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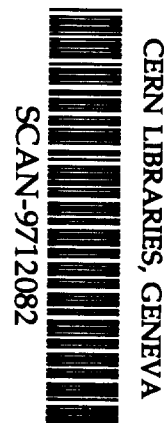## New Schemes in the Adjustment of Bendable, Elliptical Mirrors Using a Long Trace Profiler

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ABSTRACT

The Long Trace Profiler (LTP), an instrument for measuring the slope profile of long X-ray mirrors, has been used for adjusting bendable mirrors. Often an elliptical profile is desired for the mirror surface, since many synchrotron applications involve imaging a point source to a point image. Several techniques have been used in the past for adjusting the profile measured in height or slope of a bendable mirror. Underwood et al. have used collimated X-rays for achieving a desired surface shape for bent glass optics[1]. Non linear curve fitting using the simplex algorithm was later used to determine the best fit ellipse to the surface under test[2]. A more recent method uses a combination of least squares polynomial fitting to the measured slope function in order to enable rapid adjustment to the desired shape.

The mirror has mechanical adjustments corresponding to the first and second order terms of the desired slope polynomial, which correspond to defocus and coma, respectively. The higher order terms are realized by shaping the width of the mirror to produce the optimal elliptical surface when bent. The difference between desired and measured surface slope profiles allows us to make methodical adjustments to the bendable mirror based on changes in the signs and magnitudes of the polynomial coefficients. This technique gives rapid convergence to the desired shape of the measured surface, even when we have no information about the bender, other than the desired shape of the optical surface. Nonlinear curve fitting can be used at the end of the process for fine adjustments, and to determine the over all best fit parameters of the surface. This technique could be generalized to other shapes such as toroids.

1. INTRODUCTION

As manufacturing techniques developed, fabrication of aspherical surfaces became a well established technique in the optics industry and aspherical optical components became widely used for the visible wavelength range to get high quality imaging performance for a variety of fields of application from automatic cameras to large aperture telescopes. However, in the x-ray region, because of reflection characteristics and the short wavelength nature of the light, highly polished grazing incidence reflective optics are required. Reflective optics are sensitive to surface error and more so to short wavelengths. Therefore, high quality aspherical surfaces are required. It is difficult to get high quality aspherical mirror surfaces for the X-ray region by grinding and polishing to the desired figure, However, it is easier to get a high quality plane surface and bend it into the desired aspherical profile. A well known example is the bendable monolithic mirror which gives a circular cylinder[3]. A microspot X-ray photon spectroscopy (μXPS) beamline recently under construction at the Advanced Light Source (ALS) uses two elliptically bent cylindrical mirrors in a Kirkpatrick-Baez configuration to get a beam with a spot size of about 1 μm × 1 μm. The similarity between the

bending equation for a uniform beam and polynomial expansion of a mirror surface shape gives us insight into how aspheric surfaces can be realized by bending. The mirror bender is designed to mechanically adjust the first and second order terms of mirror slope function using unequal couples[4]. The width of the mirrors have a specially designed edge shape to achieve the desired high order terms of the mirror profile when bent. For the optimal bending of the mirrors, the Long Trace Profiler(LTP)[5, 6] has been used during the bending process. For quick and accurate bending of the mirror, an exact equation representing the aspheric surface is derived and used.

2. BENDING THEORY

When an elastic beam experiences unequal couples C_1 and C_2 (these are torques shown in Figure 1), the bending equation of the beam is expressed as follows[4]:

$$\frac{d^2z}{dy^2} = \frac{1}{E \cdot I_{(y)}} \left[\frac{C_1 + C_2}{2} + \frac{C_1 - C_2}{L} y \right] \tag{1}$$

where E is the Young's modulus, I(y) is the section moment, and L is the length of the beam. The surface height profile of any cylinder can be represented as a polynomial equation:

$$\frac{d^2z}{dy^2} = 2a_2 + 6a_3 y + 12a_4 y^2 + 20a_5 y^3 + \cdots \cdots . \tag{2}$$

