a) show a small t peak not described by the model. This is typical evidence for absorbed π exchange [8]. The low energy K matrix model is precisely what effectively introduces the necessary absorption.

At $\sqrt{s} = 1.83$ GeV in the region where the amplitudes are to be unitarized, for $I = 0$ the fit predicts the amplitude ratio of π to M exchange ($|\pi/M|$) to be about 7 at $t = -0.05\ (\mathrm{GeV}/c)^2$ falling to about 2 at $t = -0.5\ (\mathrm{GeV}/c)^2$. Thus it is an excellent approximation as far as the low-energy $I = 0$ channel is concerned (especially after partial wave projection) to retain only K*N production by π exchange. This is the central dynamical feature of our model, and leads to a simplification which makes it analytically tractable at no loss [*].

In the channels $\overset{(-)}{K}N \to \overset{(-)}{K}N$ only natural parity exchanges contribute $\mathbb{P}$, f^0, ρ, ω, A_2. We note for completeness (we shall not use the model) that a good description of data (except for line-reversal breaking in $K^{\pm}$ CEX below 5.5 GeV/c [14]) is readily possible with a strongly EXD Regge pole exchange model ($\mathbb{P}$+M) where $\mathbb{P}$ is represented by a simple pole of slope $\alpha' = 0.6\ (\mathrm{GeV})^{-2}$. In principle we extrapolate the M exchanges to $\sqrt{s} \approx 1.83$ GeV in $I = 0$ $KN \to KN$ for unitarization. (Presumably this unitarization helps to rebuild the $\mathbb{P}$ exchange [9],[15] and to break line reversal by generating corrections to isovector exchanges.) However, the K matrix elements thus obtained are completely negligible compared to those for $KN \to K^*N$ by π exchange. This is readily appreciated by comparing relative magnitudes of $K^{\pm}$ CEX cross-sections [14] (one or two tenths of mb) with non-CEX K* production cross-sections [12] (~mb) at larger energies where the latter are M exchange dominated.

Unitarity couples in the channel $K^*N \to K^*N$ and a model is needed. Exchanges of $\mathbb{P}$, M, π are all possible. Guided by $KN \to K^*N$, we assume only π exchange to be significant. The amplitudes are calculated as for $KN \to K^*N$, with the same VVP Lagrangian. The only surviving amplitude is

$$\pi_{11} = \frac{1}{2} \lambda \left(\frac{t}{s_0}\right) \left[1 + e^{-i\pi\alpha_\pi}\right] \left(\frac{s}{s_0}\right)^{\alpha_\pi} , \qquad (3)$$

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*) Also, we note that possible contributions from u channel baryon exchanges are even more negligible.

for initial and final K* t channel helicities equal to ±1 and no nucleon flip. The SU_6 relation [16] between $g_{K^*K\pi}$ and $g_{K^*K^*\pi}$ relates this coupling λ to the β of Eq. (2) by

$$\lambda/s_0 = \frac{4}{m_{K^*}^2 - m_K^2}\,\beta \quad , \qquad (4)$$

in the approximation $m_{K^*} = m_\rho$. The evasive pion leads to correct s channel partial wave threshold behaviour.

PHASE SHIFTS

The low energy model contains therefore in first approximation just the two π exchange terms $\pi_0(KN \to K^*N)$ and $\pi_{11}(K^*N \to K^*N)$. These have to be crossed to the s channel and projected to partial waves. This is fortunately particularly simple near $\sqrt{s} = 1.83$ GeV or $q = 0$ ($q = K^*$ c.m. momentum) because of the form of the helicity crossing matrices as $q \to 0$ [17]. The parity-conserving s channel partial wave amplitudes contain only alternate powers of q and, noting that for $\sqrt{s}$ within 150 MeV of K* threshold we have $q^3 < 1/10\ q$ (units GeV) we achieve further great simplification without loss by working to lowest order in q consistent with threshold behaviour. At the same time the model is valid near the $Z_0^*(1800)$.

