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A STUDY OF THE QUARK REARRANGEMENT MODEL OF NUCLEON-ANTINUCLEON ANNIHILATION†)

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ABSTRACT

A model of proton-antiproton annihilation via quark rearrangement suggested by Rubinstein and Stern[1]) is examined in detail. This model predicts that all final states are three-meson states, made from the set $\pi, \eta, X, \rho, \omega$, and allows a computation of their relative rates. Its predictions are compared with experimental data at rest and at several energies in flight. The model is found not to be able to account for more than 25% of the annihilations at rest, even when maximum freedom is given to certain extra parameters. The model underestimates the importance of high multiplicity states in annihilations in flight and also fails to predict the more detailed features of this data. A number of interesting aspects of the data are uncovered and possible directions along which the model might be modified are suggested.

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INTRODUCTION

Recently, Rubinstein and Stern[1] have proposed a quark rearrangement model for nucleon-antinucleon annihilation[2]. The assumption of RS is that the three quarks (3q) making up the nucleon and the three antiquarks ($3\bar{q}$) making up the antinucleon rearrange themselves into three $q\bar{q}$ pairs in a specific way, leading to the emergence of three pseudoscalar and/or vector mesons. The simplicity of the model is attractive and the assumption of the conservation of the total number of quarks and antiquarks is supported by a number of applications of the quark model in hadron physics.

In this work, we perform a detailed comparison of quark rearrangement for proton-antiproton annihilation at rest and in flight.

Annihilation at rest has particular features that are relevant for the application of the model. The reaction is believed to occur overwhelmingly from s-states of an electromagnetically bound state of the $\bar{P}P$ system. The model is therefore given a rather precise framework and this allows a detailed comparison between theory and experiment[3]. We find that quark rearrangement fails to account for the observations and we show that taking full advantage of some freedom in the model does not appreciably improve the situation.

The application of the quark rearrangement model to annihilation in flight is less straightforward. The predictions of the model are found in definite disagreement with many features of the experimental data[4], but in this case there are obvious reasons for thinking that the model in its present form should not give good experimental predictions.

In spite of the fact that the quark rearrangement model in its present form is evidently not a good description of the annihilation process, we believe that it is instructive to present our treatment in some detail. The model has the basic ingredients of any future theory, namely a set of weights for the various decay modes and a prescription for how the phase space is to be taken into account. The extraction of the experimental consequences of this relatively simple theory meets with a number of problems which another theory will almost surely encounter as well.

The first section of this paper contains a description of the quark rearrangement model and discusses the options that can be exercised when comparing the model with experiment.

The second section presents a comparison of the experimental data for annihilation at rest with the predictions of quark rearrangement. A number of extensions of the model are examined.

In the third section, the application of quark rearrangement to the annihilation in flight is discussed. A brief description of a more general model conserving the number of quarks and antiquarks is attempted.

The paper contains two appendices. In the first, we suggest that it will not be easy to preserve the idea that the number of quarks plus antiquarks is conserved in the annihilation at rest. The second contains a brief systematization of the problem of obtaining the rearrangement coefficients from the quark spin wave functions of the particles.

1. THE QUARK REARRANGEMENT MODEL

The quark rearrangement model of annihilation suggested in RS proposes that the three spin-½ quarks making up the nucleon and the three spin-½ quarks making up the antinucleon rearrange themselves into three quark-antiquark pairs, without exchanging spin, isospin, or hypercharge. The final state, therefore, contains three non-strange mesons. The quark-antiquark pairs are assumed to be $L = 0$ states, so that the mesons in the final state are restricted to the π, η, X, ρ, and ω. (The SU(4) classification in which the φ is made entirely of strange quarks is assumed.) The computation of the branching ratios relies on a factorization of the matrix element similar to what is assumed in statistical models. The analogue of the spin and isospin weights of statistical models are the "rearrangement coefficients", which are the projections of the initial $P\bar{P}$ spin wave function onto the various three-meson spin wave functions. One obtains the decay rates by multiplying the rearrangement coefficients by the magnitude of the three-body phase space, computed for a unit spatial matrix element (i.e., one assumes that the three final mesons are in relative s-waves). Of course, the absolute rates are not predicted by the model; to compare with experiment means to compare percentages.

The model has a number of attractive features, which were noticed by its proponents[1,2]. The annihilation rate into pairs of strange mesons and into φ mesons is predicted to be zero, when in fact the rates for annihilation into strange mesons in all channels are somewhat smaller and the $\varphi\, \pi^+\pi^- / \omega\, \pi^+\pi^-$ ratio is an order of magnitude smaller than naive phase space considerations would lead one to predict. The decay rate into two-meson states is predicted to be zero, while the $\pi^+\pi^-$ mode in particular is known to account for only 0.5% of the annihilations at rest and an even smaller percentage of the annihilations in flight. The four-meson states are also predicted to be absent, which is in agreement with the fact that the annihilation into $\pi^+\pi^+\pi^-\pi^-$ at rest is compatible with 100% $\rho^0 \pi^+\pi^-$ formation.

The appearance of an approximate conservation law for the total number of quarks plus antiquarks is an attractive feature of the model. This hypothesis is compatible with the observation that the baryons and mesons appear to be made of a definite number of quarks, since they fit quite well into irreducible SU(6) representations and their high energy total cross-sections obey certain elementary sum rules whose derivation requires the assumption of definite quark number[5].

The model is extremely powerful, in the sense that it produces a large number of predictions which may be compared with experiment. Before this can be done, however, there are a few questions which must be clarified.

Consider first the annihilation at rest. This annihilation is known to be preceded by the formation of an electromagnetically bound state known as protonium and is believed to proceed almost entirely from the singlet and triplet s-states. (The evidence for s-state annihilation comes primarily from the analysis of the rare decay[6] into $K^0\bar{K}^0$, with the result $s/(s + p) = 98\%$.) The s-wave $P\bar{P}$ system has negative intrinsic parity. Since the $L = 0$ $q\bar{q}$ mesons that we are considering also have negative intrinsic parity, the three mesons produced in the annihilation can be in relative s-waves. The model, by completely decoupling spin and orbital angular momentum and assuming a unit spatial matrix element, assumes that the final mesons are in relative s-waves. Whether they are in relative s-waves or not is a question which can in principle be answered by an examination of the production Dalitz plots.

The $P\bar{P}$ system is furthermore a definite superposition of $I = 0$ and $I = 1$ states. As far as selection rules are concerned, therefore, one can divide the protonium annihilation into four "sectors", labelled π, η, ρ, ω, in each of which a meson with all the quantum numbers of the corresponding particle, except for a mass just below twice the proton mass, decays by means of the strong interactions. We are interested in how the annihilations are divided among the four sectors.

If one were able to characterize all of the final states, it would be possible to determine what percentage of $P\bar{P}$ annihilations proceed from each sector. Unfortunately, more than 60% of all $P\bar{P}$ annihilations at rest produce final states with more than one neutral particle and cannot be fully analysed. Thus it appears to us that the relative strengths of the capture rates in each of the four sectors must be regarded as free parameters of the model.

To see why this is so, observe that we have enough freedom in the model to regard the annihilation as proceeding in two stages, a collapse toward annihilation, determined by peripheral strong interaction forces which are beyond the scope of the model and which could favour one sector more than another, and a subsequent quark rearrangement determining only the branching fractions within each sector. For example, it is a priori possible to imagine that the entire protonium annihilation proceeds from a singlet I = 1 state. One might have observed 100% annihilation into 3π final states in s-waves; we would then speak of 100% annihilation in the π sector.

