# Composite Model and Partially Conserved Currents in Weak Interactions

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It is shown that the idea of partially conserved currents has a natural basis in the composite model. Within this framework, the Goldberger-Treiman relation for the pion lifetime is discussed without recourse to dispersion theory.

THE universal $V$-$A$ theory of weak interactions[1] couples four types of "strong" charged currents with each other and with the charged lepton currents. The four "strong" charged currents are[2] the vector strangeness-conserving current $J^{(V)}$, the axial vector strangeness-conserving current $J^{(A)}$, the vector strangeness-nonconserving current $G^{(V)}$, and the axial vector strangeness-nonconserving current $G^{(A)}$. In order to circumvent the difficulties of computing the strong interaction "renormalization" effects on these four currents, it has been hypothesized that these currents are conserved or at least "partially conserved."

The idea that $J^{(V)}$ is conserved originated[3] with the observation that the coupling constants for muon decay and the beta decay of $O^{14}$ (Fermi transition) are nearly equal. The assumption that

$$\partial_\mu J_\mu{}^{(V)}=0 \tag{1}$$

guarantees that the strong-interaction renormalization effect is unity for $J^{(V)}$ so that equal unrenormalized coupling constants may be chosen for the vector parts of the four-fermion interactions governing muon and beta decay. It is a simple matter to construct a $J^{(V)}$ satisfying Eq. (1) in any theory for which isospin invariance holds.

The equation analogous to (1) for $J^{(A)}$ cannot be true since, among other things, it prohibits the leptonic decay of the pion.[4] On the other hand, the success of the Goldberger-Treiman (G-T) relation[5] connecting the pion-decay lifetime to the axial vector coupling constant of beta decay has led to the idea of a "partially conserved current." According to this idea, one postulates the relation[6]

$$\partial\mu J_\mu{}^{(A)}=a\pi, \tag{2}$$

where $\pi$ is the pion field operator and $a$ is a constant. By further assuming that the pion vertex operator is "gentle" (i.e., highly nonsingular), the G-T relation can easily be derived. The drawback is that within the conventional framework where all mesons and baryons are treated as independent fields, Eq. (2) can only be derived on the basis of special and rather implausible models.[6] This is particularly true when one takes account of strange particles where, as Gell-Mann has pointed out,[7] it is necessary to introduce a new scalar field $K'$ with unit strangeness (assuming that the $K$ meson is pseudoscalar) in order to satisfy Eq. (2).

As regards the strangeness-nonconserving currents $G^{(V)}$ and $G^{(A)}$, it has been proved under rather general conditions[8] that it is impossible to write down equations of the type (1) for these currents. However, it is still attractive to consider the possibility of "partially conserved" currents, namely to write down equations of the type (2):

$$\partial_\mu G_\mu{}^{(V)}=bK', \tag{3}$$

$$\partial_\mu G_\mu{}^{(A)}=cK. \tag{4}$$

But again as in the case of Eq. (2), Eqs. (3) and (4) place severe restrictions on the structure of the strong interactions if the mesons and baryons are regarded as independent fields. Bernstein's very special example[9] of an interaction satisfying Eq. (3) emphasizes the validity of this last remark.

We wish to point out that Eqs. (2)–(4) become much less restrictive if we adopt a composite model for the elementary particles. We use the Sakata model[10] for the remainder of our discussion, since in many respects it is the simplest,[11] but other composite models would do equally well. In the Sakata model, the pion and $K$ mesons are no longer elementary but are bound states of the $\Lambda$ hyperon and nucleons. On this interpretation, Eqs. (2)–(4) can be regarded as definitions of the $\pi$,

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2 See R. E. Marshak, Proceedings of the 1959 Annual International Conference on High-Energy Physics at Kiev.

3 R. P. Feynman and M. Gell-Mann, reference 1; see also S. S. Gershtein and Ya. B. Zeldovich, Zhur. Eksp. i Teoret. Fiz. **29**, 698 (1955) [translation: Soviet Phys.-JETP **2**, 576 (1956)].

4 J. C. Taylor, Phys. Rev. **110**, 1216 (1958).

5 M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958).

6 M. Gell-Mann and M. Lévy, Nuovo cimento **16**, 705 (1960); J. Bernstein, M. Gell-Mann, and L. Michel, Nuovo cimento **16**, 560 (1960); see also R. E. Marshak, *Proceedings of 1958 Annual International Conference on High-Energy Physics at* CERN (CERN Scientific Information Service, Geneva, 1958), p. 219.