Note that although in principle there is some ambiguity in interpretation of the complex π exchange terms as K matrix elements, in practice the partial wave projections are overwhelmingly real because of the nearness to the physical region of $\alpha_\pi = 0$ and the rapid exponential fall-off in t. We take $\mathrm{Re}\,\pi$, but numerically $|\pi|$ is little different. The important feature is simply the size of π exchange relative to the other amplitudes.

It is then straightforward to obtain expressions for the KN isoscalar phase shifts. For $J^P = \frac{1}{2}^\pm$ ($S_{\frac{1}{2}}$ and $P_{\frac{1}{2}}$ waves) inversion of 3×3 matrices gives when $q^2 > 0$

$$\tan\delta_s = 3ipqK_1^2 \qquad (5)$$

and

$$\tan\delta_P = \frac{i p q K_3^2 (1 - 2\sqrt{2}\, i q K_2)}{1 + q^2 K_2^2 - 2\sqrt{2}\, i q K_2} , \qquad (6)$$

where p is the $KN \to KN$ c.m. momentum and the three K matrix elements $K_{1,2,3}$ are constants given by

$$K_1 = \frac{-\beta s_0^A (1 + \cos\pi A)}{16\pi (m_N + m_{K^*})^{2A+1}} , \qquad (7a)$$

$$K_2 = \frac{\lambda}{4\sqrt{2}\pi (m_N + m_{K^*})} , \qquad (7b)$$

$$K_3 = \frac{-\beta s_0^A}{8\pi (m_N + m_{K^*})^{2A+1}} \left\{ \pi \sin\pi A + 2(1+\cos\pi A) \ln\left[(m_N + m_{K^*})/\sqrt{s_0}\right] \right\} , \qquad (7c)$$

where

$$A/\alpha' = m_\pi^2 + m_N (m_{K^*}^2 - m_K^2)/(m_{K^*} + m_N)$$

and $\alpha' \equiv$ trajectory slope $= 1.0$ GeV^{-2}(fixed). Continuation to $q^2 < 0$ is by $q \to i|q|$. For $J^P = \frac{3}{2}^\pm$ ($P_{\frac{3}{2}}$ and $D_{\frac{3}{2}}$ waves) there are more involved formulae (omitted) from inversion of 4×4 matrices. We emphasize that these expressions are valid for $1.7 \lesssim \sqrt{s} \lesssim 2.0$ GeV.

Phase shifts for $J = \frac{1}{2}, \frac{3}{2}$ are plotted in Fig. 2, where comparison is made between this theory with $\beta = 65$, λ given by SU_6 [Eq. (4)] and results of the so-called BGRT-D (Sens γ) phase shift solution [4]. Considering the crudeness of the model, the agreement is good. General features of the BGRT solution are reproduced - notably the sign of the $S_{\frac{1}{2}}$ and $P_{\frac{3}{2}}$ phase shifts below 1.83 GeV and the large counter-clockwise $P_{\frac{1}{2}}$ loop. Equation (6) shows the latter to be an inevitable feature of the model, although the exact mass of the resonance is sensitive to the coupling λ of π exchange in $K^*N \to K^*N$, through K_2. As shown in Fig. 2 the mass on a maximum speed criterion is about 1.82 GeV, and width is roughly 250-300 MeV. Note from Eq. (5) that the S wave below the resonance is always repulsive. The $P_{\frac{3}{2}}$ amplitude shows interesting rapid movement. The limited range of validity of this version of the model makes it difficult to know, but presumably the corresponding KN scattering length is negative. The same remark applies to the $D_{\frac{3}{2}}$ wave.

