π - π Scattering, Nucleon Structure and π - N Scattering +)

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The question of the effect of a $\pi - \pi$ scattering resonance in the $J = T = 1$ state on $\pi - N$ scatering has recently received a good deal of attention. This problem has first been investigated by Frazer and Fulco [1] who derived the integral equations for the $\pi + \pi \to N + \bar{N}$ reaction amplitude involving the $\pi - \pi$ phase shifts. These equations were applied by Frazer and Fulco to the problem of the isovector nucleon form factors [2].

In a recent investigation [3] we have obtained single one-dimensional representations for the $\pi - N$ scattering amplitudes by means of the Cini-Fubini method [4], [5]. This representation includes all the results previously obtained by Frazer and Fulco and furthermore allows one to obtain definite predictions for the low energy $\pi - N$ phase shifts in which the effect of the $J = T = 1$ $\pi - \pi$ resonance is taken into account. We wish to report here on the comparison between the predictions of our theory and experiment and show that a good agreement between the two is obtained.

A conclusion opposite to ours has recently been reached by Frautschi [6] who finds a $\pi - \pi$ effect much too large to fit the $P_{1/2}$ $\pi - N$ scattering lengths. In view of this we wish to discuss the point of view we have adopted in I and explain the reason for the discrepancy pointed out by Frautschi.

Let us write the $\pi - \pi$ scattering amplitude in the $J = T = 1$ state in a simple Breit-Wigner form

$$f_{\pi\pi} = \frac{e^{i\delta_{\pi\pi}} \sin \delta_{\pi\pi}}{q^3} = \frac{\gamma}{t_R - t - i\gamma q^3} \tag{1}$$

where $q = (t/4 - \mu^2)^{1/2}$ is the pion momentum in the centre of mass system and t_R and γ are the constants describing the position and width of the resonance.

Consider now the π - N scattering channel and for example the p-wave isotopic-spin-flip amplitude. It has been shown in I that, in the limit of a narrow $\pi - \pi$ resonance, the contribution to this amplitude arising from $\pi - \pi$ resonance scattering is [7]

$$f_{11}^{(\pi\pi)} - f_{31}^{(\pi\pi)} = \frac{-9M}{2Wt_R}\left[\omega C_1 F_1 + k^2\left(C_2 + \frac{C_1}{2M}\right)(F_0 - F_2)\right] \quad (2)$$

where W and k are the total energy and pion momentum in the c.m. system, $\omega = (k^2+\mu^2)^{1/2}$, μ and M are the pion and nucleon masses and the functions F_α are given by

$$F_\alpha\left(\frac{2k^2}{t_R}\right) = \int_{-1}^{+1} \frac{y^\alpha\, dy}{1 + 2k^2(1-y)/t_R}$$

This term has to be added to the contribution of the Born term and of the 33-resonance (which we shall call the "nucleon term").

The two constants C_1 and C_2 determine the size of the effect of the π - π resonance on π - N scattering [8]. The main cause of disagreement between the work in reference [6] and our own lies in the different value used for t_R and in the determination of the value of these two constants in the sense that the values used in reference [6] are much larger than ours. We wish therefore to discuss in some detail our procedure for determining these constants and, in particular, to clarify what information can be obtained from the experimental data on the electromagnetic structure of the nucleon.

The effect of the $\pi - \pi$ resonance on the absorptive parts of the isovector nucleon form factors $G_1^V(t)$ and $G_2^V(t)$ is given, in the limit of a narrow resonance by [9)]

$$\mathrm{Im}\, G_1^V(t) = -\pi e \frac{C_1 \sqrt{t_R}}{\gamma}\, \delta(t_R - t)$$

$$\mathrm{Im}\, G_2^V(t) = -\pi e \frac{C_2 \sqrt{t_R}}{\gamma}\, \delta(t_R - t) \qquad (3)$$

