REGGE POLES AND THE PRODUCTION OF HIGH ENERGY PARTICLES

BY γ RAYS

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A B S T R A C T

If exchanged particles in a peripheral collision are Regge objects, the cross-section at forward angles will differ in a specific way from what is predicted by the ordinary peripheral model. Thus experiments of peripheral collision type may be used to test the Regge pole hypothesis in particle physics. In this note we examine the photoproduction of high energy particles in the forward direction from this point of view.

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Recent measurements of nucleon-nucleon, nucleon-antinucleon and pion-nucleon scattering suggest strongly that these processes are dominated at high energy by a small number of Regge poles [1]. The hypothesis of Regge poles may turn out to be so fundamental in our understanding of particle physics [2] that it is desirable to test its validity by all possible means. In this note we should like to examine the possibility of using the photoproduction of high energy pions, kaons and (anti) baryons in the forward direction for this purpose. This process was originally proposed by Drell [3] as a possible means of generating a collimated beam of high energy particles by γ rays. The corresponding cross-sections are much larger than those predicted by the statistical model for nucleon initiated processes, in so far as the particle exchanged between the incident γ ray and the target retains its identity as an elementary particle for any incident energy in the conventional field theoretical sense. Therefore, it is interesting to examine how much this is modified if the exchanged object is a Regge pole rather than an elementary particle.

We will consider first the production of high energy charged pions. In the Regge pole approach the process will be governed by Regge trajectories associated with the π and the ρ meson (Fig. 1), but will have nothing to do with the Pomeranchuk trajectory, ω meson trajectory or P' trajectory [4]. Thus it will be particularly useful for the purpose of exploring whether the pion and/or ρ meson are elementary particles or not [5].

If we assume that the theorem of factorization [6] applies to this case, the contribution of π and ρ meson trajectories to the S matrix may be written as [7]

$$f_{\gamma\pi\pi}(t)\, f_{NN^*\pi}(t)\, \frac{1 + e^{-i\pi\alpha_\pi(t)}}{\sin \pi\alpha_\pi(t)}\, P_{\alpha_\pi(t)}\left(-x_t\right), \tag{1}$$

$$f_{\gamma\pi\rho}(t)\, f_{NN^*\rho}(t)\, \frac{1 - e^{-i\pi\alpha_\rho(t)}}{\sin \pi\alpha_\rho(t)}\, P_{\alpha_\rho(t)}\left(-x_t\right), \tag{2}$$

2.

respectively. Here, t is the square of the momentum transfer between γ and π and

$$\chi_t = \frac{2t(s - M^2) + (t - \mu^2)(t - M_n^2 + M^2)}{(t - \mu^2)\sqrt{t - (M_n - M)^2}\sqrt{t - (M_n + M)^2}}$$

is the cosine of the scattering angle in the crossed channel t, where s is the square of the total energy of γ and N in the centre of mass system, M_n is the total energy of the n outgoing particles (Fig. 1) in their own centre of mass system. Finally, $f_{\gamma\pi\pi}(t)$, etc., describe the $\gamma\pi\pi$ vertex, etc.

The contribution of pion and ρ meson trajectories to the cross-section may be written as [8]

$$d\sigma_\pi^R = F_\pi(s,t)\, d\sigma_\pi^0 , \tag{3}$$

$$d\sigma_\rho^R = F_\rho(s,t)\, d\sigma_\rho^0 . \tag{4}$$

In these equations F_π and F_ρ are given by :

$$F_\pi(s,t) \approx \left| \frac{\pi}{2}\alpha_\pi'(\mu^2)(t - \mu^2)\frac{1 + e^{-i\pi\alpha_\pi(t)}}{\sin\pi\alpha_\pi(t)}\left(\frac{s}{S_0}\right)^{\alpha_\pi(t)} \right|^2 , \tag{5}$$

