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POLARIZATION IN HIGH-ENERGY πp ELASTIC AND CHARGE EXCHANGE SCATTERING

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ABSTRACT

The phenomenological discussion of various polarization measurements in high-energy πp elastic and charge-exchange scattering is given in terms of the helicity formalism. Both the high-energy limit and the high-energy behaviour of the polarization are discussed under various assumptions suggested by the experimental data and/or theoretical models. The case when the non-spin-flip amplitude dominates the high energy elastic scattering is considered in some detail.

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1. INTRODUCTION

In this paper we would like to discuss the high-energy, fixed momentum transfer behaviour of the polarization in the elastic πp scattering, including charge-exchange scattering. The discussion is based on the following two general assumptions:

a) the elastic $\pi^+ p$ and $\pi^- p$ amplitudes do not oscillate for energy tending to infinity and fixed momentum transfer, and

b) they are analytic functions of s, for fixed t, which obey the usual crossing relations.

These two assumptions (a more precise formulation of them will be given later) imply several relations between the scattering amplitudes of the direct and crossed reactions (in this case the $\pi^+ p$ and $\pi^- p$ elastic scattering) in the high-energy limit. They have been discussed by several authors. A review of the results obtained can be found in the papers by Logunov et al.[1] and by Van Hove[2,3]. The general formulation for the scattering of particles with arbitrary spin, using the helicity formalism, was given by the present authors[4]. We would now like to apply these general formulae to the particular problem of πp scattering.

Starting from the high-energy relations for the πp scattering amplitudes, the polarization at high energy in the elastic πp scattering has been discussed by Logunov et al.[1] and by Van Hove[2]. These authors confined themselves to the discussion of the first approximation, i.e. they considered only the limiting value of the polarization at extremely high energy. The purpose of the present paper is twofold. We would like firstly to re-derive the results given by Logunov et al. and by Van Hove, using the helicity formalism, and secondly to consider the second approximation, i.e. the way the limiting values are reached. The general behaviour of the elastic cross-sections at the present machine energies shows that the asymptotic high-energy limit is not yet obtained. However, the second approximation in the asymptotic expansion in terms of inverse powers of energy is, in general, sufficient to describe the experimental data for energies > 5 GeV with quite good accuracy[5,6]. Taking into account this experience, we hope that the estimations we can make on the basis of the considered second approximation will describe correctly the main features of the polarization at high energies.

As the principal results of the present paper we consider

a) the parametrization of the high-energy πp elastic and charge-exchange scattering which we hope will be useful in the description of the present and future experimental data; and,

b) the analysis of the consequences of the optical models, in particular of the assumption that the non-spin-flip amplitude dominates the high-energy scattering for the polarization in the high-energy $\pi^+ p$ and $\pi^- p$ elastic scattering.

The next section contains a survey of the relations which express different polarization parameters in terms of the helicity amplitudes. In Section 3 we give a proper formulation of the assumptions a) and b), as well as the relations between the helicity amplitudes implied by them. In Section 4 we re-derive the results obtained by Logunov et al.[1], and in Section 5 we discuss the second approximation of the asymptotic expansion. The consequences of the optical model are discussed in Section 6. Our conclusions are listed in the last section.

An Appendix contains a derivation of the rotation angle appearing in the transformation of helicity states for a Lorentz transformation from the centre of momentum to the laboratory system; this angle enters in the polarization parameters of the laboratory system expressed in terms of the centre-of-momentum system helicity amplitudes.

2. POLARIZATION FORMALISM IN THE HELICITY REPRESENTATION

In this section we derive expressions for all kinds of polarization parameters in terms of quantities characterizing the pion-nucleon scattering. These results are already well known[7], but to our knowledge they have never previously been derived using helicity formalism[8] from the outset. Since, in our opinion, this formalism is the best one for discussing polarization phenomena, we thought it worth while to present the derivation in some detail.

In this section we shall pay no attention to the charge configuration, but only require the scattering to be kinematically elastic, i.e. that there is a pion of mass μ and a nucleon of mass m (but no other particles) in both initial and final states. We refer to Fig. 1 and to Ref. 4) for further notation.

As quantities characterizing the pion-nucleon scattering, we take the c.m.s. (centre-of-momentum system) helicity amplitudes, $T_{\kappa\lambda}$, where $\lambda = \pm \frac{1}{2}$ and $\kappa = \pm \frac{1}{2}$ are the helicities of the initial (or target) nucleon and the final (or recoil) nucleon, respectively. These amplitudes are functions of the total c.m.s. energy E and of the c.m.s. scattering angle Θ. By virtue of parity conservation [cf. Eq. (A.6) of Ref. 4)],

$$T_{-\kappa,-\lambda} = (-)^{\kappa-\lambda} T_{\kappa\lambda} \tag{2.1}$$

implying that there are only two independent helicity amplitudes, which we choose as $T_{++} = T_{1/2,\,1/2}$ and $T_{+-} = T_{1/2,\,-1/2}$. They are normalized in such a way that for scattering on an unpolarized target the differential cross-section reads

$$\frac{d\sigma}{dt} = \frac{1}{16\pi\,\lambda(s,m^2,\mu^2)} \left\{ |T_{++}|^2 + |T_{+-}|^2 \right\} \tag{2.2 a}$$

$$\lambda(s,m^2,\mu^2) = (s-m^2-\mu^2)^2 - 4m^2\mu^2 \tag{2.2 b}$$

The use of helicity states means that we attach to the initial nucleon a right-handed co-ordinate system x y z, called for short its helicity frame, with the z-axis in the direction of motion and with, in our conventions, the y-axis along the normal $\vec{n} = (\vec{k}_N \times \vec{k}_{N'})/|\vec{k}_N \times \vec{k}_{N'}|$ to the scattering plane. Correspondingly, the helicity frame $x'y'z'$ of the final nucleon has its z'-axis along the line-of-flight of this particle, while its y'-axis again is the normal $\vec{n}$. These co-ordinate systems are indicated in Fig. 1. According to its definition, then, the helicity is the spin projection along the z-axis (z'-axis) in the rest frame of the target (recoil) nucleon.

Polarization experiments are most conveniently discussed in terms of spin space density matrices, in our approach referred to as the helicity basis. If the target nucleon has a polarization vector, $\vec{P}_t$, i.e. if it is $100 \cdot |\vec{P}_t|$% polarized in the direction $\vec{P}_t/|\vec{P}_t|$, its helicity density matrix is

$$\rho = \frac{1}{2}\left(1 + \vec{P}_t \cdot \vec{\sigma}\right) \tag{2.3}$$

where $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the conventional 2×2 Pauli matrices and where all vectors are referred to the helicity frame of the initial nucleon.

By applying the rule of forming the density matrix in the final state from that in the initial state and the transition matrix, we obtain for the helicity density matrix of the outgoing nucleon

$$\rho'_{\kappa\kappa'} = \left(T \rho T^{+}\right)_{\kappa\kappa'} = \sum_{\lambda\lambda'} T_{\kappa\lambda}\, \rho_{\lambda\lambda'}\, T^{*}_{\kappa'\lambda'} \tag{2.4}$$

where a dagger denotes hermitian and a star denotes complex conjugation. Manipulating this equation, we find

$$\rho' = \frac{1}{2} I_0 \left\{ 1 + P\left(\vec{P}_t' \cdot \hat{n}\right) + \vec{\xi}^{\,cms} \cdot \vec{\sigma} \right\} \tag{2.5 a}$$

$$I_0 = |T_{++}|^2 + |T_{+-}|^2 \tag{2.5 b}$$

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$$I_0 P = 2\, \mathrm{Im}\left(T_{++} T_{+-}^{*}\right) \qquad (2.5\ c)$$

$$\vec{\xi}^{\,cms} = \left(P + \vec{P}_t' \cdot \vec{n}\right)\vec{n} + Y\left(\vec{P}_t' \times \vec{n}\right) \times \vec{n} + Z\left(\vec{P}_t' \times \vec{n}\right) \qquad (2.5\ d)$$

$$I_0 Y = |T_{++}|^2 - |T_{+-}|^2 \qquad (2.5\ e)$$

$$I_0 Z = 2\, \mathrm{Re}\left(T_{++} T_{+-}^{*}\right) \qquad (2.5\ f)$$

Here, all vectors are referred to the helicity frame of the final nucleon; in particular, $\vec{P}_t'$ is the vector, which has the same components along the $x'y'z'$-axes as does $\vec{P}_t$ along the x y z-axes. We emphasize that ρ' is obtained from ρ by a transformation involving the centre-of-momentum system helicity amplitudes.

