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Evidence for $\omega \to \pi^0 + \gamma$ decay

by

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1. Introduction

At the present time it is almost certain that the ω-meson decays mainly according to the mode $\omega \to \pi^+ + \pi^- + \pi^\circ$. Since the charge parity of the ω-meson is negative the decay mode $\omega \to 3\pi^\circ$ is forbidden, but radiative decay modes $\omega \to m\pi + \gamma$, where $m \gtrsim 1$, are permissible. As shown in the theoretical work of Kobzarev and Okun [1] the small width of the ω-meson observed experimentally means that the probability of radiative decays may be relatively high. In particular, the theoretical estimates of the ratio between the probability of the decay modes $(\omega \to \pi^\circ + \gamma)/(\omega \to \pi^+ + \pi^- + \pi^\circ)$ give a value of ~ 0.2 [2]. On the other hand, among radiative decays of the $\omega \to m\pi^\circ + \gamma$ type, the decay $\omega \to \pi^\circ + \gamma$ should be predominant [3].

The existence of ω-meson decays into neutral particles has been confirmed experimentally by several authors [4-6]. The average $(\omega \to \pi^+ + \pi^- + \pi^\circ)/(\omega \to$ neutral particles) probability ratio according to these authors is 5 ± 1. However the nature of the neutral products of ω-meson decays was not established in these experiments.

The purpose of the present work is to provide evidence for the decay $\omega \to \pi^\circ + \gamma$.

2. Theory

Let us consider the radiative decay, according to the mode $X^\circ \to \pi^\circ + \gamma \to 3\gamma$, of the neutral vectorial X°-meson with negative internal parity. This decay is described by three diagrams, one of which is reproduced in Fig. 1; the others can be obtained by permutation of the final γ-quanta.

The matrix element of the decay $X^\circ \to \pi^\circ + \gamma$ is written as [1]

$$M_X = \sqrt{\alpha}\; L_X\; \varepsilon_{\alpha\beta\gamma\delta} K_\alpha \tilde{K}_\beta e_\gamma \varphi_\delta \qquad (1)$$

where $\alpha = 1/137$, K and $\tilde{K}$ are the four dimensional momenta of the π°-meson and the photon, e_γ and φ_δ are the wave functions of the photon and the X°-meson, and L_x is the decay constant with length dimensionality. The matrix element of the decay $\pi^\circ \to 2\gamma$ can be written as follows :

$$M_\pi = \alpha L_\pi \, \varepsilon_{\alpha\beta\gamma\delta} \, K_{1\alpha} \, K_{2\beta} \, e_\gamma^{(1)} e_\delta^{(2)} \tag{2}$$

where K_1 and K_2 are the four dimensional momenta of the photons, $e^{(1)}$ and $e^{(2)}$ the wave functions of the photons, L_π the decay constant linked to the probability Γ of the decay $\pi^\circ \to 2\gamma$ by the relation

$$L_\pi = \frac{8}{\alpha \, m_\pi} \sqrt{\frac{\Gamma}{m_\pi}} \tag{3}$$

where m_π is the mass of the π°-meson.

By choosing the π°-meson propagator as follows :

$$D(K^2) = \frac{i}{(2\pi)^4 \left[K^2 - (m_\pi - i\Gamma/2)^2\right]} \tag{4}$$

where the complex addition to the mass m_π takes into account the instability of the π°-meson, one can, on the basis of diagrams of the type shown in Fig. 1 and relations (1), (2) and (4) obtain the matrix element of $X^\circ \to 3\gamma$ decay :

$$M = \sum_{j=1}^{3} M_{xj} \, M_{\pi j} \, D\left[(p-K_j)^2\right] \tag{5}$$

where the sum according to J corresponds to 3 possible $X^\circ \to 3\gamma$ decay diagrams.

The probability of $X^\circ \to 3\gamma$ decay of an X°-meson in motion is expressed by the matrix element according to the formula

$$dw = \frac{1}{2E} |M|^2 (2\pi)^4 \delta^4(p-K_1-K_2-K_3) \frac{d^3K_1 \, d^3K_2 \, d^3K_3}{8 \, (2\pi)^9 \omega_1 \omega_2 \omega_3} \qquad (6)$$

where p and E are the four-dimensional momentum and the energy of the X°-meson, K_1, K_2, K_3, and ω_1, ω_2, ω_3 are the four-dimensional momenta and the energy of the photons. Since $\Gamma/m_\pi \sim 10^{-8}$, only the real π°-meson states contribute to the probability (6) and when $\Gamma/m_\pi \to 0$ the expression (6) taking into account the equation (3), (4) and (5) can be written as follows :

$$dw = \frac{\alpha L_x^2 (m_x^2 - m_\pi^2)}{12.3!(2\pi)^3 E} \Big\{ \delta(2pK_1 - m_x^2 + m_\pi^2) + \delta(2pK_2 - m_x^2 + m_\pi^2) +$$

$$+ \delta(2pK_3 - m_x^2 + m_\pi^2) \Big\} \frac{d^3K_1 \, d^3K_2 \, d^3K_3}{\omega_1 \omega_2 \omega_3} \delta^4(p-K_1-K_2-K_3) \qquad (7)$$

Thus it is found that the probability of $X^\circ \to 3\gamma$ decay does not depend on the probability of $\pi^\circ \to 2\gamma$ decay. This result was not unexpected, since on account of the small width of Γ the π°-mesons will decay into 2 photons over distances much greater than the characteristic area for $X^\circ \to \pi^\circ + \gamma$ decay ($1/\Gamma \sim 10^8/m_\pi$, whereas $L_x \sim 1/1.6 m_\pi$[1]).