It is clear that the comparison of Eq. (3) with the experimental data on the form factors, either by means of subtracted or unsubtracted dispersion relations can only yield information on t_R and on the ratios C_1/γ and C_2/γ. The comparison was carried through in I by means of subtracted dispersion relations which lead to expressions for $G_1^V(t)$ and $G_2^V(t)$ of the Clementel-Villi [10)] form

$$G_1^V(t) = \frac{e}{2}\left(1 + \frac{\eta_1 t}{t_R - t}\right)$$

$$G_2^V(t) = \frac{g_V e}{2M}\left(1 + \frac{\eta_2 t}{t_R - t}\right) \qquad (4)$$

where $\eta_1 = -2C_1/t_R^{1/2}\gamma$ and $\eta_2 = -2MC_2/g_V t_R^{1/2}\gamma$. The expression (4) for $G_2^V(t)$ is known to give an excellent fit to the data with $\eta_2 = 1.2$ and the mean square radius $r_0 = (6\eta_2/t_R)^{1/2}$ $= .8 \times 10^{-13}$ cm. [11)]. Furthermore, the experimental near-equality of the charge and magnetic moment form-factors gives $\eta_1 \cong \eta_2$. These values yield [12)]

$$t_R = 22.4\,\mu^2, \quad C_1/\gamma = -2.66\,\mu \quad C_2/C_1 = .272\,\mu^{-1}$$

It must be emphasized that as one has no independent determination of γ one cannot obtain from electromagnetic data alone the size of the effect of the $\pi\pi$ resonance on π - N scattering.

There is a theoretical possibility of determining C_1 and C_2 in terms of the π - N coupling constant f^2 and the 33-resonance parameters. This determination is based on the work of Frazer and Fulco on the $\pi + \pi \rightarrow N + \bar{N}$ reaction amplitude [1]. These authors have obtained the following expressions for the $\pi + \pi \rightarrow N + \bar{N}$ p-wave helicity amplitudes f_+ and f_- :

$$f_{\pm}(t) = \frac{F_\pi(t)}{\pi} \int_{-\infty}^{a} \frac{\Im m\, f_{\pm}(t')\, dt'}{(t'-t)\, F_\pi(t')} \qquad (5)$$

where $\Im m\, f_{\pm}(t')$ is known in terms of f^2 and the 33-resonance parameters and $F_\pi(t')$ is the pion form factor which is known in terms of Eq. (1). Eq. (5) can in principle be used to determine C_1 and C_2. We wish to point out, however, that some of the integrals appearing in Eqs. (5) are divergent and that even those which do converge depend strongly on high t' contributions where the expressions for $\Im m\, f_{\pm}(t')$ can no longer be trusted. A determination of C_1 and C_2 along these lines has been carried out by Frautschi and Walecka who find $C_1 = -2.2$, $C_2 = -0.74\,\mu^{-1}$. This determination, together with the much smaller value of t_R ($\sim 11.5\,\mu^2$) used in reference [6] is the cause of the large disagreement with experiment pointed out by Frautschi. At the time I was being prepared, we had reached a similar conclusion as regards the s-waves and we decided for this reason to abandon the determination of C_1 and C_2 by means of Eq. (5) which is, in our opinion, the weakest point in the theory. We therefore consider C_1 and C_2 as free parameters to be fitted to experimental data on π - N scattering and nucleon structure. From the discussion of the form factors it is seen that there is only one remaining free parameter, say C_1 which we shall determine by fitting the π - N s-wave phase shifts. The procedure, which we shall discuss below, yields [13]

$$C_1 = -1.0 \qquad C_2 = -.272\mu^{-1}$$
$$\gamma = .376\mu^{-1} \qquad t_R = 22.4\mu^2 \tag{6}$$

It is obvious that this choice of C_1 is indeed compatible with the data on the electromagnetic structure of the nucleon. Thus we feel that the criticism expressed in reference [6] that the agreement with the π - N phase shifts is obtained at the expense of failing to explain the nucleon structure is not justified.