Due to the fact that polarization experiments are invariably referred to the laboratory system, we have to cast Eqs. (2.5) into a form which is suitable for analysing such experiments[9]. To this end, we must perform a pure (active) Lorentz transformation $\ell(\vec{v})$ with the velocity $\vec{v} = \vec{k}_N/m$ from the laboratory system to the c.m.s. (cf. Fig. 1c). This transformation has the property that

$$\ell(\vec{v})\, k_t = k_N\,, \qquad (2.6\ a)$$

$$\ell(\vec{v})\, k_R = k_{N'}\,. \qquad (2.6\ b)$$

Now, helicity states are invariant under Lorentz transformations along the direction of motion. The helicity of the initial nucleon is thus invariant under the transformation $\ell(\vec{v})$; this has, in fact, already been used above when we wrote down the helicity density matrix for the target nucleon. On the other hand, for a pure Lorentz transformation along directions other than that of the motion, the helicity is not invariant. Under the transformation $\ell(\vec{v})$, in particular, the helicity states of the outgoing nucleon do change according to the transformation law[10]

$$L(\vec{v})\,|\vec{k}_R, \nu\rangle = \sum_{\kappa} |\vec{k}_{N'}, \kappa\rangle\, D^{1/2}_{\kappa\nu}(r) \qquad (2.7)$$

Here, $|\vec{k}_R, \nu>$ is a helicity state in the laboratory system, $|\vec{k}_{N'}, \kappa>$ a helicity state in the c.m.s., and $L(\vec{v})$ the transformation operator on the helicity states, induced by the transformation $\ell(\vec{v})$. Finally, the rotation r entering in the spin - ½ rotation coefficients $D^{1/2}_{\kappa\nu}(r)$ is a so-called Wick rotation[10], in this particular case a rotation around the normal $\vec{n}$ by a certain angle ω,

$$D^{1/2}_{\kappa\nu}(r) = d^{1/2}_{\kappa\nu}(\omega)\ . \qquad (2.8)$$

In Appendix A we prove that, in fact,

$$\omega = \Theta_R, \qquad (2.9)$$

where Θ_R is the angle in the laboratory system between the beam and the recoil nucleon, defined to be positive for a clock-wise rotation around the normal $\vec{n}$ as is indicated in Fig. 1.

From Lorentz invariance we thus arrive at the following connection between the c.m.s. helicity density matrix ρ' and the laboratory density matrix ρ^R of the recoil nucleon

$$\rho^{R}_{\nu\nu'} = \langle \vec{k}_R, \nu | \rho | \vec{k}_R, \nu' \rangle = \langle \vec{k}_R, \nu | L^{-1} L \rho L^{-1} L | \vec{k}_R, \nu' \rangle =$$

$$= \sum_{\varkappa\varkappa'} \langle \vec{k}_R, \nu | L^{-1} | \vec{k}_{N'}, \varkappa \rangle \langle \vec{k}_{N'}, \varkappa | \rho' | \vec{k}_{N'}, \varkappa' \rangle \langle \vec{k}_{N'}, \varkappa' | L | \vec{k}_R, \nu' \rangle = \tag{2.10}$$

$$= \sum_{\varkappa\varkappa'} d^{1/2}_{\nu\varkappa}(-\omega) \, \rho'_{\varkappa\varkappa'} \, d^{1/2}_{\varkappa'\nu'}(\omega)$$

After some algebra, finally we find that ρ^R may be written

$$\rho^R = \frac{1}{2} I_0 \left\{ 1 + P(\vec{P}_t^R \cdot \vec{n}) + \vec{\xi}^R \cdot \vec{\sigma} \right\} \tag{2.11 a}$$

$$\vec{\xi}^R = (P + \vec{P}_t^R \cdot \vec{n}) \cdot \vec{n} + R(\vec{n} \times \vec{P}_t^R) \times \vec{n} + A(\vec{n} \times \vec{P}_t^R) \tag{2.11 b}$$

$$R = Y \cos\theta_R + Z \sin\theta_R \tag{2.11 c}$$

$$A = Y \sin\theta_R - Z \cos\theta_R \tag{2.11 d}$$

Here, the notation of Eqs. (2.5 b - f) has been followed. Furthermore, all vectors now refer to the co-ordinate system x_R y_R z_R of Fig. 1; in particular, $\vec{P}_t^R$ is that vector which has the same components along the x_R y_R z_R-axes as $\vec{P}_t$ has along the x y z-axes.

The expression (2.11) is the desired helicity density matrix for the recoil nucleon referred to the laboratory system. It may be used directly to relate the results of different polarization experiments to the c.m.s. helicity amplitudes T_{++} and T_{+-}. Conventionally, three such types of experiments are usually performed:

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1. The target nucleon is polarized in the direction of the normal $\vec{n}$ to the scattering plane, $\vec{P}_t = P_t \cdot \vec{n}$, and the recoil nucleon polarization is analysed along the same axis. This situation contains as a special case the experiment on an unpolarized target, $\vec{P}_t = 0$. From Eqs. (2.11) we obtain immediately that the outgoing nucleon also is polarized along $\vec{n}$ with a polarization P_n given by

$$P_n = \frac{P + P_t}{1 + P P_t} \qquad (2.12)$$

2. The target nucleon is polarized in the scattering plane perpendicular to the beam direction, $\vec{P}_t = P_t \cdot \vec{x}_t$ ($\vec{x}_t$ is the unit vector along the x_t-axis), while the polarization of the recoil nucleon is analysed along the x_R-axis, i.e. in the scattering plane perpendicular to the line-of-flight (see Fig. 1b). This is usually called the "R-experiment". From Eqs. (2.11) we discover that R of Eq. (2.11 c) is just the polarization, divided by P_t, of the outgoing nucleon along the x_R-axis.

3. The target nucleon is longitudinally polarized, $\vec{P}_t = P_t \vec{z}_t$ ($\vec{z}_t$ is the unit vector along the z_t-axis), while the recoil nucleon is analysed as in the R-experiment. This type of experiment is usually called the "A-experiment". Analogously to R, we find that A of Eq. (2.11 d) is just the polarization, divided by P_t, of the outgoing nucleon along the x_R-axis.

For pion-nucleon scattering, one is in the fortunate situation that the three polarization experiments outlined above, complemented by differential cross-section measurements on an unpolarized target, are enough to obtain a complete knowledge of the two helicity amplitudes (at a given angle Θ and energy E), apart from a common phase. For experiment (1), in this respect it is even sufficient to make the measurement using an unpolarized target, since the use of a polarized target would tell us nothing new concerning P which could not be learned from scattering on an unpolarized one. Observe also that, since

$$P^2 + Y^2 + Z^2 = 1 \tag{2.13 a}$$

$$P^2 + R^2 + A^2 = 1 \tag{2.13 b}$$

the determination of two out of the three quantities P, R, and A (or P, Y, and Z) fixes the absolute value but not the sign of the third one. For the helicity amplitudes T_{++} and T_{+-} (with relative phase φ), the sign of Y tells us which one is the biggest, while a knowledge of only $|P|$ (of $|Z|$) means that one cannot distinguish φ from $-\varphi$ (φ from $\pi - \varphi$).