By integrating expression (7) by means of the δ-functions one obtains :

$$dw = \frac{\alpha L_x^2 (m_x^2 - m_\pi^2) m_\pi^2}{288E(2\pi)^2} \Bigg\{ \frac{m_x^2 - m_\pi^2}{\left[(E - p\cos\Theta_1)(E - p\cos\Theta_2) - (m_x^2 - m_\pi^2) \sin^2 \frac{\Theta_{12}}{2} \right]^2} +$$

$$+ \frac{1}{\sin^3 \frac{\Theta_{12}}{2} \sqrt{(m_x^2 + m_\pi^2)^2 \sin^2 \frac{\Theta_{12}}{2} - 4m_\pi^2 (E - p\cos\Theta_1)(E - p\cos\Theta_2)}} \Bigg\} d\cos\Theta_1 \, d\cos\Theta_2 \, d\varphi_2 \qquad (8)$$

Where Θ_1 and Θ_2 are the angles of flight of 2 γ-quanta in relation to the direction of motion of the X°-meson, Θ_{12} the angle between these γ-quanta and φ_2 the azimuthal angle of one of the γ-quanta in relation to the plane containing the $\vec{K}_1$ and $\vec{p}$ vectors. It should be noted that the first item in (8) describes the distribution according to the angle between the γ-quantum from $X^\circ \to \pi^\circ + \gamma$ decay and one of the γ-quanta from $\pi^\circ \to 2\gamma$ decay, and the second item refers only to $\pi^\circ \to 2\gamma$ decay.

Now, assuming in (8) that $p = 0$ and integrating with respect to Θ_{12}, one obtains the angular distribution of the γ-quanta in the X°-meson rest system :

$$dw = \frac{\alpha L_x^2 \, (m_x^2 - m_\pi^2)^3}{96(2\pi)^2 m_x^3} \, d\cos\Theta_1 d\varphi_2 \qquad (9)$$

It is clear that the momenta of the 3 γ-quanta from the decay of the X°-meson at rest are in the same plane. How, supposing one returns to the system in which the X°-meson is in motion, on the basis of geometrical considerations and relativistic angle transformation formulae one obtains the relation (see annex) :

$$\cos\frac{\beta}{2} = \frac{v\sin\Theta_1\sin\varphi_2}{\sqrt{1 - v^2 + v^2\sin^2\Theta_1\sin^2\varphi_2}} \qquad (10)$$

where $v = p/E$ is the velocity of the X°-meson and β the angle of aperture of the circular cone on the surface of which are located the momenta $\vec{K}_1$ $\vec{K}_2$ and $\vec{K}_3$ of the 3 γ-quanta from the $X^\circ \to \pi^\circ + \gamma \to 3\gamma$ decay. This cone will be referred to as the "decay cone".

By using expression (10) and integrating with (9) we obtain the probability distribution according to the angle of aperture of the decay cone :

$$dw = \frac{\alpha L_x^2 \, (m_x^2 - m_\pi^2)^3}{96.(2\pi).m_x^3} \cdot \frac{\sqrt{1-v^2}}{v} \cdot \frac{d\beta}{\mathrm{Sin}^2\frac{\beta}{2}} \qquad (11)$$

as shown in the annex,

$$2 \arcsin \frac{m_x}{E} \leqslant \beta \leqslant \pi \qquad (12)$$

By integrating (11) within these limits one finds a total probability of $\omega = \alpha \, L_x^2 \, ({m_x}^2 - {m_\pi}^2)^3 / 96\pi {m_x}^3$, which agrees with the results of the work of Kobzarev and Okun[1].

From (12) it follows that the decay come has a minimum angle of aperture β_{min} for which

$$\mathrm{Sin} \frac{\beta_{min}}{2} = \frac{m_x}{E} \qquad (13)$$

Thus having established the existence of $X° \to 3\gamma$ decay experimentally, by studying the distribution of decays according to the angle β one can obtain the value β_{min} and assess the mass of the particle m_x, using the relation (13), and also compare the experimental distribution $dw/d\beta$ with the theoretical distribution (11). This is the basic method for discovering $\omega \to \pi° + \gamma$ decay, developed and applied in this paper.

3. Experimental section

$\omega \to \pi° + \gamma$ decay was studied investigating the reaction

$$\pi^- + p \to n + \omega \qquad (14)$$

For this reaction in the π^--meson-proton centre of mass system, the ω-mesons have a constant momentum which can be calculated according to the known momentum of the π^--meson in the laboratory system. Thus in the $\pi^- p$ centre of mass system the reaction (14) is a source of mono-energetic ω-mesons and in order to adapt the data measured to this system, one can use the method of identifying $\omega \to \pi^\circ + \gamma$ decay described in chapter 2.

The reaction (14) was studied with a π^--meson beam of the proton synchrotron of the Theoretical and Experimental Physics Institute. The π^- beam had a momentum resolution of ± 4% as shown by measurement using a live wire in a magnetic field. A 17-litre bubble chamber [7)] was placed in this beam; it had a useful observation volume of $15 \times 22 \times 48$ cm^3 and was filled with a mixture of C_3H_8 propane and xenon. The density of the mixture was 0.84 g/cm^3, the weight content of xenon in the mixture being 57.3%. The chamber worked without a magnetic field. The working liquid used was highly efficient for recording radiative processes. The γ-quanta were observed in the chamber upon conversion into electron-positron pairs. To increase efficiency, the π^- beam was collimated so that its section measured 4×14 cm^2. Moreover, the statistics included only those events occuring in the first three-quarters of the volume of the chamber.