Of course, the two-channel, stable-particle, approximation makes the inelastic threshold rather abrupt and renders details unreliable. However, qualitative features of relative sizes and signs of predicted amplitudes are invariant against changes of parameters γ, β, s_o which are reasonable in view of the K* production data [12]. The effects of the neglected natural parity exchange amplitudes can be estimated numerically as 5-10% perturbations to the π exchange phase shifts in $J = \frac{1}{2}$, and smaller in $J = \frac{3}{2}$. The Regge exchanges in $KN \rightarrow KN$ in particular remove the exact amplitude zero at K*N threshold, but the change is not large. Indeed, to the extent that the BGRT-D (Sens γ) phase shifts are to be relied upon, the results clearly imply that π exchange alone is a realistic first approximation to the dynamics.

CONCLUSIONS

The general phenomenon of resonance formation linked to vector production by π exchange has been investigated before [3),18)]. The calculations of Aaron et al. [3),18)] seem to favour S and D wave resonances. The present model has the advantage of simplicity and transparency. It shows how, because of the dominant amplitudes in the K*N channels, the $KN \rightarrow KN$ partial waves are forced to be small at K*N threshold. That is, phase shifts δ near $0, \pi, \ldots$ etc., at $q = 0$. These values arise from either a small excursion from KN threshold back to $\delta \approx 0$ or else a rapid loop to $\delta \approx \pi$. The pion coupling to K* helicity-one amplitudes in $K^*N \rightarrow K^*N$ puts a loop into the $P_{\frac{1}{2}}$ wave and makes the $S_{\frac{1}{2}}$ and $P_{\frac{3}{2}}$ phase shifts negative for $q^2 < 0$. We note that dispersion relations favour a $J^P = \frac{1}{2}^+$ Z_o^* resonance [19)].

Finally we point out that the dominance of π exchange in the exotic channel makes the dual approach through the K matrix expression $T = K(1-i\rho K)^{-1}$ somewhat similar to the old-fashioned dynamics of ND^{-1} equations. In that language the Z_o^* is a K*N bound state and a KN CDD pole (and it is perhaps difficult to understand why long-range π exchange should give a good estimate of the S wave !).

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REFERENCES

1) Particle Data Group - Revs.Modern Phys. 45, 51 (1973).

2) J.D. Dowell - in E.C. Fowler (Ed.) "Hyperon Resonances-73", Moore Publ. Co., Durham, N.C. (1973), p. 157.

3) R. Aaron - in E.C. Fowler (Ed.) "Hyperon Resonances-73", Moore Publ.Co., Durham, N.C. (1973), p. 221 ;

R. Aaron, M. Rich, W.L. Hogan and Y.M. Srivastava - Phys.Rev. D4, 1401 (1973).

4) G. Giacomelli et al. (BGRT Collaboration) - "Phase Shift Analysis of $K^+N \rightarrow KN$ Scattering in the $I=0$ State up to 1.5 GeV/c", CERN Preprint (August 1973), submitted to Nuclear Phys.

5) A. Hirata et al. - in E.C. Fowler (Ed.) "Hyperon Resonances-70", Moore Publ.Co., Durham, N.C. (1970), p. 429.

6) C. Lovelace - in F. Loeffler and E. Malamud (Eds.) "Proceedings Conference on $\pi\pi$ and $K\pi$ Interactions" (May 1969), Argonne-Purdue Preprint, unpublished, p. 562.

7) F. Schrempp - Phys.Letters 29B, 598 (1969) ;

F. Schrempp - Nuclear Phys. B41, 557 (1972).

8) P.D.B. Collins - Physics Reports 1C, 103 (1971).

9) H. Harari - Edinburgh Summer School Lectures (1973), to be published.

10) M.G. Olsson - in F. Loeffler and E. Malamud (Eds.) "Proceedings Conference on $\pi\pi$ and $K\pi$ Interactions" (May 1969), Argonne-Purdue Preprint, unpublished, p. 771.

11) S.D.P. Vlassopulos - Durham Preprint (May 1974).