$$F_\rho(s,t) \approx \left| \frac{\pi}{2}\alpha_\rho'(m_\rho^2)(t - m_\rho^2)\frac{1 - e^{-i\pi\alpha_\rho(t)}}{\sin\pi\alpha_\rho(t)}\left(\frac{s}{S_0}\right)^{\alpha_\rho(t) - 1} \right|^2 , \tag{6}$$

where s_0 is a constant with the dimension of mass squared. They are so chosen that $F_\pi (s, \mu^2) = F_\rho (s, m_\rho^2) = 1$ are satisfied or, in other words, $d\sigma_{\pi,\rho}^R$ agrees with $d\sigma_{\pi,\rho}^0$ at $t = \mu^2$, m_ρ^2 respectively. $d\sigma_{\pi,\rho}^0$ are the cross-sections due to one π and one ρ exchange when these particles are assumed to be elementary. Two particular reactions will be considered in detail :

$$\gamma + N \longrightarrow \pi + N \; , \tag{7a}$$

$$\gamma + N \longrightarrow \pi + N_1^* \; , \tag{7b}$$
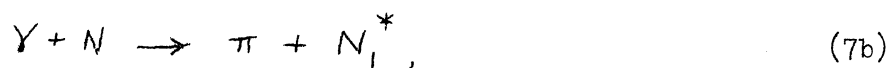
where N_1^* is the $J = T = 3/2$ pion-nucleon isobar.

For reaction (7a) the differential cross-sections $d\sigma_{\pi,\rho}^0$ at sufficiently high energy are given by :

$$d\sigma_\pi^0 = \frac{e^2}{4\pi} \frac{g^2}{4\pi} \frac{|t|}{2ks} \frac{q^3 \sin^2 \theta_q}{(t - \mu^2)^2} d\Omega_{cm} \; , \tag{8a}$$

$$d\sigma_\rho^0 = \frac{g_{\gamma\pi\rho}^2}{4\pi} \frac{g_{\rho NN}^2}{4\pi} \frac{q^3 k \sin^2 \theta_q - \frac{tq}{4ks}(t - \mu^2)^2}{(t - m_\rho^2)^2} d\Omega_{cm} \; , \tag{9a}$$

where g, $g_{\rho NN}$ are the pion-nucleon and ρ nucleon coupling constants, $g_{\gamma\pi\rho}$ is the effective coupling constant of the $\gamma\pi\rho$ vertex defined by the Lagrangian

4.

$$\mathcal{L} = g_{\gamma\pi\rho} \, \epsilon_{k\ell mn} \, \partial^k \pi_\alpha \, \partial^\ell \rho_\alpha^m \, A^n , \tag{10}$$

and θ_q is the angle of the photoproduced pion in the centre of mass system. Now to see the relative importance of pion and the ρ meson contribution at a given s, let us first consider the ratio of (8a) and (9a) at fixed $d\Omega_{CM}$. Then we obtain

$$\frac{d\sigma_\rho^0}{d\sigma_\pi^0} \simeq \frac{g_{\gamma\pi\rho}^2 \, g_{\rho NN}^2}{e^2 \, g^2} \, \frac{2k^2 s}{|t|} \left(\frac{t - \mu^2}{t - m_\rho^2} \right)^2 . \tag{11a}$$

This expression contains two poorly known coupling constants, $g_{\rho NN}$ and $g_{\gamma\pi\rho}$. If we assume that the isovector electromagnetic form factors of the nucleon are to the first approximation described by the ρ meson exchange (T = J = 1 π - π resonance at 750 MeV), then we obtain $g_{\rho NN}^2/4\pi = 1 \sim 2$ [9]. However, this interpretation is not unambiguous. Actually, it would be more appropriate for our purpose to derive $g_{\rho NN}$ from the high energy nucleon-nucleon or pion-nucleon scattering. The present experimental data analysed in the Regge pole approach seem to be not inconsistent with the above estimate but are not good enough to permit an improvement. On the other hand, all existing evidence supports the estimate $g_{\gamma\pi\rho}^2 \simeq \frac{e^2}{4\mu^2}$ of perturbation theory [10]. With this value Eq. (11a), for large s and small t of the order of $-\mu^2$ gives :

$$\frac{d\sigma_\rho^0}{d\sigma_\pi^0} \sim \frac{1}{16} \, \frac{g_{\rho NN}^2}{4\pi} \left(\frac{s}{M^2} \right)^2 . \tag{12}$$

This shows that the importance of the ρ contribution increases with s, as one expects from the fact that the spin of the ρ meson is greater than that of the pion.