Experiments were carried out with 1.25 BeV/c, 1.55 BeV/c and 2.8 BeV/c π^--mesons. 11,000, 20,000 and 60,000 stereoscopic photographs were taken respectively. Events of interest were sought by two independent scanning operations. The efficiency of this double scanning was 95 to 98%.

The events sought were those in which three or more electron-positron conversion pairs were directed towards the stopping point of a π^--meson, provided that the stop was not accompanied by any tracks of nuclear interactions (zero prong stars). The events found were interpreted as being the result of such interactions of π^--mesons with free hydrogen or with protons of C - Xe nuclei which gave rise to charge exchange of protons and generation of one or more neutral p rticles, which then decay into γ-quanta. For instance, an event with 3 γ-quanta can be brought about by the reaction $\pi^- + p \rightarrow n + \omega \rightarrow n + 3\gamma$ or the reactions $\pi^- + p \rightarrow n + m\pi^\circ \rightarrow n + 2m\gamma$ $(m \geqslant 2)$, when only 3 of the 2m γ-quanta are recorded in the chamber. It is evident that events with 4 γ-quanta also have their source in the reaction $\pi^- + p \rightarrow n + m\pi^\circ$ $(m \geqslant 2)$ or for instance the reaction $\pi^- + p \rightarrow n + \pi^\circ + \eta$ with subsequent decays of $\pi^\circ \rightarrow 2\gamma$ and $\eta \rightarrow 2\gamma$ etc. Thus for the basic process $\pi^- + p \rightarrow n + 3\gamma$ the accompanying reactions are a source of background noise. How to deduct this background will be discussed in detail later.

The majority of events found were measured on a stereo-projector in which the image of the event photographed in the chamber was reconstructed in space by means of the pairs of stereoscopic photographs. For each event with any given number of pairs we measured :

(1) Potential lengths ℓ_k, i.e. the distances from the π^--meson stop to the limit of the useful volume of the chamber along the line of flight of the γ-quanta recorded for this event.

(2) Θ_k angles between the γ-quanta and the direction of the π^--meson.

(3) Θ_{12} Θ_{13} Θ_{23} etc. angles between any given pair of γ-quanta.

The mean accuracy of measurement of the angles was $\pm$ 1°. Moreover for part of the events the distance r_k from the π^--meson stop to the beginning of the conversion pairs was measured.

The values r_k and ℓ_k were used for the experimental determination of the conversion length ℓ_o. The same method was used as for measuring the lifetime of Λ hyperons [8]. The value ℓ_o was determined from $N = 1977$ events by the equation :

$$\ell_o = \frac{1}{N}\sum_{j=1}^{N}\left(r_j + \frac{\ell_j}{e^{\ell_j/\ell_o}-1}\right) \qquad (15)$$

and was found to be 17.8 ± 2.2 cm. It is evident that the value of ℓ_o so obtained is a mean value for the energy spectrum of the γ-quanta recorded in the chamber.

For all the events measured and for each conversion pair the potential length ℓ_k was used to calculate the recording efficiency of separate γ-quanta $\eta = 1-\exp(-\ell_k/\ell_o)$ when $\ell_o = 17.8$ cm. The mean value of $\bar{\eta}$ was found to be different for events with different multiplicity. When the number of γ-quanta increases $\bar{\eta}$ decreases, since the number of events with γ-quanta at large angles to the beam grows, which on account of the shape of our chamber leads to a reduction of ℓ_k. Furthermore $\bar{\eta}$ also depends on the energy of the π^--mesons. When the energy grows $\bar{\eta}$ also increases, because the forward stretch along the chamber becomes greater. The values of $\bar{\eta}$ were found to be between 0.50 and 0.65.

The values obtained for $\bar{\eta}$ for each group of events with a given multiplicity and for a given momentum of the π^--mesons were used to calculate the probabilities of recording i γ-quanta out of K which would be needed later. These values were calculated according to the formula :

$$P_{ik} = C_k^i \, \bar{\eta}^i (1-\bar{\eta})^{k-i} \qquad (16)$$

where C_k^i is the number of combinations of K elements by i.

For the calculation in each case of the angle of aperture of the decay cone in the π^-p centre of mass system, the measured angles Θ_k were adapted to this system by means of the usual formulae. The angles between the γ-quanta were adapted to the centre of mass system according to the formula :

$$\cos\Theta'_{ik} = 1 - \frac{(1-B^2)(1-\cos\Theta_{ik})}{(1-B\cos\Theta_i)(1-B\cos\Theta_k)} \qquad (17)$$

where B is the velocity of the π^-p centre of mass system.

4. Results obtained and discussion of these results

The distribution according to the number of conversion pairs of the events found is given in Table I.

Table I

Momentum of the π-mesons in BeV/c	The number of events with K γ-quanta			
	$N_{3\gamma}$	$N_{4\gamma}$	$N_{5\gamma}$	$N_{6\gamma}$
1,25	40	13	1	-
1,55	118	25	6	2
2,80	433	136	53	24

Let n_k be the number of $\pi^- + p \rightarrow n + k\gamma$ reactions which have occurred in the chamber during the experiments. Then the number of reactions with K γ-quanta found can be written as follows :

$$\begin{aligned} N_{3\gamma} &= p_{33}n_3 + p_{34}n_4 + p_{35}n_5 + p_{36}n_6 \\ N_{4\gamma} &= p_{44}n_4 + p_{45}n_5 + p_{46}n_6 \\ N_{5\gamma} &= p_{55}n_5 + p_{56}n_6 \\ N_{6\gamma} &= p_{66}n_6 \end{aligned} \qquad (18)$$

where p_{ik} represents the probability of observing i γ-quanta out of K, calculated according to formula (16). Resolving system (18) for all three distributions of $N_{k\gamma}$ shown in Table I (with correction of the figures $N_{k\gamma}$ for scanning efficiency) gives the number of $\pi^- + p \rightarrow n + 3\gamma$ reactions which we are seeking, according to the momentum of the π^--mesons. The result expressed as a percentage of the total number of all π^--meson interactions at the given momentum is shown in figure 2. In order to determine the total number of interactions special scanning of part of the films was carried out for each of the three groups.