Equations (1) and (2) are similar, especially when the mirror has a large radius of curvature. In this case the terms higher than the cubic term in Equation (2) are negligibly small. Also, by adjusting the unequal couples C_1 and C_2, the zeroth and first order terms of Equation (1) can be controlled to match equation (2). For optimal matching between Equations (1) and (2), the section moment function $I(y)$ must change with respect to position y when the mirror is bent. This can be achieved either by varying the thickness of the mirror or by varying the width of the mirror. The mirrors that described here are examples of changing mirror width.
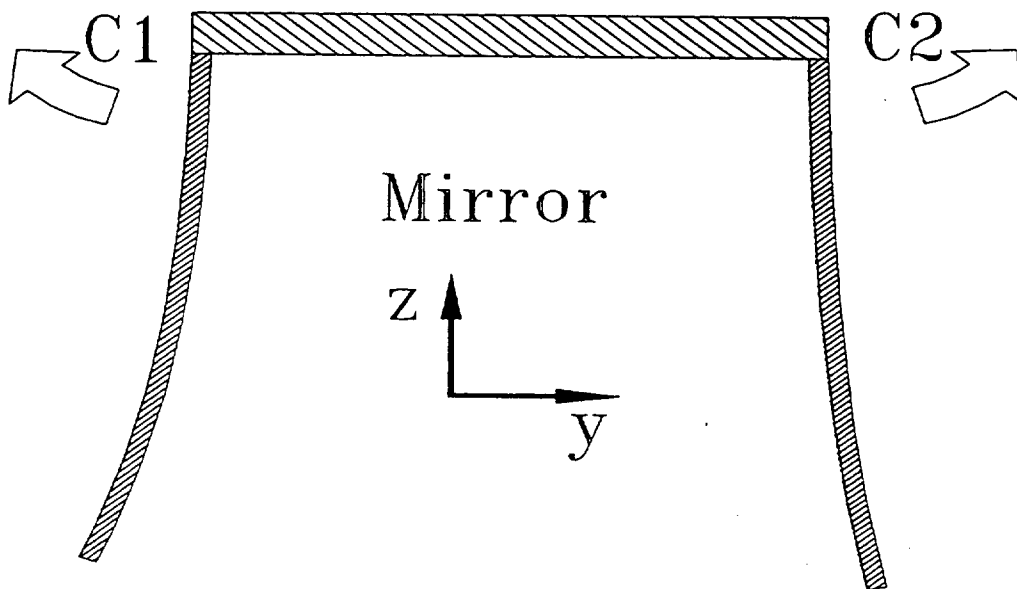
Figure 1. Schematic drawing of an elliptically bent cylindrical mirror with unequal couples.

3. REPRESENTATION OF A CONIC CYLINDER AS A POWER POLYNOMIAL AND THE EXACT EQUATION

Many synchrotron beamline designs involve imaging a point source to a point image, and thus require an ellipsoidal mirror surface. Special cases where the source point is at infinity or the image point is virtual will require a parabolic or hyperbolic surface. This section will describe the exact equation and its expansion coefficients for the conic surfaces which include ellipses, parabolas, hyperbolas, and circles.

Given the source distance r, image distance r', and the incident angle of the principal ray on the mirror measured from the surface normal θ, the ellipsoidal mirror shown in Figure 2 will have foci at $(0, -r\sin\theta, r\cos\theta)$ and $(0, r'\sin\theta, r'\cos\theta)$. The equation of the surface is represented as follows by the definition of an ellipsoid[7].

$$z = \frac{x^2 \sec^2\theta + y^2}{R_0\{(1-Ay)+\sqrt{(1-Ay)^2 - B(x^2 \sec^2\theta + y^2)}\}} \tag{3}$$