Let us finally discuss the comparison between our theory and the experimental phase shifts, restricting ourselves to the isotopic spin flip combinations which are only affected by the π - π interactions in the $T = 1$ state. The expression for the s-wave amplitude is

$$f_s^{1/2} - f_s^{3/2} = -\frac{9}{2}\frac{M}{W}\frac{\omega}{2k^2}C_1 \log\left(1 + \frac{4k^2}{t_R}\right) + b\omega \tag{7}$$

where b is an unknown parameter depending on the contributions from higher mass intermediate states. As shown in Fig. 1, the best fit is obtained for the values $C_1 = -1.0$ and $b = -.05\mu^{-2}$. There is some departure from experiment in the medium energy region which might be due to a deficiency of the experimental data. It would be very important to have more experimental information in this energy region to clarify this point. The fact that the value $C_1 = 0$ is clearly excluded shows the necessity of a $\mathcal{J} = T = 1$ $\pi - \pi$ interaction in order to explain the s-wave phase shifts.

The comparison between theory and experiment for the $f_{11} - f_{31}$ combination is shown in Fig. 2. The excellent agreement between theory and experiment is due to the presence of the pion term /Eq.(2)/ in which there are, of course, no remaining free parameters. It should be noted that the nucleon term alone even has the wrong sign.

For the $\mathcal{J} = 3/2$ p-wave amplitudes, the pion term is considerably smaller than for the $\mathcal{J} = \frac{1}{2}$ ones: no essential deviation from the usual effective range formula is to be expected.

If one considers $f^{(1/2)}_{D3/2} - f^{(3/2)}_{D3/2}$, (Fig.3) one finds that, as for p-waves, the nucleon term by itself gives a contribution of the wrong sign and that a $\pi - \pi$ interaction is definitely needed to fit experiment. Because of the 600 MeV resonance for the D_{13} wave the rescattering correction is sizeable and its inclusion improves the agreement between theory and experiment.

In conclusion, the introduction of a $T = \mathcal{J} = 1$ pion-pion resonance with the values of the parameters given in Eq. (6) gives a very satisfactory agreement with the existing experimental data on the isovector form factors and the $\pi - N$ phase shifts. The remaining problem is to explain the discrepancy between our "experimental" values of C_1 and C_2 and the "theoretical" ones given in reference [6]. We wish to point out that if the calculation in reference [6] is carried out using the value $t_R \sim 22\mu^2$ (suggested by a more careful analysis of the experimental data on the form factors [14]), the discrepancy is reduced to a factor ~ 2 [15]. This discrepancy is due, in our opinion, to imperfect knowledge of the high energy contribution to the integrals in Eq. (5) , which can cause considerable errors in their evaluation [16].

It is hoped, therefore, that a more reliable method of solution of the Frazer-Fulco equations would lead to values of C_1 and C_2 closer to those proposed in this paper.

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R E F E R E N C E S

1) W.R. Frazer and J.R. Fulco, Phys.Rev. 117, 1603 (1960).

2) W.R. Frazer and J.R. Fulco, Phys.Rev. 117, 1609 (1960).

3) J. Bowcock, W.N. Cottingham and D. Lurié, Nuovo Cimento 16, 918 (1960). Hereafter called I. See also the report by M. Cini presented at the Annual International Conference on High Energy Physics, 1960.

4) M. Cini and S. Fubini, Annals of Physics 3, 352 (1960).

5) Subsequent investigations on this question have been carried out by J. Hamilton and T.D. Spearman (preprint), S.C. Frautschi and J.D. Walecka (preprint) and by W.R. Frazer and J.R. Fulco (preprint).

6) S.C. Frautschi, Phys.Rev. Letters 5, 159 (1960).

7) As usual, f_i denotes $e^{i\delta_i}\sin\delta_i/k$.

8) C_1 and C_2 can be translated into the language of reference [6] by means of $C_1 = \gamma_1/\pi$, $C_2 = \gamma_2/M\pi$.