Thus the experimental distributions of the reactions found according to the number of conversion pairs and the recording probability already give an unambiguous indication that at energies of the π^--mesons $\gtrsim 1.2$ BeV there is a considerable source of $3\ \gamma$-quanta in π^--p collisions which are not accompanied in the photographs by tracks of nuclear interactions. It was assumed that the main source of these $3\ \gamma$-quanta is the ω-meson, namely that the phenomenon observed by us can be explained by the reaction $\pi^- + p \rightarrow n + \omega \rightarrow n + \pi^\circ + \gamma \rightarrow n + 3\gamma$, occurring in the range 1.25 - 2.28 BeV/c. The threshold of this reaction is shown by an arrow in figure 2. It can be seen that the section $\sigma_{3\gamma}$ of the process $\pi^- + p \rightarrow n + 3\gamma$ increases sharply exactly after this threshold, which is strong evidence in favour of the above hypothesis. In figure 2 is also marked the point corresponding to the result obtained at Saclay at 1.15 BeV/c [9] when the reaction $\pi^- + p \rightarrow n + \omega \rightarrow n + 3\gamma$ was not observed.

Let us consider other possible sources of the $\pi^- + p \rightarrow n + 3\gamma$ reaction apart from the ω-meson. One of the other possible reactions may be the formation of ρ°-meson, the mass of which is close to the mass of the ω-meson and its radiative decay $\rho^\circ \rightarrow \pi^\circ + \gamma \rightarrow 3\gamma$. However, in the energy range concerned, the cross-sections of the reactions $\pi^- + p \rightarrow n + \omega$ and $\pi^- + p \rightarrow n + \rho^\circ$ are about the same and for the same number of ω and ρ° mesons the decay ratio $(\rho^\circ \rightarrow \pi^\circ + \gamma)/(\omega \rightarrow \pi^\circ + \gamma) \cong \Gamma_{\gamma\rho}\ \Gamma_\omega/\Gamma_{\gamma\omega}\ \Gamma_\rho$, where Γ_ω and Γ_ρ are the widths of the ω and ρ° resonances, and $\Gamma_{\gamma\omega}$ and $\Gamma_{\gamma\rho}$ are their radiative widths. Further, $\Gamma_{\gamma\omega} \cong \Gamma_{\gamma\rho}$ and consequently $(\rho^\circ \rightarrow \pi^\circ + \gamma)/(\omega \rightarrow \pi^\circ + \gamma) \cong \Gamma_\omega/\Gamma_\rho$. But experiment shows that $\Gamma_\omega/\Gamma_\rho < 0.1$ and from theory [1] it follows that $\Gamma_\omega/\Gamma_\rho \cong 0.01$, which means that in this paper the decay $\rho^\circ \rightarrow \pi^\circ + \gamma$ can be neglected. A more appreciable source of the $\pi^- + p \rightarrow n + 3\gamma$ process may be the reaction $\pi^- + p \rightarrow \Sigma^\circ + K^\circ + \pi^\circ \rightarrow \Lambda + K^\circ + 3\gamma$ when K° and Λ-particles are not recorded in the chamber. However, at 2.8 BeV/c this reaction can simulate no more than 5% of all $\pi^- + p \rightarrow n + 3\gamma$ reactions, and even less at other energies. These conversions were made according to the number of Λ and K° events found, taking into account the recording efficiency.

In order to control the correctness of the results shown in figure 2 we proceeded as follows. Concurrently with the search for events with three or more γ-quanta, reactions were sought with 1 and 2 γ-quanta in all three series of photographs, i.e. the values $N_{1\gamma}$ and $N_{2\gamma}$ were determined. It is evident that

$$N_{1\gamma} = p_{12}n_2 + p_{13}n_3 + \ldots\ldots + p_{16}n_6 \qquad (19)$$

and

$$N_{2\gamma} = p_{22}n_2 + p_{23}n_3 + \ldots\ldots + p_{26}n_6 \qquad (20)$$

From expression (20) in combination with (18) all the n_k numbers of $\pi^- + p \rightarrow n + k\gamma$ $(K \geq 2)$ reactions were found, and the value $N_{1\gamma}$ was then calculated according to the formula (19) and compared with the number of events with one γ-quantum found, from which had been deducted the possible contribution from the $\pi^- + p \rightarrow K^\circ + \Sigma^\circ \rightarrow K^\circ + \Lambda + \gamma$ reaction, when K° and Λ are not recorded in the chamber. This contribution was estimated for 1.55 BeV/c and 2.8 BeV/c according to the number of these reactions found, taking into account the recording efficiency of strange particles. The results are given in Table II.

Table II

Momenta of π^--mesons, BeV/c	Value of $N_{1\gamma}$	
	Calculated according to formula (19)	Found by scanning (after deduction of the contribution from strange particles)
1.25	227 ± 44	247 ± 16
1.55	308 ± 55	383 ± 23
2.80	501 ± 75	611 ± 60

As can be seen from Table II both values of $N_{1\gamma}$ agree, within the error limits.

