Note on the $\Delta \cdot I=\frac{1}{2}$ selection rule in the composite model of elementary particles

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ABSTRACT

The Salam-Ward hypothesis concerning the $\Delta \cdot I=\frac{1}{2}$ selection rule in weak interactions is examined from the viewpoint of the composite model of elementary particles. In this theory, the non-leptonic weak decays have a different origin than the leptonic decays and are attributed to a small perturbation on the strong interactions.

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One is on the horns of a dilemma [1] in attempting to explain the well-established [2] $\Delta I=\frac{1}{2}$ selection rule in non-leptonic weak decays. On the one hand, if one accepts a current-current interaction as the origin of both the non-leptonic and leptonic weak decays, one is led to introduce neutral baryon currents (with [3] or without [4] intermediate bosons) without the analogous neutral lepton currents. On the other hand, if one wishes to maintain the symmetry between the baryon and lepton currents (which is attractive from several points of view [5]), one must seek the explanation of the $\Delta I=\frac{1}{2}$ selection rule in the special status of the non-leptonic weak decays compared to the leptonic decays [6]. This second approach has been adopted by Salam and Ward [7] who deduce the $\Delta I=\frac{1}{2}$ selection rule in weak interactions from a special hypothesis concerning the strong interactions, namely they assume that the K-meson field operator K possesses a very small non-vanishing vacuum expectation value in the Yukawa-type strong interaction, say $\bar{\Lambda}NK$.* The purpose of this note is to indicate in what fashion a composite model of elementary particles (of the type proposed by Sakata [8] or the present authors [5]) provides a natural basis for the hypothesis of Salam and Ward.

The strong interaction Hamiltonian containing both Λ and N (nucleon) may be written :

$$H_1 = \sum_{\lambda} g_{\lambda} (\bar{\Lambda} Q_{\lambda} \Lambda)(\bar{N} Q_{\lambda} N) \tag{1}$$

where Q_{λ} denotes the usual five covariant expressions (S,V,T,A,P), formed from the Dirac γ-matrices. Note that :

$$\bar{N} Q_{\lambda} N = (\bar{p} Q_{\lambda} p) + (\bar{n} Q_{\lambda} n) \tag{2}$$

where p and n represent the proton and neutron respectively. Then, the part containing the neutron can be transformed by means of the Fierz identity as follows :

$$
\begin{aligned}
H_n &= \sum_\lambda g_\lambda (\bar{\Lambda} Q_\lambda \Lambda)(\bar{n} Q_\lambda n) \\
&= \sum_\lambda f_\lambda (\bar{\Lambda} Q_\lambda n)(\bar{n} Q_\lambda \Lambda)
\end{aligned}
\tag{3}
$$

Now, in the usual theory, the vacuum expectation value $\langle \bar{n} Q_\lambda \Lambda \rangle_0$ must be zero, because of strangeness conservation. However, we suppose for the moment that somehow :

$$
c_\lambda = \langle \bar{n} Q_\lambda \Lambda \rangle_0 \neq 0 \quad \text{for} \quad \lambda = S \text{ or } P \tag{4}
$$

For $\lambda = V$, A or T, $c_\lambda = 0$ follows from Lorentz covariance. Then, Eq.(3) would contain an interaction [9] of the form :

$$
(f_s c_s)(\bar{\Lambda} n) + (f_p c_p)(\bar{\Lambda} \gamma_5 n) \tag{5}
$$

Thus, if Eq.(4) happened to be true, with small constants c_s and c_p (or f_s and f_p), then the desired $\Delta I=\frac{1}{2}$ selection rule in weak interactions would follow from the strong interaction Eq.(3). Note that if $f_s . f_p . c_s . c_p \neq 0$, then Eq.(5) would automatically lead to parity non-conservation. This is somewhat more natural than the original proposal of Salam and Ward [7], who postulate the existence of a scalar particle K' with unit strnageness in addition to the pseudoscalar K-meson, in order to explain parity violation in non-leptonic weak decays.

We must try to give an argument justifying Eq.(4). To do this, let us consider the theory of Nambu and Jona-Lasinio [10]. We recall their argument for deriving a finite mass for the fermion, starting with a chirality-invariant Lagrangian. In the usual perturbational treatment, the vacuum expectation value $\langle \bar{\psi}(x) \psi(x) \rangle_0$ must be zero in a chirality-invariant theory. However, Nambu and Jona-Lasinio postulate from the beginning that

$$
c = \langle \bar{\psi}(x) \psi(x) \rangle_0 \neq 0 \tag{6}
$$

and, in a self-consistent fashion, they determine the value of the non-zero c. We now apply the same type of argument to Eq.(4), and in the same way, we may obtain a non-zero value for c_λ. The desired $\Delta I=\frac{1}{2}$ weak interaction Eq.(5) would then follow.