9) We recall that $G_1^V(t)$ and $G_2^V(t)$ are normalised to

$$G_1^V(0) = \frac{e}{2}, \quad G_2^V(0) = \frac{g_V e}{2M}$$

10) E. Clementel and C. Villi, Nuovo Cimento 4, 1207 (1956).

11) R. Hofstadter, F. Bumiller and M.R. Yearean, Rev.Mod.Phys. 30, 482 (1958)

12) The use of an unsubtracted dispersion relation for $G_2^V(t)$ as done in reference [2] would lead (for a small γ) to a Yukawa form factor which corresponds to setting $\eta_2 = 1$ in Eq. (4). The fit to the data is less satisfactory and requires $r_0 \cong 1\times10^{-13}$ cm giving $t_R \cong 12\,\mu^2$. This accounts for the difference in the values of t_R obtained in references [2] and [3]. It may be pointed out that the error in t_R introduced by the narrow resonance approximation is of the order of 10 %.

13) This value of C_1 is based on a more careful analysis of the experimental data and is somewhat larger than the value given in I.

14) Evidence in favour of a $T = J = 1$ pion-pion resonance with $t_R \sim 20\,\mu^2$ can be found in high energy pion nucleon collisions: I. Derado, Nuovo Cimento 15, 853 (1960), F. Selleri, Nuovo Cimento 16, 775 (1960), P.Caruthers and H.A. Bethe, Phys. Rev. Letters 4, 536 (1960), E. Pickup et al., Phys. Rev. Letters 5, 161 (1960); and from nucleon-antinucleon annihilation - see: F. Cerulus, Nuovo Cimento 14, 827 (1959).

15) The evaluation of C_1 and C_2 in reference [6] is nearly insensitive to the value of t_R .

16) As stated in a thesis by P. Cziffra, UCRL report No. 9249, a similar conclusion was reached by D.Y. Wong. He compared the value of $f_+(0)$ calculated using equation (5) with the value as accurately known from a sum rule which gives $f_+(0)$ directly in terms of π - N total cross-sections. He found that in order to get agreement between the two values, the "partial wave" estimation of $\mathrm{Im}\, f_+(t')$ had to be used for values of t' much larger than where this method of determination is valid. This confirms that the calculation of C_1 and C_2 using the unsubtracted relations (5) and the partial wave expansion is unreliable.