To see now how Eq. (12) is modified if the pion and ρ meson are "reggeized", we will assume that for small momentum transfer $\alpha_\pi(t)$ and $\alpha_\rho(t)$ are approximated as follows :

$$\alpha_\pi(t) \simeq \frac{1}{30\mu^2}\left(t - \mu^2\right),$$ (13)

$$\alpha_\rho(t) - 1 \simeq \frac{1}{50\mu^2}\left(t - m_\rho^2\right).$$ (14)

Then, for $t = -\mu^2$ we find [11]

$$\frac{F_\rho}{F_\pi} \sim 1.4\left(\frac{s}{s_0}\right)^{-0.92}.$$ (15)

This means that, as s increases, F_ρ decreases much faster than F_π. From Eqs. (15) and (12) we obtain :

$$\frac{d\sigma_\rho^R}{d\sigma_\pi^R} \sim \frac{g_{\rho NN}^2}{4\pi}\frac{s_0 s}{12 M^4}.$$ (16)

Thus, the importance of the ρ contribution increases again with s. However, if the π and ρ are Regge objects, it follows from the last relation that the growth of the ratio $d\sigma_\rho^R/d\sigma_\pi^R$ as s increases is considerably slower than that of Eq. (12). This is due to the fact that in the region $t \sim -\mu^2$, the process of "reggeizing" the exchanged particles reduces the spin of the ρ much more than it reduces the spin of the pion. We also remark that some information on $g_{\rho NN}$ could in principle be obtained from the measurement of the energy dependence of photopion production as is seen from Eq. (16).

For reaction (7b) the differential cross-section $d\sigma^o_{\pi,\rho}$ may be written in the form :

$$d\sigma^o_\pi = \frac{e^2}{(2\pi)^3} \frac{\sqrt{(P,k-q)^2 - M^2(k-q)^2}}{(k,p)} \frac{2q^2 \sin^2(\theta_q/2)}{(t-\mu^2)^2} \sigma(\pi'N \to N^*_1) \frac{d^3q}{2\omega_q} , \quad (8b)$$

$$d\sigma^o_\rho \simeq \frac{g^2_{\gamma\pi\rho}}{(2\pi)^3} \frac{\sqrt{(P,k-q)^2 - M^2(k-q)^2}}{(k,p)} \frac{(s-M^2_{N^*_1})(s-M^2)}{2M^2_{N^*_1}} \times \quad (9b)$$

$$\times \frac{k\,q\,\sin^2(\theta_q/2)}{(t-m^2_\rho)^2} \sigma(\rho'N \to N^*_1) \frac{d^3q}{2\omega_q} ,$$

where π' and ρ' are the pion and ρ meson off the mass shell and $M_{N^*_1}$ is the mass of the N^*_1. The formula (9b) is approximate for large s and small angles. Taking the ratio of (9b) and (8b) at high s, small θ_q and fixed d^3q we obtain :

$$\frac{d\sigma^o_\rho}{d\sigma^o_\pi} \simeq r \frac{s^2}{16\mu^2 M_{N^*_1}} \left(\frac{t-\mu^2}{t-m^2_\rho}\right)^2 , \quad (11b)$$

where $r = \sigma(\rho'N \to N^*_1)/\sigma(\pi'N \to N^*_1)$. Unfortunately (11b) depends critically on the ratio $r = r(s)$ which is unknown. In general, provided that $r(s)$ does not decrease rapidly with s, the conclusions of (11b) are similar to those derived from Eq. (12). For small $|t|$ one may take

$$\sigma(\pi'N \to N^*_1) \simeq \sigma(\pi N \to N^*_1).$$

Furthermore, some information about $\sigma(\rho'N \to N^*_1)$ may be obtained from pion-nucleon charge exchange scattering. Here, the G invariance forbids one-pion exchange and therefore its high energy cross-section is expected to

be dominated by a ρ meson exchange. In the absence of better knowledge about r one might assume $r \sim \dfrac{g^2_{\rho NN}}{g^2} \lesssim \dfrac{1}{15}$. Then for the energies we are considering here the π meson trajectory will dominate at small $|t|$.

