A POSSIBLE MEANS TO OBTAIN EVIDENCE CONCERNING SPIN OF THE K' MESON

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ABSTRACT

It is shown that antiproton-proton annihilation at rest into a K' and $\bar{K}$' meson can provide information about the spin of the K' if a number of assumptions are justified. These assumptions are :
1) the K' has spin either zero or one, 2) the annihilation occurs in an S-state, 3) the K' and $\bar{K}$' decay as free particles, 4) the background of annihilation events which cannot be distinguished kinematically from K'$\bar{K}$' production is small. The method consists in showing that if the K' has spin one, it is extremely unlikely that the angular distributions **will be isotropic with** respect to all the angles specified by the production and decay of the K' and $\bar{K}$'. The degree of anisotropy is not unique. However, it would require a triple coincidence for isotropy ; i.e. three relationships would have to hold among the parameters specifying the interaction. If the K' has spin zero, the angular distribution is of course isotropic. All of the assumptions are open to some question. Nevertheless, it may be possible to obtain some evidence about the spin of the K' from the angular distributions, even if the assumptions are false.

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I. Introduction

In anti-proton annihilations at rest, it may be possible to interpret a number of events according to the reaction

$$\bar{p} + p \rightarrow K' + \bar{K}' \tag{1}$$

where K' is the meson [1] reported by Alston et al. [2] with mass 885 MeV, full width 16 MeV, and known decay mode

$$K' \rightarrow K + \pi . \tag{2}$$

It is proposed to use angular correlations in the decay of the K' produced in reaction (1) as a possible means to determine the spin of the K'. In particular, it will be shown that it is useful to measure the angular distributions of the final products with respect to three different angles, which will be defined in the next section. The point of looking at three angles is the following : if the K' has spin different from zero, the angular distribution of its decay products may still accidently be isotropic. However, by looking at the decay of both K' mesons, angular correlation measurements may be made with respect to two additional angles which might still show anisotropy even in the case the angular distribution of a single K' is isotropic. A priori, it is extremely unlikely that all three angular distributions will be isotropic if the spin of the K' is different from zero.

Another analysis, making use of only one of the angles considered here, has independently been made by d'Espagnat, Prentki and Yamaguchi [3]. Still another proposal, based on the reaction $\bar{p}+p \rightarrow K'+\bar{K}$, has been made by M. Schwartz [4].

Our detailed analysis depends on the validity of a number of assumptions which are outlined below. However, it may be possible to obtain some information even if one or more of the assumptions is incorrect.

1) We assume the K' has spin either zero or one.
2) We assume, following the arguments of Day, Snow and Sucher [5], that the annihilation at rest takes place from an S-state.
3) We assume that the K' and $\bar{K}'$ decay as free particles.

In view of the small (16 MeV) width of the K', this assumption is perhaps not too bad. Each K' from the $\bar{p}\,p$ annihilation travels at about 1/3 the speed of light, and has a mean decay length of $\sim 4 \times 10^{-13}$ cm, somewhat longer than the range of interaction. In addition to neglecting dynamical final-state interactions, we neglect any effect of Bose statistics on the two final state pions.

4) We assume there is only a small background of $K\bar{K}2\pi$ events which cannot be distinguished kinematically from the desired events.

Whether this background will indeed be small is a matter that only experiment can decide. An estimate of background based on a simple model taking into account the effect of a $\pi\pi$ resonance, was made by d'Espagnat, Prentki and Yamaguchi [3]. These authors found that there is a region in phase space in which the reaction

$$\bar{p} + p \rightarrow K + \bar{K} + \pi^{*} \rightarrow K + \bar{K} + 2\pi \qquad (3)$$

cannot be distinguished kinematically from reaction (1). They show that it is possible to select events so as to eliminate this interference. Other sources of background of course remain.

Of course, we also assume that the usual strong interaction conversation laws are valid in the production of the $K'\bar{K}'$ pair and in their decay. Also we make one very reasonable dynamical approximation that the annihilation radius is small enough to exclude orbital angular momentum $L=3$ for the outgoing $K'\bar{K}'$ pair. This approximation is made only for convenience ; a more complete treatment is given in the appendix.

