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ISSN 1343-2230

CNS-REP-7
February, 1998

CNS

CNS Report

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Due to Beam-Beam Collisions

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To appear in Physical Review Letters

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SPIN DEPOLARIZATION DUE TO BEAM-BEAM COLLISIONS

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Abstract

Effect of beam-beam interaction on spin depolarization in proton-proton collider has been studied. Employed method is based on matrix formalism for spin advance and for perturbed betatron particle motion in a ring. Calculations were done for a collider with one interaction point and two installed Siberian Snakes in each ring. Matrix for spin advance after arbitrary large number of turns is found. Performed study indicates, that spin depolarization due to beam-beam collisions is suppressed if beam-beam interaction is stable and if operation point is far enough from spin resonances. Meanwhile, in the absence of Snakes or under beam-beam instability, spin is a subject of strong depolarization. Analytical estimations are confirmed by results of computer simulations.

I. INTRODUCTION

Particle colliders with polarized beams require careful control of spin depolarization. During acceleration spin is subjected to intrinsic and imperfection resonances, resulting in depolarization. Extra source of depolarization is beam-beam collisions. Due to beam-beam interaction, particle motion become essentially nonlinear and under some circumstances, unstable. In present paper effect of beam-beam collision on spin depolarization in a proton - proton collider is studied. Betatron particle motion is defined as a linear oscillator perturbed by nonlinear beam-beam kick. Spin rotation is described by subsequent spin matrix multiplication in dipole magnet, in Siberian Snakes and in beam-beam interaction point. Analytical treatment of the problem provides choice of the collider operation point, where depolarization is suppressed. It also indicates zone of relatively strong depolarization.

II. SPIN MATRIX FORMALISM

Rotation of spin $\vec{S}$ of a particle with charge q, mass m, velocity $\vec{\beta} = \vec{v}/c$ and energy γ is governed by the Bargmann-Michel-Telegdi (BMT) equation [1]:

$$\frac{d\vec{S}}{dt} = \frac{q}{m\gamma}\vec{S} \times [(1 + G\gamma)\vec{B}_\perp + (1+G)\vec{B}_{II} + (G\gamma + \frac{\gamma}{1+\gamma})\frac{\vec{E}\times\vec{\beta}}{c}], \tag{2.1}$$

where G = 1.79285 is an anomalous magnetic momentum of the proton, $\vec{E}$ is an electrical field, $\vec{B}_\perp$ and $\vec{B}_{II}$ are components of magnetic field, perpendicular and parallel to particle velocity, respectively:

$$\vec{B}_\perp = \frac{1}{v^2}(\vec{v} \times \vec{B}) \times \vec{v}, \tag{2.2}$$

$$\vec{B}_{II} = \frac{1}{v}(\vec{v}\cdot\vec{B})\,\frac{\vec{v}}{v}. \tag{2.3}$$

Particle velocity, $\vec{v}$ is expanded in the orthonormal set of curvilinear coordinate $(\vec{x}, \vec{y}, \vec{z})$ as follow:

$$\vec{v} = \dot{z}\,[x'\,\vec{x} + y'\,\vec{y} + (1+\frac{x}{\rho})\vec{z}], \tag{2.4}$$

where ρ is a curvature radius of the reference coordinate system, dot means derivative over time, $\dot{}$ = d/dt and prime means derivative over longitudinal coordinate, ' = d/dz. Change of independent variable in Eq. (2.1) from t for z gives:

$$\frac{d\vec{S}}{dt} = \frac{d\vec{S}}{dz}\frac{dz}{dt} = \frac{d\vec{S}}{dz}\frac{v}{\sqrt{(1+\frac{x}{\rho}) + x'^2 + y'^2}}. \tag{2.5}$$

Calculation of magnetic field components (2.2), (2.3) results in the following expressions:

$$\vec{B}_\perp = [x'^2 + y'^2 + (1+\frac{x}{\rho})]^{-1}\{[((1+\frac{x}{\rho})^2 + y'^2)B_x - x'y'B_y - x'B_z\ (1+\frac{x}{\rho})]\vec{x}$$

$$+ [-x'y'B_x + ((1+\frac{x}{\rho})^2 + x'^2)B_y - y'B_z\ (1+\frac{x}{\rho})]\vec{y}$$

$$+ [-x'B_x\ (1+\frac{x}{\rho}) - y'B_y(1+\frac{x}{\rho}) + (x'^2 + y'^2)\ B_z\ ]\vec{z}\}\ , \tag{2.6}$$

$$\vec{B}_{II} = \{x'^2 + y'^2 + (1+\frac{x}{\rho})^2\}^{-1}\{x'B_x + y'B_y + (1+\frac{x}{\rho})B_z\}\{x'\vec{x} + y'\vec{y} + (1+\frac{x}{\rho})\vec{z}\}\ . \tag{2.7}$$

Vector product of electric field and particle velocity gives:

$$\vec{E} \times \vec{v} = \dot{z}\{[E_y(1+\frac{x}{\rho}) - E_z y']\vec{x} + [E_z x' - (1+\frac{x}{\rho})\ E_x]\vec{y} + [y'E_x - E_y x']\vec{z}\}\ . \tag{2.8}$$

Combining all terms, BMT equation now can be written as

$$\frac{d\vec{S}}{dz} = \vec{S} \times \vec{P}, \text{ or} \tag{2.9}$$

$$\begin{cases} \frac{dS_x}{dz} = S_y P_z - S_z P_y \\ \frac{dS_y}{dz} = S_z P_x - S_x P_z\ , \\ \frac{dS_z}{dz} = S_x P_y - S_y P_x \end{cases} \tag{2.10}$$

where vector $\vec{P} = (P_x, P_y, P_z)$ is given by the terms up to the first order by the following expressions:

$$P_x = \frac{q}{m\gamma v}\ [(1+G\gamma)(B_x - x'B_z) + (1+G)x'B_z + \frac{v}{c^2}(\frac{\gamma}{1+\gamma} + G\gamma)(E_y - y'E_z)]\ , \tag{2.11}$$

$$P_y = \frac{q}{m\gamma v}\ [(1+G\gamma)(B_y - y'B_z) + (1+G)y'B_z + \frac{v}{c^2}(\frac{\gamma}{1+\gamma} + G\gamma)(x'E_z - E_x)]\ , \tag{2.12}$$

$$P_z = \frac{q}{m\gamma v}\ [(1+G\gamma)(-x'B_x - y'B_y) + (1+G)(x'B_x + B_z + y'B_y\ ) + \frac{v}{c^2}(\frac{\gamma}{1+\gamma} + G\gamma)(y'E_x - E_y x')]\ . \tag{2.13}$$

To derive matrix of spin rotation, let us assume, that the vector $\vec{P}$ is a constant at the infinitesimal distance δz. Second derivative of spin vector is given by

$$\left\{\begin{aligned} \frac{d^2S_x}{dz^2} &= S_zP_xP_z - S_x(P_z^2 + P_y^2) + S_yP_xP_y \\ \frac{d^2S_y}{dz^2} &= S_xP_xP_y - S_y(P_x^2 + P_z^2) + S_zP_yP_z \\ \frac{d^2S_z}{dz^2} &= S_yP_yP_z - S_z(P_y^2 + P_x^2) + S_xP_zP_x \end{aligned}\right. . \quad (2.14)$$

Taking the third derivative of spin vector, equations (2.10) are reduced to the third-order differential equations:

$$\left\{\begin{aligned} S_x''' + P_o^2 S_x' &= 0 \\ S_y''' + P_o^2 S_y' &= 0 \\ S_z''' + P_o^2 S_z' &= 0 \end{aligned}\right. , \quad (2.15)$$

$$P_o^2 = P_x^2 + P_y^2 + P_z^2 \ . \quad (2.16)$$

General solution to the problem (2.15) can be written in the form:

$$\left\{\begin{aligned} S_x &= C_{x1} + C_{x2}\cos(P_o\delta z) + C_{x3}\sin(P_o\delta z) \\ S_y &= C_{y1} + C_{y2}\cos(P_o\delta z) + C_{y3}\sin(P_o\delta z) \\ S_z &= C_{z1} + C_{z2}\cos(P_o\delta z) + C_{z3}\sin(P_o\delta z) \end{aligned}\right. , \quad (2.17)$$

where constants C_{ij}, $i = (x, y, z)$, $j=(1, 2, 3)$ depend on initial conditions.

Let us express constants in Eqs. (2.17) through initial values of spin and it's derivatives. Assuming in Eq. (2.17) $\delta z = 0$, initial value of spin vector $\vec{S}_o = (S_{xo}, S_{yo}, S_{zo})$ is given by

$$\left\{\begin{aligned} S_{xo} &= C_{x1} + C_{x2} \\ S_{yo} &= C_{y1} + C_{y2} \\ S_{zo} &= C_{z1} + C_{z2} \end{aligned}\right. . \quad (2.18)$$

From Eqs. (2.17) initial values of the first $\vec{S}_o' = (S_{xo}', S_{yo}', S_{zo}')$ and of the second $\vec{S}_o'' = (S_{xo}'', S_{yo}'', S_{zo}'')$ order derivatives of spin vector are

$$\left\{\begin{aligned} S_{xo}' &= C_{x3} P_o \\ S_{yo}' &= C_{y3} P_o \\ S_{zo}' &= C_{z3} P_o \end{aligned}\right. , \quad (2.19)$$

$$\begin{cases} S''_{xo} = - C_{x2} P_o^2 \\ S''_{yo} = - C_{y2} P_o^2 \\ S''_{zo} = - C_{z2} P_o^2 \end{cases} . \qquad (2.20)$$

Combining Eqs. (2.17) - (2.20), solution for spin advance at the distance δz can be written as follow:

$$\vec{S} = \vec{S}_o + \frac{\vec{S}'_o}{P_o} \sin (P_o \delta z) + \frac{\vec{S}''_o}{P_o^2} [1 - \cos (P_o \delta z)] \ . \qquad (2.21)$$

Substitution of Eqs. (2.10) and (2.14) into Eq. (2.21) gives the following matrix of spin rotation at the distance δz [2]:

$$\begin{vmatrix} S_x \\ S_y \\ S_z \end{vmatrix} = \begin{vmatrix} 1 - a(B^2+C^2) & ABa + Cb & ACa - Bb \\ ABa - Cb & 1 - a(A^2+C^2) & BCa + Ab \\ ACa + Bb & BCa - Ab & 1 - a(A^2+B^2) \end{vmatrix} \begin{vmatrix} S_{x,o} \\ S_{y,o} \\ S_{z,o} \end{vmatrix} , \qquad (2.22)$$

$$A = \frac{P_x}{P_o}, \quad B = \frac{P_y}{P_o}, \quad C = \frac{P_z}{P_o}, \qquad (2.23)$$

$$a = 1 - \cos \varphi, \quad b = \sin \varphi, \quad \varphi = P_o \, \delta z \ . \qquad (2.24)$$

Matrix (2.22) can be used for calculation of spin rotation in arbitrary electromagnetic field, assuming field is constant at the distance δz. Below, matrix (2.22) will be applied for calculation of spin advance in bending magnet and in a beam-beam interaction point.

