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A GENERAL ALGORITHM FOR TRACK FITTING*

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We present here the details of the algorithm used for the fitting of muon tracks in the SDC detector. The method is based on the Kalman filter mechanism which is modified to avoid the inversion of matrices. This allows for the quick fitting of found tracks. This work is associated with the GEANT based simulation of the tracks. The reconstruction results do not include the effects due to lost or misassociated hits. We present the resulting resolution of the measured tracks and discuss the CPU time needed to fit a given track in the SDC detector.

1. The Solution to the Equations of Motion

The motion of a charged particle with energy E and $\overrightarrow{P}$ in a magnetic field is described by the equations

$$\frac{d\overrightarrow{P}}{dt} = \frac{q}{c}(\overrightarrow{v} \times \overrightarrow{B}) \qquad \frac{dE}{dt} = 0 \tag{1}$$

Using the relation $\overrightarrow{P} = \gamma m \overrightarrow{v}$ we can rewrite these equations in the form

$$\frac{d\overrightarrow{v}}{dt} = \overrightarrow{v} \times \overrightarrow{k_B} \tag{2}$$

where

$$\overrightarrow{k_B} = \frac{q\overrightarrow{B}}{\gamma mc} = \frac{qc}{E}\overrightarrow{B}$$

These equations can be written in matrix form

$$\frac{d\mathbf{V}}{dt} = \mathbf{AV} \tag{3}$$

where $\mathbf{A}$ and $\mathbf{V}$ are the matrices

$$\mathbf{V} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} 0 & k_z & -k_y \\ -k_z & 0 & k_x \\ k_y & -k_x & 0 \end{pmatrix}$$

The solution of this system of linear differential equations has the form

$$\mathbf{V} = e^{\mathbf{A}(t-t_0)}\mathbf{V_0}$$

$$\mathbf{R} = \mathbf{R_0} + \mathbf{A}^{-1}(e^{\mathbf{A}(t-t_0)} - 1)\mathbf{V_0} \tag{4}$$

where **R**, **V**, $\mathbf{R_0}$, and $\mathbf{V_0}$ are column matrices describing the position, velocity, initial position, and initial velocity components. The choice of parameters one uses to describe the motion varies. For example, in some cases it is advantageous to use the 5 parameters 1/P, θ, ϕ, $x_\perp$, $z_\perp$. We choose the 6 cartesian components because given the various magnetic fields in the full detector system, it leads to the simplest structure in the calculations and hence speeds up the fit.

Because $k_{z,x,y}$(t-t_0) are << 1 for most of the particle momenta that we are detecting we can expand these expressions in the form

$$\mathbf{V} = [\mathbf{I} + \mathbf{A}(t - t_0)]\mathbf{V_0} \tag{5}$$

$$\mathbf{R} = \mathbf{R_0} + \mathbf{V_0}(t - t_0) + \mathbf{A}\mathbf{V_0}\frac{(t-t_0)^2}{2} \tag{6}$$

Expressions (5) and (6) are the ones used in our fitting procedure. To justify the approximation we note

$$k_z(t - t_0) = \frac{qB}{\gamma mc}(t - t_0) \approx \frac{qB}{\gamma mc}\frac{\delta s}{v_0} = \frac{qB}{cP_0}\delta s$$

where $\delta s \approx 2$ meters represents the radial region where there is a magnetic field. For the solenoid B=1.8 Tesla, and P_0=5 Gev/c we have k_z(t-t_0)$\approx$0.2. In cases where we want to fit lower momentum tracks which are present in the central tracker we can use small sections of the track and connect them by using the track merging methods based on the Kalman filtering technique being used in our global fit techniques.

In order to use the track merging methods for track fitting and to do pattern recognition we will need the 6×6 transport matrix. The matrix is defined by the relation

$$\mathbf{Q} = \begin{pmatrix} \mathbf{R} \\ \mathbf{V} \end{pmatrix} = \mathbf{T} \begin{pmatrix} \mathbf{R}_0 \\ \mathbf{V}_0 \end{pmatrix} = \mathbf{T}\mathbf{Q}_0$$

The transport matrix becomes

$$\mathbf{T} = \begin{pmatrix} \mathbf{I} & \mathbf{I}(t-t_0) + \mathbf{A}\frac{(t-t_0)^2}{2} \\ \mathbf{0} & \mathbf{I} + \mathbf{A}(t-t_0) \end{pmatrix} \tag{7}$$

These equations form the basis of our fitting procedure. In this process we choose $t_0 = 0$ but will calculate the minimum of the χ^2 and the error matrix at shifted values to improve the accuracy in the calculations as discussed in the appendix.

2. The Global Fitting Procedure

We carry out the fitting process in two separate steps. We first do a regional fit in those sections of the detector where multiple scattering can be neglected in comparison to the effect of the spatial resolution. Including multiple scattering in those sections is straightforward but has not been carried out in this work. Then we use the progressive track matching procedure using the Kalman filter method to connect these regional fits and carry out a global fit with multiple scattering included.

The Regional Fits

For the SDC detector the natural subdivision where we can carry out the localized fit with multiple scattering neglected are:

1. The inner tracker that consists of the silicon and the straw or scintillating fibers systems.
2. The first tracking of the muon system (BW1 for the barrel and FW1, FW2 in the forward direction).
3. The outer layers of muon tracking (BW2, BW3 for the barrel, FW4, FW5 in the forward direction, and BI3, BI4 in the intermediate region).