Let us illustrate our point of view in greater detail. For definiteness, we have recourse to the Tamm-Dancoff procedure previously used by Dürr et al [11] and by the present authors [5]. Furthermore, for simplicity, we assume $f_p=0$. The equation of motion for the neutron is then :

$$(\gamma\frac{\partial}{\partial x} + m_n)n = -f_s(\bar{\Lambda} n)\,\Lambda \tag{7}$$

and we define the Green's function $K(x-y)$ by

$$K(x-y) = \langle (n(x)\,\bar{\Lambda}(y))_+\rangle_0 \tag{8}$$

Using Eq.(7), we obtain :

$$(\gamma\frac{\partial}{\partial x} + m_n)\,K(x-y) = -f_s\langle(\bar{\Lambda}(x)\,n(x)\,\Lambda(x)\,\bar{\Lambda}(y))_+\rangle_0 \tag{9}$$

where we have assumed :

$$[n(x),\ \bar{\Lambda}(y)]_+ = 0 \quad \text{for} \quad x_0=y_0 \tag{10}$$

As a first-order approximation of the Tamm-Dancoff method [11], we may replace :

$$\begin{aligned}
&\langle(\bar{\Lambda}_\alpha(x)\,n_\alpha(x)\,\Lambda_\mu(x)\,\bar{\Lambda}_\nu(y))_+\rangle_0 \\
&\simeq \langle(\bar{\Lambda}_\alpha(x)\,n_\alpha(x))_+\rangle_0\,\langle(\Lambda_\mu(x)\,\bar{\Lambda}_\nu(y))_+\rangle_0 \\
&\quad - \langle(\bar{\Lambda}_\alpha(x)\,\Lambda_\mu(x))_+\rangle_0\,\langle(n_\alpha(x)\,\bar{\Lambda}_\nu(y))_+\rangle_0
\end{aligned} \tag{11}$$

The second term of the r h.s. of Eq.(11) can be incorporated into the m_n of Eq.(9), i.e., it only yields a mass renormalization and hence we omit this term.

Thus, Eq.(11) becomes :

$$\begin{aligned} &\langle(\bar{\Lambda}_\alpha(x)\, n_\alpha(x)\, \Lambda_\mu(x)\, \bar{\Lambda}_\nu(y))_+\rangle_0 \\ &\simeq -(T_r\, K(0)) \cdot S^{(\Lambda)}_{F\mu\nu}(x-y) \end{aligned} \tag{12}$$

where, of course, $S_F^{(\Lambda)}(x-y) = \langle(\Lambda(x)\bar{\Lambda}(y))_+\rangle_0$ is the Λ propagator. Inserting Eq.(12) into Eq.(9), we obtain :

$$(\gamma\frac{\partial}{\partial x} + m_n)\, K(x-y) = f_s(T_r K(0))\; S_F^{(\Lambda)}(x-y)$$

Integrating this equation under the assumption that $K(x-y)$ does not contain any free field part, we find :

$$K(x-y) = f_s(T_r K(0)) \cdot \int d^4x'\; G^{(n)}(x-x')\; S_F^{(\Lambda)}(x'-y) \tag{13}$$

where $G^{(n)}$ is the Green's function satisfying the equation :

$$(\gamma\frac{\partial}{\partial x} + m_n)\; G^{(n)}(x-x') = \delta(x-x')$$

Equating $x=y$ in Eq.(13) and taking the trace, we get

$$c_s = f_s\, c_s \int d^4x'\; T_r\left[G^{(n)}(x-x')\; S_F^{(\Lambda)}(x'-x)\right] \tag{14}$$

where we have put

$$c_s = -T_r\, K(0)$$

in accordance with Eq.(14).

Writing out Eq.(12), we have :

$$c_s = c_s - f_s \cdot \frac{-4i}{(2\pi)^4} \int d^4p \, \frac{1}{p^2+m_n^2} \cdot \frac{1}{p^2+m_\Lambda^2} \left(m_n m_\Lambda - p^2 \right) \tag{15}$$

where the integral is divergent and actually, we should introduce a cut-off factor. Eq.(15) gives $c_s=0$ in general, unless :

$$1 = f_s \cdot \frac{-4i}{(2\pi)^4} \int d^4p \, \frac{1}{p^2+m_n^2} \, \frac{1}{p^2+m_\Lambda^2} \left(m_n m_\Lambda - p^2 \right) \tag{16}$$

The quantity c_s is then completely arbitrary, which is obviously ridiculous.

The above implies that our approximation Eq.(11) is insufficient and we must go on to the next approximation. This is exactly similar to the situation which Dürr et al [11),5)] encountered. The approximation Eq.(11) corresponds to the fact that we are calculating the bubble diagram, Fig. 1. We must calculate higher order diagrams, see Fig. 2.