7 M. Gell-Mann, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, New York, 1960), p. 508.

8 S. Okubo, Nuovo cimento **13**, 292 (1959).

9 J. Bernstein, Brookhaven National Laboratory Internal Report (unpublished).

10 S. Sakata, Progr. Theoret. Phys. (Kyoto) **16**, 686 (1956).

11 See R. E. Marshak and S. Okubo, Nuovo cimento **19**, 1226 (1961).

$K'$, and $K$ operators (this statement will be justified below) so that the "partial conservation" of the currents $J^{(A)}$, $G^{(V)}$, and $G^{(A)}$ actually follows from the Sakata model. Indeed, it has already been pointed out by Okun[12] that in the Sakata model the "absolute conservation" of the current $J^{(V)}$ [i.e., Eq. (1)] is a direct consequence of the isospin invariance of the four-baryon interactions.

Let us now prove the above statement for "partially conserved currents" within the framework of a composite model.

Several authors[13] have proved that we can assign local field operators even to composite particles. For our purposes, Haag's[13] procedure is the most convenient one and we recall his result here. Haag proves that, under certain rather general assumptions, any "almost local" field $B(x)$ can represent the pion field operator, if $B(x)$ satisfies the following conditions:

$$\langle 0|B(x)|0\rangle = 0, \quad \langle \pi|B(x)|0\rangle \neq 0. \tag{5}$$

The incoming pion field operator $\pi^{(\text{in})}(x)$ can always be defined in the usual fashion, irrespective of whether the pion is a composite particle or not. Haag then proves that

$$B_f(t) \xrightarrow[\text{weakly}]{} \alpha\pi_f^{(\text{in})}(t), \quad (\text{when } t\rightarrow -\infty) \tag{6}$$

where

$$Q_f(t) = i\int_{x_0=t} d^3x \left( Q(x)\frac{\partial f(x)}{\partial x_0} - f(x)\frac{\partial Q(x)}{\partial x_0} \right),$$

with $f(x)$ the wave function of any one pion state and $\alpha$ a normalization factor, defined by

$$\langle \pi|B(x)|0\rangle = \alpha\langle \pi|\pi^{(\text{in})}(x)|0\rangle. \tag{7}$$

Now, the quantity $\partial_\mu J_\mu^{(A)}$ obviously satisfies the condition of Eq. (5): the first condition because of parity conservation, the second because otherwise the pion would not decay into leptons.[4] Thus, it is convenient to regard $(1/\alpha)\partial_\mu J_\mu^{(A)}$ as the local pion field operator $\pi$ by virtue of Haag's theorem. Equation (2) then follows immediately where $a$ is simply the renormalization constant $\alpha$ in Eq. (7), if we identify $\partial_\mu J_\mu^{(A)}$ with $B(x)$. Of course, the derivation of the G-T relation still requires the assumption that the newly defined pion operator is "gentle." Bernstein *et al.*[14] have shown that the G-T relation can be explained without the use of Eq. (2) if one attributes the "gentle" properties to a dispersion theoretic representation of the divergence of $J_\mu^{(A)}$ [i.e., the left-hand side of Eq. (2)]. Our preference is to retain Eq. (2) as a natural consequence of a composite model and to explain the G-T relation by a hypothesis about the gentle behavior of the pion vertex operator without having recourse to dispersion theory.

In a similar fashion, Eq. (4) holds since the left-hand side may be regarded as a definition of the $K$-meson field. As for Eq. (3), the Haag theorem cannot be applied directly since $K'$, if it exists at all,[15] is unstable against strong decay into $K$ and $\pi$. However, we may still regard Eq. (3) as the definition of $K'$ field if it exists. The considerations of Bernstein and Weinberg[16] concerning the effect of the existence of $K'$ on the decay of $K_{\mu3}$ and $K_{e3}$ are similar to those of Bernstein *et al.*[14] with regard to the decay of the pion. From our viewpoint, the results of Bernstein and Weinberg[16] can be derived more directly from Eq. (3) and the "gentle" assumption concerning the $K'$ vertex operator (an assumption which, incidentally, is more difficult to justify than for the pion operator).


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*Note added in proof.* It has just come to our attention that the use of Eq. (2) as the definition of the pion field in the composite particle model has been suggested by Okun[17].

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[16] J. Bernstein and S. Weinberg, Phys. Rev. Letters **5**, 481 (1960).

[17] L. B. Okun, Zhur. Eksp. i Teoret. Fiz. **39**, 214 (1960), [translation, Soviet Phys.—JETP **12**, 154 (1961)].