12) L. Seidl et al. - Phys.Rev. D7, 621 (1973) ;

C. Fu et al. - Nuclear Phys. B28, 528 (1971) ;

W. de Baere et al. - Nuovo Cimento 51A, 401 (1967), and Preprint CERN D.Ph.II Phys. 72-31 ;

K.W.J. Barnham et al. - UCRL 20107 (1971), and LBL 736 (1972) ;

J.M. Brunet et al. - Nuclear Phys. B37, 114 (1972) ;

K. Buchner et al. - Nuclear Phys. B45, 333 (1972) ;

D. Cords et al. - Phys.Rev. D4, 1974 (1971) ;

D. Lissauer - UCRL 20644 (1971) ;

J.S. Loos et al. - SLAC PUB 144 (1972) ;

M. Aguilar Benitez et al. - Phys.Rev. D4, 2583 (1971), and BNL-15707 (1972) ;

H. Tiecke et al. - Nuclear Phys. B39, 596 (1972) ;

R. Engelmann et al. - Phys.Rev. D5, 2162 (1972) ;

A. Rougé et al. - Nuclear Phys. B46, 29 (1972) ;

D.D. Carmony et al. - Nuclear Phys. B12, 9 (1969), and Phys.Rev. D2, 30 (1970) ;

B.J. Burdick et al. - Nuclear Phys. B41, 45 (1972) ;

D. Johnson et al. - ANL Preprint and Batavia Conference paper 652 ;

K. Kang et al. - Phys.Rev. 126, 1587 (1968) ;

M. Deutschmann et al. - Nuclear Phys. B36, 373 (1972).

13) C. Michael - Nuclear Phys. B57, 292 (1973).

14) D. Cline, J. Matos and D.D. Reeder - Phys.Rev.Letters 23, 1318 (1969).

15) A.W. Reid and D.G. Sutherland - "Models for the Pomeron and Low Energy K^+p Scattering", Glasgow Preprint (1973) unpublished.

16) B. Sakita and K.C. Wali - Phys.Rev. 139, B1355 (1965).

17) A.D. Martin and T.D. Spearman - "Elementary Particle Theory", North Holland, Amsterdam (1970), pp. 328-344.

18) R. Aaron and D. Amado - Phys.Rev.Letters 27, 1316 (1971).

19) N. Hedegaard-Jensen, H. Nielsen and G.C. Oades - Phys.Letters 46B, 385 (1973).

FIGURE CAPTIONS

Figure 1 Fits to selected K^* production data [from Ref. 12] with the model described in the text.

Figure 2 Comparison between :

(a) model phase shifts calculated as described in the text (reference points at $\sqrt{s} = 1700$, 1820, 1827, 1830, 1950 MeV) ;

(b) phase shifts of the BGRT-D (Sens γ) solution from Ref. 4) (reference points at $\sqrt{s} = 1700$, 1747, 1794, 1840, 1887, 1932, 1977 MeV).