For other processes, such as nucleon-nucleon scattering, more polarization measurements have to be made to determine all amplitudes. Here it is clear from the presentation how the formal treatment in the helicity representation of such experiments should go, and that the helicity formalism is a very convenient one for discussing polarization phenomena.

3. HIGH-ENERGY BEHAVIOUR OF THE $\pi^{\pm}$p ELASTIC AMPLITUDES

In this section we shall apply the general asymptotic relations for helicity amplitudes obtained in the preceding paper[4)] to the special problem of $\pi^{\pm}$p elastic and charge-exchange scattering. Let G_{++} and G_{+-} be the helicity amplitudes for π^+p elastic scattering. We denote the amplitudes for π^-p elastic scattering by H_{++} and H_{+-}. [Reference 4) may be consulted for the further notation.]

We now repeat the assumptions we have used in Ref. 4) to derive general asymptotic relations between the helicity amplitudes. The first group of assumptions concerns the analytic properties of the amplitudes G and H. We assume that:

a) both G and H are, for fixed t, analytic functions of s in the upper-half s-plane, except possibly for a finite region. For $s \to \infty$ in the upper-half s-plane, they are bounded by a polynomial in s. Finally, they are continuous along the real axis, except possibly for a finite region.

b) G and H obey the crossing relations which, for t fixed and for large real s, take the form[11]:

$$G^{*}_{++}(s,t) = H_{++}(u,t) \tag{3.1}$$

$$G^{*}_{+-}(s,t) = H_{+-}(u,t). \tag{3.2}$$

In these relations it is understood that the analytic continuation is performed in the upper-half s-plane. The more detailed description of this process can be found in Ref. 4).

The second important assumption is that the amplitudes G and H do not oscillate very rapidly for t fixed in the physical region and $s \to \infty$ along the real axis. This assumption means that there exist asymptotic expansions of both G and H, for t fixed and $s \to \infty$, in terms of non-oscillating functions of s. A simple example of such an expansion, which we shall use henceforth, is

$$G_{\mu\lambda} \propto g_{\mu\lambda}(t)\, s^{\alpha(t)} + g^{(1)}_{\mu\lambda}(t)\, s^{\beta(t)} + \ldots \tag{3.3}$$

$$H_{\mu\lambda} \propto h_{\mu\lambda}(t)\, s^{\alpha(t)} + h^{(1)}_{\mu\lambda}(t)\, s^{\beta(t)} + \ldots \tag{3.4}$$

where the dots indicate further terms. In order to exclude oscillations we have to assume that the functions $\alpha(t)$ and $\beta(t)$ are real.

Using these assumptions, one can prove that there exist relations between the coefficients of the expansions (3.3) and (3.4). We have[4,12]

$$h^{*}_{\mu\lambda}(t) = g_{\mu\lambda}(t)\, e^{i\pi\alpha(t)} \tag{3.5 a}$$

$$h^{(1)*}_{\mu\lambda}(t) = g^{(1)}_{\mu\lambda}(t)\, e^{i\pi\beta(t)} \quad \text{if} \quad \alpha(t) - \beta(t) < 1 \tag{3.5 b}$$

The relations (3.5) represent strong constraints on the asymptotic behaviour of $\pi^{\pm}p$ elastic scattering at high energy, and show that at high energy these two reactions are indeed closely related.

We now propose to analyse the high-energy behaviour of the $\pi^{\pm}p$ elastic scattering using the asymptotic expansions (3.3) and (3.4) as a mathematical tool but without invoking any detailed dynamics. It seems reasonable that at high energy the first few terms of the expansions will be sufficient to describe the experimental data with a good accuracy. Our point is simply that, by considering the $\pi^{\pm}p$ data together, the number of phenomenological parameters needed for the description is strongly reduced by the relations (3.3) to (3.5)[13].

In the next two sections we shall discuss the first and the second approximation, i.e. the first and the second term of the expansions (3.3) and (3.4). However, before we come to that we would like to recall some special features of the πp scattering amplitudes which will be useful in further discussion. The amplitudes G and H can both be written as a superposition of the C = +1 and C = −1 exchange amplitudes, where C is the charge conjugation quantum number. We have

$$G_{\mu\lambda} = T^{C=+1}_{\mu\lambda} + T^{C=-1}_{\mu\lambda} \tag{3.6}$$

$$H_{\mu\lambda} = T_{\mu\lambda}^{C=+1} - T_{\mu\lambda}^{C=-1} \tag{3.7}$$

where the superscripts denote C = 1 and C = -1 exchange amplitudes, respectively. Accordingly, the coefficients $g_{\mu\lambda}$ and $h_{\mu\lambda}$ also split into the sum and the difference of C = +1 and C = -1 exchange terms

$$g = t^{C=+1} + t^{C=-1} \tag{3.8}$$

$$h = t^{C=+1} - t^{C=-1} \tag{3.9}$$

For simplicity, we have omitted here all indices. It is easy to see that the relations (3.5 a) imply

$$\left[t^{C=+1}\right]^* = t^{C=+1} e^{i\pi\alpha} \tag{3.10}$$

$$\left[t^{C=-1}\right]^* = -t^{C=-1} e^{i\pi\alpha} \tag{3.11}$$

with analogous relations following from Eq. (3.5 b).

In the special case of πp scattering, the determination of the charge conjugation quantum number exchanged also fixes other exchanged quantum numbers[14]. Thus, C = +1 exchange corresponds to I = 0 and P = +1 exchange, where I and P are isotopic spin and parity, respectively. Correspondingly, C = -1 exchange amplitudes correspond to I = 1 and P = -1 exchange.

4. HIGH-ENERGY LIMIT OF THE CROSS-SECTIONS AND POLARIZATIONS

In this section we discuss the consequences of the analytic and non-oscillatory character of the scattering amplitudes for the high-energy limit of the cross-sections and polarizations in πp elastic and charge-exchange scattering. To this end we shall substitute the asymptotic expansions (3.3) and (3.4) into the formulae (2.2) and (2.5) and calculate the measurable quantities in terms of the helicity amplitudes.

Since we are interested only in the high-energy limit, it is enough to keep the highest power of s in the expansions (3.3) and (3.4), i.e. to put

$$G_{++} = g_{++}(t)\, s^{\alpha(t)} \qquad G_{+-} = g_{+-}(t)\, s^{\alpha(t)} \tag{4.1}$$

$$H_{++} = h_{++}(t)\, s^{\alpha(t)} \qquad H_{+-} = h_{+-}(t)\, s^{\alpha(t)} \tag{4.2}$$

where the functions g_{++}, g_{+-}, h_{++} and h_{+-} obey the relations

$$h^{*}_{++} = g_{++}\, e^{i\pi\alpha} \qquad h^{*}_{+-} = g_{+-}\, e^{i\pi\alpha} \tag{4.3}$$

Since α is a real function of t we conclude

$$|h_{++}| = |g_{++}| \qquad |h_{+-}| = |g_{+-}| \tag{4.4}$$

and therefore

$$\lim_{E\to\infty} \left(\frac{d\sigma^+}{dt} \Big/ \frac{d\sigma^-}{dt} \right) \longrightarrow 1 \tag{4.5}$$

where superscripts + and - refer to the $\pi^+ p$ and $\pi^- p$ scattering, respectively. This is a well-known condition[15], which extends the Pomeranchuk theorem[16] to the elastic scattering.