In section II, we give a detailed analysis, treating the assumptions as true. In section III, we briefly discuss the validity of the assumptions and indicate how the analysis must be modified if they are incorrect.

II. The angular distributions

If the antiproton and proton annihilate in an S-state, the parity of the system is odd, and the $K'\bar{K}'$ pair must be emitted in a state of odd orbital angular momentum L. Suppose first that the K' has spin zero. Then the reaction cannot take place in the singlet state of the $\bar{p}\,p$ system (total angular momentum $J=0$), since then L would be even (zero) and parity would not be conserved. The reaction can go in the triplet state, and of course, the angular distribution of the decay products must be isotropic.

Next, suppose that the K' has spin $s=1$. Then the reaction can go either via the singlet or triplet states of $\bar{p}\,p$. Since the reaction amplitudes in these states are unknown, the angular distribution is not unique.

Let us consider the case $s=1$ in greater detail. We distinguish 3 directions.

1) The line of flight of the $K'\bar{K}'$ pair,
2) the line of flight of the K and π in the rest system of the K', and
3) the line of flight of the $\bar{K}$ and π in the rest system of the $\bar{K}'$.

First, consider that the reaction takes place from the singlet state of $\bar{p}\,p$. The total spin S of the $K'\bar{K}'$ system must be $S=1$ and must be combined with orbital angular momentum $L=1$ to give total angular momentum $J=0$. Since the K and π from the decay of the K' each have spin 0, the quantum numbers (ℓ,m) of their orbital angular momentum will be the same as the quantum numbers $(s=1,\, m_s)$ of the spin of the K'. (The same is of course true for the $\bar{K}'$ decay into $\bar{K}+\pi$.) Thus the angular part of the final state wave function will be an eigenfunction of three orbital angular momentum operators, each orbital angular momentum being equal to unity, and all three combining to form $J=0$. This eigenfunction can easily be constructed from products of three spherical harmonics.

The amplitude A_0 for the reaction (1) is then of the form

$$A_0 = a \sum_{\mu m} (1 1 \mu, m-\mu | 1 m)(1 1 m, -m | 0 0) Y_1^{-m}(1) Y_1^{\mu}(2) Y_1^{m-\mu}(3) \quad (4)$$

where $(j_1 j_2 m_1 m_2 | jm)$ are appropriate Clebsh-Gordan coefficients to make $J=0$, a is a complex number specifying the amplitude of the reduced matrix element for the reaction, and the Y_1^m's are spherical harmonics with arguments 1,2,3 specifying the angles $\theta_1 \varphi_1$, $\theta_2 \varphi_2$, $\theta_3 \varphi_3$ of the three directions defined previously with respect to an arbitrary z axis.

In the triplet state, the expression for the transition amplitude may be simplified by making use of the invariance of the system under charge conjugation C [6]. The $\bar{p}\,p$ system is an eigenstate of C with eigenvalue $(-1)^{L+S}$. Then the $K'\bar{K}'$ must also be an eigenstate. If annihilation occurs in the ${}^3S_1$ state, $C = -1$. Since the $K'\bar{K}'$ system has L odd, this implies that S is even. Thus if the K' and $\bar{K}'$ have spin 1, they can combine to form $S=0$ or 2, but not $S=1$. Likewise, if annihilation is from the ${}^1S_0$ state, the total spin of the $K'\bar{K}'$ must be $S=1$. (This is another reason for the reaction being forbidden in the ${}^1S_0$ state if the K' has spin $s=0$).

If $J=1$ (annihilation from triplet state), the orbital angular momentum L of the $K'\bar{K}'$ must be $L=1$ or $L=3$. Since the relative momentum of the $K'\bar{K}'$ pair is low, $L=3$ cannot occur with an appreciable amplitude for a reasonable interaction radius. An analysis including

L=3 is relegated to the appendix ; we neglect this amplitude in the following.