In this equation the origin of the coordinates is at the intersection of the principal ray and the mirror. The parameters A, B, and R_0 are defined as follows:

$$\frac{\cos^2\theta}{r} + \frac{\cos^2\theta}{r'} = \frac{2\cos\theta}{R_0} \tag{4}$$

$$A = \frac{\sin\theta}{2}\left(\frac{1}{r'} - \frac{1}{r}\right) \tag{5}$$
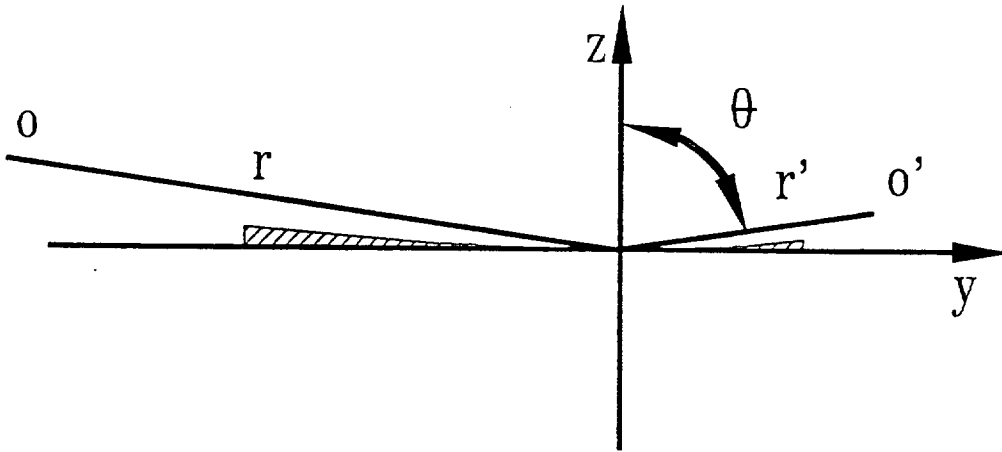
$$B = A^2 + \frac{1}{rr'} \tag{6}$$

Figure 2. Coordinates for the representation of elliptical mirror.

In Equations (4), (5), and (6), r, r' and θ have positive values. If either r or r' has a negative value, then Equation (3) represents a hyperboloid. When either r or r' is set to infinity, it represents a paraboloid. When $r = r'$ and $\theta = 0$, then it is the equation of a sphere. Setting $x = 0$ in Equation (3) will yield a conic cylinder, which could be any of an elliptical, hyperbolic, parabolic, or circular cylinder.

A power expansion of Equation (3) for the cylindrical mirror surface gives the coefficients a_i of Equation(2)

$$a_2 = \frac{1}{2R_o}, \tag{7}$$

$$a_3 = a_2 A, \tag{8}$$

$$a_4 = a_2 \frac{4A^2 + B}{4}, \tag{9}$$

$$a_5 = a_2 A \frac{4A^2 + 3B}{4}, \tag{10}$$

$$a_6 = a_2 \frac{8A^4 + 12A^2 B + B^2}{8}, \tag{11}$$

$$a_7 = a_2 A \frac{8A^4 + 20A^2 B + 5B^2}{8}, \tag{12}$$

$$a_8 = a_2 \frac{64A^6 + 240A^4 B + 120A^2 B^2 + 5B^3}{64} \tag{13}$$

$$a_9 = a_2 A \frac{64A^6 + 336A^4 B + 280A^2 B^2 + 35B^3}{64} \tag{14}$$

$$a_{10} = a_2 \frac{128A^8 + 896A^6 B + 1120A^4 B^2 + 280A^2 B^3 + 7B^4}{128} \tag{15}$$

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Since the LTP measures the slope of a mirror, we represent the mirror profile with slope equations. Taking the derivative of Equation (3) gives the following slope function of a conic cylinder:

$$\frac{\partial z}{\partial y} = \frac{1}{BR_0} \{ \frac{A - A^2 y + By}{\sqrt{(1 - Ay)^2 - By^2}} - A \} . \tag{16}$$

4. DESCRIPTION OF THE μXPS MIRRORS

By using two elliptically bent cylindrical mirrors in a Kirkpatrick-Baez configuration, the beamline designers intend to get a spot in the image plane having a size of 1 μm × 1 μm. The source is an elliptically shaped pinhole with dimensions 40 μm (horizontal) × 20 μm (vertical), and the parameters of the two mirrors are listed in Table 1.