It is apparent that this example could only be realizable in the presence of forces which provide transitions between the different sectors. For example, in the absence of electromagnetic forces, the proton-neutron doublet would be degenerate in mass and the I = 1 annihilation would result in the simultaneous production of the neutron-antineutron state, until a pure I = 0 state remained, a superposition of $P\bar{P}$ and $N\bar{N}$, which would never annihilate. (This is the analogue of the $K^0\bar{K}^0$ problem with a stable K_2.) In fact this is the opposite of the experimental situation; the presence of the Coulomb forces, which bring about the formation of the protonium in the first place, assures ample mixing between the I = 0 and I = 1 states. Desai[7)] has estimated that the oscillation time between I = 0 and I = 1 states is one or two orders of magnitude shorter than the time in which the annihilation proceeds.

On the other hand, the transition between singlet and triplet s-states of protonium is a forbidden atomic transition and probably does not have time to operate. If indeed the only capture was from the singlet state (returning to our example), the forbidden atomic transitions would eventually occur, and that fraction of the annihilations proceeding from the triplet state would have a time dependence characteristic of a two-stage radioactive decay. In fact the decay occurs from both singlet and triplet states at a rate which makes it unlikely that singlet-triplet s-state transitions should play any significant role.

Moreover, neither the capture process leading to the formation of protonium in a state of large principal quantum number, nor the "spiralling down" process which culminates in annihilation is expected to alter the singlet or triplet nature of the initial $P\bar{P}$ spin state. Thus, prior to annihilation it is believed that there are three times as many $^3S_1$ protonium atoms as $^1S_0$ ones[8].

Accordingly, we are inclined not to leave the singlet-triplet ratio as a free parameter, but to impose the condition that 75% of the annihilations at rest should proceed from the triplet state. In our first derivation of the branching ratios, we force this result to emerge by normalizing the singlet and triplet modes separately. It is our impression that a different procedure was followed in RS: they weighted the triplet rearrangement coefficients by a factor of three and then let phase space determine the triplet-singlet ratio[9]. Clearly, in the limit of degenerate meson masses (or, equivalently, infinite $P\bar{P}$ centre-of-mass energy) the two procedures are equivalent. It is an amusing accident that in spite of the huge differences between the phase space available for various channels, the rearrangement coefficients happen to conspire to make the two procedures almost identical: the weighting procedure of RS yields the result that 72% of the annihilations at rest proceed from the triplet state. The distinction between the two procedures is thus academic for the case at hand, but the same issue is likely to arise in future treatments of annihilation at rest, which is why we have discussed it in some detail.

In dealing with the superposition of the I = 1 and I = 0 states of protonium, our initial procedure is to bury our ignorance of the capture mechanism by letting the rearrangement coefficients and the available phase space determine what fraction of the annihilations proceeds from each state.

This was also done in RS. It turns out that the 25% of singlet annihilations contains 4% I = 0 and 21% I = 1 (most of which is the abundant 3π mode), while the 75% of triplet annihilations contains 40% I = 0 and 35% I = 1.

Obviously, these procedures for apportioning the annihilations among the four sectors involve quite a bit of guess-work. Accordingly, our first attempt to modify the calculations of Section 2 considers the effect of varying these proportions.

When we turn to annihilation in flight, we observe two principal differences, the presence of higher angular momentum states and the availability of reaction channels for elastic scattering and production. The unitarity upper bound on the s-wave contribution to annihilation[1c] is 0.6 mb at 5.7 GeV/c, compared with a total annihilation cross-section of 22 mb[4]. For higher angular momenta there is the possibility of coupling between the quark spins and the orbital momenta, a coupling which is implicitly neglected in the quark rearrangement model in its present form.

Evidently, the neglect of spin-orbit coupling again leaves us with four orthogonal sectors through which the annihilation can occur (singlet and triplet, I = 0 and I = 1). The rearrangement theory is again compatible with any assignment of the relative strength of annihilation in each sector. In this instance, we have no reason to believe that 75% of the annihilations must proceed from triplet states, because of the presence of elastic scattering and meson production channels. We have followed the procedure of RS described above, weighting the individual triplet channels by a factor of three relative to the singlet channels. Otherwise, we have let the available phase space and the rearrangement coefficients determine the per cent of annihilation in each sector. If the agreement with experiment were more promising, and if annihilation from higher angular momentum states were better understood, we might have introduced the extra parameters corresponding to the relative capture strengths and determined a best fit to experiment. In view of the evident inapplicability of the model in its present form, we have not done so.

The rearrangement coefficients are given in Table I. Their derivation is sketched briefly in Appendix II. A few errors and omissions in the Table given in RS have been corrected.

The rearrangement coefficients for each initial $P\bar{P}$ state (S = 0 and S = 1) sum to unity. This is a consequence of the fact that the three-meson states span the space of three quarks and three antiquarks. The reader may also verify that for each value of the initial spin, the partial sums over the coefficients for the channels with G-parity +1 and -1 are each $\frac{1}{2}$; this is a consequence of the fact that the initial state is a superposition of I = 0 and I = 1 with equal weights.

We have considered various quark assignments for the η and X. We have found that the variation of the singlet-octet mixing angle over its entire range produces an average variation of less than one per cent in the different categories of Tables III and VII, and hence does not affect the agreement with experiment. We present in our tables of results only those which follow from the mixing in which η has a pure octet character.

2. ANNIHILATION AT REST

The procedures we employ to obtain branching ratios for annihilation at rest have been described in the previous section. We force the ratio of the total triplet annihilation rate to the total singlet annihilation rate to be 3 : 1. We allow the rearrangement coefficients and the phase space integrals to determine the ratio of the annihilation rates in the $I = 0$ and $I = 1$ sectors. Table II presents the distribution of final states for $P\bar{P}$ annihilation at rest which follow from these assumptions.

The comparison of Table II with experiment is involved. Many of the channels cannot be separated experimentally because they decay into a final state with several neutral particles. Furthermore, the identification of resonant production is not straightforward in all instances. Ambiguities arise from the multiplicity of combinations of pions in the final state which can form the same resonance.

A first and rather weak comparison that can be performed is the following. There are eight "categories" of events (topologies) that can be separated almost unambiguously. The measurement of the tracks of charged pions (prongs) emerging from the annihilation determines a missing mass that corresponds to one of the three following possibilities: absence of any neutral particle, production of one π^0 [11], or production of a heavy neutral system. The number of prongs and the nature of the associated missing mass determines the category. The decay modes and the branching ratios of the resonances being known, it is straightforward to transform Table II into a set of predictions for the distribution of annihilations among eight categories, as shown in Table III. The experimental distribution[3] is also presented in this Table.

One sees that the discrepancies between the predictions and the observations are large and are distributed over the whole range of multiplicities.

It is possible to make some more specific comparisons on the basis of Tables II and III:

1) The rate for the $3\pi^0$ channel (11%) is in definite disagreement with the number of zero-prong events ($3.2 \pm 0.5\%$) that represents the upper limit for this process.

2) Although the prediction for the $\pi^+\pi^-\pi^0$ topology appears to be confirmed, this is fortuitous because this decay channel has been experimentally resolved into two contributions[12], the uncorrelated 3π production and the $\rho\pi$ production. The latter accounts for more than half of the rate ($4.3 \pm 0.6\%$)[13]. This shows that the $\pi^+\pi^-\pi^0$ rate is overestimated by the quark model. It also suggests that the smallness of the $\pi^+\pi^-$ rate may be misleading, since another two-body rate appears to be much more appreciable.