III MODEL OF COLLIDER WITH POLARIZED PARTICLES

A. Particle betatron motion

Let us consider a collider ring with two installed Siberian Snakes. We use a two-dimensional particle model in coordinates (x, $p_x = \beta_x^* \frac{dx}{dz}$), ($y$, $p_y = \beta_y^* \frac{dy}{dz}$), where β_x^*, β_y^* are beta-functions of the ring. Particle motion between subsequent collisions combines linear matrix transformation, perturbed by beam-beam interaction:

$$\begin{pmatrix} x_{n+1} \\ p_{x,n+1} \\ y_{n+1} \\ p_{y,n+1} \end{pmatrix} = \begin{pmatrix} \cos \bar{\theta}_x & \sin \bar{\theta}_x & 0 & 0 \\ -\sin \bar{\theta}_x & \cos \bar{\theta}_x & 0 & 0 \\ 0 & 0 & \cos \bar{\theta}_y & \sin \bar{\theta}_y \\ 0 & 0 & -\sin \bar{\theta}_y & \cos \bar{\theta}_y \end{pmatrix} \begin{pmatrix} x_n \\ p_{x,n} + \Delta p_{x,n} \\ y_n \\ p_{y,n} + \Delta p_{y,n} \end{pmatrix} , \qquad (3.1)$$

where $\overline{\theta}_x = 2\pi Q_x$, $\overline{\theta}_y = 2\pi Q_y$ are betatron angle, Q_x and Q_y are betatron tunes. Beam-beam kicks $\Delta p_{x,n}$, $\Delta p_{y,n}$ are expressed as a result of interaction of particles with opposite beam with Gaussian distribution function

$$\Delta p_{x,n} = 4\pi\xi x_n \frac{1 - \exp(-\frac{r_n^2}{2\sigma_n^2})}{(\frac{r_n^2}{2\sigma_n^2})} , \tag{3.2}$$

and similar for $\Delta p_{y,n}$. Parameter ξ is a beam-beam parameter, which characterizes strength of beam-beam interaction

$$\xi = \frac{N r_o \beta^*}{4\pi \sigma^2 \gamma} , \tag{3.3}$$

where N is a number of particles per bunch, $r_o = q^2 / (4\pi\varepsilon_o mc^2)$ is a classical particle radius and σ is a transverse standard deviation of the opposite beam size.

B. Spin matrix

Rotation of spin vector $\vec{S} = (S_x, S_y, S_z)$ is described by subsequent matrix transformation in a lattice arc, in Siberian Snakes and in interaction point.

1. Dipole magnet

Spin rotation in an ideal lattice arc is described as a spin precession in dipole magnet with bending angle υ. Assume, that field of dipole magnet has only one vertical component:

$$B_x = 0, \quad B_z = 0, \quad B_y = B \ . \tag{3.4}$$

Therefore, components of vector $\vec{P}$, Eqs. (2.11) - (2.13), and corresponding matrix coefficients, Eqs. (2.23) -(2.24), are given by:

$$P_x = 0, \ P_y = \frac{(1+G\gamma)}{\rho}, \ P_z = 0, A = 0, B= 1, C = 0, \tag{3.5}$$

$$P_o \cdot \delta z = \frac{(1+G\gamma)}{\rho} \delta z = (1+G\gamma)\,\upsilon \ . \tag{3.6}$$

Matrix of spin rotation in dipole magnet is [3]:

$$D_\upsilon = \begin{vmatrix} \cos(P_o\delta z) & 0 & -\sin(P_o\delta z) \\ 0 & 1 & 0 \\ \sin(P_o\delta z) & 0 & \cos(P_o\delta z) \end{vmatrix} . \tag{3.7}$$

2. Siberian Snakes

Siberian Snakes rotate any spin vector by angle π around axis [4]. Two types of Snakes are used, which matrixes are given by:

$$S_1 = \begin{vmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix}, \tag{3.8}$$

$$S_2 = \begin{vmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{vmatrix}. \tag{3.9}$$

3. Interaction point

Spin advance after crossing an interaction point is described by matrix (2.22), where δz is an interaction distance, defined below. Vector $\vec{P}$, Eqs. (2.11)-(2.13), in case of head-on beam-beam collision is as follow:

$$P_x = \frac{1}{B\rho}\,[(1+G\gamma)B_x + (G\gamma + \frac{\gamma}{1+\gamma})\frac{\beta E_y}{c}], \tag{3.10}$$

$$P_y = \frac{1}{B\rho}\,[(1+G\gamma)B_y - (G\gamma + \frac{\gamma}{1+\gamma})\frac{\beta E_x}{c}], \tag{3.11}$$

$$P_z = 0 \quad , \tag{3.12}$$

where small terms x', y' are neglected, $B\rho = mc\beta\gamma/q$ is a rigidity of particles, $\vec{E} = (E_x, E_y, 0)$ is an electrical field and $\vec{B} = (B_x, B_y, 0)$ is a magnetic field of the opposite bunch. Due to Lorentz transformations, components of electromagnetic field of the opposite bunch are connected via relationships

$$B_x = \beta\,\frac{E_y}{c}, \qquad B_y = -\,\beta\,\frac{E_x}{c}. \tag{3.13}$$

Assuming, that interacted particles are ultra relativistic $\beta \approx 1, \gamma >> 1$, the vector $\vec{P}$ is simplified:

$$P_x = \frac{q\,E_y}{mc^2\gamma}\,[(1+G\gamma) + (G\gamma + \frac{\gamma}{1+\gamma})] \approx 2\,G\,\frac{q\,E_y}{mc^2}, \tag{3.14}$$

$$P_y = \frac{q\,E_x}{mc^2\gamma}\,[-(1+G\gamma) - (G\gamma + \frac{\gamma}{1+\gamma})] \approx -\,2\,G\,\frac{q\,E_x}{mc^2}. \tag{3.15}$$

Let us express matrix parameter φ, Eq. (2.24), via beam-beam parameter ξ. Electrostatic field of the opposite round Gaussian bunch with length l and peak current $I = \frac{q N \beta c}{l}$, is:

$$E_r = \frac{q N}{2\pi\varepsilon_o l r} [1 - \exp(-\frac{r^2}{2\sigma^2})] = \frac{I}{2\pi\varepsilon_o \beta c r} [1 - \exp(-\frac{r^2}{2\sigma^2})] , \quad (3.16)$$

$$E_x = E_r \frac{x}{r} , \qquad E_y = E_r \frac{y}{r} . \quad (3.17)$$

Substitution of the expression of electrostatic field into Eqs. (3.14), (3.15) gives expression for vector $\vec{P}$

$$P_x = 4G \frac{I}{I_c} \frac{y}{r^2} [1 - \exp(-\frac{r^2}{2\sigma^2})] , \quad (3.18)$$

$$P_y = -4G \frac{I}{I_c} \frac{x}{r^2} [1 - \exp(-\frac{r^2}{2\sigma^2})] , \quad (3.19)$$

where $I_c = 4\pi\varepsilon_o m c^3/q = (A/Z)\cdot 3.13\cdot 10^7$ Amp is a characteristic value of the beam current. Beam-beam parameter ξ in Eq. (3.3) can be rewritten as follow:

$$\xi = \frac{\beta^*}{4\pi} \frac{I}{I_c} \frac{l}{\gamma \sigma^2} . \quad (3.20)$$

To define the interaction distance δz, let us suppose, that at the time moment $t = 0$ test particle enters the opposite bunch (see Fig. 1). Equation of motion of test particle is $z_1 = v_1 t$. Equation of motion of the right edge of the bunch is $z_2 = l - v_2 t$. Test particle will leave opposite bunch when $z_1 = z_2$, or after time interval $t = \frac{l}{v_1 + v_2}$. Coordinate of test particle at this moment, $z_1 = v_1 t$, is equal to the interaction distance δz:

$$\delta z = v_1 t = l \frac{v_1}{v_1 + v_2} = \frac{l}{2} . \quad (3.21)$$

Taking into account Eqs. (3.18), (3.19), parameters of spin matrix $P_x \delta z$, $P_y \delta z$ can be expressed as follow

$$P_x \delta z = P_x \frac{l}{2} = 4\pi G \gamma \xi \frac{y}{\beta^*} [\frac{1 - \exp(-\frac{r^2}{2\sigma^2})}{(\frac{r^2}{2\sigma^2})}] , \quad (3.22)$$

$$P_y \delta z = P_y \frac{l}{2} = -4\pi G \gamma \xi \frac{x}{\beta^*} [\frac{1 - \exp(-\frac{r^2}{2\sigma^2})}{(\frac{r^2}{2\sigma^2})}] . \quad (3.23)$$

Finally, parameter φ is given by

$$\varphi = \sqrt{(P_x \delta z)^2 + (P_y \delta z)^2} = 4\pi G \gamma \xi \frac{r}{\beta^*} \left[\frac{1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)}{\left(\frac{r^2}{2\sigma^2}\right)} \right]. \qquad (3.24)$$

Parameter φ is typically much smaller than 2π, which gives us possibility to simplify matrix of spin rotation in the interaction point and to provide analytical treatment of the problem (see next Section).

Model developed in this section was incorporated into numerical code BEAMPATH [5]. Typical parameters of numerical model are summarized in Table 1.

IV. ANALYTICAL TREATMENT OF SPIN DEPOLARIZATION

A. Simplified spin matrix in the interaction point

To make an analytical treatment of spin depolarization, let us simplify the suggested model. Consider collider with two Siberian Snakes and one interaction point. Matrix of spin advance after one revolution in the ring between beam-beam interaction is

$$M_{ring} = D_{\pi/2} \cdot S_2 \cdot D_\pi \cdot S_1 \cdot D_{\pi/2} = \begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix}. \qquad (4.1)$$

Suppose, that betatron angels in x and y directions are equal each other $\theta_x = \theta_y = \theta$. We consider particle motion far enough from low order resonances, therefore, particle trajectory can be expressed as a linear oscillator with perturbed betatron tune θ:

$$x = r \cos(n\theta + \Psi), \qquad y = r \sin(n\theta + \Psi), \qquad \theta = \bar{\theta} + \Delta\theta, \qquad (4.2)$$

where Ψ is an initial phase of betatron particle oscillations and $\Delta\theta \ll 2\pi$ is tune perturbation due to beam-beam collisions. In Fig. 2 example of particle trajectories in presence of stable beam-beam interaction is given. Particle trajectories in phase space are slightly deformed ellipses. In this case beam envelopes and beam emittances are also stable (see Fig. 3). Beam-beam instability and it's effect on spin depolarization will be considered in Section V.

Parameters A and B, Eq. (2.23), at the interaction point can be expressed as follow:

$$A = \frac{P_x}{P_o} = \frac{y}{r} = \sin(n\theta + \Psi) \ , \tag{4.3}$$

$$B = \frac{P_y}{P_o} = -\frac{x}{r} = -\cos(n\theta + \Psi) \ . \tag{4.4}$$

Let us take into account, that parameter φ is small:

$$\varphi = P_o\delta z = 4\pi G \gamma \xi \frac{r}{\beta^*} [1 - \frac{r^2}{4\sigma^2} + \ldots] \ll 2\pi \ . \tag{4.5}$$

Hence, the matrix parameters, a and b, are as follow:

$$a = 1 - \cos\varphi \approx \frac{\varphi^2}{2}, \qquad b = \sin\varphi \approx \varphi \ . \tag{4.6}$$

Finally, matrix of spin advance of a particle in interaction point at the n-th turn is given by:

$$M_{b\text{-}b}(n) = \begin{vmatrix} 1 - \frac{\varphi^2}{2}\cos^2(n\theta+\Psi) & -\frac{\varphi^2}{4}\sin 2(n\theta+\Psi) & \varphi\cos(n\theta+\Psi) \\ -\frac{\varphi^2}{4}\sin 2(n\theta+\Psi) & 1 - \frac{\varphi^2}{2}\sin^2(n\theta+\Psi) & \varphi\sin(n\theta+\Psi) \\ -\varphi\cos(n\theta+\Psi) & -\varphi\sin(n\theta+\Psi) & 1 - \frac{\varphi^2}{2} \end{vmatrix} . \tag{4.7}$$

B. Spin matrix after arbitrary number of turns

Now let us derive matrix of spin advance after arbitrary number of turns. Due to small value of parameter φ, we will leave in the resulting matrix only terms, proportional to φ and φ^2, while neglecting terms with φ^3, φ^4 and higher order.