The best fit in each of these subdetectors is carried out by minimizing the Chi-Square. Since one neglects the multiple scattering in each of these initial fits, the associated covariant error matrix is simple and the fits are very fast. The fit minimizes the expression

$$\chi^2 = \sum_{i=1}^{N} \frac{(\overline{r}_i - \overline{r}_{fit}(a, t_i))^2}{\sigma_i^2} \tag{8}$$

where "a" represents the parameters one fits to; these parameters describe the position and direction of the track at some initial point, namely

$$a = (x_0, v_{0x}, y_0, v_{0y}, z_0, v_{0z})$$

σ_i is the measurement error for the ith point; and $\overline{r}$ describes the trajectory of the track in each of the subdetectors.

The regional fits are carried out to get the best initial estimates of the track parameter "a" and the error (covariance) matrix at a specific point in each region. This specific point is so chosen to make the calculations simpler. The details of this procedure and the calculations are presented in the Appendix.

We then use a transport matrix, presented in the Appendix, to calculate the position and direction of the track, and the associated error matrix at the following six well defined locations:

1. The coordinates and direction at the interaction point and at the entrance to the solenoid.
2. Similarly the position and direction of the track at the exit to the calorimeter and the entrance to the iron toroid.
3. The position and direction of the track at the exit of the iron toroid and the last point in BW3 for the barrel tracking, BI2 for the intermediate tracker, and BF5 for the forward tracking.

Hence, the results of the regional fits give us the initial estimate for the regional track parameters (3 position and 3 directions or momentum components) at the ith location

$$\mathbf{Q_i} = \begin{pmatrix} \mathbf{R}_i \\ \mathbf{V}_i \end{pmatrix} = \mathbf{T}_{0\to i} \begin{pmatrix} \mathbf{R}_0 \\ \mathbf{V}_0 \end{pmatrix}$$

and the associated error (covariance) matrix

$$\mathbf{E}_i = \mathbf{T}_{0\to i}\mathbf{E}_0\mathbf{T}^T_{0\to i} \tag{9}$$

where $\mathbf{R}_0$, $\mathbf{V}_0$, and $\mathbf{E}_0$ are the parameters and errors in the parameters at the best location discussed above and $\mathbf{T}_{0\to i}$ is the appropriate transport matrix to the ith point.

The Connection of the Regional Fits. The Overall Fit.

We now proceed to describe the progressive track fitting technique[1] using the Kalman filter[2] method. This leads to the complete solution. Within the framework of the filter theory, the progressive track fitting method can be considered as an extended Kalman filter, and the smoothing (discussed below) part of the procedure makes the method more powerful and flexible.

There are three types of operations to be performed in the fitting algorithm:

1. The Prediction This is the estimation of the track position and direction at the point $t_{(i-1)}$. We make two estimates; the first uses the fitted parameters from the position measurements at times $\geq t_i$, initially from the local fits associated with these times, constructs the transport matrix, and projects the track position and direction at time t_i to that at time $t_{(i-1)}$. The second uses the local fit associated with the measurements at times $< t_{(i-1)}$, constructs its transport matrix, and generates the track position and direction at time $t_{(i-1)}$.

2. The Filtering This is the estimation of the optimal parameters at the point $t_{(i-1)}$, by means of a Chi-Square minimization process using the two predictions of the track position and direction at that point, based on the calculations discussed above.

3. The Smoothing This is the recalculation of the track parameters that describe the track position and direction at the points t_k $(k = i-1, i, i+1, \ldots)$ using the new best fit. These new parameters are then used for more accurate future projections towards the collision point.

We should point out that one of the drawbacks of the Kalman filter process, namely the lack of an error matrix at the start of the process, is avoided here by the local fits that generate this matrix. In addition, the local fits give us the parameters necessary for the initial Kalman filter state vector (including the curvature of the trajectory). As a result we avoid the calculations with the "measurement matrix"[2].

We start with the parameters of the track and its corresponding error matrix (as obtained from the local fit) at the track location just outside the toroid and transport the solution at this point to the solution at a point just inside the toroid using the present knowledge of the momentum and the field map inside the toroid.

To carry this out we use the transport matrix in eq. 7 and we include the effects of multiple scattering described by eqs. 38-41. The predicted positions and directions become:

$$\mathbf{Q}^{\mathbf{pr}}_{(\mathbf{i-1})} = \mathbf{T}_{(\mathbf{i\to i-1})}\mathbf{Q}_{\mathbf{i}}$$

$$\mathbf{E}^{\mathbf{pr}}_{(\mathbf{i-1})} = \mathbf{T}_{(\mathbf{i\to i-1})}\mathbf{E}_{\mathbf{i}}\mathbf{T}^{T}_{(\mathbf{i\to i-1})} + \mathbf{E}_{\mathbf{MS}}$$