In order at this stage to apply the method of identifying $\omega \rightarrow \pi^\circ + \gamma$ decay described in Chapter 2 it is necessary to plot the distribution of events with 3 γ-quanta according to the angular aperture of the decay cone β in the $\pi^- p$ centre of mass system. This is shown in figures 3a and 4a for 1.55 BeV/c and 2.8 BeV/c π^--mesons. When plotting the graphs in figures 3a and 4a the recording efficiency of each separate event was taken into consideration. Instead of one event, a value of $1/\eta_1\ \eta_2\ \eta_3$ was taken along the Y axis. The areas of the spectra were normalized for the number of events measured.

It is obvious that the distributions shown in figures 3a and 4a are due not only to the reaction $\pi^- + p \rightarrow n + 3\gamma$ but also to processes with higher multiplicity, namely, including background. For the experimental determination of the shape of the background distributions we made combinations of three with all the events with 4, 5 and 6 γ-quanta, calculating the β angles for these combinations, and we then plotted distributions for them according to the aperture of the decay cone. As a correction for efficiency we introduced factors of the type $(1 - \eta_m)/\eta_i\ \eta_k\ \eta_\ell\ \eta_m$ for $ik\ell$ combinations of events with 4 γ-quanta, $(1 - \eta_m)(1 - \eta_n)/\eta_i\ \eta_k\ \eta_\ell\ \eta_m\ \eta_n$ for a similar combination of events with 5 γ-quanta etc. The results are shown in figures 3b, 4b and 4c. The distributions of combinations of three of the events with 5 and 6 γ-quanta were amalgamated since the number of $\pi^- + p \rightarrow n + 5\gamma$ reactions is very small and in practive all the events with 5 γ-quanta have their source in the reaction $\pi^- + p \rightarrow n + 6\gamma$. This result is obtained from the resolution of system (18). The distributions of combinations for 5 and 6 γ-quanta at 1.55 BeV/c were not plotted, on account of the small amount of statistical data.

Figure 5 shows the results of deducting the background, namely the distributions according to the β angle for the reaction $\pi^- + p \to n + 3\gamma$. The graph shows only statistical errors. When deducting the background for 2.8 BeV/c the distributions in figures 4c and 4d were deducted from those in figure 4a with the corresponding standardisations. The distribution 4d was obtained as the difference between 4d and 4c. According to calculations the contributions of 4c and 4d to 4a were 13% and 33%, which means that the background in the distribution for 3 γ-quanta at 2.8 BeV/c was 46%. The background for 1.55 BeV/c was 36%. When deducting this background it was assumed that the shape of the distributions for combinations of 5 and 6 γ-quanta was the same as for 4 γ-quanta, namely as shown in figure 3b. This supposition is fairly probable since the contribution of 5 and 6 γ-quanta to 3b distribution is high (47%). Furthermore, the fact of ignoring the actual shape of the distribution for 5 and 6 γ-quanta cannot substantially distort the result at 1.55 BeV/c since the contribution of this distribution to the graph 3a is only 8%.

The arrows in figure 5 show the values of the minimum angles of aperture β_{min} of the decay cone for a ω-meson with a mass of 782 MeV. These values are 119° for 1.55 BeV/c and 82° 30' for 2.8 BeV/c. At the top of the graph are shown the mass values in BeV calculated from the relation $\mathrm{Sin}\,(\beta_{min}/2) = m_x/E$. It can be seen that the graphs in figure 5 differ from the background distributions shown in 3b, 4b and 4c. The number of events in the graphs in figure 5a between 0° and 120° is 12 ± 6 and, between 120 and 180°, 50 ± 9. In the graph in figure 5 there are 20 ± 8 events between 0° and 80° and 172 ± 23 between 80° and 180°. It is evident that this sharply asymmetric character of the distributions of the $\pi^- + p \to n + 3\gamma$ reactions in relation to the angle β_{min} proves that these distributions are the result of $\omega \to \pi^\circ + \gamma \to 3\gamma$ decays.

In order to compare the experimental results in figures 5a and 5b with the theoretical dependence $dw/d\beta$, the possible alignment of a ω-meson generated in the $\pi^- + p \to n + \omega$ reaction must be taken into account. Let us define the distribution of γ-quanta from $\omega \to \pi^\circ + \gamma$ decay as $a + b\cos^2\Theta$, where Θ is the angle between the γ-quantum from $\omega \to \pi^\circ + \gamma$ decay and the direction of motion of the ω-meson in its rest system. Then, by using expression (11) and defining the ratio a/b, which is an unknown function of the parameters of the $\pi^- + p \to n + \omega$ reaction, one can obtain the distribution according to β taking into account the alignment. The results are given in figure 6. The comparison of the curves in figure 6 with the results in figure 5 shows that the curves which take into account the alignment are in better agreement with the experimental data. Evidently it is not out of the question that the increased number of events observed for large β angles might be due not only to the alignment of the ω-meson, but also to the presence of a particle with a greater mass than the ω-meson and the same mode of decay.

The graph in figure 5 shows a small number of events produced at angles $\beta < \beta_{min}$. Such events may be the result of statistical fluctuations but they may also be partly due to the background reaction $\pi^- + p \to K^\circ + \Sigma + \pi^\circ$, mentioned above, or to systematic errors which were not taken into account.