In Table I we give a few numerical values to indicate the order of magnitude of the reduction factor F_{π}[11a]. We have used $\alpha'_{\pi}(\mu^2) = \dfrac{1}{30\,\mu^2}$[12] and $s_0 \sim 2M^2$ in accordance with the analysis of nucleon-nucleon, Ref.[13], and pion-nucleon[14] scattering data. Since $F_{\pi} \sim 1$ for $k = 25$ GeV and $\theta_q \simeq 0°$, the main effect of the Regge hypothesis is simply to reduce the width of the near-forward peak of the peripheral model[15]. Thus, the integrated flux of high energy pions emitted in the forward direction will be reduced in a corresponding manner.

For exchange of Regge objects, the effective angle opening of the forward cone in which the production of energetic pions takes place, will be determined from the factor $\left(\dfrac{s}{s_0}\right)^{2\alpha_{\pi}}$ rather than the propagator $(t - \mu^2)^{-2}$. For given s we may, then, expect to have significant flux of pions for θ_q such that

$$\left(\frac{s}{s_0}\right)^{2\left(\alpha_{\pi}(\theta_q) - \alpha_{\pi}(0)\right)} \geq \frac{1}{e} \,. \tag{17}$$

With a pion Regge trajectory of slope $\alpha'_{\pi}(\mu^2) = \dfrac{1}{30\,\mu^2}$ and $s_0 \sim 2M^2$ we find that

$$\theta_{q,max} \sim \sqrt{\frac{15\,\mu^2}{k\,\omega_q\,\log\left(\frac{s}{s_0}\right)}} \,. \tag{18}$$

Numerical values for this quantity are given in Table I. For the values of k, q of Table I the order of magnitude of $\theta_{q,max}$ turns out to be the same as in Ref.[3] $(\theta_{q,max} \sim \mu/\omega_q)$.

If we assume an universal slope for all Regge trajectories, the process of "reggeizing" reduces the magnitude of various peripheral graphs more drastically when the mass of the exchanged particle is larger. We saw it already in the case of the pion and the ρ meson. We expect an even greater reduction of perturbation theoretical predictions for the exchange of kaons and antibaryons.

For instance, in the case of the photoproduction of energetic anti-nucleons (Fig. 2), a straightforward application of the rules for "reggeizing" the two-body scattering amplitude with no anomalous thresholds gives [16],[17]

$$F_N\left(s,t\right) = \left| \frac{\frac{\pi}{2}\alpha'_N(M^2)\left(t-M^2\right)}{\sin\left[\pi\left(\alpha_N(t)-\frac{1}{2}\right)\right]}\left(\frac{s}{s_0}\right)^{\alpha_N(t)-\frac{1}{2}} \right|^2 . \tag{19}$$

Using a nucleon Regge trajectory with the slope $1/50\ \mu^2$ and $s_o \sim 2M^2$, we find that $F_N(s,t) \sim 10^{-3}$ for the case $k = 25$ GeV, $\omega_q = 20$ GeV and $\theta_q = 1^\circ$. Similar large reduction of cross-section is found in the case of photoproduced kaon, too. Thus, if there is any range of energy and angle where kaons or antinucleons may be produced copiously according to the peripheral model such a region will be completely wiped out if the kaon or antinucleons are reggeized.