Suppose, initial position of particles is just before interaction point. After interaction point, matrix of spin advance is the matrix (4.7), where n = 0:

$$M_{b\text{-}b}(1) = \begin{vmatrix} 1 - \frac{\varphi^2}{2}\cos^2\Psi & -\frac{\varphi^2}{4}\sin 2\Psi & \varphi\cos\Psi \\ -\frac{\varphi^2}{4}\sin 2\Psi & 1 - \frac{\varphi^2}{2}\sin^2\Psi & \varphi\sin\Psi \\ -\varphi\cos\Psi & -\varphi\sin\Psi & 1 - \frac{\varphi^2}{2} \end{vmatrix} . \tag{4.8}$$

After interaction point particles perform one revolution in the ring and spin matrix after first turn, $M_{1/o}$, is a product of matrixes (4.1) and (4.8):

$$M_{1/0} = \begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} \begin{vmatrix} 1-\frac{\varphi^2}{2}\cos^2\Psi & -\frac{\varphi^2}{4}\sin 2\Psi & \varphi\cos\Psi \\ -\frac{\varphi^2}{4}\sin 2\Psi & 1-\frac{\varphi^2}{2}\sin^2\Psi & \varphi\sin\Psi \\ -\varphi\cos\Psi & -\varphi\sin\Psi & 1-\frac{\varphi^2}{2} \end{vmatrix} =$$

$$= \begin{vmatrix} -1+\frac{\varphi^2}{2}\cos^2\Psi & \frac{\varphi^2}{4}\sin 2\Psi & -\varphi\cos\Psi \\ -\frac{\varphi^2}{4}\sin 2\Psi & 1-\frac{\varphi^2}{2}\sin^2\Psi & \varphi\sin\Psi \\ \varphi\cos\Psi & \varphi\sin\Psi & -1+\frac{\varphi^2}{2} \end{vmatrix}. \qquad (4.9)$$

Analogously, after second turn spin matrix is

$$M_{2/0} = \begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} \cdot \begin{vmatrix} 1-\frac{\varphi^2}{2}\cos^2(\theta+\Psi) & -\frac{\varphi^2}{4}\sin 2(\theta+\Psi) & \varphi\cos(\theta+\Psi)] \\ -\frac{\varphi^2}{4}\sin 2(\theta+\Psi) & 1-\frac{\varphi^2}{2}\sin^2(\theta+\Psi) & \varphi\sin(\theta+\Psi) \\ -\varphi\cos(\theta+\Psi) & -\varphi\sin(\theta+\Psi) & 1-\frac{\varphi^2}{2} \end{vmatrix} \cdot$$

$$\cdot \begin{vmatrix} -1+\frac{\varphi^2}{2}\cos^2\Psi & \frac{\varphi^2}{4}\sin 2\Psi & -\varphi\cos\Psi \\ -\frac{\varphi^2}{4}\sin 2\Psi & 1-\frac{\varphi^2}{2}\sin^2\Psi & \varphi\sin\Psi \\ \varphi\cos\Psi & \varphi\sin\Psi & -1+\frac{\varphi^2}{2} \end{vmatrix} =$$

$$\begin{vmatrix} 1-\frac{\varphi^2}{2}[\cos\Psi+\cos(\theta+\Psi)]^2 & -\frac{\varphi^2}{4}[\sin 2\Psi-\sin 2(\theta+\Psi)+...] & \varphi[\cos\Psi+\cos(\theta+\Psi)] \\ \frac{\varphi^2}{4}[-\sin 2\Psi+\sin 2(\theta+\Psi)+....] & 1-\frac{\varphi^2}{2}[\sin\Psi-\sin(\theta+\Psi)]^2 & \varphi[\sin\Psi-\sin(\theta+\Psi)] \\ -\varphi[\cos\Psi+\cos(\theta+\Psi)] & -\varphi[\sin\Psi-\sin(\theta+\Psi)] & 1-\varphi^2-\varphi^2\cos(\theta+2\Psi) \end{vmatrix}. \qquad (4.10)$$

Every element of the matrix (4.10) has a specific dependence on turn number. Let us assume, that after (n+1) turns the resulting matrix of spin rotation will be as follow:

$M_{(n+1)/0} =$

$$\begin{vmatrix} (-1)^{n+1}\{1-\frac{\varphi^2}{2}[\sum_{i=0}^{n}\cos(i\theta+\Psi)]^2\} & \frac{\varphi^2}{4}[\sum_{i=0}^{n}(-1)^{i+n}\sin 2(i\theta+\Psi)+...] & (-1)^{n+1}\varphi\sum_{i=0}^{n}\cos(i\theta+\Psi) \\ \frac{\varphi^2}{4}[\sum_{i=0}^{n}(-1)^{i+1}\sin 2(i\theta+\Psi)+...] & 1-\frac{\varphi^2}{2}\{\sum_{i=0}^{n}(-1)^{i}\sin(i\theta+\Psi)\}^2 & \varphi\sum_{i=0}^{n}(-1)^{i}\sin(i\theta+\Psi) \\ (-1)^{n}\varphi\sum_{i=0}^{n}\cos(i\theta+\Psi) & \varphi\sum_{i=0}^{n}(-1)^{i+n}\sin(i\theta+\Psi) & (-1)^{n+1}[1-\frac{n+1}{2}\varphi^2+...] \end{vmatrix}. \quad (4.11)$$

Then, multiplying the suggested matrix (4.11) by matrix of spin advance in the next beam-beam interaction, Eq. (4.7), and by the matrix of spin advance in a ring, Eq. (4.1), the matrix after (n+2) turns is obtained:

$$M_{(n+2)/0}= \begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} \begin{vmatrix} 1-\frac{\varphi^2}{2}\cos^2[(n+1)\theta+\Psi] & -\frac{\varphi^2}{4}\sin 2[(n+1)\theta+\Psi] & \varphi\cos[(n+1)\theta+\Psi] \\ -\frac{\varphi^2}{4}\sin 2[(n+1)\theta+\Psi] & 1-\frac{\varphi^2}{2}\sin^2[(n+1)\theta+\Psi] & \varphi\sin[(n+1)\theta+\Psi] \\ -\varphi\cos[(n+1)\theta+\Psi] & -\varphi\sin[(n+1)\theta+\Psi] & 1-\frac{\varphi^2}{2} \end{vmatrix}.$$

$$\begin{vmatrix} (-1)^{n+1}\{1-\frac{\varphi^2}{2}[\sum_{i=0}^{n}\cos(i\theta+\Psi)]^2\} & \frac{\varphi^2}{4}[\sum_{i=0}^{n}(-1)^{i+n}\sin 2(i\theta+\Psi)+...] & (-1)^{n+1}\varphi\sum_{i=0}^{n}\cos(i\theta+\Psi) \\ \frac{\varphi^2}{4}[\sum_{i=0}^{n}(-1)^{i+1}\sin 2(i\theta+\Psi)+...] & 1-\frac{\varphi^2}{2}\{\sum_{i=0}^{n}(-1)^{i}\sin(i\theta+\Psi)\}^2 & \varphi\sum_{i=0}^{n}(-1)^{i}\sin(i\theta+\Psi) \\ (-1)^{n}\varphi\sum_{i=0}^{n}\cos(i\theta+\Psi) & \varphi\sum_{i=0}^{n}(-1)^{i+n}\sin(i\theta+\Psi) & (-1)^{n+1}[1-\frac{n+1}{2}\varphi^2+...] \end{vmatrix} =$$

$$\begin{vmatrix} (-1)^{n+2}\{1-\frac{\varphi^2}{2}[\sum_{i=0}^{n+1}\cos(i\theta+\Psi)]^2\} & \frac{\varphi^2}{4}[\sum_{i=0}^{n+1}(-1)^{i+n+1}\sin 2(i\theta+\Psi)+...] & (-1)^{n}\varphi\sum_{i=0}^{n+1}\cos(i\theta+\Psi) \\ \frac{\varphi^2}{4}[\sum_{i=0}^{n+1}(-1)^{i+1}\sin 2(i\theta+\Psi)+...] & 1-\frac{\varphi^2}{2}\{\sum_{i=0}^{n+1}(-1)^{i}\sin(i\theta+\Psi)\}^2 & \varphi\sum_{i=0}^{n+1}(-1)^{i}\sin(i\theta+\Psi) \\ (-1)^{n+1}\varphi\sum_{i=0}^{n+1}\cos(i\theta+\Psi) & \varphi\sum_{i=0}^{n+1}(-1)^{i+n+1}\sin(i\theta+\Psi) & (-1)^{n+2}[1-\frac{n+2}{2}\varphi^2+...] \end{vmatrix}. \quad (4.12)$$

Resulting matrix (4.12) can be written as the matrix (4.11), where index (n) is substituted by index (n+1). Therefore, suggestion (4.11) is correct and gives the matrix of spin advance after arbitrary number of turns.