The amplitudes A_1^M for the reaction in the triplet state proceeding from an initial state J=1, with projection M may be written

$$A_1^M = b\sum_m (11\,m,-m|00)(010M|1M)Y_1^M(1)\,Y_1^m(2)Y_1^{-m}(3)$$

$$+c\sum_{m\mu}(11\,m,\mu-m|2\mu)(21\mu,M-\mu|1M)Y_1^{M-\mu}(1)\,Y_1^m(2)Y_1^{\mu-M}(3) \quad (5)$$

where b and c are complex numbers specifying the reduced matrix elements of the reaction corresponding to total spin S=0 and S=2 respectively. The transition probability W(123) for the reaction $\bar{p}p \rightarrow K'+\bar{K}' \rightarrow (K+\pi) + (\bar{K}+\pi)$ will be proportional to

$$W(123) \propto |A_0|^2 + \sum_M |A_1^M|^2. \quad (6)$$

It is convenient to take direction 3 as the z axis and to integrate over the angles φ_1 and φ_2. The resulting transition probability is denoted by $W(\theta_1\theta_2)$ where θ_1 is the angle between the line of flight of the $\bar{K}'$ and the line of flight of its decay products in its own rest system ; and θ_2 is the angle between the lines of flight of the decay products of the K' and $\bar{K}'$, each measured in the appropriate rest system. The quantity $W(\theta_1\theta_2)$ is given by

$$W(\theta_1\theta_2) \propto \frac{1}{2} + \frac{9}{20}Rx + \left(\frac{9}{20}Rx - \frac{1}{2}\right)\cos^2\theta_1$$

$$+ \left(2R - \frac{1}{2} - \frac{41}{20}Rx - 2Ry\right)\cos^2\theta_2 \tag{7}$$

$$+ \left(\frac{1}{2} + \frac{3}{4}Rx + 6Ry\right)\cos^2\theta_1\cos^2\theta_2 .$$

This expression depends on three real parameters R, x and y which are functions of the complex amplitudes a, b, and c as follows

$$R = \frac{|b|^2 + |c|^2}{|a|^2}, \quad x = \frac{|c|^2}{|b|^2 + |c|^2}, \quad y = 2\left(\frac{x - x^2}{5}\right)^{\frac{1}{2}}\cos\delta \tag{8}$$

where δ is the relative phase between b and c. Note that these parameters have restricted ranges

$$0 \leq R \leq \infty, \quad 0 \leq x \leq 1, \quad -1 \leq \cos\delta \leq 1. \tag{9}$$

Integrating $W(\theta_1\theta_2)$ over θ_2, we get the distribution in the variable θ_1 :

$$W(\theta_1) \equiv \int W(\theta_1,\theta_2)\, d(\cos\theta_2) \propto \left(\frac{1}{2} + R - \frac{7}{20}Rx - Ry\right)\sin^2\theta_1$$

$$+ R\left(1 + \frac{7}{10}x + 2y\right)\cos^2\theta_1 \tag{10}$$

It can be readily seen that this expression can be isotropic for allowed values of the parameters. For this reason we also consider $W(\theta_1\theta_2)$ integrated over θ_1 to obtain the correlation function in θ_2. We get

$$W(\theta_2) \equiv \int W(\theta_1\theta_2)\, d(\cos\theta_1)$$
$$\propto \left(\tfrac{1}{2} + \tfrac{9}{10}Rx\right)\sin^2\theta_2 + 3R\left(1-\tfrac{3}{5}x\right)\cos^2\theta_2 . \qquad (11)$$

For both these angular distributions to be isotropic, a double coincidence must occur, i.e. two independent relations between R, x and δ must hold. Nevertheless, since isotropy can occur, it may be useful to consider $W(\theta_1\theta_2)$. This expression (Eq. 7) can be used by dividing the events into two groups depending on whether θ_2, say, is greater or less than 45^o, and observing whether there is anisotropy in θ_1 for either group. For the above expression to be isotropic the coefficients of $\cos^2\theta_1$, $\cos^2\theta_2$ and $\cos^2\theta_1\cos^2\theta_2$ must separately vanish. This actually happens to be possible for allowed values of R, x and $\cos\delta$; the unique solution is,

$$R = 7/6, \quad x = 20/21. \quad \cos\delta = -1 \qquad (12)$$

<u>A priori</u>, such a situation is extremely unlikely.