In addition we have the track position and direction for the (i-1) point from the local solution using the BW1 measurements. Call these solutions $\mathbf{Q}^{l}_{(\mathbf{i-1})}$ and its associated error matrix $\mathbf{E}^{l}_{(\mathbf{i-1})}$. We then form a Chi-Square

$$\chi^2 = (\mathbf{Q}^{\mathbf{pr}}_{(\mathbf{i-1})} - \mathbf{Q}^{\mathbf{b}}_{(\mathbf{i-1})})^T \mathbf{W}^{\mathbf{pr}}_{(\mathbf{i-1})}(\mathbf{Q}^{\mathbf{pr}}_{(\mathbf{i-1})} - \mathbf{Q}^{\mathbf{b}}_{(\mathbf{i-1})}) + (\mathbf{Q}^{l}_{(\mathbf{i-1})} - \mathbf{Q}^{\mathbf{b}}_{(\mathbf{i-1})})^T \mathbf{W}^{l}_{(\mathbf{i-1})}(\mathbf{Q}^{l}_{(\mathbf{i-1})} - \mathbf{Q}^{\mathbf{b}}_{(\mathbf{i-1})}) \tag{10}$$

where $\mathbf{W} = \mathbf{E}^{-1}$ and $\mathbf{Q}^{\mathbf{b}}_{(\mathbf{i-1})}$ is the functional form of the track trajectory. We minimize with respect to the parameters in the equation for $\mathbf{Q}^{\mathbf{b}}_{(\mathbf{i-1})}$. The solution to the minimization is

$$\mathbf{Q}^{\mathbf{b}}_{(\mathbf{i-1})} = \mathbf{Q}^{\mathbf{pr}}_{(\mathbf{i-1})} + \mathbf{K}_{(\mathbf{i-1})}(\mathbf{Q}^{l}_{(\mathbf{i-1})} - \mathbf{Q}^{\mathbf{pr}}_{(\mathbf{i-1})}) \tag{11}$$

where **K** is the Kalman gain matrix given by

$$\mathbf{K}_{(\mathbf{i-1})} = \mathbf{E}^{\mathbf{pr}}_{(\mathbf{i-1})}(\mathbf{E}^{l}_{(\mathbf{i-1})} + \mathbf{E}^{\mathbf{pr}}_{(\mathbf{i-1})})^{-1}$$

This gives the new more accurate fits of the track using both the information from BW1 and BW2-BW3. These become our new track parameters. Their new associated error matrix becomes

$$\mathbf{E}^{\mathbf{b}}_{(\mathbf{i}-\mathbf{1})} = (\mathbf{I} - \mathbf{K}_{(\mathbf{i}-\mathbf{1})})\mathbf{E}^{\mathbf{pr}}_{(\mathbf{i}-\mathbf{1})} \tag{12}$$

Now, with these new parameters, we carry out the same process between the outside of the calorimeter and the inside of the calorimeter using these fits and the local solution of the central tracker. The final process is then the transport to the vertex point.

Finally we would like to mention that by the process of smoothing we constantly gain an improved set of track parameters in the muon part of the system. This then becomes a powerful tool for matching the tracks between the muon system and the inner detector by looking for convergence in the final solution of the match.

3. Results

We have done a study of the resolution achieved and the time needed to fit a given track by following the process described here. We present the achieved resolution in the three tables and the associated graphs. We found that we needed .02 CPU sec. on the VAX 8800 to fit a given track that was generated with PRTGUN. This time is only the time needed for the fit; it does not include the time needed for the generation of the track using GEANT which of course takes much longer.

APPENDIX

Determination of the Parameters and the Error Matrix

Here we derive the solution for the values of the parameters and the error matrix for both types of trajectories we encounter in the SDC detector. These are the trajectories in the muon chambers where the magnetic field is zero, and the trajectories in the central tracker. We also show some techniques that simplify and improve the accuracy of the solutions.

A.1. Solutions of the Trajectories in the Muon Trackers

The equation of motion (eq. 6), for $t_0 = 0$ can be written as 3 equations:

$$x = x_0 + v_{0x} t_x$$

$$y = y_0 + v_{0y} t_y$$

$$z = z_0 + v_{0z} t_z$$

The χ^2 function which we want to minimize is given by

$$\chi^2 = \sum_{i=1}^{N_x} \{x_i - [x_0 + v_{0x} t_{xi}]\}^2 w_i + y \quad term + z \quad term$$

where N_x is the number of x measurements, and $w_i = \frac{1}{\sigma_i^2}$ and σ_i is the error in the x_i measurement. w_i is known as the weight of the x_i measurement.

We show the detailed calculation for the x term. The solution for the y and z terms are identical. The solution to these equations are best expressed in matrix notation. We proceed by defining the four matrices

$$\mathbf{W} = \begin{pmatrix} 1/\sigma_1^2 & 0 & 0 & 0 & \dots \\ 0 & 1/\sigma_2^2 & \dots & & \\ 0 & 0 & 1/\sigma_3^2 & \dots & \\ \vdots & & & & \end{pmatrix}$$

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_x} \end{pmatrix} \qquad \mathbf{P} = \begin{pmatrix} x_0 \\ v_{0x} \end{pmatrix} \qquad \mathbf{H} = \begin{pmatrix} 1 & t_{x1} \\ 1 & t_{x2} \\ \vdots & \vdots \\ 1 & t_{xN_x} \end{pmatrix}$$

The χ^2 equation can now be written

$$\chi^2 = (\mathbf{X} - \mathbf{HP})^T \mathbf{W} (\mathbf{X} - \mathbf{HP}) \tag{A.1}$$