Let us now discuss the behaviour of the polarization coefficient P [see Section 2, in particular Eq. (2.5 c) for its definition], which gives the polarization of the recoil proton in the scattering on an unpolarized target. We have

$$\mathrm{Im}\left(G_{++} G_{+-}^{*}\right) = s^{2\alpha}\, \mathrm{Im}\left(g_{++} g_{+-}^{*}\right) \tag{4.6}$$

$$\mathrm{Im}\left(H_{++} H_{+-}^{*}\right) = s^{2\alpha}\, \mathrm{Im}\left(h_{++} h_{+-}^{*}\right) =$$

$$= s^{2\alpha}\, \mathrm{Im}\left(g_{++}^{*} g_{+-}\right) = -\mathrm{Im}\left(G_{++} G_{+-}^{*}\right) \tag{4.7}$$

If follows from Eqs. (4.6), (4.7) and (2.5 c) that in the high-energy limit the polarization P changes sign when we pass from $\pi^+ p$ to $\pi^- p$ scattering[17]:

$$\lim_{E\to\infty} P^{+} = -\lim_{E\to\infty} P^{-} \tag{4.8}$$

Note, however, that in general the condition

$$\lim_{E\to\infty} \left(\frac{P^{+}}{P^{-}}\right) = -1 \tag{4.9}$$

is not valid. The reason is that in the case when

$$\lim_{E\to\infty} P^{+} = \lim_{E\to\infty} P^{-} = 0 \tag{4.10}$$

the important contributions to the polarization come from the further terms in the expansions (3.3) and (3.4). We postpone the discussion of this problem to the next section.

Let us now see how the different assumptions concerning the dominance of exchange quantum numbers in the asymptotic $\pi^{\pm}p$ elastic amplitudes influence the values $P^{\pm}_{\infty} = \lim_{E\to\infty} P^{\pm}$. If one assumes that the elastic amplitudes are dominated at high-energy by pure C = 1 or pure C = −1 exchange (as being opposite to a linear combination of both of them), we have from Eqs. (3.8) and (3.9):

$$H_{++} = \pm G_{++} \qquad H_{+-} = \pm G_{+-} \tag{4.11}$$

where the upper and lower signs correspond to C = 1 and C = −1, respectively. It follows from Eq. (4.11) and from the formula (2.5 c) for P, that in this case

$$\lim_{E\to\infty} P^{+} = \lim_{E\to\infty} P^{-} \tag{4.12}$$

Combined with formula (4.8), this gives

$$\lim_{E\to\infty} P^{+} = \lim_{E\to\infty} P^{-} = 0 \tag{4.13}$$

The assumption of the dominance of a given parity (+ or −) exchange at high energy gives precisely the same condition because, as we have indicated in the preceding section, it is equivalent to the assumption of C = 1 or C = −1 dominance. The condition (4.13) will therefore be satisfied in the Regge-pole model, which assumes that the high-energy behaviour of the amplitudes is dominated by the exchange of the vacuum (C = 1, parity +), Pomeranchuk Regge-pole.

In order to have a non-vanishing polarization in the high-energy limit, both C = 1 and C = -1 exchange amplitudes must give contributions to the high-energy scattering. This case has been studied recently by Dosch and Friedman[18], within the framework of a particular model. These authors calculated the polarization at high energy in $\pi^{\pm}p$ elastic scattering under the assumption that the amplitude is dominated by two terms: a) a C = 1 exchange, spin independent part, which was assumed to be purely imaginary, and b) the elementary ρ-meson exchange contribution (corrected for unitarity), i.e. the exchange of a particle with C = -1 and negative parity. Both these terms behave like $s \cdot f(t)$ at high energy and therefore according to Eq. (4.8) the calculations give the opposite sign for the polarization in the $\pi^{+}p$ and $\pi^{-}p$ scattering.

Let us now consider the behaviour of the other polarization parameters Y and Z, as they are defined in Eqs. (2.5). They are directly measurable in the A and R experiments, as explained in Section 2. Using the relations (4.3) and the formulae (2.5), one easily gets

$$\lim_{E\to\infty} Y^{+}(t) = \lim_{E\to\infty} Y^{-}(t) \tag{4.14}$$

and

$$\lim_{E\to\infty} Z^{+}(t) = \lim_{E\to\infty} Z^{-}(t) \tag{4.15}$$

where the superscripts + and - refer to the $\pi^{+}p$ and $\pi^{-}p$ scattering, respectively. The relations (4.14) and (4.15) show that the high-energy limits of the polarization parameters R and A are the same for $\pi^{+}p$ and $\pi^{-}p$ elastic scattering.

Because of isotopic spin invariance, the charge-exchange scattering is related to the elastic $\pi^{+}p$ and $\pi^{-}p$ scattering: the amplitude for charge-exchange is proportional to the difference between the amplitudes for $\pi^{+}p$ and $\pi^{-}p$ elastic scattering. Denoting the amplitude for charge-exchange by M, we have

$$M = \frac{1}{\sqrt{2}}(G - H) \tag{4.16}$$

It follows from this formula that the charge-exchange scattering is described only by the C = -1 exchange amplitude of the elastic scattering. Accordingly, we write

$$M_{\mu\lambda} = m_{\mu\lambda}(t)\, s^{\gamma(t)} + \ldots \tag{4.17}$$

where $\gamma(t)$ is the exponent in the leading term of the C = -1 exchange amplitude. Since the functions m_{++} and m_{+-} satisfy [cf. Eq. (3.11)]

$$m_{++}^{*} = - m_{++}\, e^{i\pi\gamma} \qquad m_{+-}^{*} = - m_{+-}\, e^{i\pi\gamma} \tag{4.18}$$

we have

$$\begin{aligned} \mathrm{Im}\,(M_{++} M_{+-}^{*}) &= s^{2\gamma}\, \mathrm{Im}\,(m_{+-}^{*}\, m_{++}) = \\ &= s^{2\gamma}\, \mathrm{Im}\,(m_{++}^{*}\, m_{+-}) = -\mathrm{Im}\,(M_{++} M_{+-}^{*}) \end{aligned} \tag{4.19}$$

and therefore

$$\lim_{E\to\infty} P^{ex} = 0 \tag{4.20}$$

that is, the polarization of the recoil neutron in the charge-exchange scattering on an unpolarized target vanished in the limit of extremely high energy[1,2]).

5. HIGH-ENERGY BEHAVIOUR OF THE POLARIZATION

To discuss the behaviour of the polarization at high, but finite energies (as opposed to the high-energy limit discussed in the preceding section), one should go beyond the first approximations (4.1) and (4.2). In this section we shall consider the second approximation, i.e. we shall also keep the second term in the expansions (3.3) and (3.4). Furthermore, we shall assume that the high-energy, small momentum transfer elastic $\pi^{\pm}p$ amplitude is dominated by the C = 1 exchange. This assumption is strongly suggested by the present experimental data which show that the energy dependence of the cross-section for elastic πp scattering (where both C = 1 and C = -1 exchange may contribute) is very weak, whereas the cross-section for charge-exchange scattering (where only C = -1 exchange occurs) decreases rather quickly with increasing energy (approximately in the region above 5 GeV/c, $\sigma_{ch.ex.} \propto 1/s$)[6,19-21].

