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SOME REMARKS ABOUT LOW-ENERGY PION-PION SCATTERING

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One of the most important question in pion physics is the problem of pion-pion interaction at low energy. The knowledge of low-energy pion-pion phase shifts is of fundamental importance in the interpretation of many low-energy phenomena involving pions and nucleons and in the study of the main features of high-energy collisions.

Recently a p-wave pion-pion resonance has been found[1] with position and width not far from those predicted theoretically[2]. On the other hand, the situation concerning pion-pion interaction at low energy in the two s-wave states is not yet settled, since we do not yet have a unique choice for the phase shifts which is satisfactory both from the theoretical and the experimental point of view.

One interesting proposal has recently come from the interpretation of the observed anomaly in the He^3 spectrum in the p + d reaction[3], which has led to the hypothesis of a strong pion-pion interaction at threshold in the T = 0 state. Using for the low-energy pion-pion amplitude the Chew - Mandelstam formula[4]

$$A_0 = \left(\frac{\nu+\mu^2}{\nu}\right)^{1/2} e^{i\delta_0} \sin\delta_0 =$$

$$= \frac{a_0}{1 + \frac{2}{\pi} a_0 \left(\frac{\nu}{\nu+\mu^2}\right)^{1/2} \ln\left[\frac{\nu^{1/2} + (\nu+\mu^2)^{1/2}}{\mu}\right] + i\, a_0 \left(\frac{\nu}{\nu+\mu^2}\right)^{1/2}} \tag{1}$$

(ν = square of the c.m. momentum; μ = pion mass) a good fit of the experimental data has been obtained in correspondence of the scattering length $a_0/\mu = 2.5 \pm 0.4\ 1/\mu$. This very large value of a_0 looks rather surprising because it means thresholds π-π cross-sections of the order of the barn which then decrease rapidly with energy.

On the other hand, such a value of a_0 seems hard to reconcile with the indications coming from the theoretical interpretation of the τ decay leading to $a_2 - a_0$ small and positive[5].

In this letter we want to point out another theoretical difficulty related to the effect of s-wave pion-pion interaction in pion-nucleon production processes. Let us consider the reaction $\pi^- + p \to \pi^- + \pi^+ + n$. In the hypothesis of a large pion-pion interaction the dominant contribution to the production cross-section will be due to the effect of the interaction of the incoming pion with the pions in the cloud of the target nucleon[6].

Disregarding the possible corrections due to rescattering which in any case do not alter appreciably[7] the order of magnitude of the result, the π-π contribution to production is simply given by formula[8]

$$d\sigma = \frac{8}{9}\frac{f^2}{\pi\mu^2}\frac{\Delta^2}{\Delta^2+\mu^2}\left(\frac{\nu}{\nu+\mu^2}\right)^{1/2}\frac{M}{(p^2+M^2)^{1/2}}\frac{d^3p}{q}\int\left|2A_0 + A_2 + 9A_1\cos\vartheta\right|^2 d\cos\vartheta$$

$$A_T = \left(\frac{\nu+\mu^2}{\nu}\right)^{1/2} e^{i\delta_T}\sin\delta_T \qquad (2)$$

where q is the momentum of the incoming pion,
p is the momentum of the recoil nucleon both in lab.,
ν is the square of the outgoing pions momentum in their own c.m. system,
ϑ is the angle of π-π scattering in the same system, and finally
$\Delta^2 = 2M(\sqrt{M^2+p^2} - M)$, M = nucleon mass.

Since, as shown by Eq. (1), the large value of the threshold π-π cross-section decreases very rapidly with energy, the contribution of s-wave pion-pion interaction to Eq. (2) will be particularly important for energies of the incoming pion not too far from threshold. We shall therefore consider explicitly the energy range below 300 MeV. Before comparing the consequences of Eqs. (1) and (2) with experiment we want to discuss briefly the possible effects of A_1 and A_2 in Eqs. (1) and (2). For what concerns A_1 the contribution is positive definite without any interference with A_0 and A_2; reasonable estimates lead to a negligible contribution in our energy range.

We discuss the T = 2 amplitude more in detail because of the possibility of a large T = 2 scattering length a_2/μ interfering destructively with a_0/μ.

The possibility of a good estimate of a_2 is given by the analysis of the experimental results of $\pi^{\pm} + p \to p + \pi^{\pm} + \pi^0$ at 1.03 BeV[9]. Both using the peripheral formula in the physical region and by performing a Chew - Low extrapolation in the energy range $5\mu^2 < 4(\nu + \mu^2) < 8.2\mu^2$ one obtains similar values for the low-energy $(\pi^{\pm},\pi^0)$ cross-section[10] ~ 40 mb leading to a T = 2 scattering length $|a_2/\mu| \cong 0.54\ 1/\mu$. This, naturally, excludes the possibility of a strong interference if one takes for a_0 the large value proposed by ABC.

Let us now finally compare with experiment the theoretical results coming from Eqs. (1) and (2) in correspondence of different choices of a_0 and a_2. Such a comparison is shown in Table I. The large disagreement in order of magnitude between theory and experiment seems to exclude values of a_0 larger than 1. One might, of course, hope that the possible corrections to Eq. (2) coming for example from rescattering might attenuate such a large discrepancy; however, we think this to be very unlikely especially in the low-energy region.

We conclude that the interpretation of the ABC experiment in terms of a strong s-wave pion-pion interaction gives rise to very serious theoretical problems and that the situation concerning that experiment has to be considered by no means settled.

Therefore we think that a careful experimental analysis is very desirable. In particular, it would be very important to observe directly the charged pions in the reaction $p + d \rightarrow He^3 + \pi^+ + \pi^-$ in order to study the mass spectrum of the $(\pi^+\pi^-)$ system. We also think that a study of the reaction $\pi^- + p \rightarrow \pi^+ + \pi^- + n$ for small masses of the $(\pi^+\pi^-)$ system will be very desirable because it would lead, through the peripheral formula, to a termination of a_0 analogous to that of a_2 discussed in this paper[11].

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10) Using the peripheral formula one obtains $\sigma_t = 35 \pm 4$ mb from $\pi^+\pi^0$ and $\sigma \cong 27$ mb from $\pi^-\pi^0$. Finally, if one interprets the data by means of a Chew - Low extrapolation one obtains $\sigma = 40 \pm 20$ mb. The effect of the T = 1 interaction in that energy range can be estimated to about 3 mb.

11) An analysis of this kind has been made by Erwin et al., Ref. 1). Unfortunately, the number of events found for low $(\pi\pi)$ masses is too small to allow any estimate of a_0.

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Table I

a_0	a_2 \ Incident energy (lab)	225 MeV	245 MeV	260 MeV	280 MeV	290 MeV	317 MeV
2.8	0.54	1.30	1.84	2.22	2.68	2.88	3.30
	-0.54	0.83	1.16	1.39	1.65	1.76	1.98
2	0.54	0.90	1.33	1.65	2.03	2.20	2.59
	-0.54	0.51	0.73	0.89	1.08	1.17	1.35
1.5	0.54	0.65	0.98	1.23	1.54	1.70	2.06
	-0.54	0.30	0.46	0.57	0.70	0.77	0.91
1	0.54	0.39	0.62	0.78	1.00	1.10	1.37
	-0.54	0.133	0.21	0.26	0.33	0.36	0.45
experiment [12]		(a) 0.03±0.02	(c) 0.10±0.04	(b) 0.14±0.10	(c) 0.3±0.2	(c) 0.7±0.2	(b) 0.71±0.17

Total $\pi^- + p \to \pi^- + \pi^+ + n$ cross-section in mb.