It was also possible to determine experimentally the upper limit of the ratio of decay probabilities $(\omega \to \pi^+ + \pi^- + \pi^\circ)/(\omega \to \pi^\circ + \gamma)$. For this purpose we scanned part of the photographs from the 1.55 and the 2.8 BeV/c series in search of $\pi^- + p \to n + \pi^+ + \pi^- + m\gamma$ reactions where $m \geqslant 2$. The distributions obtained were used for calculating the number of $\pi^- + p \to n + \pi^+ + \pi^- + \pi^\circ$ reactions which naturally includes both $\omega \to \pi^+ + \pi^- + \pi^\circ$ decays and non-resonant states of 3 π-mesons. In this way it was found that the ratio of the number of events $(\pi^- + p \to n + \pi^+ + \pi^- + \pi^\circ)/(\pi^- + p \to n + \omega \to n + \pi^\circ + \gamma)$ $= 7.3 \pm 1.7$ for 1.55 BeV/c and 8.6 ± 2.3 for 2.8 BeV/c. It follows that $(\omega \to \pi^+ + \pi^- + \pi^\circ)/(\omega \to \pi^\circ + \gamma) < 8 \pm 1.5$. If one considers that the $\omega \to \pi^- + \gamma$ decay is the mean decay mode of the ω-meson into neutral particles, then the result obtained agrees with the value $(\omega \to \pi^+ + \pi^- + \pi^\circ)/(\omega \to \text{neutral particles}) = 5 \pm 1$ known from experiments [4-6].

Let us also consider the question of $\omega \to 2\pi^\circ + \gamma$ decay. The resolutions of system (18) show that the maximum possible value of the ratio of the number of reactions $(\pi^- + p \to n + 5\gamma) / (\pi^- + p \to n + 3\gamma)$, averaged out for the whole series of measurements, is 0.09 i.e. $r = (\omega \to \pi^\circ + \gamma)/(\omega \to \pi^\circ + \gamma) \lesssim 0.1$. This result agrees with the theoretical estimate of Singer [3], who obtained $r = 1/9$ on the hypothesis that the radiative decays of the ω-meson pass through intermediate states with a ρ°-meson.

Let us now deduce from the data obtained information concerning the cross-section of the $\pi^- + p \to n + \omega \to n + \pi^\circ + \gamma$ reaction. It can be calculated according to the formula

$$\sigma_\omega = \frac{S_1}{S} \cdot \frac{n_3}{\sum\limits_{k=2}^{6} n_k} \sigma_\circ \qquad (21)$$

where S_1 is the area of the spectrum shown in figures 5a or 5b in the range from β_{min} to 180°, S the area of the whole spectrum, n_k the number of $\pi^- + p \to n + K\gamma$ reactions calculated as shown above, and $\sigma_\circ$ is the cross-section of the reactions $\pi^- + p \to n +$ neutral particles known from experiments [10,11]. The values of $\sigma_\circ$ were taken as: 4.6 mb at 1.25 BeV/c [10], 4.0 mb at 1.55 BeV/c [10] and 2.2 mb for 2.8 BeV/c [11]. When using the formula (21) it was assumed that the $n_3/\Sigma n_k$ ratio was the same for π^--meson interactions with free protons and bound protons of the nuclei. It should be noted that the fraction of quasi hydrogen events for the xenon-propane mixture used in the present work is 35%, as can be shown by calculation.

The results of the calculations are given in figure 7. The correction S_1/S was not introduced for 1.25 BeV/c. Both statistical errors and the error in the value of ℓ_0 were taken into account. In addition to our points, figure 7 includes data from other work [12,13], reduced by a factor of 5 in order to adapt them to the decay mode ($\omega \rightarrow$ neutral particles). As can be seen, the result at 1.25 BeV/c obtained for the charge symmetrical reaction $\pi^+ + n \rightarrow p + \omega$ [12] agrees well with our result. Two other points at 1.9 BeV/c and 2.2 BeV/c [13] can be considered to agree satisfactorily with our data, since these results are purely tentative and are given by authors without indication of the measurement error. The foregoing comparison between the values of the cross-sections $\sigma(\pi^- + p \rightarrow n + \omega)/5$ and $\sigma(\pi^- + p \rightarrow n + \omega \rightarrow n + \pi^\circ + \gamma)$ leads to the conclusion that among ω-meson decays into neutral particles $\omega \rightarrow \pi^\circ + \gamma$ decay is predominant.

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A N N E X

Kinematics of $X^\circ \rightarrow \pi^\circ + \gamma \rightarrow 3\gamma$ decay

Figure 8 shows the plane F formed in the system of rest of the X°-meson by the vectors of the momenta of the 3 γ-quanta from $X^\circ \rightarrow \pi^\circ + \gamma \rightarrow 3\gamma$ decay. The direction of motion of the X°-meson in the centre of mass system of the $\pi^- + p \rightarrow n + X^\circ$ reaction is represented in figure 8 by the vector $\vec{p}$, and the directions of motion of 2 γ-quanta by the vectors $\vec{K}_1$ and $\vec{K}_2$. Plane OCB is perpendicular to plane F, plane ACB is perpendicular to OCB, plane ABD is perpendicular to AOD, the straight line OO' is perpendicular to plane F, point B' is situated on the continuation of the straight line OB. The vector $\vec{p}$ forms with the plane F an angle BOC = ξ and with the vector $\vec{K}_1$ an angle AOC = Θ_1. Angle ACB = φ_1 is the azimuthal angle of the vector $\vec{K}_1$, and angle BAD = φ_2 is the azimuthal angle of $\vec{K}_2$ in relation to the plane AOD. The following relations can be easily obtained from geometrical considerations :

$$\mathrm{tg}\ \xi = \mathrm{tg}\ \Theta_1 \ \mathrm{Cos}\ \varphi_1 \qquad (1)$$

$$\mathrm{Sin}\ \xi = \mathrm{Sin}\ \Theta_1 \ \mathrm{Sin}\ \varphi_2 \qquad (2)$$

Let us now carry out the relativistic transformation of the angles Θ_1, ξ and $\phi = \pi - \xi$ in a system where the X°-meson is in motion. Let the directions OA, OB, OB^1 and OO^1 in figure 8 be represented in that system by the vectors $\vec{K}_1'$, $\vec{\xi}'$, $\vec{\phi}'$ nd $\vec{n}$. Let the Z axis of the system be directed along the vector $\vec{p}$. Since the transformation is carried out along $\vec{p}$, the vectors $\vec{\xi}'$ $\vec{\phi}'$ and $\vec{n}$ will be situated in the same plane as $\vec{p}$.