The assumption that C = 1 exchange dominates is expressed by the equalities [see Eq. (4.11)]

$$h_{++} = g_{++} \qquad h_{+-} = g_{+-} \tag{5.1}$$

The coefficients $g^{(1)}$ and $h^{(1)}$ contain, in general, contributions from both C = 1 and C = -1 exchange. It is therefore convenient to write

$$g^{(1)} = b + d \tag{5.2}$$

$$h^{(1)} = b - d \tag{5.3}$$

where for simplicity we have suppressed the helicity indices. It is clear from Eqs. (3.8) and (3.9) that b and d describe the C = 1 and C = -1 exchange, respectively. The general formulae (3.10) and (3.11), have the following consequences for the coefficients h = g, b and d

$$g^{*} = g\, e^{i\pi\alpha} \tag{5.4}$$

$$b^{*} = b\, e^{i\pi\beta} \;, \qquad d^{*} = -d\, e^{i\pi\beta} \tag{5.5}$$

or, equivalently,

$$\mathrm{Re}\, g = -\mathrm{ctg}\,\frac{\pi\alpha}{2}\,\mathrm{Im}\, g \tag{5.4'}$$

$$\mathrm{Re}\, b = -\mathrm{ctg}\,\frac{\pi\beta}{2}\,\mathrm{Im}\, b \qquad \mathrm{Re}\, d = \mathrm{tg}\,\frac{\pi\beta}{2}\,\mathrm{Im}\, d \tag{5.5'}$$

The formulae (5.4) and (5.5) show that the present approach describes the elastic and charge-exchange πp scattering in terms of eight parameters (which are functions of t): α, β, Im g_{++}, Im g_{+-}, Im b_{++}, Im b_{+-}, Im d_{++} and Im d_{+-}. The number of free parameters being rather large, it is impossible to determine all of them from the present experimental data without further simplifying the assumptions. We shall therefore give only a qualitative discussion of some possibilities which arise in this context.

In the most general case, if we make no further restrictions on the eight parameters entering our formulae, the situation is very similar to that in the Regge-pole model of high-energy elastic scattering. As is well known, the experimental data suggest the presence of at least three Regge-poles in the high-energy πp scattering[6,22]. They are the two vacuum Regge-poles P and P′ and the reggeized ρ. In our approach we also have three different contributions to the amplitude, i.e. g, b, and d amplitudes, which describe the exchange of the same quantum numbers as P, P′ and ρ, respectively. The only difference is that according to our philosophy it is more natural to assume that the parameter $\beta(t)$ (i.e. in the Regge-pole language, the trajectory of the Regge-pole in the physical region of the s-channel) is common for the b and d amplitudes, whereas in the Regge-pole approach one takes two different trajectories[23]. It is interesting to note that in the actual cross-section calculations in the Regge-pole model for elastic πp, Kp and pp scattering, which have been done recently by Phillips and Rarita[6], the parameters of these two trajectories appear to be very close to each other. Therefore, the general discussion in our case would be very similar to that given by Phillips and Rarita[7] and we shall not try to present it here.

6. CONSEQUENCES OF THE OPTICAL MODEL FOR THE HIGH-ENERGY BEHAVIOUR OF THE POLARIZATION IN THE $\pi^{\pm}p$ ELASTIC SCATTERING

As pointed out by Phillips and Rarita[7], and by Leader[22], the polarization measurements represent a very important test of the Regge-pole model of the elastic scattering at high-energy. The reason is that the Regge-pole model predicts a rather large spin-flip part, comparable to the non-spin-flip contribution, in the amplitudes dominating the elastic scattering in the limit of extremely high energy (i.e. in the Pomeranchuk exchange term). On the other hand, it seems that the optical models rather favour the situation in which the spin-flip part of the amplitude is negligible compared to the non-spin-flip part in the high-energy limit[24]. It is therefore interesting to investigate the consequences of that second possibility, and see whether one can get out of it some definite predictions for the present machine energies. In terms of our parameters, this assumption reads

$$g_{+-} = 0 \tag{6.1}$$

We shall also assume that, for t fixed and energy tending to infinity, the elastic scattering tends to a finite, non-vanishing limit. This assumption (which is in contrast to the Regge-pole model prediction), seems more natural in the framework of the optical model, and is also strongly suggested as the simplest extrapolation of the present experimental data[25, 26]. The existence of a finite, non-vanishing limit of the elastic differential cross-section implies that

$$\alpha(t) = 1 \tag{6.2}$$

The immediate consequence of the condition (6.2) and the relation (5.4) is

$$Re\, g_{++} = 0 \tag{6.3}$$

i.e.

$$g_{++} = i\, c \tag{6.4}$$

where c is a real function of t.

Taking into account the conditions (6.1) and (6.4), we arrive at the following formulae for the cross-sections and the polarization parameters in the $\pi^+ p$ and $\pi^- p$ elastic scattering:

$$\frac{d\sigma^\pm}{dt} = \frac{1}{16\pi} |c|^2 \left\{ 1 + 2 \frac{\mathrm{Im}(b_{++} \pm d_{++})}{c} s^{\beta-1} \right\} \tag{6.5}$$

$$P^\pm = 2 s^{\beta-1} \frac{\mathrm{Re}(b_{+-} \pm d_{+-})}{c} \tag{6.6}$$

$$Y^\pm = 1 \tag{6.7}$$

$$Z^\pm = 2 s^{\beta-1} \frac{\mathrm{Im}(b_{+-} \pm d_{+-})}{c} \tag{6.8}$$

From these formulae we first see that the assumption (6.1) enables us to estimate the ratio $|d_{++}/d_{+-}|^2$ in terms of the differential cross-sections. We have

$$\Lambda^2 \equiv \left|\frac{d_{++}}{d_{+-}}\right|^2 = \frac{\left(\frac{d\sigma^+}{dt} - \frac{d\sigma^-}{dt}\right)^2}{2\Omega \frac{d\sigma^{ex}}{dt}\left(\frac{d\sigma^+}{dt} + \frac{d\sigma^-}{dt}\right) - \left(\frac{d\sigma^+}{dt} - \frac{d\sigma^-}{dt}\right)} \tag{6.9}$$

where

$$\Omega = 2 \cos^2 \frac{\pi\beta}{2} \tag{6.10}$$

We expect this ratio to be rather small in the region $0.05 \lesssim -t \lesssim 0.4$. First of all, the curved shape of the charge-exchange elastic cross-section for small t in the logarithmic scale suggests the presence of a rather large spin-flip amplitude in this process[6,20,21,27,28]. Furthermore, the difference between elastic π^+p and π^-p differential cross-sections changes sign when $-t$ increases from zero to higher values: $(d\sigma^+/dt) - (d\sigma^-/dt)$ is negative for $t = 0$, and definitely positive for $-t > 0.2\ (\mathrm{GeV}/c)^2$. In the case we discuss, this change of sign can be explained only by the fact that d_{++} passes through zero in this region because the assumption (6.1) rules out possible cancellations from terms involving g_{+-}[29]. In fact, the experimental data at 13 GeV/c primary momentum suggest

$$\Lambda^2 \lesssim 0.5 \qquad\qquad 0.05 \lesssim -t \lesssim 0.4 \tag{6.11}$$

We are now in a position to discuss the possible value of the difference $p^+ - p^-$ between the polarization in the π^+p and π^-p elastic scattering in the t region indicated by the formula (6.11). The sign of this difference depends on the sign of the Re d_{+-} which is unknown. For the absolute value we have

$$|P^+ - P^-| = 4\ \frac{s^{\beta-1}\,|\mathrm{Re}\, d_{+-}|}{c} \tag{6.12}$$

This formula can be re-written in terms of the measurable quantities in the following way

$$|P^+ - P^-| = 4 \sin\frac{\pi\beta}{2} \sqrt{\frac{d\sigma^{ex}/dt}{d\sigma^+/dt + d\sigma^-/dt}}\ (1+\Lambda^2)^{-\frac{1}{2}} \tag{6.13}$$

where we have used the relations (5.5). For $t \to 0$ at fixed energy the parameter $\Lambda \to \infty$ because $d_{+-} = 0$ for $t = 0$, and therefore $P^+ - P^- \to 0$, as expected[30].