The equations for the y and z trajectories can be incorporated either by increasing the sizes of the matrices appropriately or by carriying out the solutions separately if there are no error correlations between them. The minimization condition is given by the equations

$$\frac{\partial \chi^2}{\partial x_0} = 0 \qquad \frac{\partial \chi^2}{\partial v_{0x}} = 0$$

leading to the matrix equation

$$\mathbf{H}^T \mathbf{W} (\mathbf{X} - \mathbf{HP}) + (\mathbf{X} - \mathbf{HP})^T \mathbf{W} \mathbf{H} = 0$$

which can be solved for the parameters. Noting that the second term is the transpose of the first, we get in matrix notation

$$\mathbf{P} = (\mathbf{H}^T\mathbf{W}\mathbf{H})^{-1}(\mathbf{H}^T\mathbf{W}\mathbf{X}) \tag{A.2}$$

where the matrix terms are

$$\mathbf{H}^T\mathbf{W}\mathbf{X} = \begin{pmatrix} \sum_{i=1}^{N_x} x_i w_i \\ \sum_{i=1}^{N_x} x_i t_{xi} w_i \end{pmatrix} \tag{A.3}$$

$$\mathbf{H}^T\mathbf{W}\mathbf{H} = \begin{pmatrix} \sum_{i=1}^{N_x} w_i & \sum_{i=1}^{N_x} t_{xi} w_i \\ \sum_{i=1}^{N_x} t_{xi} w_i & \sum_{i=1}^{N_x} t_{xi}^2 w_i \end{pmatrix} \tag{A.4}$$

The inverse of $\mathbf{H}^T\mathbf{W}\mathbf{H}$ is obtained by calculating the co-factor of each term and dividing by the determinant. Once this is obtained the solution for the parameters x_0, v_{0x} is straightforward.

The error matrix consists of the following terms

$$\mathbf{E_P} = \begin{pmatrix} \delta x_0 \delta x_0 & \delta x_0 \delta v_{0x} & \delta x_0 \delta y_0 & \delta x_0 \delta v_{0y} & \delta x_0 \delta z_0 & \delta x_0 \delta v_{0z} \\ \delta v_{0x} \delta x_0 & \delta v_{0x} \delta v_{0x} & \delta v_{0x} \delta y_0 & \delta v_{0x} \delta v_{0y} & \delta v_{0x} \delta z_0 & \delta v_{0x} \delta v_{0z} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \delta v_{0z} \delta x_0 & \delta v_{0z} \delta v_{0x} & \delta v_{0z} \delta y_0 & \delta v_{0z} \delta v_{0y} & \delta v_{0z} \delta z_0 & \delta v_{0z} \delta v_{0z} \end{pmatrix}$$

The calculation of each term follows assuming that we can consider the x,y and z motion separately. This is an approximation at the moment and can be improved on.

$$\mathbf{E_P} \equiv < (\mathbf{P} - \mathbf{P_{ave}})(\mathbf{P} - \mathbf{P_{ave}})^T >$$

Using eq. A.2 this becomes

$$\mathbf{E_P} = (\mathbf{H}^T\mathbf{W}\mathbf{H})^{-1}(\mathbf{H}^T\mathbf{W} < (\mathbf{X} - \mathbf{X_{ave}})(\mathbf{X} - \mathbf{X_{ave}})^T > \mathbf{W}\mathbf{H})(\mathbf{H}^T\mathbf{W}\mathbf{H})^{-1}$$

Since

$$< (\mathbf{X} - \mathbf{X_{ave}})(\mathbf{X} - \mathbf{X_{ave}})^T >= \mathbf{E_{meas.}} = \mathbf{W}^{-1}$$

we get

$$\mathbf{E_P} = (\mathbf{H}^T\mathbf{W}\mathbf{H})^{-1}(\mathbf{H}^T\mathbf{W}\mathbf{W}^{-1}\mathbf{W}\mathbf{H})(\mathbf{H}^T\mathbf{W}\mathbf{H})^{-1} = (\mathbf{H}^T\mathbf{W}\mathbf{H})^{-1} \quad \text{(A.5)}$$

These equations lead to the following results:

$$\mathbf{E_P} = (\mathbf{H}^T\mathbf{W}\mathbf{H})^{-1} = \frac{1.}{Det.}\begin{pmatrix} \sum_{i=1}^{N_x} t_{xi}^2 w_i & -\sum_{i=1}^{N_x} t_{xi} w_i \\ -\sum_{i=1}^{N_x} t_{xi} w_i & \sum_{i=1}^{N_x} w_i \end{pmatrix} \quad \text{(A.6)}$$

where

$$Det. = \sum_{i=1}^{N_x} t_{xi}^2 w_i \sum_{i=1}^{N_x} w_i - (\sum_{i=1}^{N_x} t_{xi} w_i)^2$$

These lead to the following equations for the parameters

$$x_0 = \frac{1.}{Det.}(\sum_{i=1}^{N_x} t_{xi}^2 w_i \sum_{i=1}^{N_x} x_i w_i - \sum_{i=1}^{N_x} t_{xi} w_i \sum_{i=1}^{N_x} x_i t_{xi} w_i) \tag{A.7}$$

$$v_{0x} = \frac{1.}{Det.}(\sum_{i=1}^{N_x} x_i t_{xi} w_i \sum_{i=1}^{N_x} w_i - \sum_{i=1}^{N_x} t_{xi} w_i \sum_{i=1}^{N_x} x_i w_i) \tag{A.8}$$

From a theoretical standpoint these equations give the solutions for the parameters of the trajectory and the corresponding error matrix. Nevertheless, in practice, the elements of the matrix are the result of differences of large numbers and hence are sensitive to round off errors in the intermediate calculations. This leads to inaccuracies in the final fits to the parameters of the tracks.