The mirrors are made of 17-4PH stainless steel[8] and have constant thickness. They have a specially designed edge shape in order for the mirror to have an elliptical surface when it is bent. To get the 1 μm spot size (2 σ width), it is necessary to adjust the mirrors into the desired surface shape with a maximum of 0.97 and 2.1 μrad RMS of surface slope error, respectively. To get the proper performance of the mirrors, ± 0.01 mm of width accuracy and ± 0.1 mm thickness accuracy are required. Also, the back and front surface should be parallel to better than ± 0.002 mm. Given the long source distance, short image distance, and long mirror length, the mirrors' surfaces correspond to that part of the ellipse where there is much curvature change over the length of the mirror. A few lower order terms in the polynomial expansion of the surface shape are not enough to represent the desired shape within the acceptable slope error. Therefore, either more polynomial terms or an exact equation must be used. Before bending, the mirrors were installed on the mirror bender and the surfaces were measured by both the LTP and a ZYGO interferometer with a150

Mirror	Parameters	Aperture size
M1 (Vertical Focusing Mirror)	$r = 3880mm$ $r' = 220mm$ $\theta = 88.4$	$80mm(L) \times 2mm(W)$
M2 (Horizontal Focusing Mirror)	$r = 4000mm$ $r' = 100mm$ $\theta = 88.4$	$50mm(L) \times 2mm(W)$

Table 1. Parameters of the mirrors bent.

mm diameter aperture. The mirrors have errors for polishing flat mirror. Mirror M1 (clear aperture of 80 mm (L) × 5 mm (W)) has 3.5 µrad RMS slope error and mirror M2 (clear aperture of 50 mm (L) × 3 mm (W)) has 3.7 µrad RMS slope error when they are flat.

5. BENDING OF MIRRORS

As is mentioned previously, there are two adjustments for controlling C_1 and C_2. When the section moment $I(y)$ is a slowly varying function of position y, then the coefficients a_2 and a_3 have a close relationship with C_1 and C_2 as follows:

$$2a_2 = \frac{1}{2EI}(C_1 + C_2) \tag{17}$$

$$6a_3 = \frac{1}{EIL}(C_1 + C_2) \tag{18}$$

The Equations (17) and (18), gives us enough information for a rough adjustment of C_1 and C_2, in the bending procedure. However, Figures 3 and 4 shows that the two terms are not enough to explain the mirror shape, for the final stage of bending. Equation (19) below defines a function to check the bending error as the difference D_{sl} between measured slope $(\frac{dz}{dy})_m$ and the desired slope $\frac{dz}{dy}$ that was represented in Equation (16):