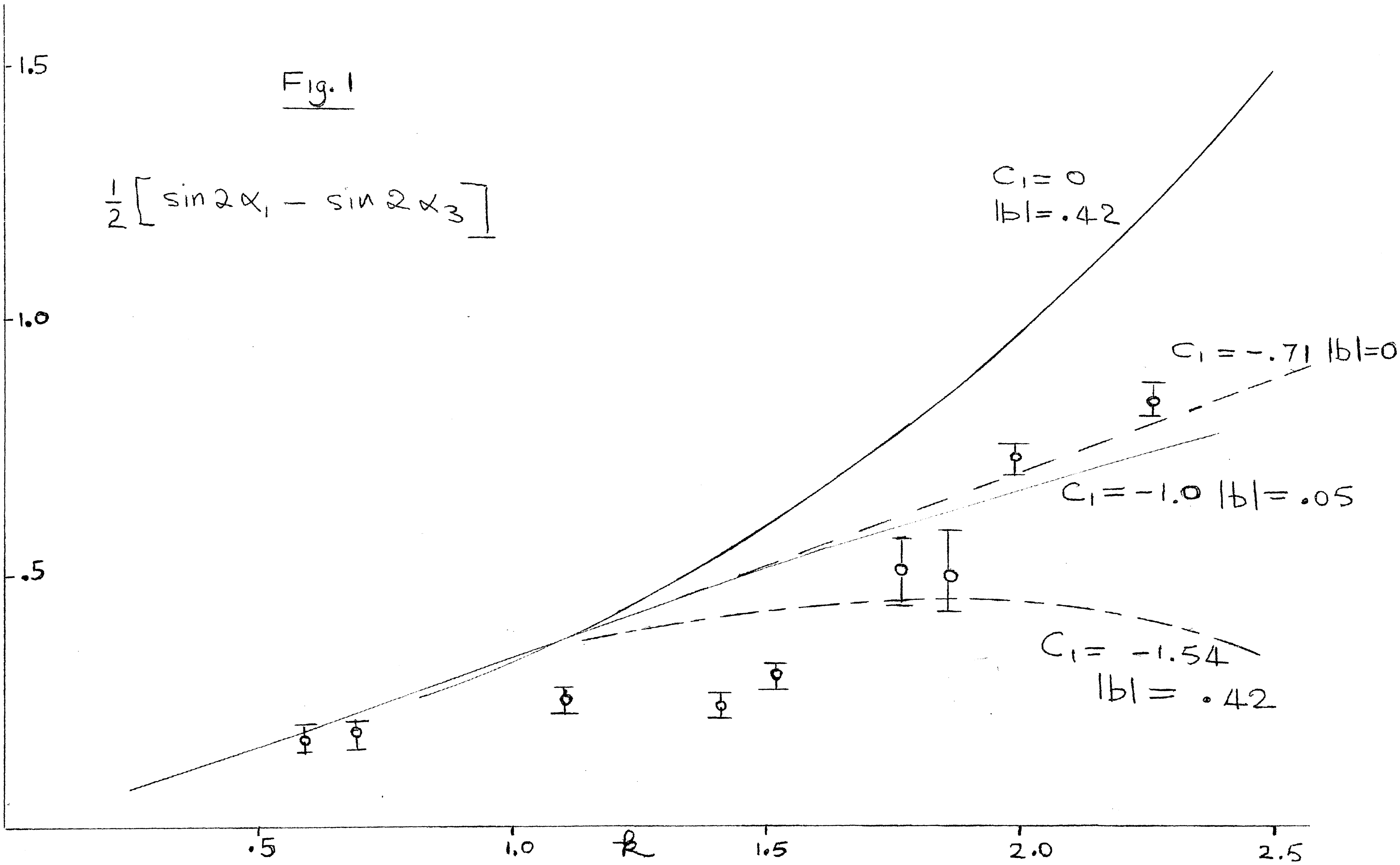
Figure Captions

Fig. 1 - The isotopic spin-flip combination for s-waves for several values of C_1 and b.
The experimental points are from:
Annual International Conference on High-Energy Physics 1958-1959.
A. Stanghellini, Nuovo Cimento X, 398 (1958).
W.D. Walker et al., Phys.Rev. 118, 1612 (1960).

Fig. 2 - The isotopic spin-flip combination for the $P_{\frac{1}{2}}$ waves. The theoretical curve is drawn taking $C_1 = -1.0$. The experimental points are from the same data as Fig. 1.