C. Spin components after n turns

Developed approach gives us possibility to predict effect of beam-beam interaction on spin depolarization after large number of turns. Suppose, initial spin vector has only one transverse component $S_y = 1$ and other components are equal to zero $S_x = S_z = 0$ (see Fig. 4). Spin advance is as follow

$$\begin{vmatrix} S_x \\ S_y \\ S_z \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix} , \qquad (4.13)$$

therefore, only matrix elements a_{12}, a_{22}, a_{32} are essential to determine the values of spin components after n turns:

$$S_x = \frac{\varphi^2}{4}\Big[\sum_{i=0}^{n-1} (-1)^{i+n-1} \sin 2(i\theta+\Psi) + ...\Big] = (-1)^{n-1} \frac{\varphi^2}{4} \frac{\sin\big[\frac{n\,(2\theta+\pi)}{2}\big]}{\cos\theta} \sin\Big[2\Psi + \frac{n-1}{2}(2\theta+\pi)\Big] + ... \quad , (4.14)$$

$$S_y = 1 - \frac{\varphi^2}{2}\Big\{\sum_{i=0}^{n-1} (-1)^i \sin(i\theta+\Psi)\Big\}^2 = 1 - \frac{\varphi^2}{2} \frac{\sin^2\big[\frac{n\,(\theta+\pi)}{2}\big]}{(\cos\frac{\theta}{2})^2} \sin^2\Big[\Psi + \frac{n-1}{2}(\theta+\pi)\Big] , \qquad (4.15)$$

$$S_z = (-1)^{n-1}\,\varphi \sum_{i=0}^{n-1} (-1)^i \sin(i\theta+\Psi) = (-1)^{n-1}\,\varphi \frac{\sin\big[\frac{n\,(\theta+\pi)}{2}\big]}{\cos\frac{\theta}{2}} \sin\Big[\Psi + \frac{n-1}{2}(\theta+\pi)\Big] . \qquad (4.16)$$

Average values of spin components are achieved by integration of Eqs. (4.14) - (4.16) over all initial phases:

$$\overline{S}_x = \frac{1}{2\pi}\int_0^{2\pi} S_x\, d\Psi = 0 , \qquad (4.17)$$

$$\overline{S}_y = \frac{1}{2\pi}\int_0^{2\pi} S_y\, d\Psi = 1 - \frac{\varphi^2}{4} \frac{\sin^2\big[\frac{n\,(\theta+\pi)}{2}\big]}{(\cos\frac{\theta}{2})^2} , \qquad (4.18)$$

$$\overline{S}_z = \frac{1}{2\pi}\int_0^{2\pi} S_z\, d\Psi = 0 . \qquad (4.19)$$

Root-mean-square values of spin components are given by

$$<S_x^2> = \frac{1}{2\pi}\int_0^{2\pi} S_x^2 \, d\Psi = \frac{\varphi^4}{16}\left[\frac{\sin^2\left[\frac{n(2\theta+\pi)}{2}\right]}{2(\cos\theta)^2} + \ldots\right] , \qquad (4.20)$$

$$<S_y^2> = \frac{1}{2\pi}\int_0^{2\pi} (S_y - \bar{S}_y)^2 \, d\Psi = \frac{\varphi^4}{32}\frac{\sin^4\left[\frac{n(\theta+\pi)}{2}\right]}{(\cos\frac{\theta}{2})^4} , \qquad (4.21)$$

$$<S_z^2> = \frac{1}{2\pi}\int_0^{2\pi} S_z^2 \, d\Psi = \frac{\varphi^2}{2}\frac{\sin^2\left[\frac{n(\theta+\pi)}{2}\right]}{(\cos\frac{\theta}{2})^2} . \qquad (4.22)$$

Introduced average and rms spin component parameters characterize spin depolarization. From formulas (4.17) - (4.22) it follows, that they are turn dependent. Turn number, n, appears as an argument in trigonometric functions, providing oscillation of average and rms spin parameters. Therefore, spin depolarization is suppressed. Taking average values of trigonometric functions

$$\overline{\sin^2\left[\frac{n(\theta+\pi)}{2}\right]} = \frac{1}{2} , \qquad (4.23)$$

$$\overline{\sin^4\left[\frac{n(\theta+\pi)}{2}\right]} = \frac{3}{8} , \qquad (4.24)$$

and average values of parameters φ, θ among all particles,

$$\tilde{\varphi} \approx 4\pi G \gamma \xi \frac{\sigma}{\beta^*} , \qquad (4.25)$$

$$\tilde{\theta} \approx 2\pi (Q - \frac{\xi}{2}) , \qquad (4.26)$$

the turn-independent average and rms spin parameters are

$$\bar{\bar{S}}_x = 0, \quad \bar{\bar{S}}_y = 1 - \frac{\tilde{\varphi}^2}{8(\cos\frac{\tilde{\theta}}{2})^2}, \quad \bar{\bar{S}}_z = 0 , \qquad (4.27)$$

$$\overline{<S_x^2>} = \frac{\tilde{\varphi}^4}{16}\left[\frac{1}{4(\cos\tilde{\theta})^2} + \ldots\right] , \quad \overline{<S_y^2>} = \frac{3\tilde{\varphi}^4}{256(\cos\frac{\tilde{\theta}}{2})^4}, \quad \overline{<S_z^2>} = \frac{\tilde{\varphi}^2}{4(\cos\frac{\tilde{\theta}}{2})^2} . \qquad (4.28)$$

Attained formulas (4.27), (4.28) indicate, that spin depolarization due to beam-beam collisions is suppressed and depends on betatron tune in a ring. The most dangerous working point is close to half-integer value, because in that case the value of $\cos\frac{\tilde{\theta}}{2}$ is close to zero and spin depolarization parameters become large. The matrix (4.11) was obtained in linear approximation to betatron particle motion and to beam-beam forces, therefore it cannot treat higher order nonlinear spin resonances. Due to small value of φ, depolarization effects, proportional to φ^4 are negligible as compare with that, proportional to φ^2. Among possible depolarization effects the most pronounced are change of the values of $\overline{S}_y$ and $\overline{<S_z^2>}$.

V. NUMERICAL SIMULATION OF BEAM-BEAM EFFECT ON SPIN DEPOLARIZATION

A. Spin depolarization as function of betatron tune

Computer simulations utilizing numerical model of Section III were performed for the beam parameters, presented in Table 2. For that combination of collider parameters, the values of matrix parameters are as follow:

$$\tilde{\varphi} = 4\pi G \gamma \xi \frac{\sigma}{\beta^*} = 7.2 \cdot 10^{-3} \ , \qquad (5.1)$$

$$\tilde{\theta} = 2\pi (Q - 0.00625) \ . \qquad (5.2)$$

Initial particle distribution in phase space was chosen to be Gaussian:

$$f = f_o \exp - \left(\frac{p_x^2 + p_y^2}{2p_o^2} + \frac{x^2 + y^2}{2\sigma^2}\right) . \qquad (5.3)$$

During simulations, the average and rms values of spin parameters were calculated according to the formulas:

$$\overline{S}_x = \frac{1}{N} \sum_{i=1}^{N} S_x (i) \ , \qquad (5.4a)$$

$$\overline{S}_y = \frac{1}{N} \sum_{i=1}^{N} S_y (i), \qquad (5.4b)$$

$$\overline{S}_z = \frac{1}{N} \sum_{i=1}^{N} S_z (i), \qquad (5.4c)$$

$$\sqrt{<S_x^2>} = \sqrt{\sum_{i=1}^{N} \frac{1}{N} [S_x(i) - \bar{S}_x]^2} \,, \tag{5.4d}$$

$$\sqrt{<S_y^2>} = \sqrt{\sum_{i=1}^{N} \frac{1}{N} [S_y(i) - \bar{S}_y]^2} \,, \tag{5.4e}$$

$$\sqrt{<S_z^2>} = \sqrt{\sum_{i=1}^{N} \frac{1}{N} [S_z(i) - \bar{S}_z]^2} \,. \tag{5.4f}$$

In a storage ring spin is a subject of intrinsic resonances, obeying resonance condition

$$G\gamma = k_o + k_x Q_x + k_y Q_y \,, \tag{5.5}$$

where k_o, k_x, k_y are integers. Average and rms spin components as function of tune values are presented in Fig. 5. In that simulations horizontal and vertical tunes were taken to be equal each other $Q_x = Q_y$. As seen, spin depolarization is most significant, if fractional part of the tune is close to 1/2, as it was predicted by Eqs. (4.27) - (4.28). Also depolarization is observed, if higher order spin resonances are exited. Nonlinear spin resonances are not treated by analytical formulas of Section IV due to assumptions of the linear model. If tunes are far enough from that values, spin depolarization is suppressed.

In Figs. 6, 7 results of suppressed spin depolarization for $Q_x = Q_y = 14.43$ are presented. Average values of S_x and S_z are close to zero, as expected from Eqs. (4.27). Average value of S_y is slightly less, than initial value of 1, and oscillates around stable value of 0.99987. Rms values of spin components are also oscillatory functions of turn number. Numerical values of average and rms values of spin components are close to analytical estimations (see Table 3). Depolarization provides small tail of distribution of S_y component, which lasts from 1 to 0.9991. Distribution of S_x component is much narrow than that of S_z component. It also follows from Eqs. (4.28), where $<S_x^2>$ is proportional to φ^4, while $<S_z^2>$ is proportional to φ^2. Numerical simulations confirm analytical prediction, that spin depolarization due to beam-beam interaction is suppressed, if particle trajectories are stable and spin resonance conditions are avoided.

In Figs. 8, 9 results of strong spin depolarization for $Q_x = Q_y = 14.505$ are presented. Average value of S_y is less than 0.5. Rms values of S_y and S_z spin components are several order of magnitude larger, than that for previous case. Spin distribution has a spread from -1 to 1. It indicates significant depolarization, as expected from results of previous Section.

B. Spin depolarization in a ring without Siberian Snakes

To estimate effect of Siberian Snakes on spin depolarization in presence of beam-beam interaction, consider a ring without Snakes. Derivation of spin matrix rotation after arbitrary number of turns results in awkward expressions, so we have to rely on computer simulations. In Figs. 10, 11 results of spin depolarization in a ring without Snakes are presented. Simulations were performed for the same values of betatron tunes $Q_x = Q_y = 14.43$ as in Figs. 6, 7, where spin depolarization was suppressed. As seen, in the absence of Snakes beam-beam collisions result in steady spin depolarization.

C. Spin depolarization in presence of beam-beam instability

Up to now we have considered particle motion in presence of stable beam-beam interaction. There are several mechanisms, which lead to beam-beam instability. Excitation of nonlinear resonances and unstable stochastic particle motion due to overlapping of resonance islands is the fundamental phenomena in beam-beam interaction [6]. Another mechanism of unstable particle motion is a diffusion created by random fluctuations in distribution of the opposite beam. In Ref. [7] noise beam-beam instability was studied for the case of random fluctuations in opposite beam size

$$\sigma_n = \sigma_o \left(1 \pm \frac{u \cdot u_n}{2}\right), \tag{5.6}$$

where u is a noise amplitude and u_n is a uniform random function with unit amplitude. It was shown, that in presence of noise, beam emittance is increased with time as

$$\frac{\varepsilon_n}{\varepsilon_o} = \sqrt{1 + D\,n}\ , \tag{5.7}$$

where diffusion coefficient D is a function of beam-beam parameter ξ, noise amplitude u and ratio of beam size, a, to opposite beam size, 2σ:

$$D = \pi^2 (\xi u)^2 \left(\frac{a}{2\sigma}\right)^4 . \tag{5.8}$$

Noise in the beam-beam collision always induces instability if beam-beam kick is a nonlinear function of the coordinate. Due to diffusion character, noise beam-beam instability does not have a threshold character and can exist at any value of beam-beam parameter.

Increase of beam emittance is accompanied with increase of beam size. In Figs. 12 - 14 results of beam dynamics study and spin depolarization in presence of noisy beam-beam interaction are given. The value of noise amplitude u = 0.025 was chosen arbitrary, to demonstrate the main features of diffusion beam-beam instability. In contrast with Fig. 2 particle trajectories at phase planes are not closed (See Fig. 12). Beam emittances and beam envelopes are monotonous increasing functions of turn number (see Fig. 13). Increasing of beam sizes results in steady spin depolarization (see Fig. 14). It is also expected from analytical formulas (4.27), (4.28), where average and rms beam parameters are proportional to the powers of parameter φ, which, in turns, is proportional to beam size according to Eq. (4.5). Therefore, beam-beam instability is a source of spin depolarization.

Spin depolarization due to beam-beam interaction was observed experimentally at the electron-positron collider PETRA [8]. Below beam-beam limit, where particle motion was stable, spin depolarization was negligible. Above beam-beam limit, a significant depolarization was observed, which was strongly correlated to beam blow up due to electron-positron collisions.