Geometrical considerations and the use of the formulae of the relativistic transformation of angles and of relation (1) gives :

$$\mathrm{Cos}\,\omega = \frac{v \,\mathrm{Sin}\,\xi}{\sqrt{1-v^2\mathrm{Cos}^2\xi}} \qquad (3)$$

where ω is the angle between the vectors $\vec{K}_1'$ and $\vec{n}$, and $v = p/E$ is the velocity of the X°-meson. By writing the angle $(\vec{\xi}', \vec{n})$ = angle $(\vec{\phi}', \vec{n}) = \beta/2$, one also obtains the relation :

$$\mathrm{Cos}\frac{\beta}{2} = \frac{v \,\mathrm{Sin}\,\xi}{\sqrt{1-v^2\mathrm{Cos}^2\xi}} \qquad (4)$$

After comparing (3) and (4) we find that the vectors of the momenta of γ-quanta in the system where the X°-meson is in motion, are on the surface of a circular cone with an angle of aperture β. The vector $\vec{n}$ is the axis of this cone.

As can be seen from figure 8 the angle ξ is within limits $0 \leqslant \xi \leqslant \pi$ i.e. the angle of aperture of the decay cone, in agreement with (4), varies within the following limits :

$$2 \arcsin \sqrt{1-v^2} \leqslant \beta \leqslant \pi \qquad (5)$$

Furthermore by using formulae (2) and (4) one can obtain :

$$\mathrm{Cos}\frac{\beta}{2} = \frac{v \,\mathrm{Sin}\Theta_1 \,\mathrm{Sin}\varphi_2}{\sqrt{1-v^2+v^2\mathrm{Sin}^2\Theta_1\mathrm{Sin}^2\varphi_2}} \qquad (6)$$

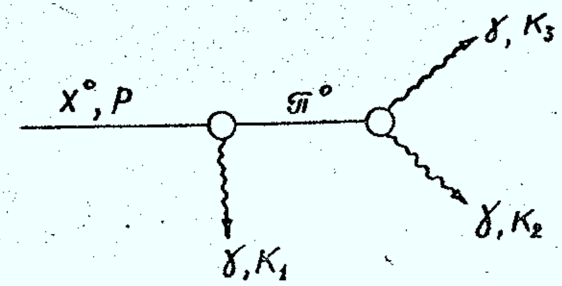

Fig. 1

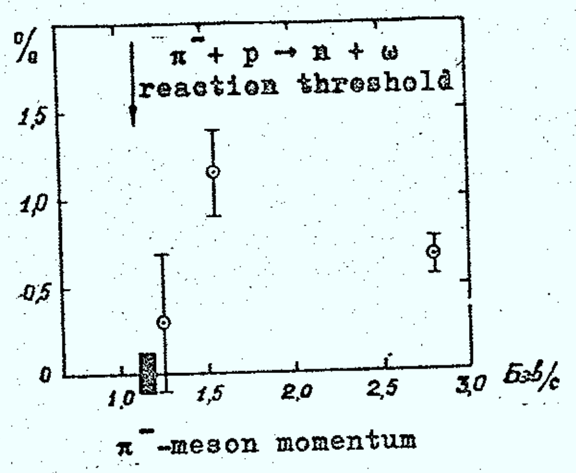

Fig. 2

Output of $\pi^- + p \rightarrow n + 3\gamma$ reactions as a percentage of total number of interactions. ▩ - obtained from the results of work [9].

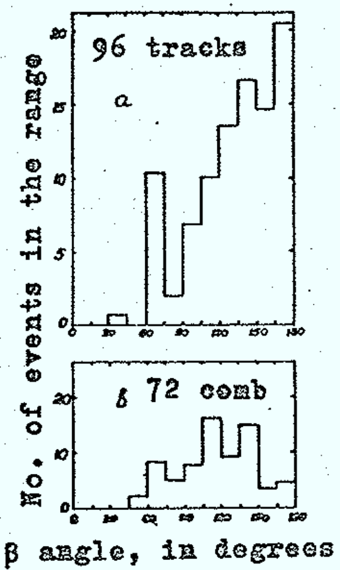

Fig. 3

Histogram for 1.55 BeV/c.
a - events with 3 γ-quanta;
b - combinations by 3 of events with 4 γ-quanta.