Although the results given in Table I must not be trusted numerically, it will be very interesting to measure the cross-section of forward particle production as a function of the incident energy $\sqrt{s}$ since some of the features of this cross-section will be recognizable with relative ease and do not depend critically on the exact values of r and s_o. Some data at $k \sim 6$ GeV from the new Cambridge accelerator may appear in the very near future.

ACKNOWLEDGEMENTS

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5)   Contogouris, Frautschi and Wong analyzed the reaction $N+N \rightarrow N+N^*$, where
        N^* is one of the pion-nucleon isobars. In the case N^* is the
        3-3 isobar they showed that the existing experimental evidence
        favors a non-elementary pion. The process considered in our paper
        will serve as an indepedent check of their conclusion.
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6)   M. Gell-Mann, Phys.Rev.Letters $\underline{8}$, 263 (1962);
        V.N. Gribov and I.Ya. Pomeranchuk, id. $\underline{8}$, 343 (1962).

7)   In the present treatment the asymptotic form of the amplitude is taken
        to be essentially the same as in potential scattering. In a more
        complete treatment, the amplitude, e.g., for $\gamma N \rightarrow \pi N$ is, at
        first, analyzed in its 4 invariant components, next the partial
        wave analysis is carried out and finally the Sommerfeld-Watson
        transform is applied to each of the 4 partial wave series. Note
        that this procedure satisfies gauge invariance in each step.

8)   The exchange of a Regge pole introduces to the amplitude a term of the form

$$\frac{\beta}{\sin \pi \alpha(t)} \left[1 \pm e^{-i\pi\alpha(t)} \right] P_{\alpha(t)} \left(-x_t \right).$$

Due to lack of detailed information we will assume that the structure of the residue function β is essentially the same as in perturbation theory. Under this assumption the interference term of the graphs (α) and (β) of Fig. 1 will be small for spin-averaged cross-sections.

9)   J. Bowcock, N. Cottingham and D. Lurié, Nuovo Cimento 16, 918 (1960) and Phys.Rev.Letters 5, 386 (1960).

10) For an estimate of $g_{\gamma\pi\rho}$ from analysis of photoproduction data see, e.g., J.S. Ball, Phys.Rev. 124, 2014 (1961); also

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On the other hand, Gramenitski, Ivanovskaja, Kanarek, Martinov, Ochrimenko, Prokesh and Tikhonova reported the measurement of the cross-section for the process $\pi^- \rightarrow \pi^- \pi^0$ in the Coulomb field of Xe at the 1962 International Conference on High Energy Physics at CERN. From this they extract the cross-section of 0.6 mb for the process $\gamma\pi^- \rightarrow \pi^0 \pi^-$ at the c.m. energy of 2.8 GeV. Since the cross-section for $\pi\pi \rightarrow \pi\pi$ is about 120 mb at the ρ resonance, this result will be in agreement with $g^2_{\gamma\pi\rho} \sim e^2/4\,\mu^2$. The order of magnitude of $g_{\gamma\pi\rho}$ is discussed, e.g., by F. Hadjioannou (CERN preprint) and A.P. Contogouris. Nuovo Cimento 25, 104 (1962).

11) For the process $\gamma N \rightarrow \pi N$ the square of the minimum momentum transfer is $|t|_{min} \simeq \mu^4 M^2/s^2$. Note that the corresponding $\cos\theta_t$ is one and some of the asymptotic forms usually employed in the application of the Regge pole hypothesis are invalid. For $t \sim -\mu^2$ and $k \sim 10$ GeV, however, it turns out to be $\cos\theta_t \gg 1$.

11a) Although the ρ contribution is larger than the pion contribution in the elastic case at the energies considered (see (16)), the one pion exchange will be more important when all channels are opened as in Fig. 1 α. See Eq. (11b) for example.

12) Contogouris, Frautschi and Wong (Ref. [5]) estimated that for small $|t|$

$$\frac{1}{35\mu^2} \lesssim \alpha'_\pi \lesssim \frac{1}{25\mu^2} \, .$$

13) S.D. Drell, see Ref. [4].