Alternatively to considering $W(\theta_1\theta_2)$, we can consider the angular distribution with respect to still another angle φ. This angle is defined as follows : the direction of the K' and the direction of its decay products establish a plane. The production and decay of the $\bar{K}'$ establish another plane. Then φ is the angle between these two planes. This angular distribution in this variable can be found by returning to Eq. (5) for $W(123)$. Letting direction 1 be the z axis, and integrating over θ_2 and θ_3, we obtain an expression which depends on $\varphi_2 - \varphi_3$, which is just the angle φ defined above. The expression is given by

$$W(\varphi) \propto 1 + 3R + \left(2R - 1 - 2Ry - \frac{8}{5}Rx\right)\cos 2\varphi. \quad (13)$$

Unfortunately, if $W(\theta_1\theta_2)$ is isotropic, so is $W(\varphi)$, i.e. they are not independent. Nevertheless, it may be useful to measure both $W(\theta_1\theta_2)$ and $W(\varphi)$ to give a check against experimental bias.

III. Discussion

In conclusion, we may say the following. If our assumptions are justified, a completely isotropic distribution means that it is very likely the K' has spin zero. If the K' has spin one, there is some anisotropy in the variables θ_1, θ_2 or φ, unless the parameters specifying the interaction have the unique values given in Eq. (12). In addition, it is apparent that if the K' has spin one, measurement of the degree of anisotropy provides enough information to determine the parameters R, x, and $\cos \delta$ which characterize the annihilation into $K'\bar{K}'$. If the amplitude for orbital angular momentum L=3 cannot be neglected, the conclusions of the last two sentences must be modified. In this case (see appendix) the three angular distributions are given in terms of five, rather than three parameters. It is apparent that for isotropy with respect to all three angles, three relationships would still have to be satisfied by these five parameters.

We now discuss validity of our assumptions. If the K' has spin greater than one, or if the $\bar{p}\,p$ annihilation sometimes occurs in a P-state, the angular distributions are not given by the expressions obtained here. Nevertheless, it remains true that $W(\theta_1)$, $W(\theta_2)$ and $W(\varphi)$ are independent in the sense that the isotropy of one does not imply the isotropy of the others. Therefore, it is still unlikely that $W(\theta_1)$, $W(\theta_2)$ and $W(\varphi)$ will all be isotropic unless the K' has spin zero. We also remark that d'Espagnat[7)] has made a proposal to check whether $\bar{p}\,p$ annihilation occurs from the S-state.

Evidence as to whether the K' decays as a free particle can be obtained from experiment. If the decay is free, conservation of parity requires that equal number of π's must be emitted forward and backward with respect to the direction of the K'. Any forward-backward asymmetry indicates either that the decay is not free or that there are background events. The events in which the decay is not free will in general not only alter the observed angular distribution, but will alter the momentum spectrum as well. In other words, the width of the K' will appear to be greater than 16 MeV if there are an appreciable number of non-free decays. For this reason, these events may be considered together with other background events.

Next consider a background of $K\bar{K}\pi\pi$ events. These events can be grouped into two classes : those arising from $\bar{p}+p \rightarrow K+\bar{K}+\pi^*$, and those arising from other causes. As mentioned previously, d'Espagnat, Prentki and Yamaguchi [3] have shown that it is possible to eliminate events arising from the $\pi\pi$ resonance. The interested reader is referred to their paper for details [8].

Finally, consider a background of "other" $K\bar{K}\pi\pi$ events. These background events will have a different momentum spectrum from the $K'\bar{K}'$ events, so the fraction of background events can be determined. Unless this fraction is very small, it requires a rather elaborate analysis to obtain information about the spin of the K'. We merely sketch one possible method of analysis.

Away from the region of phase space corresponding to $K'\bar{K}'$ production, there will be an angular distribution due entirely to background events. This will have the form, say, $\left|\sum_i \epsilon_i P_i(\cos\theta)\right|^2$, where the ϵ_i are parameters. In the resonance region of phase space, the distribution will be $\left|\sum (\alpha_i + \epsilon_i) P_i(\cos\theta)\right|^2$, where the α_i are known in terms of at most three unknown parameters if assumptions 1) and 2) are valid and if L=3 is excluded. These parameters are R, x and δ if the K' has spin 1 ; if the spin is zero, only α_0 is different from zero. Since the ϵ_i can be determined in the non-resonant region, some information can be obtained about the α_i if the ϵ_i are assumed to vary smoothly through the resonance. It may be necessary to repeat this process for the angles θ_2 and φ, but in a favourable case one should be able to obtain enough information to decide between spin zero and one.