The parameter $\beta(t)$ has been estimated by Logan[31], by Phillips and Rarita[6] and by Höhler[21], to be not very different from 0.5 in the region $|t| \lesssim 0.4$ $(\mathrm{GeV/c})^2$. This information, together with the estimation (6.11), enables us to estimate the possible values of $|P^+ - P^-|$ in that region. Assuming that $\beta(t)$ is actually equal to 0.5, and taking into account the estimation (6.11), we get

$$|P^+ - P^-| \simeq 2\sqrt{2}\left\{\frac{d\sigma^{ex}/dt}{d\sigma^+/dt + d\sigma^-/dt}\right\}^{1/2} \qquad (6.14)$$

The value of the right-hand side of the Eq. (6.14) is ~ 0.3 at 6 GeV/c and then decreases approximately as $p^{-1/2}$ where p is the incident laboratory momentum. Let us also remark that, independently of the values of the parameters β and Λ, the formula (6.13) gives an upper limit for the value $|P^+ - P^-|$ which is $\sim \sqrt{2}$ times larger than the value given by (6.14).

Similar estimations can be obtained for the difference between the polarization coefficients Z^+ and Z^-. The formulae (6.6) and (6.8) as well as the condition (5.5′), imply

$$Z^+ - Z^- = (P^+ - P^-)\, \mathrm{tg}\, \frac{\pi\beta}{2} \qquad (6.15)$$

Assuming, as before, $\beta \simeq 0.5$, we get

$$Z^+ - Z^- \approx P^+ - P^- \qquad (6.16)$$

which shows that the difference $Z^+ - Z^-$ must also be rather small at the present machine energies.

The parameters Y^+ and Y^- are equal to 1 up to terms of order less than $s^{\beta - \alpha}$, therefore we expect their difference to be very small.

Very little can be said on the sum of the polarization parameters in the $\pi^+ p$ and $\pi^- p$ elastic scattering, because their values depend on b_{+-}, which we have no possibility of estimating from the present data. Let us only note the relation between $(P^+ + P^-)$ and $(Z^+ + Z^-)$ which follows from the condition (5'5):

$$Z^+ + Z^- = -\,tg\,\frac{\pi\beta}{2}\left(P^+ + P^-\right) \qquad (6.17)$$

In conclusion, we see that the conditions (6.1) and (6.2) give rather strong implications for the behaviour of the polarization in the elastic π^+p and π^-p elastic scattering at present machine energies[32]. One can summarize them as follows:

a) There exist simple relations (6.15) and (6.17) between the polarization parameters $P^{\pm}$ and $Z^{\pm}$ of the elastic scattering [see Section 2, in particular Eqs. (2.5) and (2.11) for the physical meaning of P and Z], and the parameter β, which determine the rate of change of the π^-p charge-exchange cross-section with energy.

b) The present data on the π^-p charge-exchange scattering provide an upper limit for the absolute value of the difference of the polarization parameters $P^+ - P^-$ and $Z^+ - Z^-$ in the π^+p and π^-p elastic scattering.

c) The polarization parameter Y (cf. Section 2 for its definition) is expected to be very close to 1 for both π^+p and π^-p elastic scattering.

If the experimental data in the region, say around 10 GeV/c, will show that any of the statements a), b), or c) is incorrect, this will strongly suggest that the assumption (6.1) is not valid[32], i.e. that some spin-flip amplitude is present even at highest energies.

We now turn to the discussion of the charge-exchange scattering. As has been shown in Section 5, the polarization P^{ex} vanishes in the first approximation. To estimate the second approximation, one has to introduce further terms in the expansion of the C = −1 exchange amplitude. As at present we know absolutely nothing about these fine effects, we can give no serious estimation of the polarization in the high-energy charge-exchange scattering. There are no compelling reasons why P^{ex} has to be zero at the present machine energies. It has, however, to decrease with energy approximately like $p^{-1/2}$ where p is the primary laboratory momentum[33].

The limiting values of the polarization parameters Y^{ex} and Z^{ex} for extremely high energy can be expressed in terms of the ratio Λ^2, defined by the Eq. (6.9), in the following way:

$$Y^{ex} \simeq \frac{\Lambda^2 - 1}{\Lambda^2 + 1} \tag{6.18}$$

$$Z^{ex} = \frac{2\Lambda}{\Lambda^2 + 1} \tag{6.19}$$

It seems that the corrections to the formulae (6.18) and (6.19), induced by the higher order terms should not be larger than 10% for 10 GeV/c primary momentum[33].

7. CONCLUSIONS

Assuming the existence of an asymptotic expansion in terms of real powers of the c.m. energy, and the "usual" analyticity properties (see Section 3) of the elastic π^+p and π^-p amplitudes, we have discussed the behaviour of the polarization in π^+p and π^-p elastic scattering at high energy. A considerable simplification, as compared to the standard treatment[1,2] has been obtained by using consequently the helicity formalism[4,8]. We have considered both the first and the second approximations in the asymptotic expansion, i.e. (a) the limiting values of the polarization, and (b) the way these limiting values are reached.

The problem (a) has been discussed before by Logunov et al.[1] and by Van Hove[2]. We have re-derived their results in a form which is more convenient in the direct application to the experimental data. For the reader's convenience, we repeat here the main results one can obtain by this method.

c 1. In the limit of extremely high energy, the polarization of the recoil proton in the scattering plane is the same for π^+p and π^-p elastic scattering. The polarization for scattering on an unpolarized target has the same absolute value for π^+p and π^-p elastic scattering, but with the opposite signs.

2. The additional assumption that the charge-exchange cross-section tends to zero in the high-energy limit implies that the polarization of the recoil proton in π^+p and π^-p elastic scattering on an unpolarized target also vanishes in that limit.

3. In the π^-p charge-exchange scattering on an unpolarized target, the polarization of the recoil neutron vanishes in the limit of extremely high energy.

In the investigation of the problem (b) we have analysed in some detail the consequences of the assumption that the non-spin-flip elastic amplitude dominates the πp elastic scattering in the high-energy limit, which corresponds to the diffraction model of the elastic scattering. It appears that this assumption, combined with the present experimental data on charge-exchange and elastic πp scattering, provides some interesting, experimentally measurable consequences for the polarization in the πp elastic scattering in the few GeV/c region. They are listed at the end of the Section 6. It seems that the experimental verification of these consequences would be of some importance for our understanding of the elastic scattering at high energies.

Acknowledgements

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APPENDIX

The Wick Rotation

In this Appendix we prove the relation (2.9) for the Wick rotation angle ω entering in the Lorentz transformation of the recoil nucleon's helicity states in going from the laboratory system to the c.m.s.

Let us introduce the following notation, besides that of Figs. 1 and 2:

$\cosh\tau = \gamma(v) \equiv \frac{1}{\sqrt{1-v^2}}$ $\quad$ $v = |\vec{k}_N|/m$ = velocity of c.m.s. in the laboratory system

$\sinh\tau = v\cdot\gamma(v)$,

$\cosh\sigma = \gamma(u)$, $\quad$ $u = |\vec{k}_R|/m$ = velocity of recoil nucleon in the laboratory system,

$\cosh\sigma' = \gamma(u')$, $\quad$ $u' = |\vec{k}_{N'}|/m$ = velocity of the recoil nucleon in the c.m.s.