VI. CONCLUSIONS

Effect of beam-beam interaction on spin depolarization in proton-proton collider has been studied. Employed method is based on matrix formalism for spin advance and for perturbed betatron particle motion in a ring. Analytical calculations were done for a collider with one interaction point and two installed Siberian Snakes in each ring. Matrix for spin advance after arbitrary number of turns is accomplished. Performed study indicates, that spin depolarization due to beam-beam collisions is suppressed, if beam-beam interaction is stable and spin resonances are avoided. Depolarization depends on collider operation point. Unstable beam-beam interaction provides steady depolarization.

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Table 1. Parameters of numerical model

Number of modeling particles, N	5000
Number of turns	10^6
CPU time (for VAX Alpha)	5 hours

Table 2. Parameters of the interacted beams

Particle energy, γ	260
Rms beam size at interaction point (IP), σ	0.08 mm
Beam-beam tune shift per collision ξ	- 0.0125
Beta function, β^*	0.65 m

Table 3. Average and rms spin components for $Q_x = Q_y = 14.43$.

	analytical	numerical
$\bar{S}_y$	0.99988	0.99988
$\bar{S}_x$	0	0
$\bar{S}_z$	0	0
$\sqrt{<S_y^2>}$	$1.06 \cdot 10^{-4}$	$1.7 \cdot 10^{-4}$
$\sqrt{<S_x^2>}$	$7 \cdot 10^{-6}$	$5 \cdot 10^{-5}$
$\sqrt{<S_z^2>}$	$1.54 \cdot 10^{-2}$	$1.6 \cdot 10^{-2}$

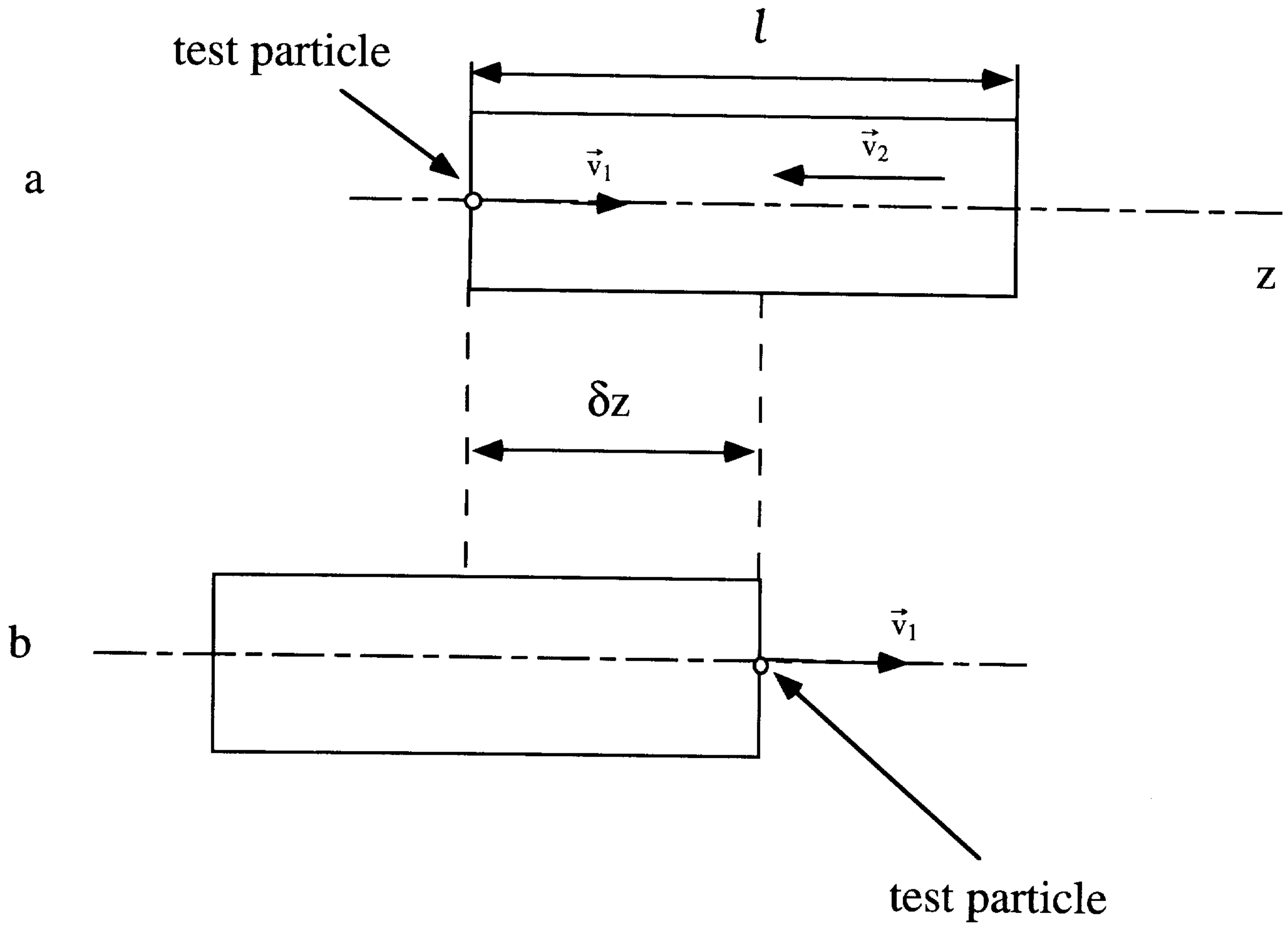


Fig. 1. Position of test particle with respect to opposite bunch: (a) before interaction; (b) after interaction.

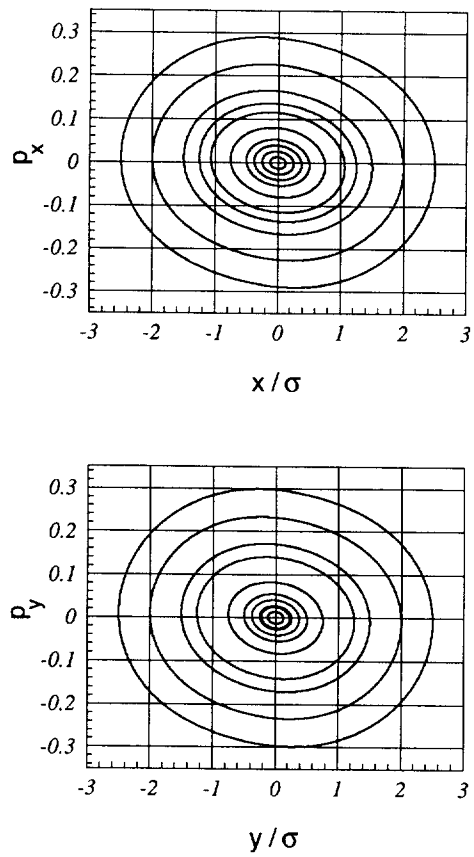


Fig. 2. Stable particle trajectories in phase space in presence of beam-beam interaction without noise.

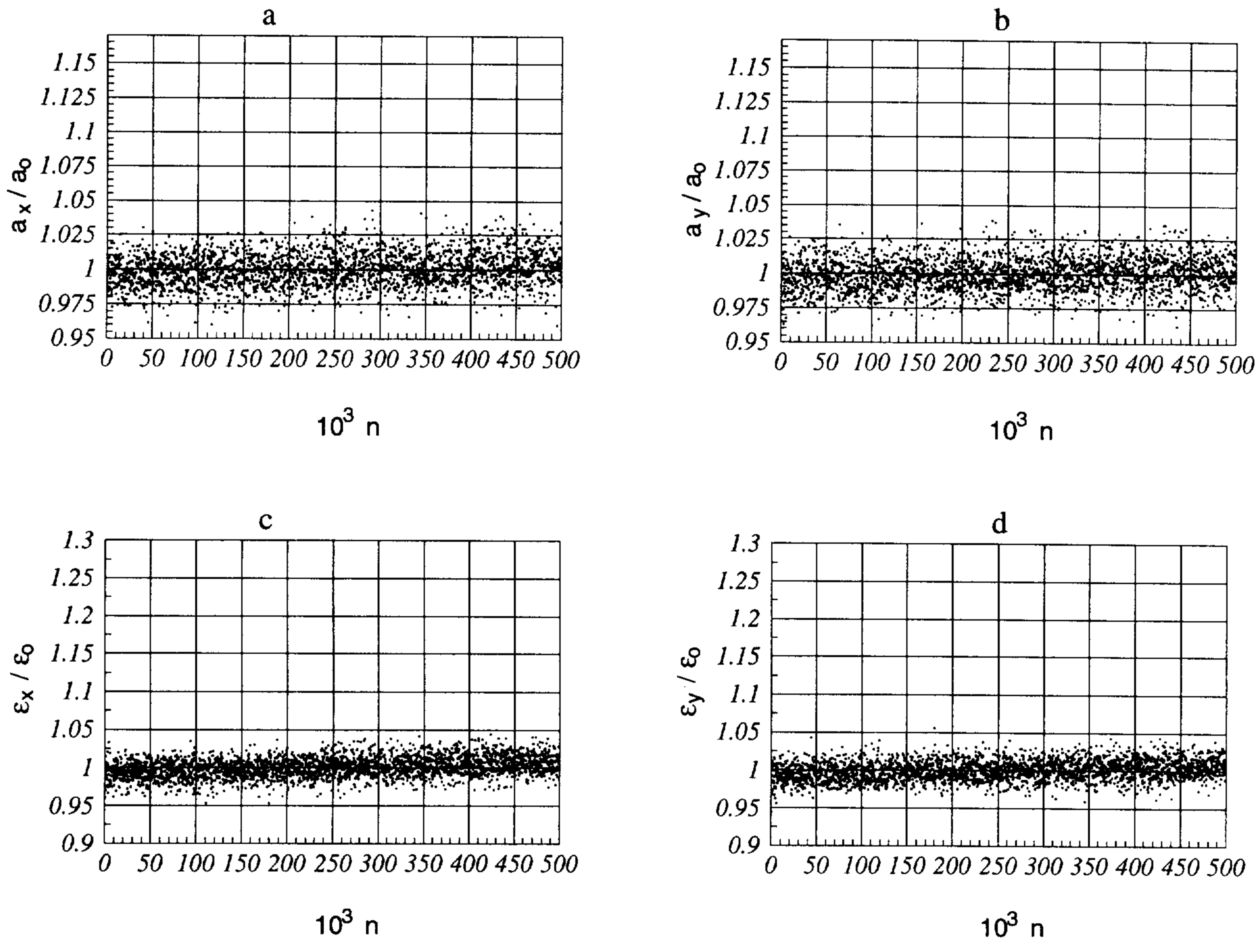


Fig. 3. (a), (b) Beam envelopes and (c), (d) beam emittances in presence of stable beam-beam interaction.