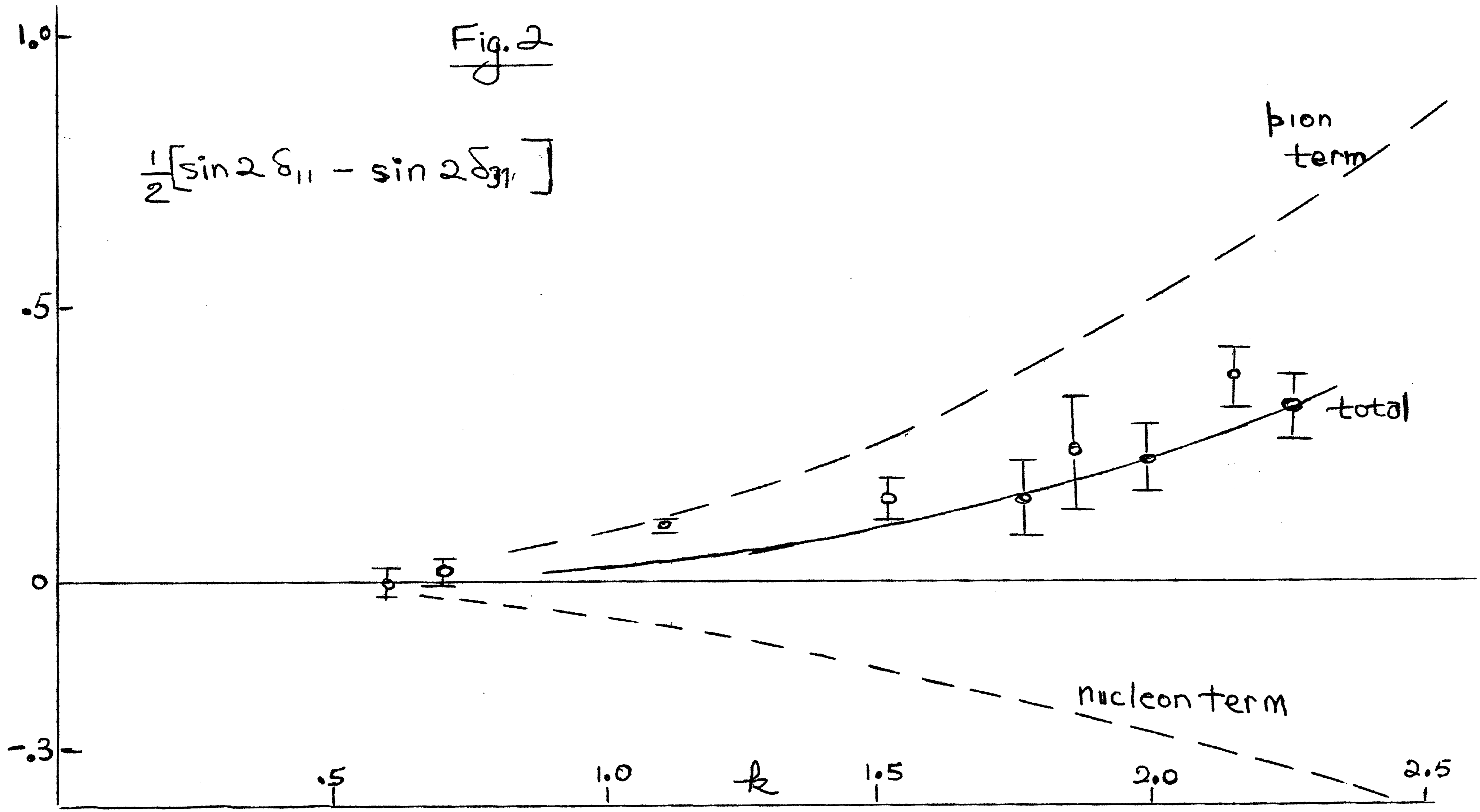
Fig. 3 - The isotopic spin-flip combination for the $D_{3/2}$ waves with $C_1 = -1.0$. The experimental points are from the same data as Fig. 1.