From the sinus and cosinus theorems for the Wick triangle[10] of Fig. 2b, we obtain

$$\frac{\sin\omega}{\sinh\tau} = \frac{\sin\theta_R}{\sinh\sigma'} \tag{A.1}$$

$$\cos\omega = \frac{\cos\sigma\cosh\sigma' - \cosh\tau}{\sinh\sigma\sinh\sigma'} = \tag{A.2}$$
$$= -\cos\theta\cos\theta_R + \sin\theta\sin\theta_R\cosh\tau$$

the last equality following easily from the Lorentz transformation formulae

$$-|\vec{k}_R|\cos\theta_R = \gamma(u)\left\{|\vec{k}_{N'}|\cos\theta - uE_{N'}\right\} \tag{A.3 a}$$

$$|\vec{k}_R| \sin\theta_R = |\vec{k}_{N'}| \sin\theta \tag{A.3 b}$$

$$E_R = \gamma(u)\left\{E_{N'} - u|\vec{k}_{N'}|\cos\theta\right\} \tag{A.3 c}$$

If we now observe that u′ = v, since the reaction considered is truly elastic, we get immediately from Eqs. (A.1), (A.2) and (A.3)

$$\sin\omega = \sin\theta_R \tag{A.4 a}$$

$$\cos\omega = \cos\theta_R \tag{A.4 b}$$

The equality (2.9) is thus proved.

In terms of the invariant kinematical variables [see Ref. 4], we have

$$\lambda^{\frac{1}{2}}(s, m^2, \mu^2)\ \lambda^{\frac{1}{2}}(t, m^2, \mu^2)\cos\theta_R = -t(s+m^2-\mu^2) \tag{A.5}$$

where the function λ is defined in Eq. (2.2 b). At high energy, in particular,

$$\cos\theta_R = \frac{\sqrt{-t}}{\sqrt{4m^2-t}}\left\{1 + O(s^{-1})\right\} \tag{A.6}$$

for $s \to \infty$, t fixed.

REFERENCES AND FOOTNOTES

1) A.A. Logunov, Nguyen Van Hieu and I.T. Todorov, Ann.Phys. 31, 203 (1965).

2) L. Van Hove, "Theory of high-energy scattering. Some consequences of analyticity in energy", Trieste preprint (May 1965).

3) L. Van Hove, "Theory of strong interactions of elementary particles in the GeV region", Invited talk, International Symposium on Electron and Photon Interactions at High Energies (June 8 - 12, 1965) (CERN -TH. 577, June 1965).

4) A. Białas and B.E.Y. Svensson, "High-energy relations for scattering of particles and antiparticles with arbitrary spin following from analyticity and crossing relations", CERN Preprint TH. 633, December 1965.

5) A. Białas and E. Białas, Nuovo Cimento 37, 1686 (1965).

6) R.J.N. Phillips and W. Rarita, Phys.Rev. 139, B1336 (1965), from which earlier works on Regge poles may be traced.

7) R.J.N. Phillips and W. Rarita, "High-energy polarization experiments and Regge poles" (UCRL-16185, June 1965, to be published in Phys.Rev.), which also contains references to previous work.

8) M. Jacob and G.C. Wick, Ann.Phys. 7, 404 (1959).

9) How this transformation should be done in the usual polarization formalism is described in H.P. Stapp, Phys.Rev. 103, 425 (1956). Cf. also P.D. Grannis, "Measurements of the polarization parameters in proton-proton scattering from 1.7 to 6.1 GeV" (Ph.D. Thesis, UCRL-16070, August 1965).

10) G.C. Wick, Ann.Phys. 18, 65 (1962), in particular the Appendix.

11) In Ref. 4), all crossing relations were obtained up to a helicity-independent phase factor. To determine this phase in the case under study, it is simplest to take a particular field-theoretical example (e.g. ρ exchange). The result is that of Eqs. (3.1) and (3.2).

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12) For $\alpha \geqslant 1+\beta$ the relation (3.5 b) is more complicated[4].

13) Such a treatment is not based on any particular model of the high-energy scattering and therefore seems to be more useful in the phenomenological discussion.

14) See, for example, V. Singh, Phys.Rev. 129, 1889 (1963).

15) L. Van Hove, Physics Letters 5, 252 (1963).

16) I.Ia. Pomeranchuk, Zh. Eksperim.i.Teor.Fiz. 34, 725 (1958), English translation: Soviet Phys. JETP 7, 499 (1958).

17) This is a special case of a more general relation, see Eq. (6.7) of Ref. 4). Cf. also Refs. 1) and 2).

18) H.G. Dosch and A. Friedman, "Polarization in high-energy pion-proton scattering", Heidelberg preprint (1965).

19) A.V. Stirling, P. Sonderegger, J. Kirz, P. Falk-Vairant, O. Guisan, C. Bruneton, P. Borgeaud, M. Yvert, J.P. Guillard, C. Caverzasio and B. Amblard, Phys.Rev.Letters 14, 763 (1965).

20) I. Manelli, A. Bigi, R. Carrara, M. Wahlig, and L. Sodickson, Phys. Rev.Letters 14, 408 (1965).

21) G. Höhler, J. Baacke and H. Schaile, Preprint, November 1965.

22) E. Leader, "The present phenomenological status of the Regge pole theory" (UCRL-16361, August 4, 1965).

23) We assumed also that energy dependence is the same for the spin-flip and non-spin-flip amplitudes. As pointed out recently by Van Hove (private communication), a small difference in the energy dependence of these two amplitudes can possibly explain the shrinkage with energy of the forward peak in the $\pi^- p$ charge-exchange scattering. Such small effects are, however, unimportant for our conclusions.

24) It is rather difficult to define precisely what is really the high-energy optical model. Here, we mean by "optical model" any model of the high-energy elastic scattering based on the idea of diffraction. See Ref. 22) for a more detailed discussion of the problem.

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25) K.J. Foley, S.J. Lindenbaum, W.A. Love, S. Ozaki, J.J. Russel and L.C.L. Yuan, Phys.Rev.Letters 11, 425 (1963).

26) This condition simplifies the discussion considerably, but it seems not very crucial for the conclusions at which we arrived. This is due to the fact that as shown, for example by Phillips and Rarita[6], even in the Regge pole model, the change of α with t is not very large in the small t region [approximately $\alpha(t) \simeq 1 + 0.3\ t$]. Therefore, the corresponding corrections are small.

27) G. Cohen-Tannoudji, A. Morel and H. Navelet, Physics Letters 19, 62 (1965).

28) H. Högaasen and J. Högaasen, Nuovo Cimento 39, 941 (1965).

29) See Reference 6), Section 3.

30) For t = 0, the spin-flip amplitude must vanish and therefore the polarization also vanishes. For any $t \neq 0$, however, the condition $P^+ = 0$ (or $P^- = 0$) is a dynamical assumption.

31) R.K. Logan, Phys.Rev.Letters 14, 414 (1965).

32) As explained in footnote 26), the condition (6.2) is rather inessential for our conclusions.

33) To estimate the order of magnitude of the corrections to the first approximation, one can use the following crude argument. The ratio of the second order corrections to the first order terms is of the order 0.1 at 10 GeV/c, as seen, for example, from the ratio of the charge exchange to the elastic scattering. One could therefore expect the same ratio between the third and the second order corrections. This gives P^{ex} of the order 0.1 at 10 GeV/c.

Figure captions

Fig. 1 : Illustrating the kinematics and the notation:

a) in the c.m.s;

b) in the laboratory system; as well as

c) the Lorentz transformation from the laboratory system to the c.m.s.

Fig. 2 : Illustrating the derivation of the Wick rotation angle ω:

a) the momentum vectors of the recoil nucleon in the c.m.s. and in the laboratory system;

b) the corresponding Wick triangle.