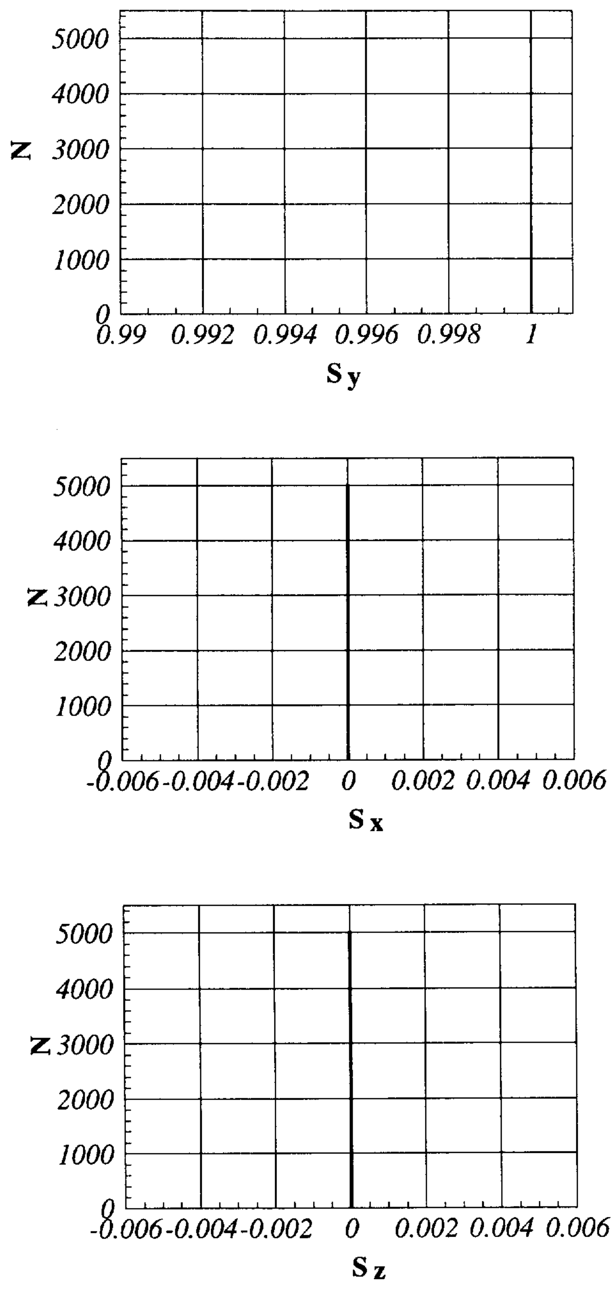


Fig. 4. Initial spin distribution.

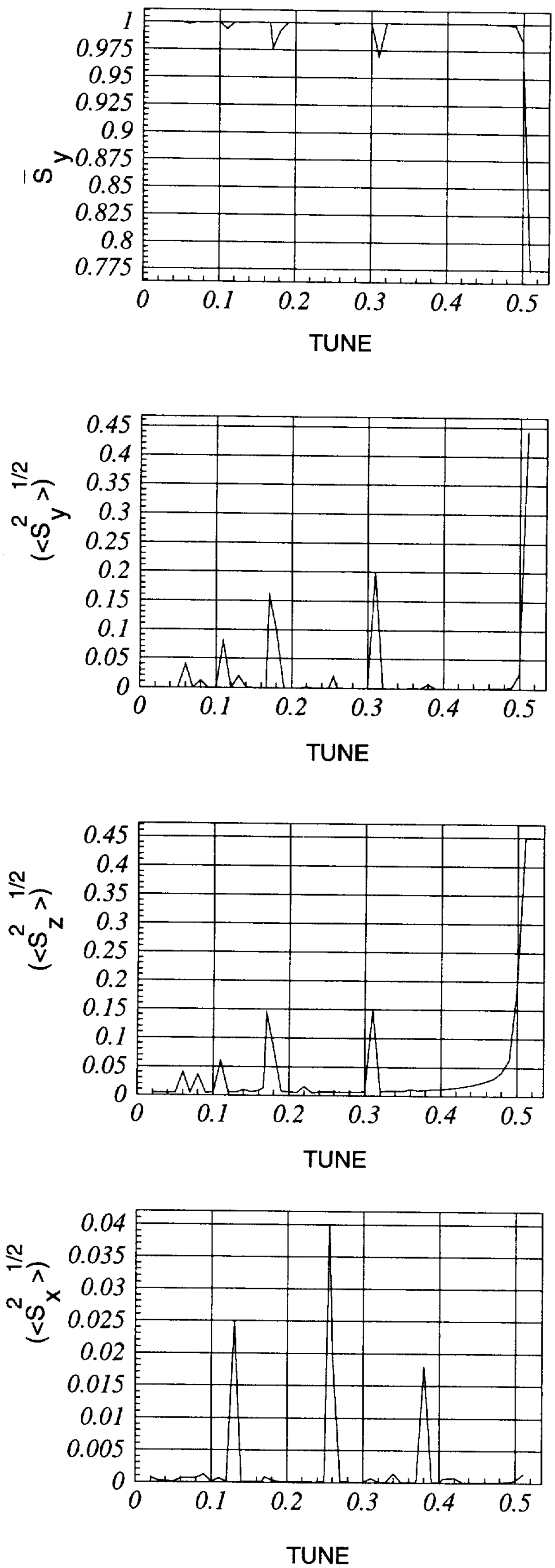


Fig. 5. Spin depolarization as a function of betatron tune.

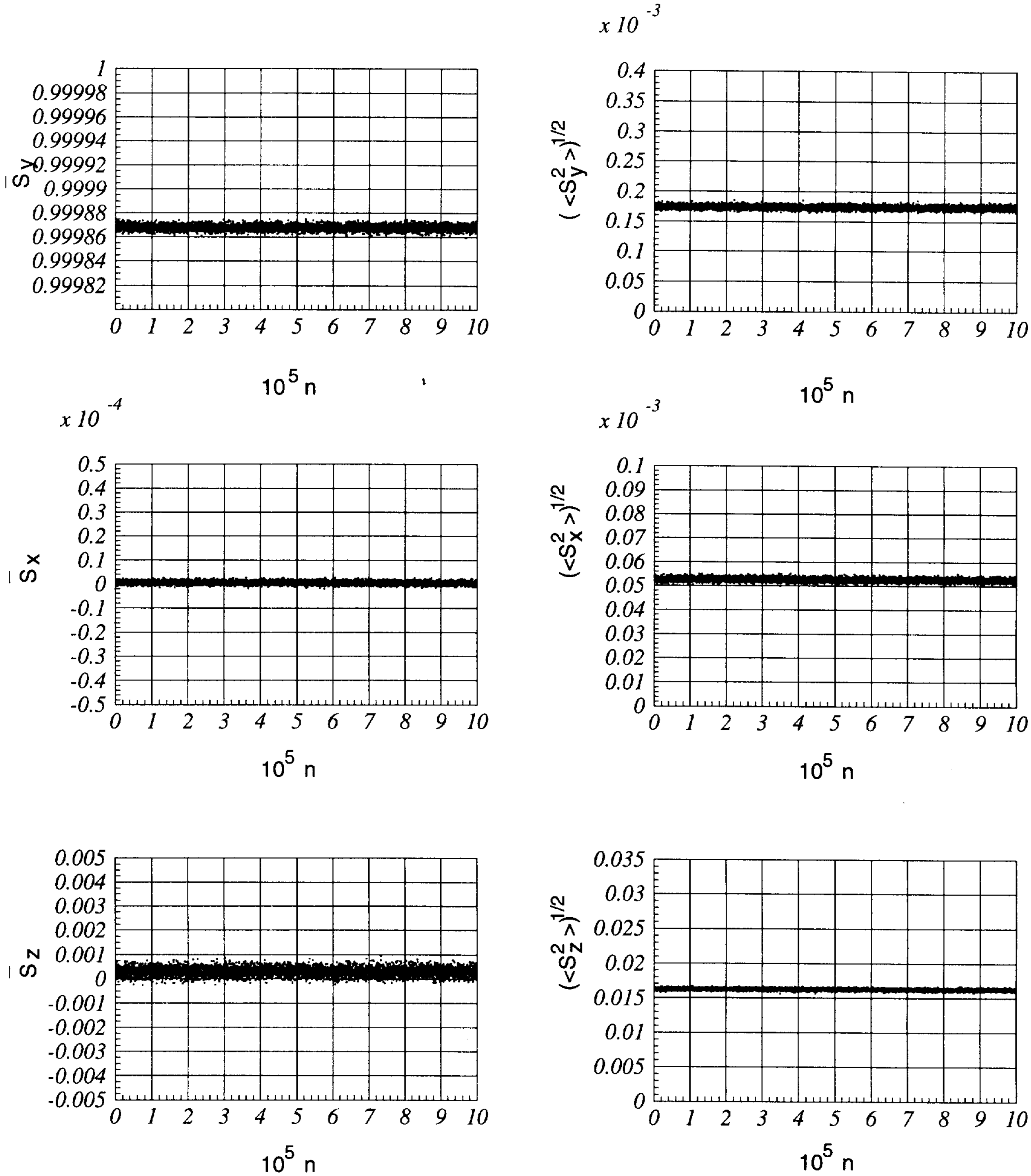


Fig. 6. Average and rms values of spin components as function of turn number, $Q_x = Q_y = 14.43$.

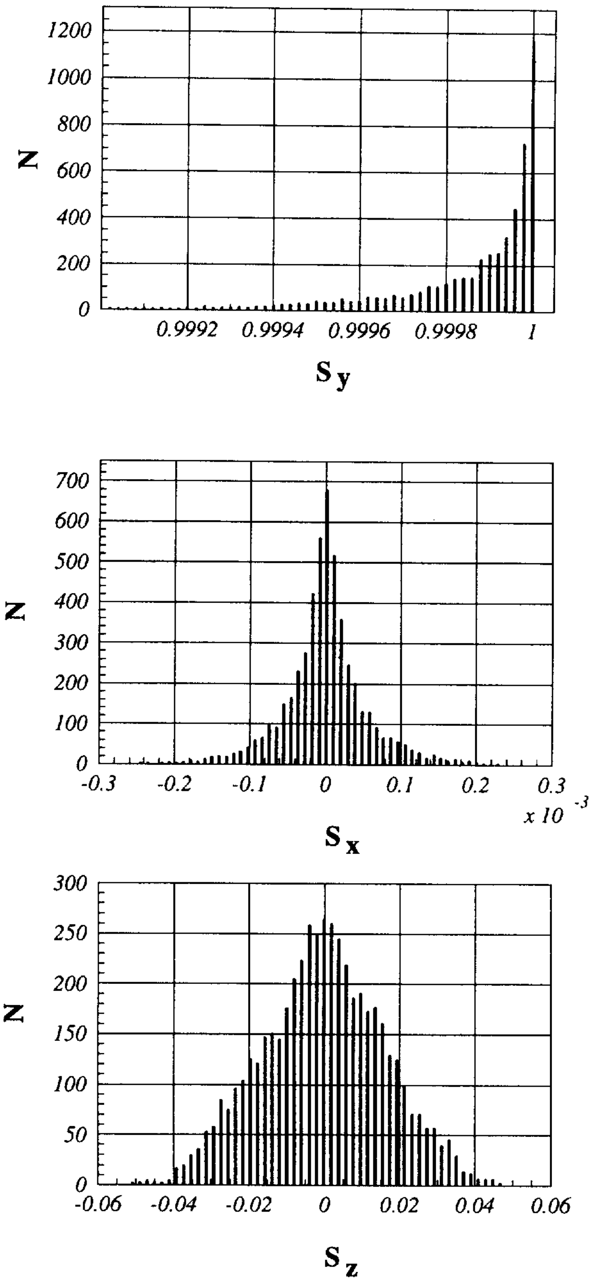


Fig. 7. Spin distribution after 10^6 turns, $Q_x = Q_y = 14.43$.