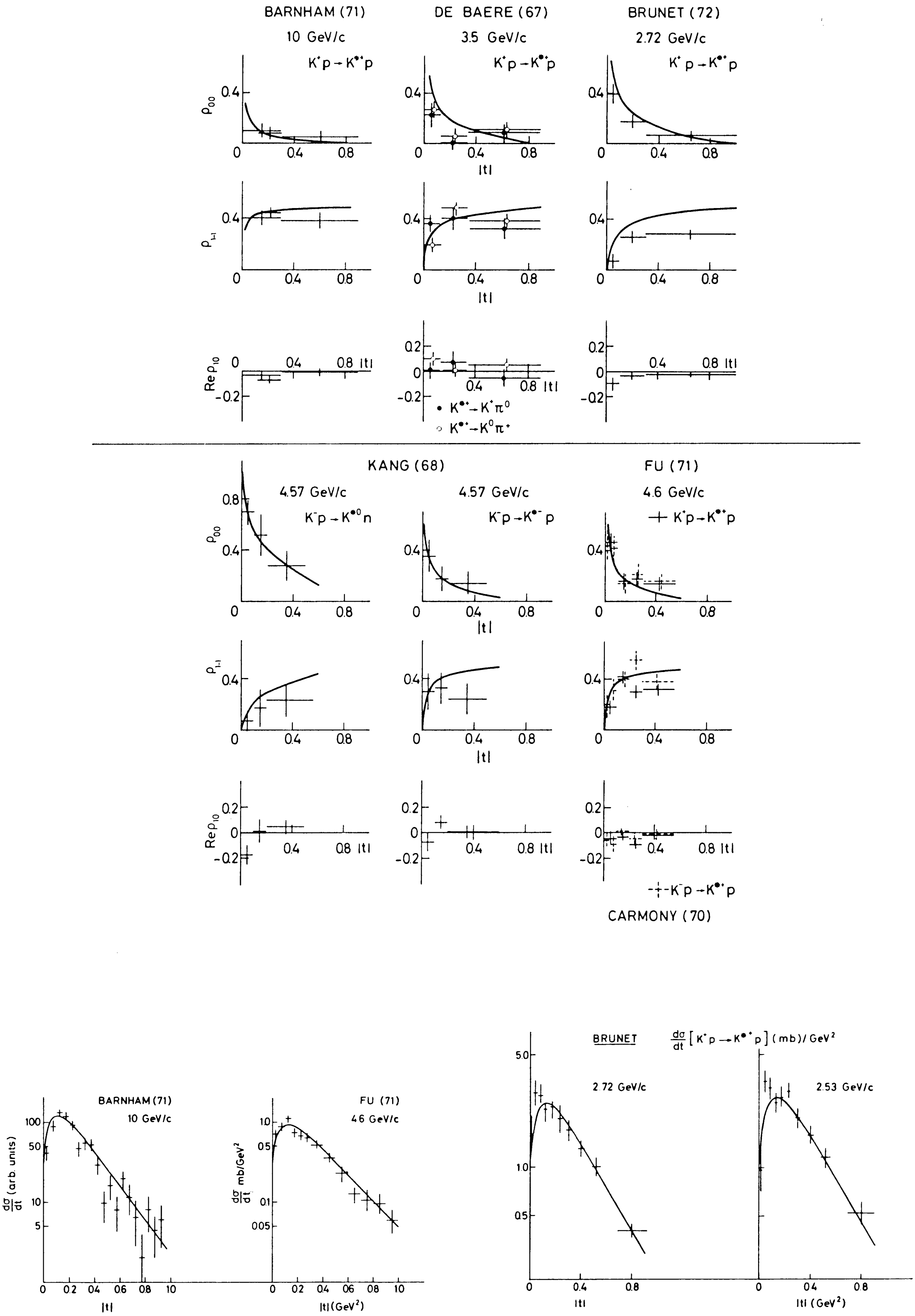
BARNHAM (71)
10 GeV/c
$K^+p \to K^{*+}p$
DE BAERE (67)
3.5 GeV/c
$K^+p \to K^{*+}p$
BRUNET (72)
2.72 GeV/c
$K^+p \to K^{*+}p$
ρ_{00}
ρ_{1-1}
$Re\,\rho_{10}$
|t|
• $K^{*+} \to K^+\pi^0$
○ $K^{*+} \to K^0\pi^+$
KANG (68)
4.57 GeV/c
$K^-p \to K^{*0}n$
4.57 GeV/c
$K^-p \to K^{*-}p$
FU (71)
4.6 GeV/c
$K^+p \to K^{*+}p$
$K^-p \to K^{*+}p$
CARMONY (70)
BARNHAM (71)
10 GeV/c
$\frac{d\sigma}{dt}$ (arb. units)
|t|
FU (71)
4.6 GeV/c
$\frac{d\sigma}{dt}$ mb/GeV^2
|t| (GeV^2)
BRUNET
$\frac{d\sigma}{dt}[K^+p \to K^{*+}p]$ (mb)/GeV^2
2.72 GeV/c
2.53 GeV/c
|t|
|t| (GeV^2)