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APPENDIX

If the amplitude for $\bar{p}\,p$ annihilation into $K'\bar{K}'$ with $I=3$ is not negligibly small, the analysis is more complicated. Let the $J=1$ amplitude be a_1^M. Then

$$a_1^M = A_1^M + B_1^M$$

where A_1^M is given by Eq. (5) and B_1^M is the contribution from the $I=3$ state. It is

$$B_1^M = d\sum_{m\mu}(1\,1\,m,\mu-m|2\,\mu)(3\,2\,\mu,M-\mu|1\,M)\,Y_3^{M-\mu}(1)\,Y_1^{m}(2)\,Y_1^{\mu-m}(3)$$

where d is an additional complex parameter.

The angular distribution $W(\theta_1)$ becomes

$$W(\theta_1) \propto \left(\tfrac{1}{2} + R - \tfrac{7}{20}Rx + \tfrac{3}{5}R' - Ry + z + \tfrac{1}{10}z'\right)\sin^2\theta_1$$

$$+\left(R + \tfrac{7}{10}Rx + \tfrac{9}{5}R' + 2Ry - 2z - \tfrac{1}{5}z'\right)\cos^2\theta_1$$

where

$$R' = \frac{|d|^2}{|a|^2}\ , \quad z = 2\left(\frac{3RR'(1-x)}{10}\right)^{\frac{1}{2}}\cos\delta' , \quad z' = 2\left(\frac{3}{2}RR'x\right)^{\frac{1}{2}}\cos(\delta-\delta')$$

and δ' is the phase difference between b and d. The angular correlation $W(\theta_2)$ is

$$W(\theta_2) \propto \left(\frac{1}{2} + \frac{9}{10}Rx + \frac{9}{10}R'\right)\sin^2\theta_2$$
$$+ \left(3R - \frac{9}{5}Rx + \frac{6}{5}R'\right)\cos^2\theta_2 .$$

Finally, $W(\varphi)$ becomes

$$W(\varphi) \propto 1 + 3R + 3R'$$
$$-\left(1 - 2R + \frac{8}{5}Rx - \frac{3}{5}R' + 2Ry - 2z + \frac{2}{5}z'\right)\cos 2\varphi .$$

These angular distributions may all be isotropic for certain values of the parameters, but it would require a three-fold coincidence for this to occur, i.e. three relationships would have to be satisfied by the 5 parameters $R, R^1, x, \delta, \delta'$ for isotropy.

R E F E R E N C E S

1) We use the term "meson" for convenience. Alternatively, we may say "resonant state of K and π". That the resonance really occurs in a single state has not been established, but we assume it to be true.

2) M. Alston et al., Phys. Rev. Letters 6, 300 (1961).

3) B. d'Espagnat, J. Prentki and Y. Yamaguchi, to be published. We wish to thank these authors for illuminating discussions about their work.

4) M. Schwartz, Phys. Rev. Letters 6, 556 (1961).

5) T. Day, G. Snow and J. Sucher, Phys. Rev. Letters 3, 61 (1959). The argument for $\bar{p}\,p$ interactions proceeding from an S-state is not as good as for K p interactions, G. Snow (private communication). See also B.P. Desai, Phys. Rev. 119, 1385 (1960).

6) Note that both isobaric spin states of the nucleon-antinucleon system are permitted, so that the relative $K'^{+}K'^{-}$ and $K'^{0}\bar{K}'^{0}$ ratio is arbitrary.

7) B. d'Espagnat, Nuovo Cimento 20, 1217 (1961).

8) This selection of events is of two kinds. The first makes use of the properties of the wave function in charge space, the second in physical space. Clearly, the first type of selection of events can be combined with this analysis. The second will in general distort the angular distributions. However, it is possible to select so that $W(\theta_1)$ remains undistorted.