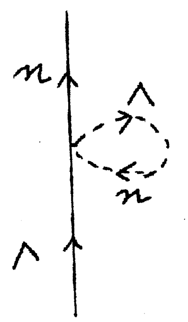

Fig. 1

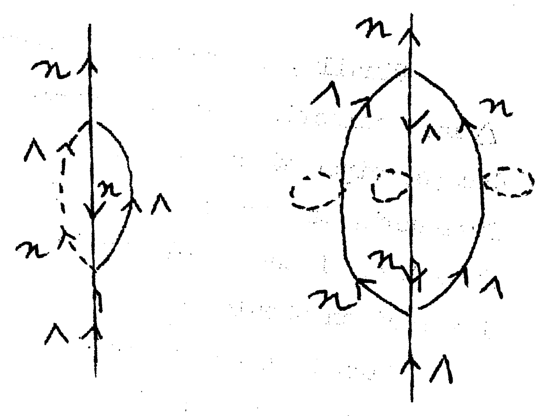

Fig. 2

In Fig. 1 and 2 the broken line represents the new propagator $K(x-y)$ given by Eq.(8) and the solid line represents the usual propagator $S_F^{(\Lambda)}$ or $S_F^{(n)}$.

In principle these contributions can be calculated by means of the method of Dürr et al [11], but we do not give the explicit results because they are not particularly interesting. The important point is that Eq.(13) will then be transformed into a more complicated expression and that, roughly speaking, we shall have an equation of the form :

$$c_s = a_o\, c_s + a_1\, c_s^3 + a_2\, c_s^5 + \ldots \qquad (17)$$

instead of Eq.(15), where a_o is given by the r.h.s. of Eq.(16) in the lowest order in f_s. Note that in general there is no guarantee that c_s will be extremely small. A sufficient condition would be $1 \doteq a_o$, i.e. Eq.(14) should be satisfied to a good approximation. This means that a small non-zero c_s can arise when there is a suitable restriction in the coupling constant f_s in connection with the non-perturbative solution of the non-linear equation with which one starts. There is no a priori reason why this possibility should not occur but it must also be admitted that a demonstration has not been given.

In this connection, we should remark that the idea of deriving a weak interaction as a small correction to a strong interaction is neither new, nor difficult. In a simple model, Van Hove [12] has shown how such a programme can be carried out.

Finally, it should be emphasized that the proposed explanation for the $\Delta I=\frac{1}{2}$ selection rule in non-leptonic weak decays does not work for the leptonic decays, since there are no strong interactions involving leptons. Within this framework, the origin of the leptonic weak interactions must be entirely different from that of the non-leptonic weak interactions and the universal V-A weak interaction should then apply only to the leptonic interactions. This differentiation between the non-leptonic and leptonic weak interactions has one attractive feature : the parity violation of the leptonic interaction can be ascribed to the two component character of the neutrino. Furthermore, this distinction is in line with the idea of the present authors [5] that the occurrence of β-decay may be attributed to the break-down of the orthogonality of the baryon and lepton Hilbert spaces resulting from the switching-on of the electromagnetic interaction since the same argument cannot be used for the non-leptonic weak interactions.

R E F E R E N C E S

1) Cf. L.B. Okun, Proc. of 1960 Ann. Inter. Conf. on High Energy Physics at Rochester (Interscience Publishers), p. 743;
Also, R.E. Marshak, Proc. of 1959 Ann. Inter. Conf. on High Energy Physics at Kiev, Session VIII, p. 269.

2) Cf. M. Schwarz, Proc. of 1960 Ann. Inter. Conf. on High Energy Physics at Rochester (Interscience Publishers), p. 726.

3) T.D. Lee and C.N. Yang, Phys.Rev. 119, 1410 (1960);
See however, B. d'Espagnat, Proc. of 1960 Ann. Inter. Conf. on High Energy Physics at Rochester (Interscience Publishers), p. 589.

4) A. Pais, Nuovo Cimento 18, 1003 (1960).

5) Cf. R.E. Marshak and S. Okubo, Nuovo Cimento (to be published).

6) The consequence of the $\Delta I=\frac{1}{2}$ selection rule in strangeness-violating leptonic decays are identical with the $I=\frac{1}{2}$ current hypothesis (see R.E. Marshak, ref. [1]).

7) A. Salam and J C. Ward, Phys.Rev. Letters 5, 390 (1960).

8) S. Sakata, Prog.Theor.Phys. 16, 686 (1956).

9) Such interactions have been already considered by several authors :
R.F. Sawyer, Phys.Rev. 112, 2135 (1958);
H. Obayashi, Prog.Theor.Phys. 22, 835 (1959);
S. Okubo, Nuovo Cimento 16, 963 (1960);
S. Oneda, J.C. Pati and B. Sakita, Phys.Rev. 119, 482 (1960).

10) Y. Nambu and G. J na-Lasinio, preprint (University of Chicago).

11) H.P. Dürr, W. Heisenberg, H. Mitter, S. Schlieder and K. Yamazaki, Z. Naturforschung 14a, 441 (1959).

12) L. Van Hove, Physica 25, 365 (1959).