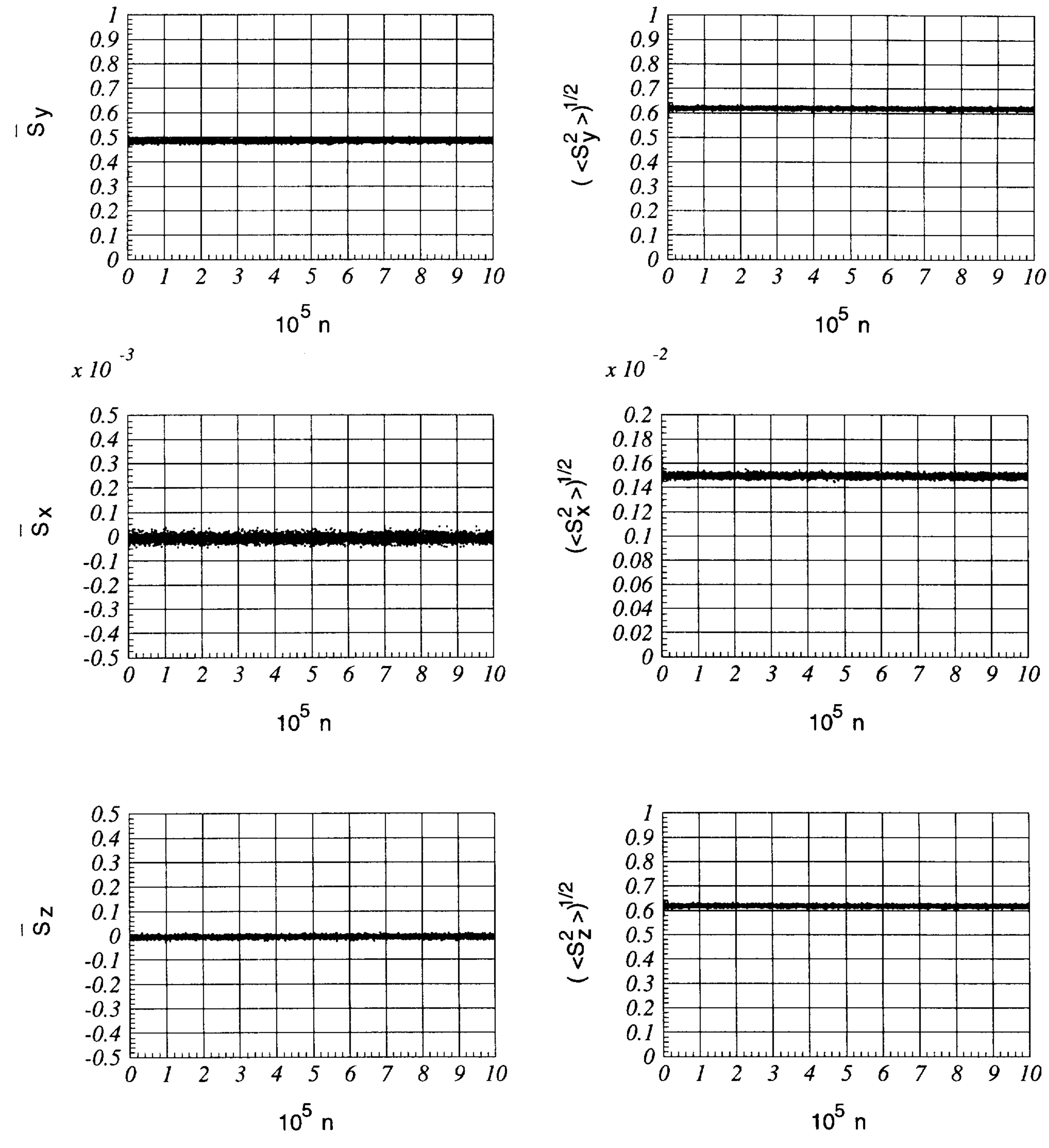


Fig. 8. Average and rms values of spin components as function of turn number, $Q_x = Q_y = 14.505$.

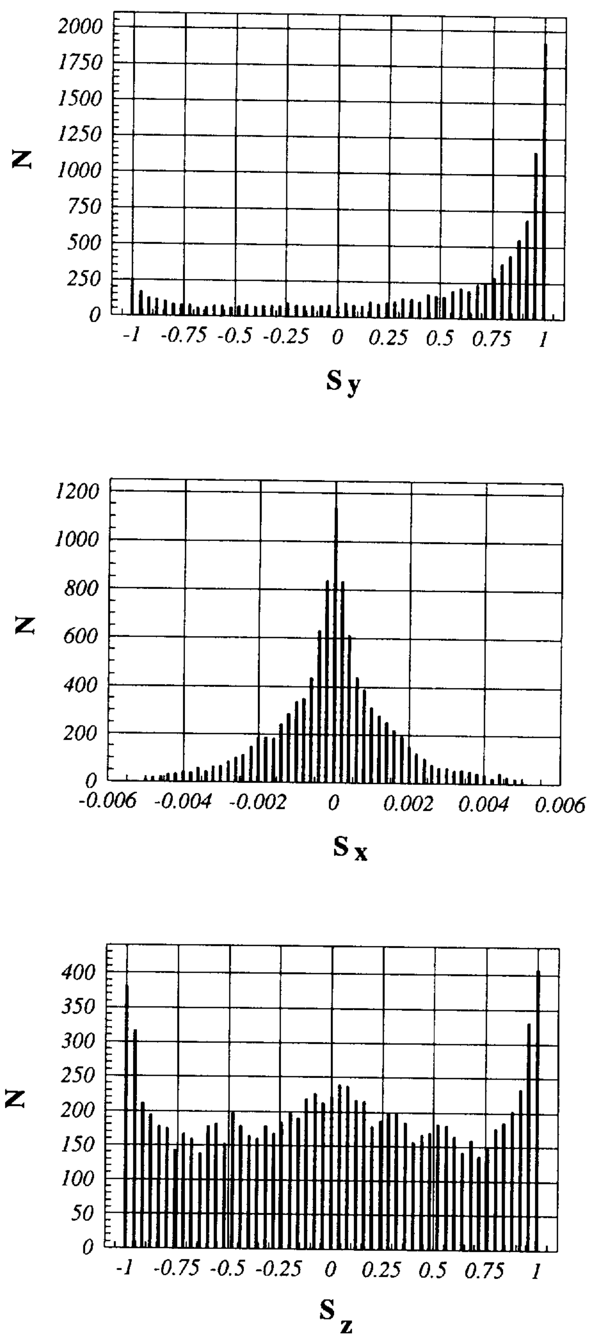


Fig. 9. Spin distribution after 10^6 turns, $Q_x = Q_y = 14.505$.

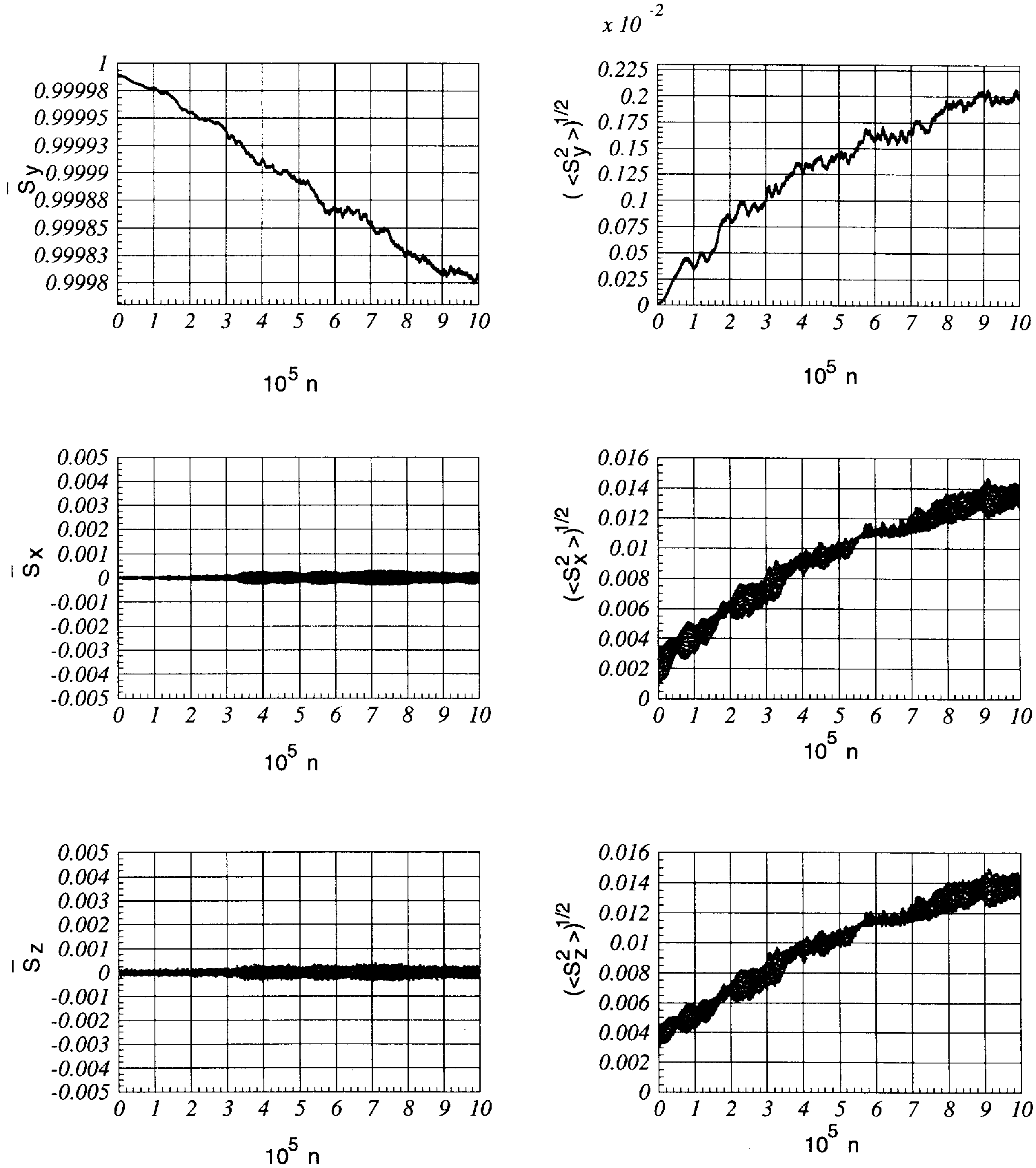
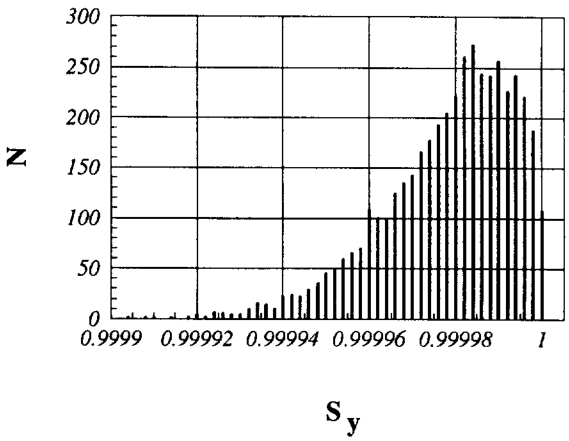
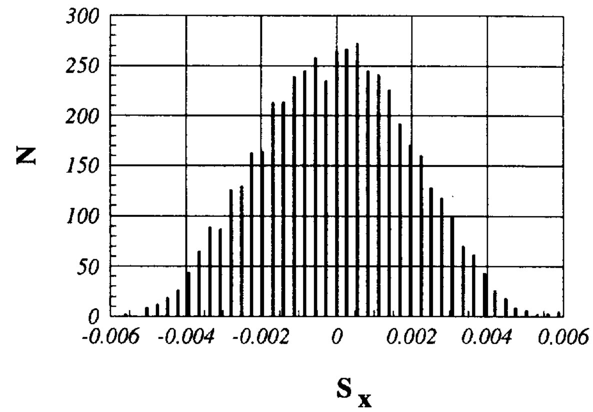


Fig. 10. Average and rms values of spin component as function of turn number for a ring without Siberian Snakes, $Q_x = Q_y = 14.43$.