3) Two channels in which one vector meson is produced, $\rho^0\pi^+\pi^-$ and $\omega\pi^+\pi^-$, are, fortunately, easily accessible experimentally, but the results are destructive to the model. These channels have large rearrangement coefficients and the predicted fractions are again too high: 25.6% against $5.8^{+0.3}_{-1.3}$ observed for $\rho^0\pi^+\pi^-$ and 24% against $3.8 \pm 0.4\%$ for $\omega\pi^+\pi^-$[14].

4) The only other prediction from Table II which can be compared with experiment at the present time is the prediction for the $\eta\pi^+\pi^-$ channel. It is amusing that this is in agreement with the data (prediction: 1.4% observation: $1.2 \pm 0.3\%$).

5) The model makes it difficult to understand the experimental result that there are six times as many states with $\pi^+\pi^-x^0$ as with $2\pi^+2\pi^-$. Obviously, either $\pi^+\pi^-2\pi^0$ is much more numerous than $2\pi^+2\pi^-$ or $\pi^+\pi^-x^0$ consists mainly of five or more mesons. The first possibility has difficulties which are engendered by the quark model. The $\rho\pi\pi$ rearrangement coefficients have just the effect of depleting the $\pi^+\pi^-2\pi^0$ state relative to the $\pi^+\pi^+\pi^-\pi^-$ state by a factor of 13 : 50. This ratio is one of the strong predictions of the rearrangement model (from isospin alone there are two independent couplings so this ratio is undetermined). If it were correct we would be forced to the conclusion that less than 5% of $\pi^+\pi^-x^0$ consists of four-meson states.

This conclusion, in turn, is not confirmed by several simple models which we have examined. The statistical distribution of five π mesons in isospin space gives almost twice as many $2\pi^+2\pi^-\pi^0$ states as $\pi^+\pi^-3\pi^0$ states[15]. Combined with Table III this gives a $\pi^+\pi^-3\pi^0$ rate of about 10%. The fully symmetric s-wave 5π state with I = 1, on the other hand, contains the three different charge distributions in the proportions[16] $2\pi^+2\pi^-\pi^0 : \pi^+\pi^-3\pi^0 : 5\pi^0 = 8 : 12 : 15$; while this gives the desirable value $\pi^+\pi^-3\pi^0 = 32\%$, it also predicts annihilation into $5\pi^0$ (zero prong events) 40% of the time!

In view of these problems, let us see what we can save of the model. We first try to obtain a better agreement between the model and the data by allowing complete freedom to the relative weighting of the four sectors. However, we have to take into account the constraints of four known reaction rates: $\eta\pi^+\pi^- = 1.2\%$, $\pi^0\pi^0\pi^0 \leq 3.2\%$, $\rho^0\pi^+\pi^- = 5.8\%$, and $\omega^0\pi^+\pi^- = 3.8\%$. These fortunately occur each in a different sector. We compute the contributions of each of these reactions in its sector and then find what percentage of the total annihilation rate can be granted to each sector to give the rate for each reaction correctly. For example, the $\omega\pi^+\pi^-$ channel competes with $\omega\pi^0\pi^0$ in the ratio 2 : 1; in addition there are smaller contributions from a few channels with higher thresholds or smaller rearrangement coefficients. The result is that 60% of the annihilations in the ω sector are predicted to be $\omega\pi^+\pi^-$, and that, therefore, the ω sector should account for $3.8\%/0.60 \approx 6\%$ of the annihilations. Table IV presents the results of the same analysis in each sector.

We see from Table IV that even if the relative strengths of the sectors are left completely free, not more than about 25% of the annihilations can be ascribed to quark rearrangement.

It might still be argued that the rearrangement annihilations <u>are</u> only 25% of the total, and that the remainder are the result of other processes. To examine this hypothesis we proceed as follows. We maximize the allowed contribution of rearrangement annihilations by using the values obtained in Table IV for the per cent of rearrangement annihilations in each sector. We distribute the final states which these annihilations account for among the eight categories of Table III and subtract from the total observed distribution given there. We obtain the distribution among categories presented in Table V. This distribution must be accounted for by other processes.

We might imagine that the quark rearrangement annihilations are all the three-body annihilations and that the rest are mainly annihilations into four or more mesons. (Such a model would arise, for example, if part of the time bremsstrahlung pions were given off before rearrangement annihilation, as suggested in RS.) Table V, however, reveals that we must explain the presence of 30% annihilation into $\pi^+\pi^-x^0$ and the virtual absence of annihilations into $2\pi^+2\pi^-$. The latter suggests the absence of uncorrelated $\pi^+\pi^-2\pi^0$ states in the 30% $\pi^+\pi^-x^0$. Thus we are again forced to imagine that the $\pi^+\pi^-x^0$ mode contains almost entirely states with five or more mesons, with the difficulties mentioned above. We cannot exclude the possibility that most of the decays in categories 4 and 6 are decays into the various charge modes of the four-meson state $\rho\pi\pi\pi$.

In the course of this examination of the data, we have noticed one puzzling regularity. The production of one ρ in association with pions seems to be favoured over uncorrelated pion production, in the sense that the following inequalities in decay rates seem to be satisfied: $\rho\pi > \pi\pi$, $\rho\pi\pi > \pi\pi\pi$, $\rho 3\pi > 4\pi$. It seems that this feature is also a characteristic of annihilations in flight.

3. ANNIHILATIONS IN FLIGHT

Two assumptions underlie the calculation of branching ratios for P$\bar{P}$ annihilation in flight presented in RS and repeated in this Section: 1) that the rearrangement coefficients for singlet and triplet decays are the same as in the annihilation at rest, and 2) that the integration of the matrix element over phase space is a straightforward procedure. The first assumption would be spoiled if spin-orbit coupling became important at high energies. As already explained in Section 1, the great majority of annihilations in flight are not s-wave annihilations. Thus the first assumption requires the hypothesis that there is no coupling of the quark spins to the orbital angular momenta present in both initial and final states, so that one can still speak of a singlet or triplet annihilation[17].

We implement the second assumption by assuming as an elementary working hypothesis that the integral of the true matrix element over phase space should be proportional to the integral of the unit matrix element over phase space, with the same constant of proportionality for all decays. Thus the same computer program was used for annihilation at rest and for annihilation in flight, varying only the total centre-of-mass energy. Our results are presented for three incident laboratory momenta where extensive experimental data are available, 3.3, 5.7 and 7.0 GeV/c[4]. Table VI gives the branching ratios for all channels which contribute at least 0.5%. Table VII gives the corresponding distributions over eleven topological categories[18], and gives the experimental distributions as well.

The model may be compared with experiment at three levels of detail. The coarsest feature of the data is the single quantity $\langle n_{ch} \rangle$, the average number of charged pions emitted per pionic annihilation. This number is underestimated by about 0.4 at the two higher energies that we have considered. The smallness of the discrepancy suggests that another three-meson model with additional channels might account for the data.

To compare the distribution of annihilations over our eleven categories is to compare at a second level of detail. Two of these categories, $2\pi^+2\pi^-$ and $3\pi^+3\pi^-$, have contributions from only one three-meson channel, $\rho^\circ\pi^+\pi^-$ and $\rho^\circ\rho^\circ\rho^\circ$ respectively. In these two channels the disagreement with experiment is particularly serious. The eight-prong events in the model come only from final states containing the X meson. Actually, many more events are observed in this category than those channels account for. The categories involving neutral particles receive contributions from a number of decay channels ranging from four to more than thirty; in these channels the disagreement tends to soften as the energy increases.