$$D_{sl} = (\frac{dz}{dy})_m - \frac{dz}{dy} \tag{19}$$

Equation (19) gives us much information about the mirror being bent. The Equation (16) is exact equation, representing a conic cylinder which could be a parabola, hyperbola, ellipse or circle. And, the Equation (19) will not give us term truncation errors for the calculation of the desired mirror surface. The shape, or the polynomial expansion coefficient of the Equation (19), gives the signed magnitude for the adjustment of couples, similar to equations (17) and (18). The RMS value of the function D_{sl} evaluated over the length of the mirror has a close relationship with image quality of the mirror. Use of equation (19) is therefore needed to get the mirror surfaces with least RMS slope error, by adjusting the couples C_1 and C_2, further more. For these reason, even if there is no information about the amount of required couples for the desired shape of the mirror, it is still possible to get the right shape of mirror if the mirror shape parameters are known. By the iterative application of adjustments, measurement and analysis of surface slope, the bending process shows good convergence and the desired curve can be achieved within an acceptable amount of error boundary as long as the mechanical adjustment systems are working properly.
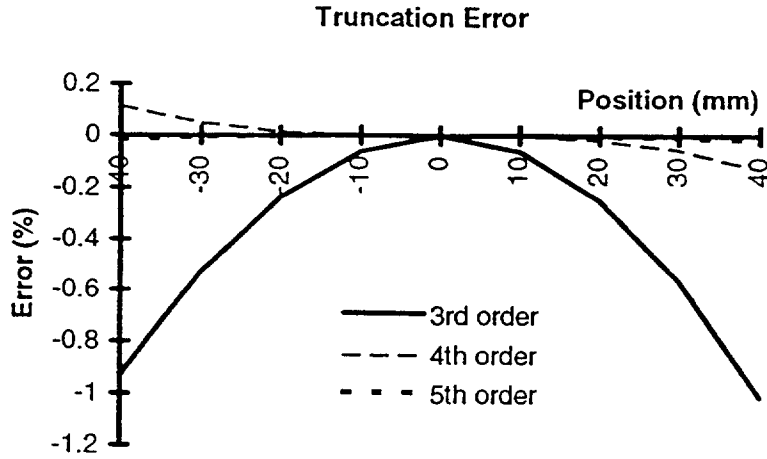
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Truncation Error

Figure 3. Surface height truncation error of mirror M1 after taking up to 3rd, 4th, and 5th order terms of polynomial expansion.

After bending the mirrors, mirror M1 has 3.1 μrad RMS of slope error for a 76 mm long clear aperture and mirror M2 has 2.5 μrad RMS of slope error for a 40 mm long clear aperture. The slope errors before and after bending are similar. The slope errors for each case are shown in Figures 5 and 6. These figures clearly show the expected "S" shape, which predicts that spherical aberration will be the dominant aberration in the image. The mirrors are installed in the beamline and show a beam spot of approximately 2 μm (horizontal) x 3 μm (vertical).

Truncation Error
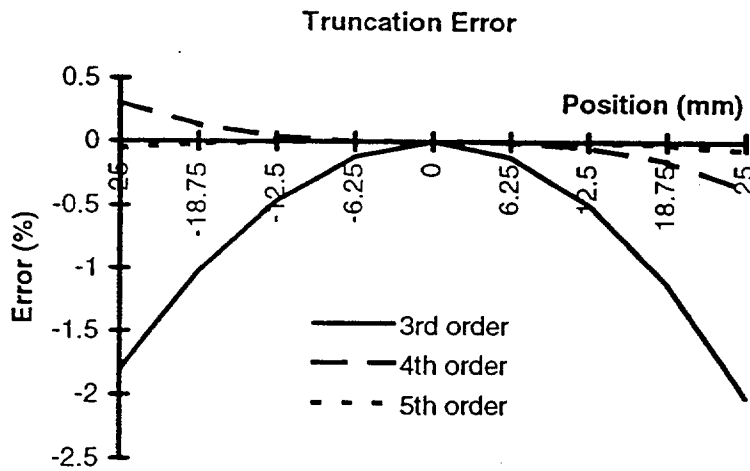

Figure 4. Surface height truncation error of mirror M2 after taking up to 3rd, 4th, and 5th order terms of polynomial expansion.