The solution to this problem is achieved by making a transformation in the value of x_0, y_0, z_0 to a location where the calculation of the error matrix is simple. We show the procedure for the x component of the solution. We redefine x_0 as follows:

$$x'_0 = x_0 + v_{0x} t_{x0}$$

leading to the new equation of motion

$$x = x'_0 + v_{0x}(t_x - t_{x0})$$

We have the freedom to choose the location x'_0 by defining t_{x0}. The following definition

$$t_{x0} \equiv \frac{\sum_{i=1}^{N_x} t_{xi} w_i}{\sum_{i=1}^{N_x} w_i} \tag{A.9}$$

is useful because the error matrix **E** of the parameters is diagonal when calculated in this manner. This location is the "weighted mean" of all the measurement locations.This method of finding a special set of locations in which to calculate the parameters that simplies the calculation and hence improves the accuracy of the parameters and the error matrix is called in the literature "the alternative linear regression method". Once the values are solved for in these particular locations we can the find the values anywhere else by means of the transport matrix.

We show this new solution process. The matrices now become

$$\mathbf{W} = \begin{pmatrix} 1/\sigma_1^2 & 0 & 0 & 0 & \dots \\ 0 & 1/\sigma_2^2 & \dots & & \\ 0 & 0 & 1/\sigma_3^2 & \dots & \\ \vdots & & & & \end{pmatrix}$$

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_x} \end{pmatrix} \qquad \mathbf{P} = \begin{pmatrix} x_0' \\ v_{0x} \end{pmatrix} \qquad \mathbf{H} = \begin{pmatrix} 1 & t_{x1} - t_{x0} \\ 1 & t_{x2} - t_{x0} \\ \vdots & \vdots \\ 1 & t_{xN_x} - t_{x0} \end{pmatrix}$$

The equivalent relations to eq. A.3 and A.4 now become

$$\mathbf{H}^T\mathbf{W}\mathbf{X} = \begin{pmatrix} \sum_{i=1}^{N_x} x_i w_i \\ \sum_{i=1}^{N_x} x_i (t_{xi} - t_{x0}) w_i \end{pmatrix} \tag{A.10}$$

$$\mathbf{H}^T\mathbf{W}\mathbf{H} = \begin{pmatrix} \sum_{i=1}^{N_x} w_i & \sum_{i=1}^{N_x} (t_{xi} - t_{x0}) w_i \\ \sum_{i=1}^{N_x} (t_{xi} - t_{x0}) w_i & \sum_{i=1}^{N_x} (t_{xi} - t_{x0})^2 w_i \end{pmatrix} \tag{A.11}$$

Because of our definition of t_{x0} in eq. A.9 simplifies to

$$\mathbf{H}^T\mathbf{W}\mathbf{H} = \begin{pmatrix} \sum_{i=1}^{N_x} w_i & 0 \\ 0 & \sum_{i=1}^{N_x} (t_{xi} - t_{x0})^2 w_i \end{pmatrix} \tag{A.12}$$

The inversion of this new matrix is simpler and does not depend on differences of large numbers leading to more accurate results. The error matrix becomes

$$\mathbf{E_P} = (\mathbf{H}^T\mathbf{W}\mathbf{H})^{-1} = \begin{pmatrix} (\sum_{i=1}^{N_x} w_i)^{-1} & 0 \\ 0 & [\sum_{i=1}^{N_x}(t_{xi} - t_{x0})^2 w_i]^{-1} \end{pmatrix} \tag{A.13}$$

The solution to the parameters becomes

$$x_0' = \frac{\sum_{i=1}^{N_x} x_i w_i}{\sum_{i=1}^{N_x} w_i} \tag{A.14}$$

$$v_{0x} = \frac{\sum_{i=1}^{N_x} x_i(t_{xi} - t_{x0})w_i}{\sum_{i=1}^{N_x}(t_{xi} - t_{x0})^2 w_i} \tag{A.15}$$

The equations A.10-A.13 are simpler and therefore the calculations lead to more accurate results.

To calculate the parameters and the error matrix at any other location represented by t_x (for example the time of the arrival at the outer layer of the toroid for a track in BW2, BW3) we write the transport matrix

$$\mathbf{T_P} = \begin{pmatrix} 1 & t_x - t_{x0} \\ 0 & 1 \end{pmatrix}$$

and solve for the parameters and the error matrix at the new location

$$\begin{pmatrix} x \\ v_x \end{pmatrix} = \mathbf{T_P} \begin{pmatrix} x_0' \\ v_{0x} \end{pmatrix}$$

$$\mathbf{E_{t_x}} = \mathbf{T_P}\mathbf{E_{t_{x0}}}\mathbf{T_P}^T$$

The resultant error matrix for t_x=0 is the same as eq. A.6.

Our procedure to obtain the values of the parameters and the error matrix are based on this alternative linear regression method.