The fine structure of the theory is contained in Table VI. Where direct comparison with experiment is possible, the predictions of the rearrangement model are generally an order of magnitude too large, and the disagreement increases with the energy. We give two examples, the $\rho^\circ\pi^+\pi^-$ and $\omega\pi^+\pi^-$ modes, in Table VIII.

It seems clear that the quark rearrangement model in its present form is in serious disagreement with experiment. It would seem that in fact many more modes are present in the annihilations at high energies than are included in the assumption of simple quark rearrangement into three L = 0 mesons.

We can suggest one way in which, while retaining the conservation of the total number of quarks plus antiquarks, the rearrangement model might be altered to include a much larger number of final states. One could consider the possibility that the angular momentum present in the higher partial waves of the initial $P\bar{P}$ system is transferred to $L \geq 1$ $q\bar{q}$ and $qq\bar{q}\bar{q}$ mesons with masses greater than one GeV. There is recent evidence that the number of mesons in this mass region is quite considerable[19)], and there is also evidence for at least one of these resonances in annihilations at 5.7 GeV/c[20)].

We conclude with a few general remarks. In our view the statistical model of annihilation into uncorrelated pions and the quark rearrangement model in its present form stand at two ends of a spectrum of possible theories of $P\bar{P}$ annihilation. The statistical model fails to account for resonance production, and whenever a detailed analysis is possible, resonance production is found to be copious. The quark rearrangement model, on the other hand, seriously overestimates the amount of resonance production in its major channels. A similar bracketing of the physical situation appears when the energy dependence of $\langle n_{ch} \rangle$ is examined, as shown in Fig. 1. The statistical model overestimates the rate of increase of $\langle n_{ch} \rangle$ with energy. The quark rearrangement model, on the other hand, underestimates this rate of increase and also predicts a finite asymptotic value of $\langle n_{ch} \rangle = 4.2$ at infinite energy, 0.2 less than its value at 7.0 GeV/c.

There thus seems to be a connection between resonance formation and the multiplicity of final pions, with resonance formation having the effect of reducing this multiplicity. We regard the tendency to form resonances as evidence of the operation of a selection rule which tends to minimize the change in the total number of quarks plus antiquarks in each stage of a strong interaction process. Because this selection rule is only partially operative (ρ decay is after all a fast process) the experimental data lie between the predictions of a statistical model of uncorrelated pions and a theory in which this selection rule is made absolute.

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APPENDIX I

CONSIDERATIONS BASED ON THREE-MESON PHASE SPACE

Although abandoning the quark rearrangement scheme, one may try to preserve the idea of three-meson production, the different channels being granted arbitrary weights, apart from the restrictions resulting from isospin conservation. In order to help the reader to carry on his own investigations, we display in Table IX the value of the phase space integrals for the different three-meson channels able to contribute to the annihilation at rest, as well as their percentage contributions to the different categories. We retain what is obviously an artificial restriction to pseudoscalar and vector mesons.

With the help of Table IX, we have made several attempts to gain a feeling for the content of the high multiplicity channels. We have allowed for liberal contributions from a number of hitherto undetected modes and we have found that the rescue of the three-meson scheme would require abnormally large weights for certain modes; we have the impression that it is unreasonable to ascribe more than 75% of all annihilation to the three-meson final states considered. If we enlarge the set of possible final states to include those containing $L \neq 0$ mesons, we find that the $A_2\ \pi\pi$ states might be capable of explaining another 4% of the annihilations. Perhaps other $L \neq 0$ resonances account for the rest.

Of course modes violating quark conservation are worth attention. The $\rho^+\rho^-$ mode in particular might contribute significantly to category 4. This would not be in contradiction with the low upper limit for the $\rho^0\rho^0$ mode, because the latter can only proceed from the singlet initial state, as do $\rho^0\omega^0$ and $\omega^0\omega^0$ as well. The best place to look for the $I = 1$ $\rho\rho$ final state is probably in annihilation in deuterium.

APPENDIX II

CALCULATION OF THE REARRANGEMENT COEFFICIENTS

In the model we have been employing in this work, spin and isospin are treated as if they were internal symmetries, decoupled from space-time. The physical particles are assigned definite spin and isospin wave functions, corresponding to particular members of the irreducible representations of the group SU(4), which is the group of unitary transformations in the spin-isospin space. [We never need the full SU(6) in this problem, since we do not need to introduce strangeness.] In this space the quark transforms like the basic four-dimensional representation; we denote its four spin-isospin states by p(+), p(-), n(+), n(-), where p, n mean $I_z = \frac{1}{2}, -\frac{1}{2}$, and +, - mean $S_z = \frac{1}{2}, -\frac{1}{2}$.

The proton is identified with the three-quark object whose spin-isospin wave function is fully symmetric under the exchange of any pair of quarks[24]. The proton wave function with $S_z = \frac{1}{2}$ is constructed by beginning with the $S, S_z = \frac{1}{2}, \frac{1}{2}$ object

$$p(+)[p(+)n(-) - p(-)n(+)],$$

(the order in which the quarks appear indicates their momenta), symmetrizing by summing over the six permutations of the quarks, and normalizing, obtaining[25]:

$$\begin{aligned} P(+) = (18)^{-\frac{1}{2}}[&2p(+)p(+)n(-) + 2p(+)n(-)p(+) + 2n(-)p(+)p(+) \\ &- p(+)p(-)n(+) - p(-)p(+)n(+) - p(+)n(+)p(-) \\ &- p(-)n(+)p(+) - n(+)p(+)p(-) - n(+)p(-)p(+)]. \end{aligned} \quad (A.1)$$

The wave function for a proton with $S_z = -\frac{1}{2}$ is obtained by reversing all quark spins, and for an antiproton with $S_z = \frac{1}{2}$ by changing all quarks to antiquarks.

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The mesons are constructed from a quark and an antiquark. The $4 \times 4^*$ representation is completely filled by the sixteen charge and spin polarization states of the ρ, ω', π, and η'. Here ω' and η' are the states made only of non-strange quarks and are linear combinations of the physical (ω,φ) and (η,X) states:

$$\omega' = \omega \cos \vartheta_1 + \varphi \sin \vartheta_1$$

$$\eta' = \eta \cos \vartheta_0 + X \sin \vartheta_0 \quad .$$

The choice $\cos \vartheta_1 = 1$, $\cos \vartheta_0 = 1/\sqrt{3}$ is the canonical one.

The set of all $(16)^3$ three-meson states $|n>$ made from ρ, ω', π, η' therefore completely spans the space of three quarks and three antiquarks. We want to find the projections of the $P\bar{P}$ states with spin 0 and 1 onto each of these states. If we work with normalized states, the sum of the squares of these projections for each $P\bar{P}$ spin state will be unity, since schematically:

$$\sum_n <P\bar{P}|\, n> \; <n|\, P\bar{P}> \; = \; <P\bar{P}\,|\,P\bar{P}> \; = 1 .$$

The problem of computing the $2 \times (16)^3$ projections is not as terrible as it may seem. If one chooses $P\bar{P}$ states with $S_z = 0$:

$$\frac{1}{\sqrt{2}}\left[P(+)\bar{P}(-) \pm P(-)\bar{P}(+)\right], \qquad \begin{matrix} + & S = 1 \\ - & S = 0 \end{matrix} , \qquad \text{(A.2)}$$

one only needs to consider projections onto $S_z = 0$ three-meson states with zero charge. We explicitly calculate the rearrangement coefficient for $\pi^+\pi^-\pi^0$ to demonstrate the techniques which are involved.