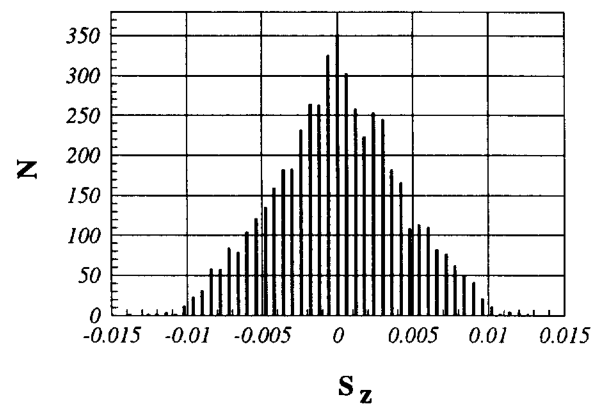


Fig. 11. Spin distribution after 10^6 turns in a ring without Siberian Snakes, $Q_x = Q_y = 14.43$.

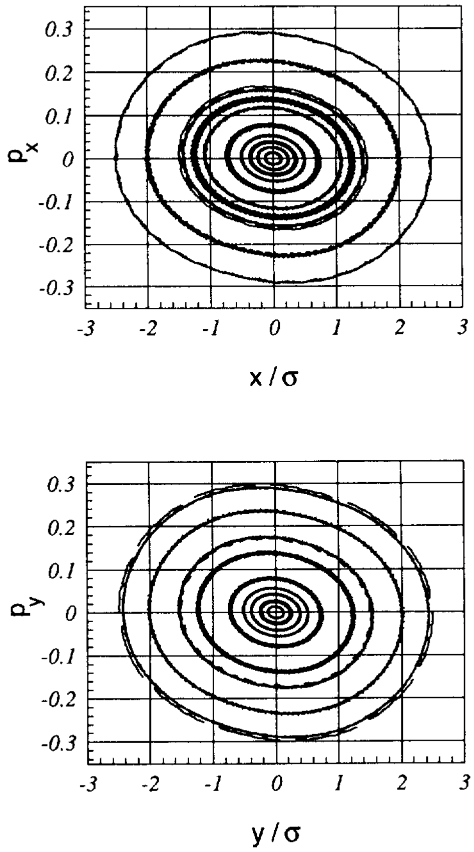


Fig. 12. Distorted particle trajectories in presence of 2.5 % noise in parameter σ in beam-beam kick, Eq. (3.2).

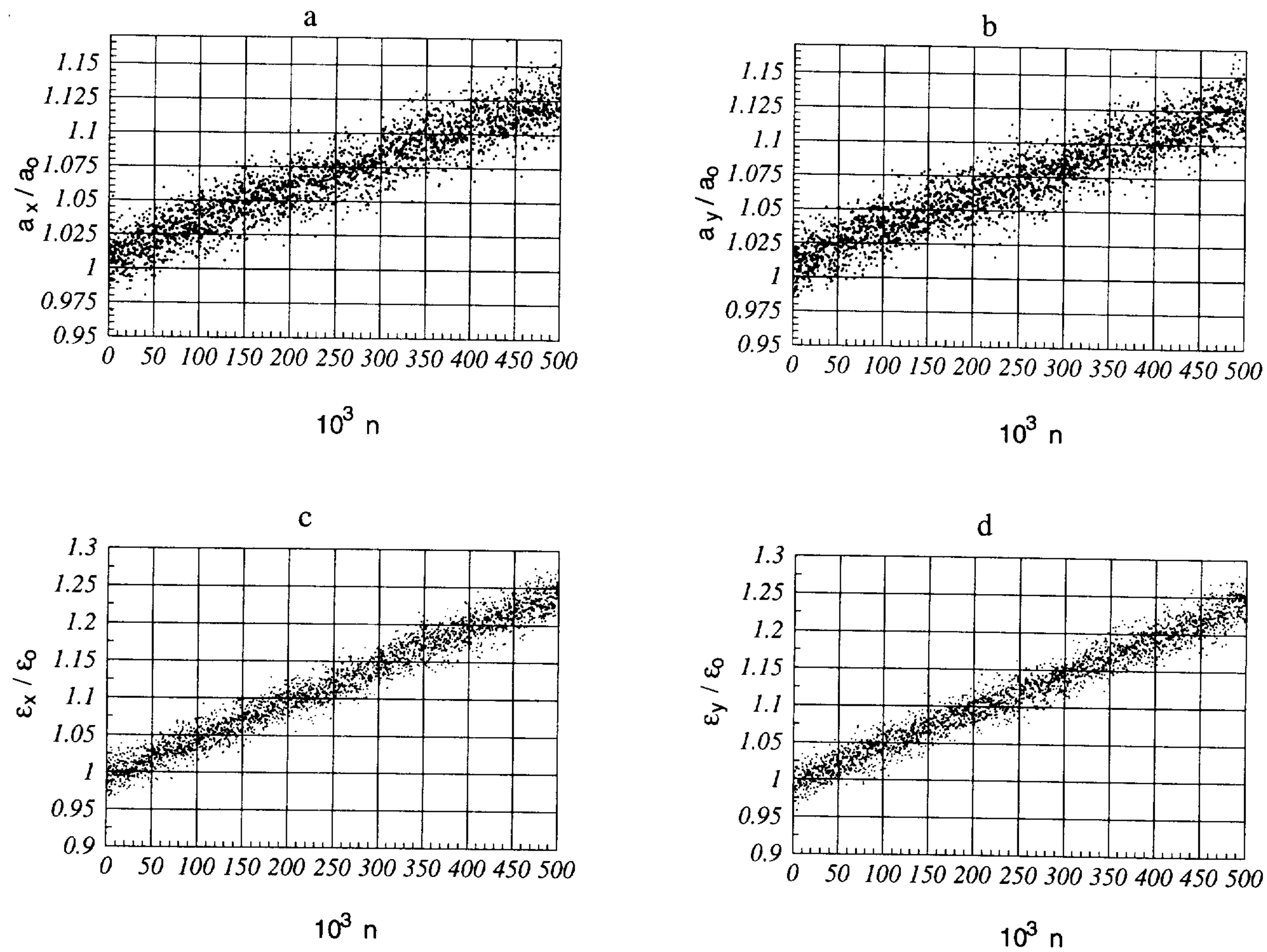


Fig. 13. (a), (b) Beam envelopes and (c), (d) beam emittances in presence of beam-beam interaction with 2.5% noise in parameter σ of beam-beam kick.

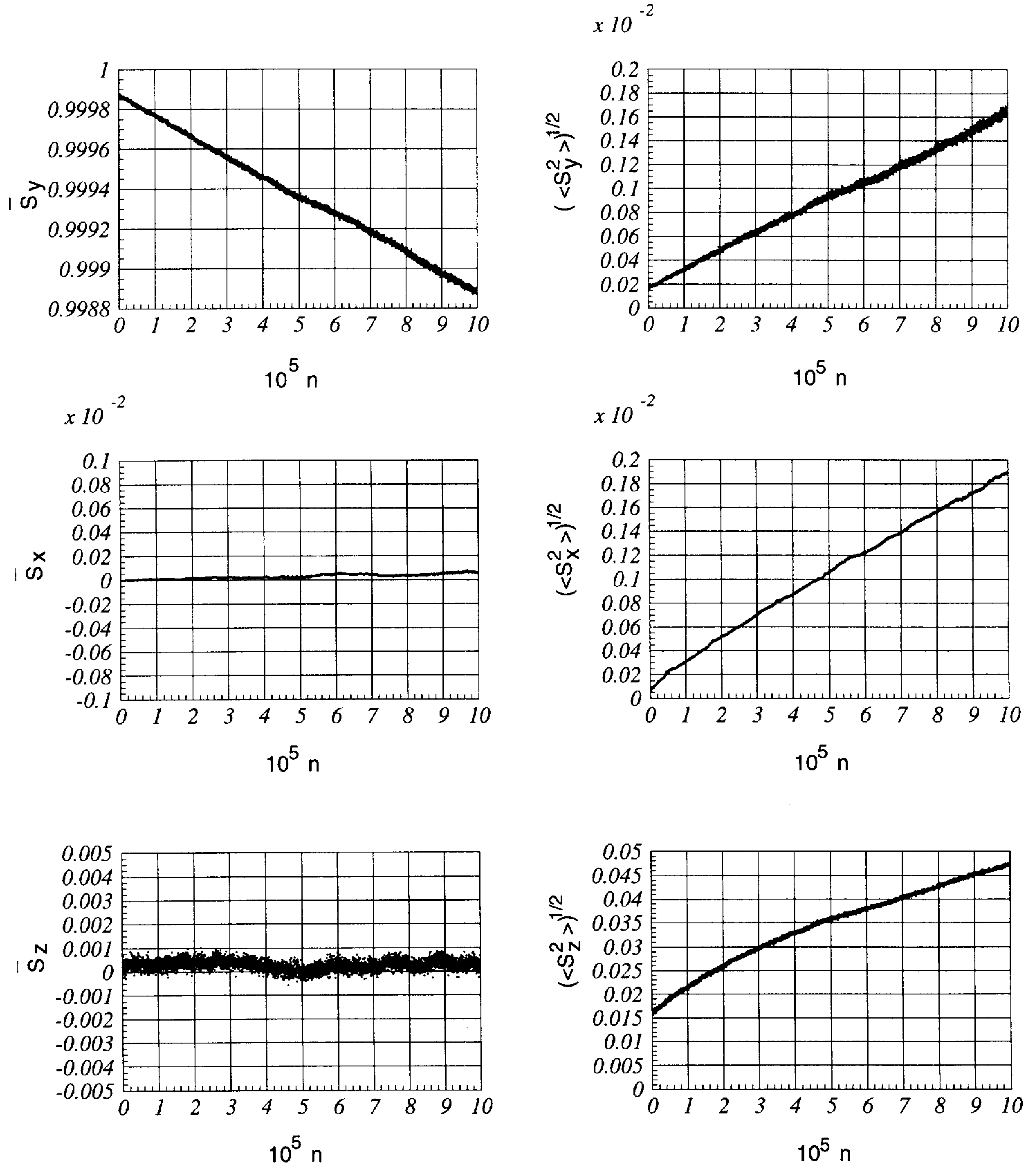


Fig. 14. Average and rms values of spin components as functions of turn number for noisy beam-beam interaction, $Q_x = Q_y = 14.43$.