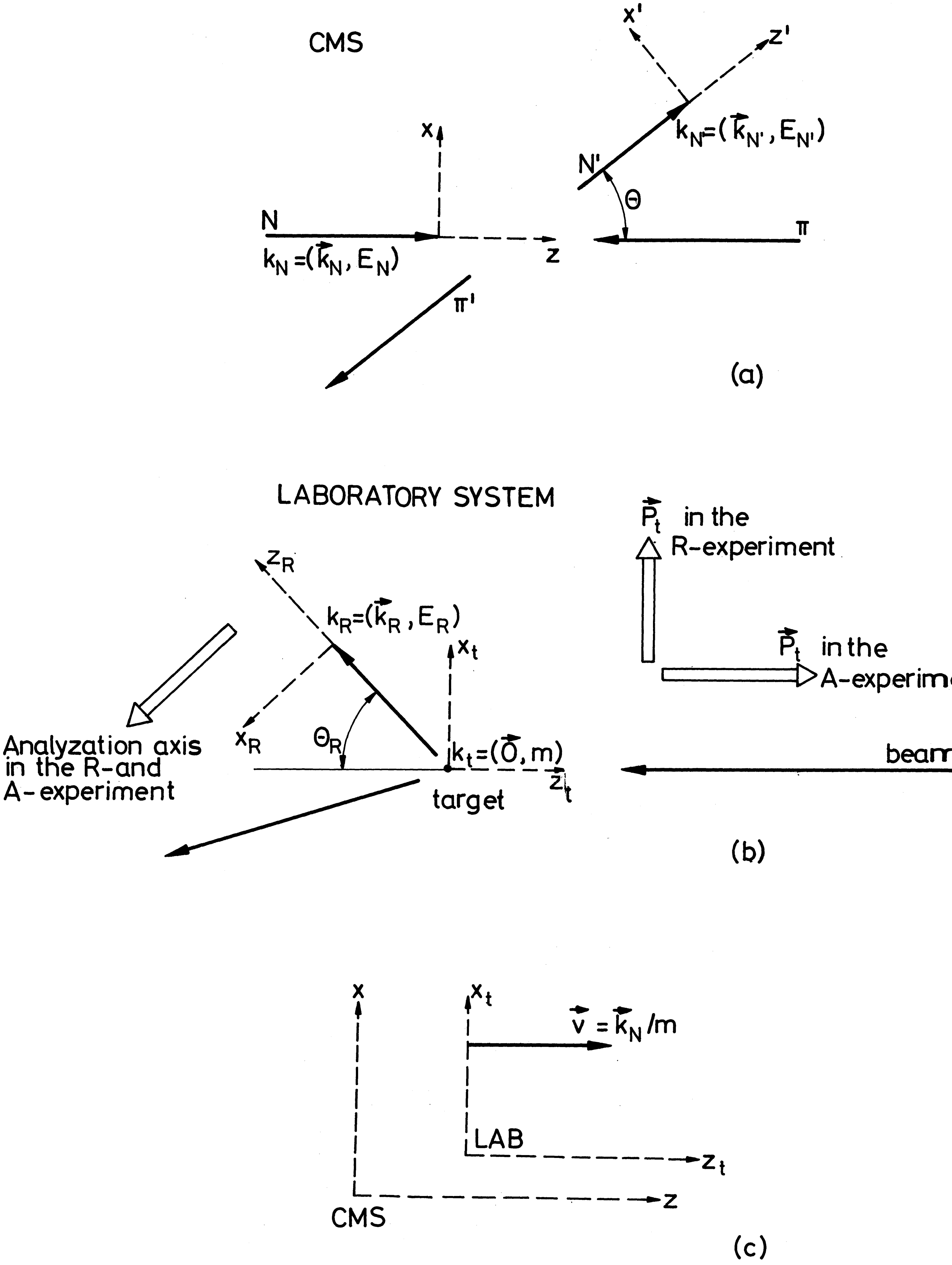

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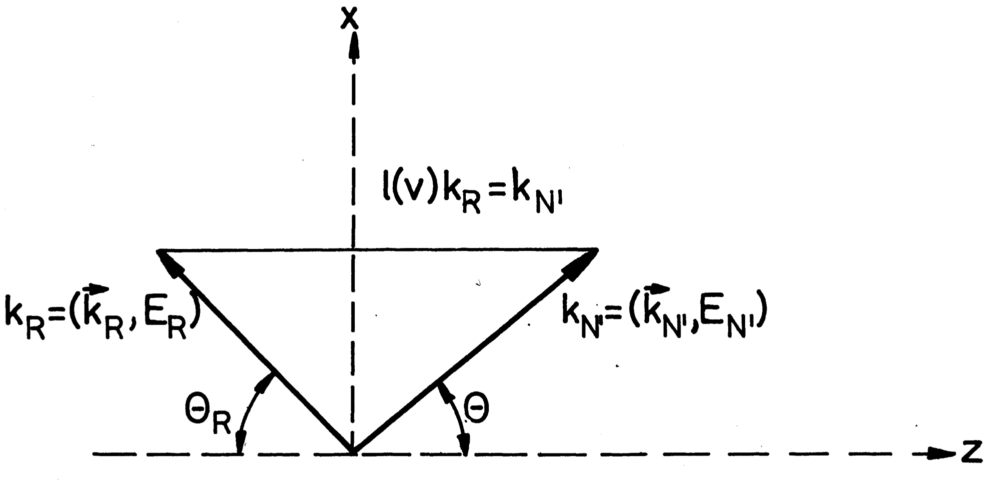
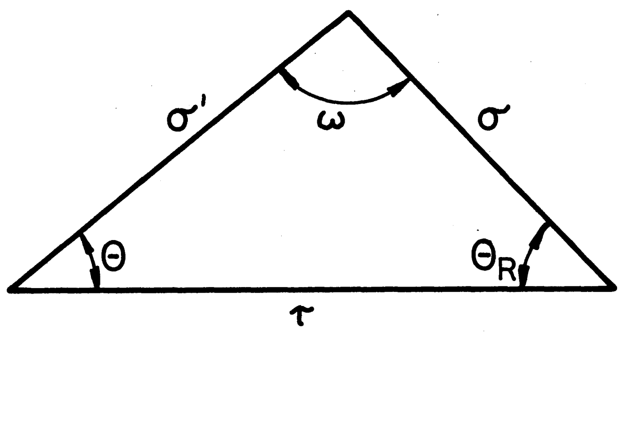
FIG.1



(a)



(b)

FIG. 2

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Erratum

A. Białas and B.E.Y. Svensson, Polarization in high-energy πp elastic and charge exchange scattering (CERN preprint TH.635, 17 December 1965)

The following misprints have been found:

p.2, line 7: read "optical model" instead of "optical models"

p.4, line 7: read "referred to the" instead of "referred to as the"

p.5, line 13: read "analyzing" instead of "analysing"

p.5, line 15: read " $\vec{v} = \vec{k}_N / E_N$ " instead of " $\vec{v} = \vec{k}_N / m$ "

p.9, last line: read " $|s| \to \infty$ " instead of " $s \to \infty$ "

p.10, line 11: delete "very rapidly"

p.21, Eq. (6.9): the expression $\left(\frac{d\sigma^+}{dt} - \frac{d\sigma^-}{dt} \right)$ in the denominator should be squared.

p.22, line 15: read "difference $P^+ - P^-$ " instead of "difference $p^+ - p^-$ "

p.27-28: the definitions of v, u and u' should read, respectively:

$$v = |\vec{k}_N| / E_N \ , \qquad u = |\vec{k}_R| / E_R \ , \qquad u' = |\vec{k}_{N'}| / E_{N'} \ ;$$

moreover, " u " should be changed to " v " at all places in Eqs. (A.3 a-c) .

Fig. 1 (c): read " $\vec{v} = \vec{k}_N / E_N$ " instead of " $\vec{v} = \vec{k}_N / m$ "

Fig. 2 (a): read " $l(\vec{v})\ k_R$ " instead of " $l(v)\ k_R$ "

(14.1.1966)