Because the $P\bar{P}$ wave function is fully symmetric in the quarks and antiquarks separately, one only needs to compute the projection for one ordering of the $\pi^+\pi^-\pi^0$ mesons. The rearrangement coefficient is then obtained by squaring the projection and multiplying by six to take into account the six ways in which these mesons can have momenta p_1, p_2, p_3. The ordered $\pi^+\pi^-\pi^0$ state is:

$$(2)^{-1/2}(2)^{-1/2}(4)^{-1/2}\left[p(+)\bar{n}(-) - p(-)\bar{n}(+)\right]\left[n(+)\bar{p}(-) - n(-)\bar{p}(+)\right]\times$$

$$\times\left[p(+)\bar{p}(-) - p(-)\bar{p}(+) + n(+)\bar{n}(-) - n(-)\bar{n}(+)\right]. \qquad (A.3)$$

Only the $p\bar{p}$ part of the π^0 will contribute, as the initial proton-antiproton state contains two p and one n. Therefore the projection of $P\bar{P}$ onto two charged mesons and one π^0 is equal to the projection onto two charged mesons and one η', since the π^0 and η' wave functions differ only by $n \longleftrightarrow -n$. (The same remark applies to the states containing two charged mesons and either a ρ^0 or ω'.) There are then eight terms when the multiplication in (A.3) is performed. Grouping quarks and antiquarks separately, one gets:

$$\begin{aligned}(1/4)\Big[&p(+)n(+)p(+)\bar{n}(-)\bar{p}(-)\bar{p}(-) - p(+)n(+)p(-)\bar{n}(-)\bar{p}(-)\bar{p}(+)\\ &- p(+)n(-)p(+)\bar{n}(-)\bar{p}(+)\bar{p}(-) + p(+)n(-)p(-)\bar{n}(-)\bar{p}(+)\bar{p}(+)\\ &- p(-)n(+)p(+)\bar{n}(+)\bar{p}(-)\bar{p}(-) + p(-)n(+)p(-)\bar{n}(+)\bar{p}(-)\bar{p}(+)\\ &+ p(-)n(-)p(+)\bar{n}(+)\bar{p}(+)\bar{p}(-) - p(-)n(-)p(-)\bar{n}(+)\bar{p}(+)\bar{p}(+)\Big].\end{aligned} \qquad (A.4)$$

The first and last terms of (A.4) do not contribute since they correspond to a total quark spin of $3/2$. As seen from (A.1), the other six terms contribute a factor of +2 or −1 from their quark (antiquark) part, depending on whether the two $p(\bar{p})$ quarks are spinning parallel or anti-parallel. As seen from (A.2), there is another factor of ±1 for terms where the total quark spin is $-1/2$. Thus presenting as four factors

1) the sign in (A.4), 2) the sign in (A.2), 3) the quark factor, and 4) the antiquark factor, one obtains:

$$< \pi^+\pi^-\pi^0 \mid P\bar{P}, J = {1 \atop 0} >$$

$$= C\Big[-(+1)(-1)(-1) - (+1)(2)(-1) + (\pm1)(-1)(2)$$

$$-(+1)(-1)(2) + (\pm1)(2)(-1) + (\pm1)(-1)(-1)\Big]$$

$$= \begin{cases} 0, & J = 1 \\ 6C, & J = 0 , \end{cases}$$

where $C^{-1} = 4 \times \sqrt{18} \times \sqrt{18} \times \sqrt{2}$ is the normalization factor. The S = 0 $\pi^+\pi^-\pi^0$ rearrangement coefficient is therefore $6 \times (6C)^2 = 18/864$, as entered in Table I. The S = 1 coefficient comes out automatically zero, which serves as a check.

The other projections for states with two charged particles are computed in the same way. For three $S_z = 0$ mesons, one gets the same eight terms as in (A.3), except with different signs. For states with one vector meson, this leads to the rearrangement coefficients directly. For states with two and three vector mesons, one must be careful to sum over all the distinct polarization states which can combine to give total $S_z = 0$. For the $\rho^+\rho^-\pi^0$ coefficient, for example, it is necessary to sum over the projections onto $\rho_1^+ \rho_{-1}^- \pi^0$, $\rho_{-1}^+ \rho_1^- \pi^0$, and $\rho_0^+ \rho_0^- \pi^0$, where the subscript is S_z. The first two projections involve terms different from those that appear in (A.4), but they are found in the same way. The charged states with two and three vector mesons, furthermore, have non-zero rearrangement coefficients for both the singlet and triplet $P\bar{P}$ annihilation.

The computation of the rearrangement coefficients for the final states containing all neutral particles is a bit more tedious, because the expression corresponding to (A.4) has more terms. For the states with more than one vector meson it is again necessary to take

66/1109/5
p/gs

into account all of the combinations of polarizations. The rearrangement coefficients for states with two (three) neutral vector mesons are non-zero only for singlet (triplet) $P\bar{P}$ annihilation, because initial and final states are eigenstates of C-conjugation.

FOOTNOTES AND REFERENCES

1) H.R. Rubinstein and H. Stern, Physics Letters 21, 447 (1966), hereafter called RS.

2) The predictions of the model for annihilations at rest are equivalent to those which follow from a model proposed by N.P. Chang and S. Shpiz, Phys.Rev.Letters 14, 617 (1965) and by R. Delburgo, Y.C. Leung, M.A. Rashid and J. Strathdee, Phys.Rev.Letters 14, 609 (1965), in the framework of a U(12) theory.

3) (a) C. Baltay, P. Franzini, G. Lütjens, J.C. Severiens, D. Tycko and D. Zanello, Phys.Rev. 145, 1103 (1966).

(b) M. Cresti, A. Grigoletto, S. Limentani, A. Loria, L. Peruzzo, R. Santangelo, G.B. Chadwick, W.T. Davies, M. Derrick, C.J.B. Hawkins, P.M.D. Gray, J.H. Mulvey, P.B. Jones, D. Radojicic and C.A. Wilkinson, Proc. of the Sienna Conference on Elementary Particles 1963, p.263.

The two sets of results do not show any significant disparities. We consistently quote the figures from Baltay et al. which are based on larger statistics.

4) We consider results at three different laboratory moments:

(a) 3.3 GeV/c:

C. Baltay, T. Ferbel, J. Sandweiss, M.D. Taft, B.B. Culwick, W.B. Fowler, M. Gailloud, J. Kopp, R. Louttit, T. Morris, J. Sandford, R. Shutt, D. Stonehill, R. Stump, A. Thorndike, M. Webster, W. Willis, A. Backmann, P. Baumel and R.M. Lea, Nucleon Structure - Proc. of the International Conference at Stanford University, 1963; see also T. Ferbel, Ph.D. Thesis, Yale (1963) (unpublished).

(b) 5.7 GeV/c:

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(c) 7.0 GeV/c:

T. Ferbel, A. Firestone, J. Johnson, J. Sandweiss and H.D. Taft, Nuovo Cimento 38, 12 (1965).

5) E.M. Levin and L.L. Frankfurt, Zhur.Eksp. i. Teoret. Fiz. Pisma v Redak. **2**, 105 (1965). [English translation Soviet Phys. - JETP Letters **2**, 65 (1965)].

H.J. Lipkin and F. Scheck, Phys.Rev.Letters **16**, 71 (1966).