A.2. Determination of the Error Matrix for the Quadratic Equations of Motion

Using eq. 6, for $t_0 = 0$, we get the three components of the trajectory

$$x = x_0 + v_{0x}t_x + (k_z v_{0y} - k_y v_{0z})\frac{t_x^2}{2}$$

$$y = y_0 + v_{0y}t_y + (k_x v_{0z} - k_z v_{0x})\frac{t_y^2}{2} \qquad \text{(A.16)}$$

$$z = z_0 + v_{0z}t_z + (k_y v_{0x} - k_x v_{0y})\frac{t_z^2}{2}$$

These equations are very coupled and hence difficult to minimize. This is a problem faced by all such equations. We proceed by making an approximation by writing

$$x = x_0 + v_{0x}t_x + \beta_x t_x^2$$

$$y = y_0 + v_{0y}t_y + \beta_y t_y^2 \qquad \text{(A.17)}$$

$$z = z_0 + v_{0z}t_z + \beta_z t_z^2$$

and minimizing assuming the equations are decoupled.

To carry out the χ^2 minimization, we proceed in the same manner as in the previous case. We define the 4 matrices

$$\mathbf{W} = \begin{pmatrix} 1/\sigma_1^2 & 0 & 0 & 0 & \dots \\ 0 & 1/\sigma_2^2 & \dots & & \\ 0 & 0 & 1/\sigma_3^2 & \dots & \\ \vdots & & & & \end{pmatrix}$$

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} \qquad \mathbf{P} = \begin{pmatrix} x_0 \\ v_{0x} \\ \beta_x \end{pmatrix} \qquad \mathbf{H} = \begin{pmatrix} 1 & t_{x1} & t_{x1}^2 \\ 1 & t_{x2} & t_{x2}^2 \\ \vdots & \vdots & \vdots \\ 1 & t_{xN} & t_{xN}^2 \end{pmatrix}$$

The minimization equations are now

$$\frac{\partial \chi^2}{\partial x_0} = 0 \qquad \frac{\partial \chi^2}{\partial v_{0x}} = 0 \qquad \frac{\partial \chi^2}{\partial \beta_x} = 0$$

These lead to the same equation as eq. A.2

$$\mathbf{P} = (\mathbf{H}^T\mathbf{W}\mathbf{H})^{-1}(\mathbf{H}^T\mathbf{W}\mathbf{X}) \tag{A.18}$$

where now the matrix terms are

$$\mathbf{H}^T\mathbf{W}\mathbf{X} = \begin{pmatrix} \sum_{i=1}^{N_x} x_i w_i \\ \sum_{i=1}^{N_x} x_i t_{xi} w_i \\ \sum_{i=1}^{N_x} x_i t_{xi}^2 w_i \end{pmatrix}$$

$$\mathbf{H}^T\mathbf{W}\mathbf{H} = \begin{pmatrix} \sum_{i=1}^{N_x} w_i & \sum_{i=1}^{N_x} t_{xi} w_i & \sum_{i=1}^{N_x} t_{xi}^2 w_i \\ \sum_{i=1}^{N_x} t_{xi} w_i & \sum_{i=1}^{N_x} t_{xi}^2 w_i & \sum_{i=1}^{N_x} t_{xi}^3 w_i \\ \sum_{i=1}^{N_x} t_{xi}^2 w_i & \sum_{i=1}^{N_x} t_{xi}^3 w_i & \sum_{i=1}^{N_x} t_{xi}^4 w_i \end{pmatrix} \tag{A.19}$$

The inverse of $\mathbf{H}^T\mathbf{W}\mathbf{H}$ is obtained by calculating the co-factor of each term and dividing by the determinant. Once this is obtained the solution for the parameters

x_0, v_{0x}, and β_x and the error matrix is, in principle, straightforward. Nevertheless, for the same reasons as before, these solutions suffer from round-off errors and inaccuracies.

To reduce these inaccuracies we proceed by redefining the function we are fitting in such a way that some of the elements of this matrix are zero. This procedure is again "the alternative linear regression method". We can choose one of two methods each one more accurate than the other.

The first method is to redefine the value of t_{x0} as we did in eq. A.9, namely

$$t_{x0} = \frac{\sum_{i=1}^{N_x} t_{xi} w_i}{\sum_{i=1}^{N_x} w_i}$$

Using this condition makes the 1-2 and 2-1 elements of the matrix in eq. A.19 zero. Hence it reduces the number of terms in the inversion process and improves the accuracy of the procedure.

The most accurate method, which is the one we are using, is to proceed as follows (we describe the method for the x component only):

We rewrite eq. A.17 in the form

$$x = x_0' + v_{0x}(t_x - \overline{t_x}) + \beta_x(t_x^2 - \overline{t_x^2}) \tag{A.20}$$

where the various terms are

$$x_0' \equiv x_0 + v_{0x}\overline{t_x} + \beta_x \overline{t_x^2}$$

$$\overline{t_x} \equiv \frac{\sum_{i=1}^{N_x} t_{xi} w_i}{\sum_{i=1}^{N_x} w_i} \qquad \overline{t_x^2} \equiv \frac{\sum_{i=1}^{N_x} t_{xi}^2 w_i}{\sum_{i=1}^{N_x} w_i} \tag{A.21}$$

We then proceed to solve for the parameters x_0', v_{0x}, and β_x. The matrices **W**,