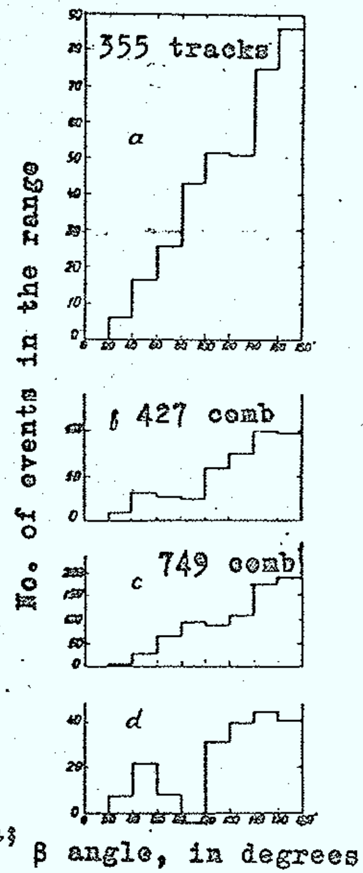

Fig. 4

Histogram for 2.8 BeV/c.
a - events with 3 γ-quant
b - combinations by 3 of events with 4 γ-quanta;
c - ditto of events with and 6 γ-quanta;
d - difference between b and c.

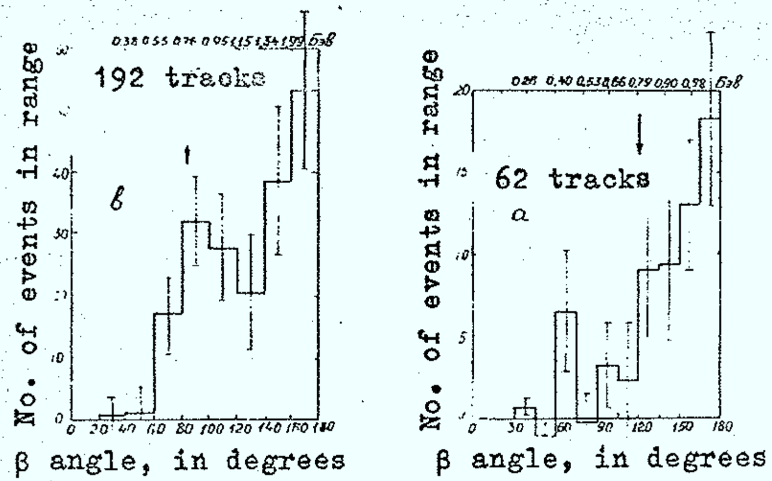

Fig. 5

Histogram of $\pi^- + p \rightarrow n + 3\gamma$ reaction after deducting background. a - 1.55 BeV/c; b - 2.8 BeV/c.

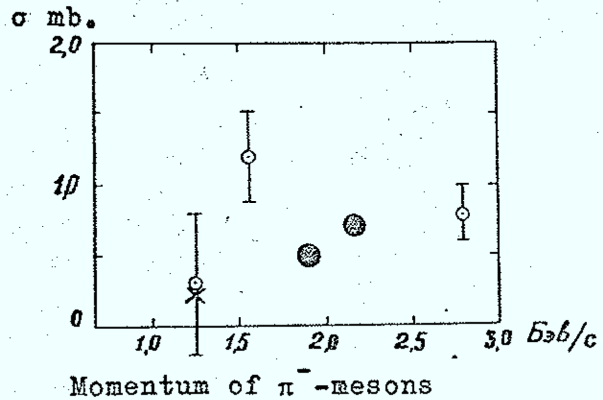

Fig. 7

Cross-sections of $\pi^- + p \rightarrow n + \omega \rightarrow n + \pi^\circ + \gamma$ reac

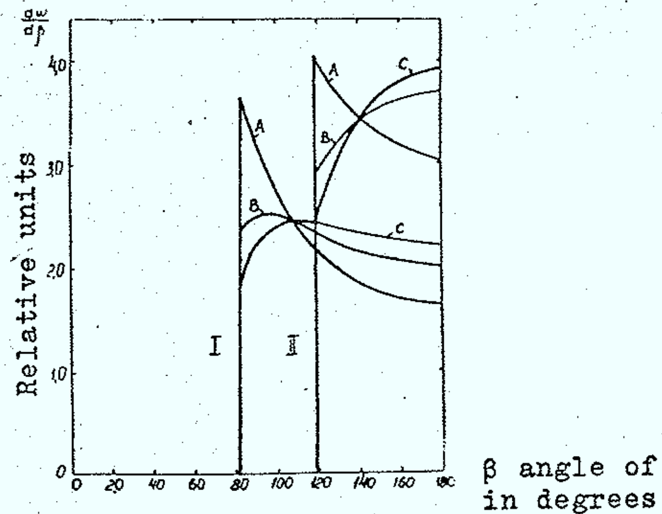

Fig. 6

Theoretical curves $dw/d\beta$. 1 - 1.55 BeV/c; 2 - 2.8 BeV/c.
A: a = 1 b = 0; B: a = 0.5 b = 1;
C: a = 0.25 b = 1. a and b - distribution parameters $a + b \cos^2\theta$

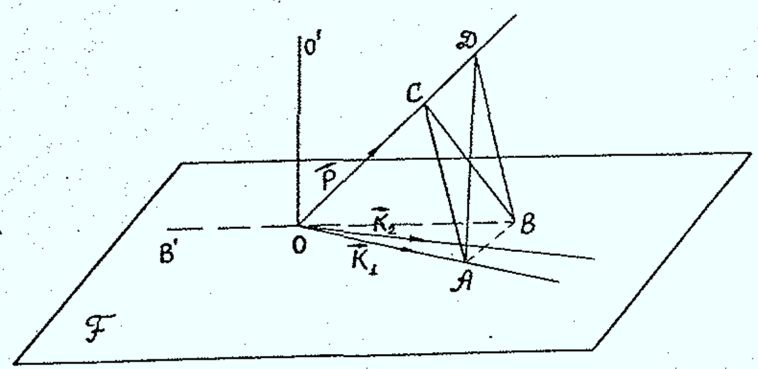

Fig. 8