6) C. Baltay, N. Barash, P. Franzini, N. Gelfand, L. Kirsch, G. Lütjens, D. Miller, J.C. Severiens, J. Steinberger, T.H. Tan, D. Tycko, D. Zanello, R. Goldberg and R.J. Plano, Phys.Rev. Letters **15**, 532, 537(E) (1965);

R. Armenteros, L. Montanet, D.R.O. Morrison, S. Nilsson, A. Shapira, J. Vandermeulen, Ch. d'Andlau, A. Astier, J. Ballam, C. Ghesquière, B.P. Gregory, D. Rahm, P. Rivet and F. Solmitz, Proc. of the International Conference on High-Energy Physics, CERN, Geneva, 1962, p.351.

7) B.R. Desai, Phys.Rev. **119**, 1390 (1960).

8) We are grateful to Prof. G.A. Snow for a discussion of these points.

9) A simple example will illustrate the distinction. Suppose a model allowed only the 3π singlet decay and the $\rho\pi\pi$ triplet decay, with a unit Clebsch-Gordan coefficient in each case. The RS procedure would give a $\rho\pi\pi$ branching fraction (the ratio of $\pi\pi\pi$ phase space to $\rho\pi\pi$ phase space is 2.8 to 1) of $(3 \times 1)/(3 \times 1 + 1 \times 2.8) = 52\%$. The procedure we prefer would force the $\rho\pi\pi$ branching fraction to be 75%.

10) We thank Dr. K. Zalewski for pointing out the existence of this unitarity bound.

11) This category probably includes a small proportion of γ's.

12) C. Baltay, P. Franzini, N. Gelfand, G. Lütjens, J.C. Severiens, J. Steinberger, D. Tycko and D. Zanello, Phys.Rev. **140**, B 1039 (1965).

13) On general grounds, we must expect the $\pi^+\pi^-\pi^0$ mode to contain correlated π-mesons. The reason is that the ratio of $\pi^0\pi^0\pi^0$ to $\pi^+\pi^-\pi^0$ is predicted to be 3:2 not only by the quark rearrangement model but simply from I-spin conservation, Bose symmetrization and the assumption of an s-wave final state. The analysis of $\pi^+\pi^-\pi^0$ events in Ref. 12 is in agreement with an s-wave final state. Therefore, even if the entire zero-prong mode were $\pi^0\pi^0\pi^0$, we would have an upper limit of $(2.2 \pm 0.4)\%$ uncorrelated $\pi^+\pi^-\pi^0$. The fact that the remaining 3π decays are not all ascribed to $\rho\pi$ is a hint that other resonant channels (for instance $f^0\pi^0$) are also present.

66/1109/5
p/em

14) Thanks to the narrow width of the ω^o, the $\pi^+\pi^-\pi^o$ combinations associated with its decay show up clearly above background and the determination of the $\omega^o\pi^+\pi^-$ rate from category 6 is therefore rather straightforward. The figure of 5.8% for $\rho^o\pi^+\pi^-$ corresponds to 100% of the $\pi^+\pi^-\pi^+\pi^-$ channel.

15) F. Cerulus, Suppl. al Nuovo Cimento 15, 402 (1960), Table XI.

16) A. Pais, Annals of Physics 9, 548 (1960), Table III.

17) As long as such spin-orbit coupling is absent, the quark rearrangement model is quite compatible with the non-isotropic production observed in annihilations in flight.

18) We have three more categories than in the analysis at rest because we divide the now plentiful six-prong events into three categories according to the value of the missing mass, and add a category for the eight-prong events as well.

19) G. Chikovani, L. Dubal, M.N. Focacci, W. Kienzle, B. Levrat, B.C. Maglic, M. Martin, C. Nef, P. Schübelin and J. Seguinot, Physics Letters 22, 233 (1966).

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21) J. McConnel and J. Shapiro, Nuovo Cimento 28, 1272 (1963).

22) S. Goldhaber, G. Goldhaber, W.M. Powell and R. Silberberg, Phys.Rev. 121, 1525 (1961).

23) G.R. Lynch, Rev.Mod.Phys. 33, 395 (1961).

24) This is not the only possible assumption for the proton wave function. It might be interesting to investigate the consequences of assigning the proton to a reducible SU(4) representation, with a non-vanishing $(4q + \bar{q})$ component.

25) The state obtained in this way must have $I = \frac{1}{2}$, because the symmetrization leads us to a member of the 20-representation of SU(4), which contains only two particles, one with $S = I = \frac{3}{2}$, the other with $S = I = \frac{1}{2}$.

26) For the study of the four and five-pion channels at 5.7 GeV/c, see:

A. Accensi, V. Alles-Borelli, B. French, A. Frisk, J.M. Howie, W. Krischer, L. Michejda, W.G. Moorhead, B.W. Powell, P. Seyboth and P. Villemoes, Physics Letters 20, 557 (1966).

V. Alles-Borelli et al., CERN TC.Pre-print, August 1966.

Table I

Rearrangement coefficients of the quark rearrangement model. The denominator for all coefficients is 864, and is suppressed. S = 0, 1 represent the singlet and triplet initial state. η' is the linear combination of η and X states which contains no strange quarks.

Channel	S = 0	S = 1
$\pi^+\pi^-\pi^0$	18	
$\pi^0\pi^0\pi^0$	27	
$\pi^+\pi^-\eta'$	18	
$\pi^0\pi^0\eta'$	9	
$\pi^0\eta'\eta'$	9	
$\eta'\eta'\eta'$	27	
$\pi^+\pi^-\rho^0$		50
$\pi^0\pi^0\rho^0$		9
$\pi^+\pi^0\rho^-$		2
$\pi^-\pi^0\rho^+$		2
$\pi^+\pi^-\omega$		50
$\pi^0\pi^0\omega$		25
$\pi^0\eta'\rho^0$		2
$\pi^+\eta'\rho^-$		2
$\pi^-\eta'\rho^+$		2
$\pi^0\eta'\omega$		2
$\eta'\eta'\rho^0$		25
$\eta'\eta'\omega$		9

contd. above right

Channel	S = 0	S = 1
$\pi^0\rho^+\rho^-$	150	36
$\pi^0\rho^0\rho^0$	27	
$\pi^+\rho^0\rho^-$	6	36
$\pi^-\rho^0\rho^+$	6	36
$\eta'\rho^+\rho^-$	150	36
$\eta'\rho^0\rho^0$	75	
$\pi^0\rho^0\omega$	6	
$\pi^+\rho^-\omega$	6	36
$\pi^-\rho^+\omega$	6	36
$\eta'\rho^0\omega$	6	
$\pi^0\omega\ \omega$	75	
$\eta'\omega\ \omega$	27	
$\rho^+\rho^-\rho^0$	108	126
$\rho^0\rho^0\rho^0$		45
$\rho^+\rho^-\omega$	108	126
$\rho^0\rho^0\omega$		63
$\rho^0\omega\ \omega$		63
$\omega\ \omega\ \omega$		45

Table II

Predicted distribution of final states for $P\bar{P}$ annihilation at rest.