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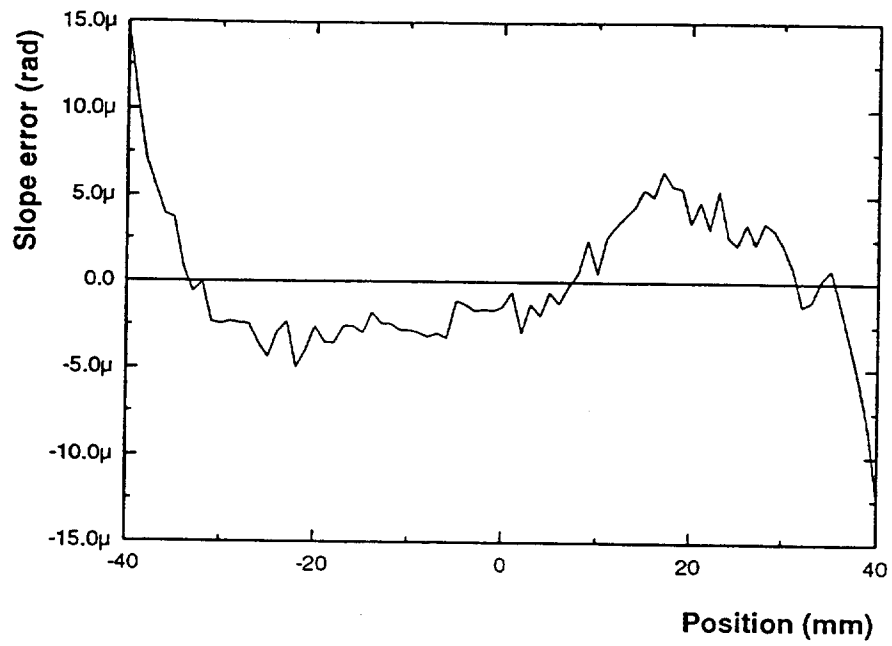
Figure 5. Slope error of mirror M1, after bending. The RMS error is 3.1 μrad for a 76 mm aperture.
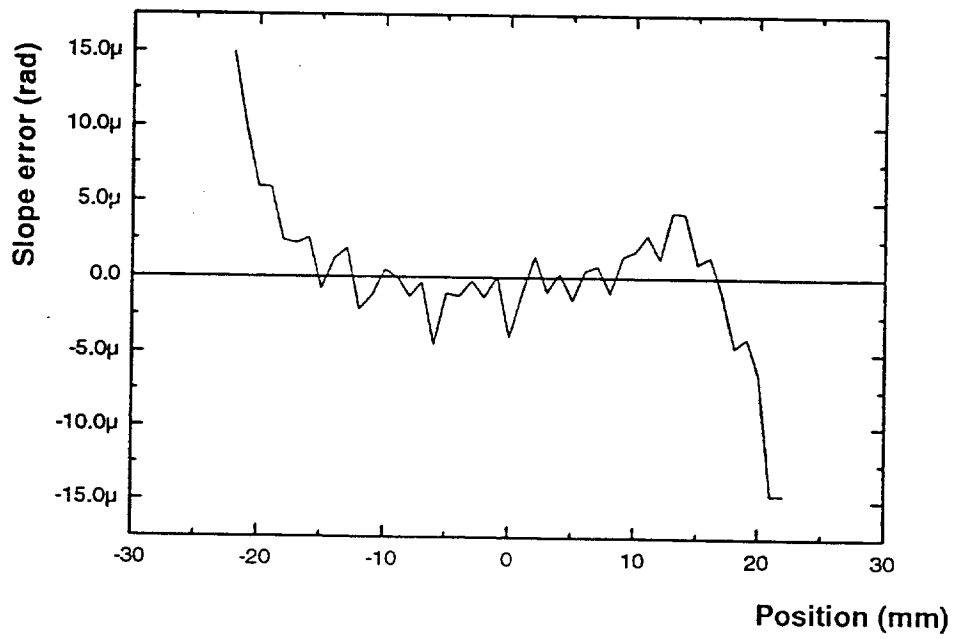

Figure 6. Slope error of mirror M2, after bending. The RMS error is 2.5 μrad for a 40 mm aperture.

6. CONCLUSION

The long trace profiler, an instrument measuring the slope profile of mirrors, has been used for adjusting bendable mirrors into elliptical cylinders. In order to quickly adjust the mirrors, exact equations representing a conic surface shape and its derivatives are derived. Comparing the slope from the equation with the slope from the measurement gives us directional and quantitative information for bending the mirror. By iterative application of bending and measuring the mirror, the desired mirror shape is achieved. It is possible to bend mirrors into the desired shape even if the mechanical characteristics of the mirror bender are not known.

ACKNOWLEDGMENTS

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