$\mathbf{P}$, $\mathbf{H}$, $\mathbf{H}^T\mathbf{W}\mathbf{X}$, and $\mathbf{H}^T\mathbf{W}\mathbf{H}$ for this new definition of the parameters become

$$\mathbf{W} = \begin{pmatrix} 1/\sigma_1^2 & 0 & 0 & 0 & \dots \\ 0 & 1/\sigma_2^2 & \dots & & \\ 0 & 0 & 1/\sigma_3^2 & \dots & \\ \vdots & & & & \end{pmatrix}$$

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} \qquad \mathbf{P} = \begin{pmatrix} x_0{}' \\ v_{0x} \\ \beta_x \end{pmatrix} \qquad \mathbf{H} = \begin{pmatrix} 1 & t_{x1} - \overline{t_x} & t_{x1}^2 - \overline{t_x^2} \\ 1 & t_{x2} - \overline{t_x} & t_{x2}^2 - \overline{t_x^2} \\ \vdots & \vdots & \vdots \\ 1 & t_{xN_x} - \overline{t_x} & t_{xN_x}^2 - \overline{t_x^2} \end{pmatrix}$$

$$\mathbf{H}^T\mathbf{W}\mathbf{X} = \begin{pmatrix} \sum_{i=1}^{N_x} x_i w_i \\ \sum_{i=1}^{N_x} x_i (t_{xi} - \overline{t_x}) w_i \\ \sum_{i=1}^{N_x} x_i (t_{xi}^2 - \overline{t_x^2}) w_i \end{pmatrix}$$

$$\mathbf{H}^T\mathbf{W}\mathbf{H} = \begin{pmatrix} \sum_{i=1}^{N_x} w_i & \sum_{i=1}^{N_x}(t_{xi} - \overline{t_x})w_i & \sum_{i=1}^{N_x}(t_{xi}^2 - \overline{t_x^2})w_i \\ \sum_{i=1}^{N_x}(t_{xi} - \overline{t_x})w_i & \sum_{i=1}^{N_x}(t_{xi}^2 - \overline{t_x}\ \overline{t_x})w_i & \sum_{i=1}^{N_x}(t_{xi}^3 - \overline{t_x^2}\ \overline{t_x})w_i \\ \sum_{i=1}^{N_x}(t_{xi}^2 - \overline{t_x^2})w_i & \sum_{i=1}^{N_x}(t_{xi}^3 - \overline{t_x^2}\ \overline{t_x})w_i & \sum_{i=1}^{N_x}(t_{xi}^4 - \overline{t_x^2}\ \overline{t_x^2})w_i \end{pmatrix} \qquad \text{(A.22)}$$

Given the conditions specified in eq. A.21, the last matrix becomes

$$\mathbf{H}^T\mathbf{W}\mathbf{H} = \begin{pmatrix} \sum_{i=1}^{N_x} w_i & 0 & 0 \\ 0 & \sum_{i=1}^{N_x}(t_{xi}^2 - \overline{t_x}\ \overline{t_x})w_i & \sum_{i=1}^{N_x}(t_{xi}^3 - \overline{t_x^2}\ \overline{t_x})w_i \\ 0 & \sum_{i=1}^{N_x}(t_{xi}^3 - \overline{t_x^2}\ \overline{t_x})w_i & \sum_{i=1}^{N_x}(t_{xi}^4 - \overline{t_x^2}\ \overline{t_x^2})w_i \end{pmatrix} \qquad \text{(A.23)}$$

The solution for the parameters and the error matrix become

$$x_0' = \frac{\sum_{i=1}^{N_x} x_i w_i}{\sum_{i=1}^{N_x} w_i}$$

$$v_{0x} = \frac{\sum_{i=1}^{N_x}(t_{xi}^4 - \overline{t_x^2}\ \overline{t_x^2})w_i \sum_{i=1}^{N_x} x_i(t_{xi} - \overline{t_x})w_i - \sum_{i=1}^{N_x}(t_{xi}^3 - \overline{t_x^2}\ \overline{t_x})w_i \sum_{i=1}^{N_x} x_i(t_{xi}^2 - \overline{t_x^2})w_i}{Den.}$$

$$\beta_x = \frac{-\sum_{i=1}^{N_x}(t_{xi}^3 - \overline{t_x^2}\ \overline{t_x})w_i \sum_{i=1}^{N_x} x_i(t_{xi} - \overline{t_x})w_i + \sum_{i=1}^{N_x}(t_{xi}^2 - \overline{t_x}\ \overline{t_x})w_i \sum_{i=1}^{N_x} x_i(t_{xi}^2 - \overline{t_x^2})w_i}{Den.} \tag{A.24}$$

$$\mathbf{E_P} = \begin{pmatrix} (\sum_{i=1}^{N_x} w_i)^{-1} & 0 & 0 \\ 0 & [\sum_{i=1}^{N_x}(t_{xi}^4 - \overline{t_x^2}\ \overline{t_x^2})w_i]/Den. & -[\sum_{i=1}^{N_x}(t_{xi}^3 - \overline{t_x^2}\ \overline{t_x})w_i]/Den. \\ 0 & -[\sum_{i=1}^{N_x}(t_{xi}^3 - \overline{t_x^2}\ \overline{t_x})w_i]/Den. & [\sum_{i=1}^{N_x}(t_{xi}^2 - \overline{t_x}\ \overline{t_x})w_i]/Den. \end{pmatrix}$$

where

$$Den. = \sum_{i=1}^{N_x}(t_{xi}^2 - \overline{t_x}\ \overline{t_x})w_i \sum_{i=1}^{N_x}(t_{xi}^4 - \overline{t_x^2}\ \overline{t_x^2})w_i - [\sum_{i=1}^{N_x}(t_{xi}^3 - \overline{t_x^2}\ \overline{t_x})w_i]^2 \tag{A.25}$$