$\pi^+\pi^-\pi^0$	7.4	$\pi^+\pi^-\rho^0$	25.6	$\pi^0\rho^+\rho^-$	2.9[a)]
$\pi^0\pi^0\pi^0$	11.2	$\pi^0\pi^0\rho^0$	4.7	$\pi^+\rho^0\rho^-$	1.4[b)]
$\pi^+\pi^-\eta$	1.4	$\pi^+\pi^0\rho^-$	1.0	$\pi^-\rho^0\rho^+$	1.4[b)]
$\pi^0\pi^0\eta$	0.7	$\pi^-\pi^0\rho^+$	1.0	$\pi^+\rho^-\omega$	1.0[b)]
$\pi^+\pi^-X$	1.0	$\pi^+\pi^-\omega$	24.0	$\pi^-\rho^-\omega$	1.0[b)]
$\pi^0\pi^0X$	0.5	$\pi^0\pi^0\omega$	12.2	$\pi^0\omega\,\omega$	0.5

a) 1.3 from triplet b) all from triplet

Table III

Predicted and observed distribution of final states among eight topological categories for $P\bar{P}$ annihilation at rest. The experimental percentages sum to 95.4%, the remainder being kaonic annihilations. The experimental data come from Ref. 3a.

Category	1	2	3	4	5	6	7	8
	0 prong	$\pi^+\pi^-$	$\pi^+\pi^-\pi^0$	$\pi^+\pi^-x^0$	$2\pi^+2\pi^-$	$2\pi^+2\pi^-\pi^0$	$2\pi^+2\pi^-x^0$	6 prong
Predicted(%)	12.9	0.0	7.4	22.3	25.6	28.5	3.1	0.1
Observed(%)	3.2 ± 0.5	0.5 ± 0.1	7.3 ± 0.9	34.8 ± 1.2	5.8 ± 0.3	18.7 ± 0.9	21.3 ± 1.1	3.8 ± 0.2

66/1109/5
p/gs

Table IV

Computation of the contribution of quark rearrangement annihilation at rest to each annihilation sector. The contributions sum to 23 ± 2%.

Sector	Channel	Exp.rate(%)	Per cent of channel in sector	Rearrangement annihilation in sectors (%)
η	$\pi^+\pi^-\eta$	1.2 ± 0.3	50	2.5 ± 0.6
π	$\pi^0\pi^0\pi^0$	3.2 ± 0.5[a)]	52	6.2 ± 1.0
ω	$\pi^+\pi^-\omega$	3.8 ± 0.4	60	6.3 ± 0.7
ρ	$\pi^+\pi^-\rho^0$	$5.8\,^{+0.3}_{-1.3}$	75	$7.7\,^{+0.4}_{-1.7}$

a) upper limit

Table V

Distribution of annihilations at rest not due to quark rearrangement. The percentages sum to 75%, as 25% are attributed to quark rearrangement.

Category	1	2	3	4	5	6	7	8
	0 prong	$\pi^+\pi^-$	$\pi^+\pi^-\pi^0$	$\pi^+\pi^-x^0$	$2\pi^+2\pi^-$	$2\pi^+2\pi^-\pi^0$	$2\pi^+2\pi^-x^0$	6 prong
Rate after subtraction	0.0	0.5	5.0	29.5	0.0	16.0	20.5	3.5

66/1109/5
p/gs

Table VI

Predicted distribution of final states for $P\bar{P}$ annihilation at three energies in flight.

Channel	Per cent of annihilation		
	3.3 GeV/c	5.7 GeV/c	7.0 GeV/c
$\pi^+\pi^-\pi^0$	2.4	1.4	1.2
$\pi^0\pi^0\pi^0$	3.6	2.1	1.8
$\pi^+\pi^-\eta$	0.6	0.4	0.3
$\pi^+\pi^- X$	0.8	0.6	0.5
$\pi^+\pi^-\rho^0$	12.3	8.4	7.6
$\pi^0\pi^0\rho^0$	2.2	1.5	1.4
$\pi^+\pi^0\rho^-$	0.5	0.3	0.3
$\pi^-\pi^0\rho^+$	0.5	0.3	0.3
$\pi^+\pi^-\omega$	12.0	8.3	7.4
$\pi^0\pi^0\omega$	6.0	4.2	3.7
$\pi^0\rho^+\rho^-$	10.6	9.5	9.1
$\pi^0\rho^0\rho^0$	1.1	1.0	1.0
$\pi^+\rho^0\rho^-$	4.7	4.2	4.0
$\pi^-\rho^0\rho^+$	4.7	4.2	4.0
$\pi^+\rho^-\omega$	4.5	4.1	4.0
$\pi^-\rho^+\omega$	4.5	4.1	4.0
$\pi^0\omega\ \omega$	2.9	2.7	2.6
$\eta\ \rho^+\rho^-$	1.9	2.3	2.4
$\eta\ \rho^0\rho^0$	0.6	0.7	0.7
$X\ \rho^+\rho^-$	0.9	2.6	3.0
$X\ \rho^0\rho^0$	0.3	0.8	0.9
$\rho^+\rho^-\rho^0$	6.0	10.0	10.9
$\rho^0\rho^0\rho^0$	1.8	2.8	3.1
$\rho^+\rho^-\omega$	5.5	9.6	10.6
$\rho^0\rho^0\omega$	2.3	3.8	4.2
$\rho^0\omega\ \omega$	2.1	3.7	4.1
$\omega\ \omega\ \omega$	1.3	2.5	2.8

66/1109/5
p/gs

Table VII

Predicted and observed distribution of final states among eleven topological categories, for $P\bar{P}$ annihilation at three energies in flight. The experimental percentages sum to 95%, which allows for 5% kaonic annihilations. The experimental data come from Refs. 4a, b and c. All eight-prong events observed are attributed to annihilations.

Category		3.3 GeV/c		5.7 GeV/c		7.0 GeV/c	
		Predicted (%)	Observed (%)	Predicted (%)	Observed (%)	Predicted (%)	Observed (%)
1	0 prong	4.5	5.4 ± 2.7	2.7	1.3 ± 1.0	2.4	0.8 ± 0.8
2	$\pi^+\pi^-$	0.0	< 0.1	0.0	< 0.2	0.0	< 0.1
3	$\pi^+\pi^-\pi^0$	2.4	1.9 ± 1.0	1.4	< 1.3	1.2	} 20.0 ± 8.0 (categories 3 and 4)
4	$\pi^+\pi^-x^0$	14.9	23.8 ± 4.5	12.4	19.7 ± 5.2	11.8	
5	$2\pi^+2\pi^-$	12.3	1.9 ± 0.2	8.4	0.5 ± 0.1	7.6	0.4 ± 0.25
6	$2\pi^+2\pi^-\pi^0$	32.5	11.6 ± 1.6	26.7	4.1 ± 0.5	25.2	2.8 ± 0.6
7	$2\pi^+2\pi^-x^0$	25.8	32.7 ± 3.0	34.7	35.3 ± 6.1	36.7	38.0 ± 8.0
8	$3\pi^+3\pi^-$	1.8	2.7 ± 0.2	2.8	1.1 ± 0.3	3.1	0.7 ± 0.2
9	$3\pi^+3\pi^-\pi^0$	2.4	6.6 ± 0.6	4.0	} 27.6 ± 1.7 (categories 9 and 10)	4.3	7.6 ± 1.2
10	$3\pi^+3\pi^-x^0$	3.4	7.3 ± 0.7	6.5		7.3	19.6 ± 3.9
11	8 prong	0.1	0.9 ± 0.3	0.3	5.2 ± 1.7	0.4	5.2 ± 2.0
$\langle n_{ch} \rangle$		3.65	3.6 ± 0.2	3.9	4.35 ± 0.2	3.95	4.35 ± 0.2

66/1109/5
p/gs

Table VIII

Predicted and observed rates for two annihilation channels in flight. The experimental data come from Refs. 4a (3.3 GeV/c), 20 (5.7 GeV/c), and 4c (7.0 GeV/c).