14) L.W. Jones, M.L. Perl and C.C. Peck, report at the 1962 International Conference on High Energy Physics at CERN.

15) Note that at $\theta_q = 0$ the cross-sections (8) vanish while the cross-sections for ρ exchange become very small.

16) S.C. Frautschi, M. Gell-Mann and F. Zachariasen, Phys.Rev. **126,** 2204 (1962).

17) According to V.N. Gribov (report at the 1962 International Conference on High Energy Physics at CERN), fermion Regge trajectories of same j and opposite parity coincide as $t \rightarrow +0$ and then become complex conjugate to each other as t becomes negative. Accordingly, the slope of Re $\alpha_N(t)$ at $t < 0$ will, in general, be different from that of $t > 0$. This means that even if we know the latter, it is not enough to give $F_N(s,t)$ for $t < 0$. Equation (19) should, therefore, be regarded as a very rough and oversimplified formula which is written down for the mere purpose of getting some idea of the order of magnitude.

FIGURE CAPTIONS

Fig. 1   The amplitude of the process $\gamma + N \to \pi$ + (n particles) .
approximated by the one-pion and one-ρ-meson exchange diagrams.

Fig. 2   The amplitude of the process $\gamma + N \to \bar{N}$ + (n particles)
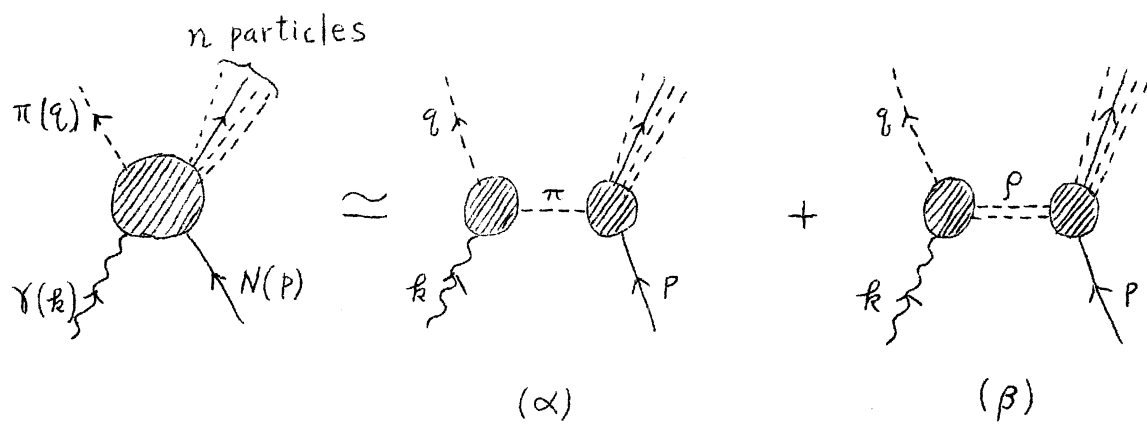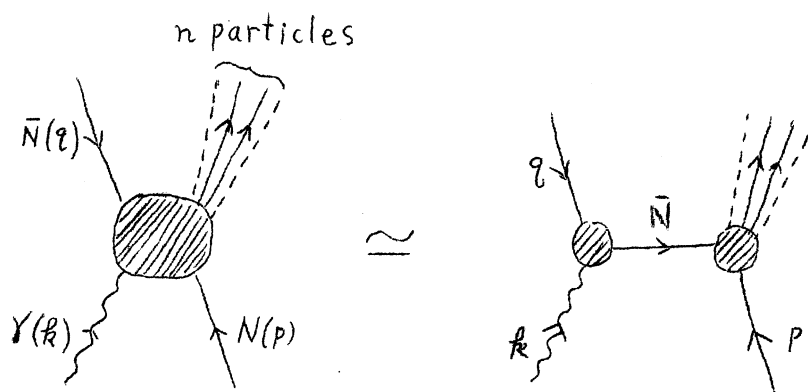approximated by the one-nucleon exchange diagram.

Fig. 1

Fig. 2