Fig. 1



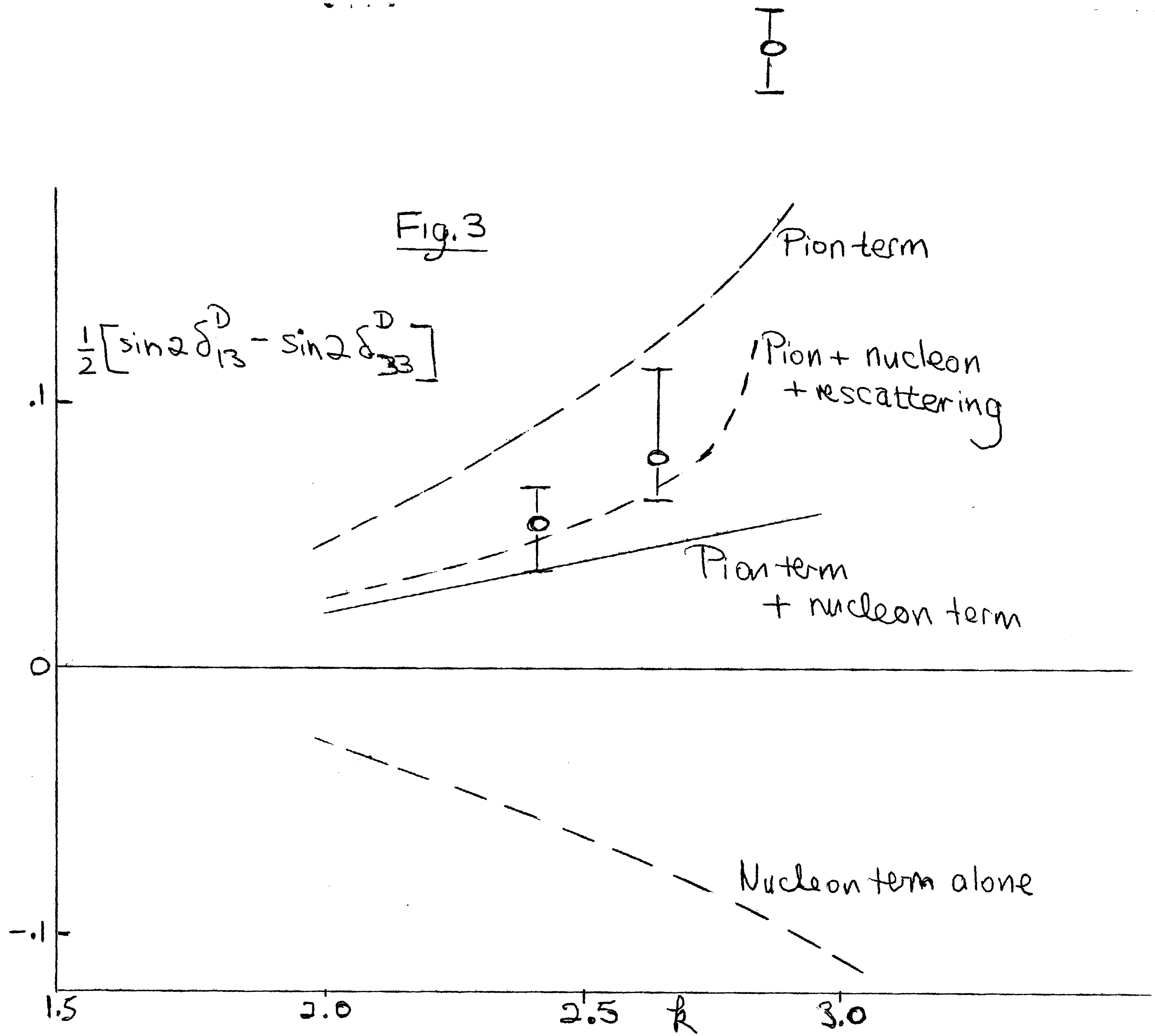
$\frac{1}{2}\left[\sin 2\alpha_1 - \sin 2\alpha_3\right]$
$C_1 = 0$ $|b| = .42$
$C_1 = -.71$ $|b| = 0$
$C_1 = -1.0$ $|b| = .05$
$C_1 = -1.54$ $|b| = .42$
1.5
1.0
.5
.5
1.0
k
1.5
2.0
2.5

Fig. 2



$\frac{1}{2}[\sin 2\delta_{11} - \sin 2\delta_{31}]$
1.0
.5
0
-.3
.5
1.0
1.5
2.0
2.5
k
pion term
total
nucleon term

Fig. 3



$\frac{1}{2}\left[\sin 2\delta_{13}^{D} - \sin 2\delta_{33}^{D}\right]$
.1
0
-.1
1.5
2.0
2.5
3.0
k
Pion term
Pion + nucleon
+ rescattering
Pion term
+ nucleon term
Nucleon term alone