Channel	3.3 GeV/c		5.7 GeV/c		7.0 GeV/c	
	Predicted (%)	Observed (%)	Predicted (%)	Observed (%)	Predicted (%)	Observed (%)
$\pi^+\pi^-\omega$	12.0	1.2 ± 0.2	8.3	0.38 ± 0.12	7.4	< 0.6
$\pi^+\pi^-\rho^0$	12.3	0.6 ± 0.1	8.4	0.19 ± 0.06	7.6	≤ 0.4 ± 0.25 [a)]

a) upper limit ≡ number of $2\pi^+2\pi^-$

66/1109/5
p/gs

Table IX

Numerical values of phase space integrals, in $(\mathrm{GeV})^2$, for annihilation at rest, and contributions of the final states to the topological categories.

Channel	Phase space integral	1 0 prong	2 $\pi^+\pi^-$	3 $\pi^+\pi^-\pi^0$	4 $\pi^+\pi^-x^0$	5 $2\pi^+2\pi^-$	6 $2\pi^+2\pi^-\pi^0$	7 $2\pi^+2\pi^-x^0$	8 6 prong
$\pi^0\pi^0\pi^0$	3.62 E 0	100							
$\pi^+\pi^-\pi^0$	3.60 E 0			100					
$\pi^0\pi^0\eta$	2.08 E 0	72			28				
$\pi^+\pi^-\eta$	2.06 E 0				72		28		
$\pi^0\eta\ \eta$	8.96 E-1	51			41			8	
$\pi^0\pi^0X$	7.29 E-1	18			68			14	
$\pi^+\pi^-X$	7.14 E-1				18		25	43	14
$\eta\ \eta\ \eta$	1.45 E-1	36			44			17	2
$\pi^0\eta\ X$	1.08 E-1	13			54			29	4
$\pi^0\pi^0\rho^0$	1.32 E 0				100				
$\pi^+\pi^-\rho^0$	1.30 E 0					100			
$\pi^\pm\pi^0\rho^\mp$	1.28 E 0				100				
$\pi^0\pi^0\omega$	1.23 E 0	9			91				
$\pi^+\pi^-\omega$	1.22 E 0				9			91	
$\pi^0\eta\ \rho^0$	4.08 E-1				72			28	
$\pi^\pm\eta\ \rho^\mp$	3.83 E-1				72			28	
$\pi^0\eta\ \omega$	3.61 E-1	8			67			25	
$\pi^0\rho^0\rho^0$	1.09 E-1						100		
$\pi^\pm\rho^\mp\rho^0$	9.64 E-2				100				
$\pi^0\rho^+\rho^-$	9.18 E-2				100				
$\pi^0\rho^0\omega$	7.60 E-2			9				91	
$\pi^\pm\rho^\mp\omega$	7.39 E-2			9				91	
$\pi^0\omega\ \omega$	6.38 E-2	1		17				82	

66/1109/5
p/gs

Figure Caption

Energy variation of average multiplicity of charged pions in pionic annihilations. The experimental data points are from Ref. 3 (1.88 GeV), Ref. 22 (2.1 GeV), Ref. 23 (2.3 and 2.44 GeV), Ref. 4a (2.86 GeV), Ref. 4b (3.55 GeV), and Ref. 4c (3.87 GeV). Curve (a) gives the predictions of the quark rearrangement model. Curves (b) and (c) give the statistical model predictions for $\Omega = 4\Omega_0$ and $\Omega = 5\Omega_0$ respectively, where $\Omega_0 = 4\pi/(3m_\pi^3)$, and are taken from Ref. 21.

66/1109/5
p/gs

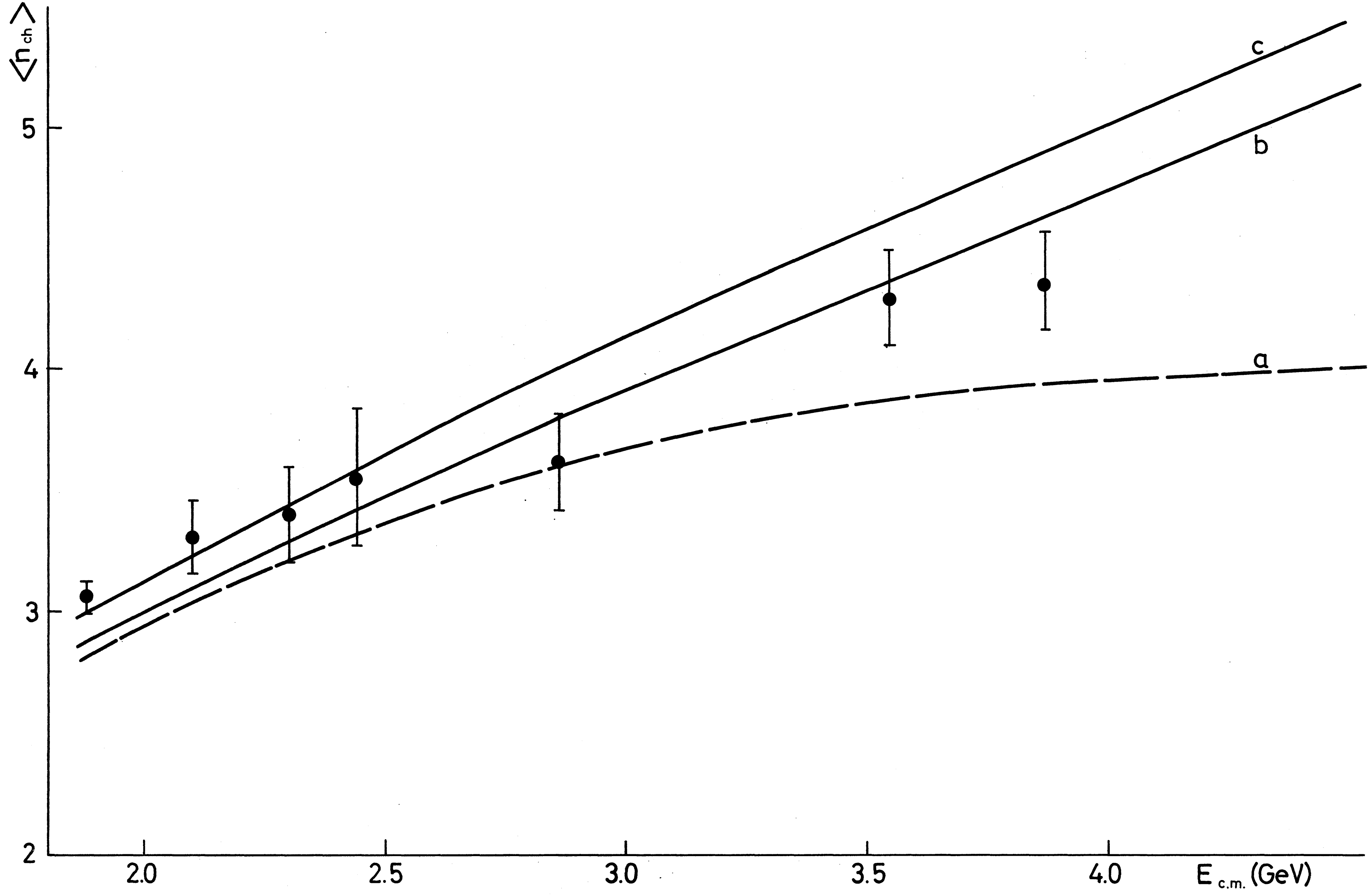

FIG.1