FIG 1

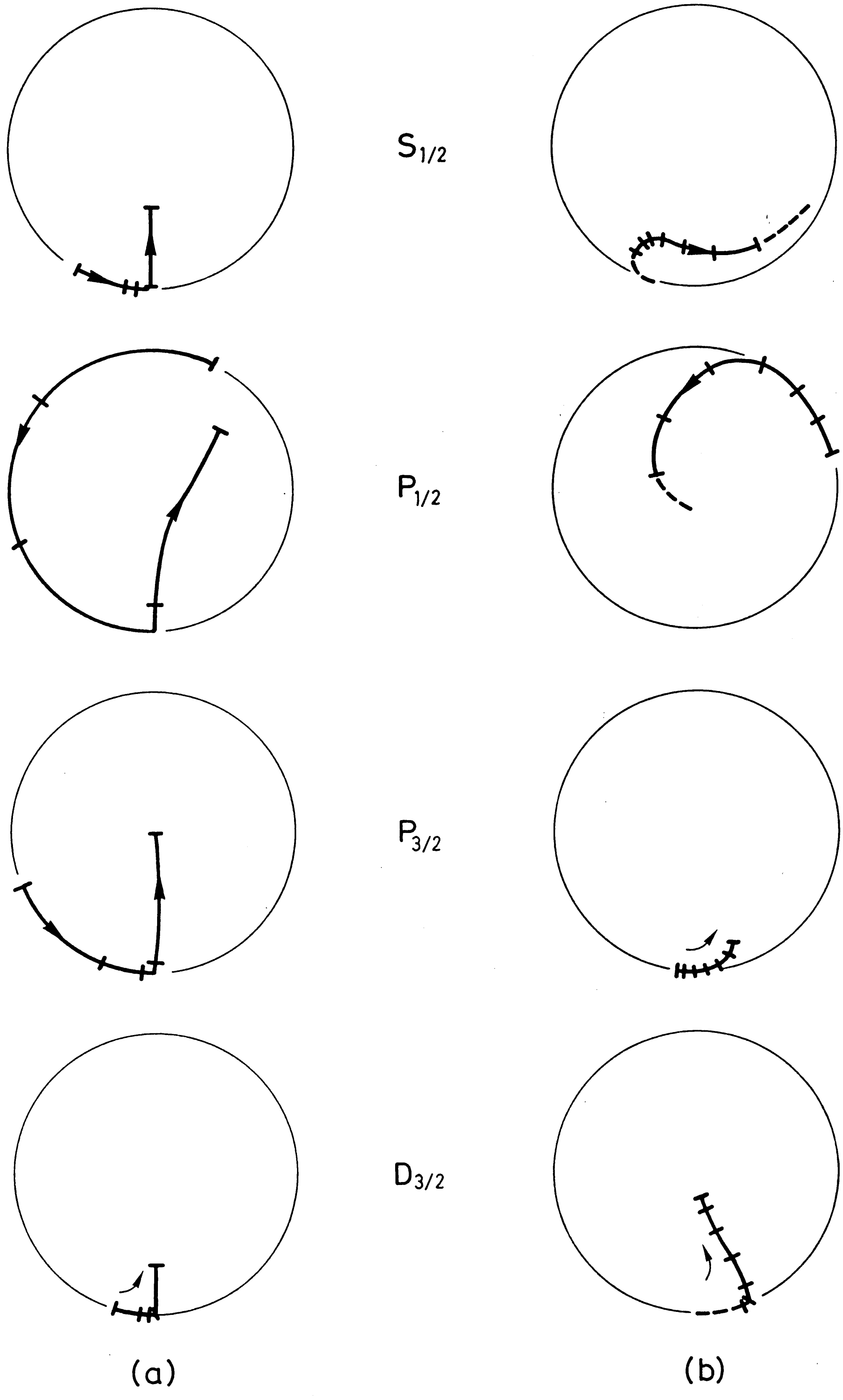

FIG. 2