To find the solution for the parameters and the error matrix at a different location in the track, for example, the trajectory at the end of the central tracking region as defined by a value of t_x, we use the transport matrix

$$\mathbf{T_P} = \begin{pmatrix} 1 & t_x - \overline{t_x} & t_x^2 - \overline{t_x^2} \\ 0 & 1 & 2(t_x - \overline{t_x}) \\ 0 & 0 & 1 \end{pmatrix}$$

and get

$$\mathbf{P}_{t_x} = \mathbf{T_P}\mathbf{P}_{(\overline{t_x},\overline{t_x^2})}$$

$$\mathbf{E}_{t_x} = \mathbf{T_P}\mathbf{E}_{(\overline{t_x},\overline{t_x^2})}\mathbf{T_P}^T$$

A.3. The Contribution to the Error Matrix Due to Multiple Scattering

When using the progressive track matching by means of the Kalman filter method to carry out the global fit we need to include the effect of multiple scattering. In this work we only include the multiple scattering due to the toroids and due to the calorimeter.

Since we are only projecting between the entrance and exit points of the scattering material we need to consider only the scattering errors at the surface of the scatterer.

The scattering angle due to a thickness "dx" of the material is

$$< d(\theta^2) >= (\frac{0.014}{P})^2 \frac{dx}{X_{rad}} \equiv \alpha_{MS} dx$$

The angular and displacement errors (in 1 dimension) at the end of the scatterer of length "l" can be determined by integrating over the effect due to an element "dx". We obtain

$$< \theta^2 >= \int_0^l d(\theta^2) = \alpha_{MS} l$$

$$< x^2 >= \int_0^l x^2 d(\theta^2) = \frac{\alpha_{MS} l^3}{3}$$

$$< x\theta >= \int_0^l x d(\theta^2) = \frac{\alpha_{MS} l^2}{2}$$

These equations can be extended to 3 dimensions[3]. The equivalence of $< d(\theta^2) >$ is

$$< \delta v_i \delta v_j >= \alpha_{MS}(v^2 \delta ij - v_i v_j) \tag{A.26}$$

The other two terms become

$$< \delta x_i \delta x_j >= \frac{l^2}{3} < \delta v_i \delta v_j > \tag{A.27}$$

$$< \delta x_i \delta v_j >= \frac{l}{2} < \delta v_i \delta v_j > \tag{A.28}$$

These are the values we use in the multiple scattering error matrix

$$\mathbf{E_{MS}} = \begin{pmatrix} \delta x \delta x & \delta x \delta v_x & \delta x \delta y & \delta x \delta v_y & \delta x \delta z & \delta x \delta v_z \\ \delta v_x \delta x & \delta v_x \delta v_x & \delta v_x \delta y & \delta v_x \delta v_y & \cdots & \\ \vdots & \vdots & \vdots & \vdots & & \end{pmatrix} \tag{A.29}$$

A.4. The Sequential Algorithm for Fast Matrix Inversion

We discuss here the algorithm used to avoid the lengthy computations needed to determine the inverse of a matrix. The direct calculation of the inverse of a real, symmetric N×N matrix requires at least a factor of N more operations than the algorithm presented here. This method is known as the "orthogonal reduction method".

We describe the algorithm using as an example our calculation of the parameters. We use eqs. A.2-A.6 for the linear trajectories or eqs. A.19-A.23 for the quadratic case. We want to solve the equation

$$(\mathbf{H}^T\mathbf{W}\mathbf{H})\mathbf{P} = \mathbf{H}^T\mathbf{W}\mathbf{X}$$

which we rewrite

$$\mathbf{A}\mathbf{P} = \mathbf{B}$$

with solution

$$\mathbf{P} = \mathbf{A}^{-1}\mathbf{B}$$

The matrix **A** is a real symmetric N×N matrix. In our case N=2 for each of the components (x,y,z) of the linear trajectories and N=3 for the quadratic. The other components are included diagonally down the matrix forming a total of N=6 and N= 9 respectively. To find the inverse of **A** we proceed by constructing the (N+3)×(N+3) symmetric matrix (one additional for each component). For the linear case

$$\mathbf{C} = \begin{pmatrix} c_{00} & b_{x1} & b_{x2} & 0 & 0 & \dots & 0 \\ b_{x1} & a_{x11} & a_{x12} & 0 & 0 & \dots & 0 \\ b_{x2} & a_{x21} & a_{x22} & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & c_{33} & b_{y1} & \dots & 0 \\ 0 & 0 & 0 & b_{y1} & a_{y11} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ 0 & 0 & 0 & 0 & 0 & \dots & a_{z22} \end{pmatrix} \equiv \begin{pmatrix} c_{00} & c_{01} & \dots & c_{0N} \\ c_{10} & c_{11} & \dots & c_{1N} \\ c_{20} & c_{21} & \dots & c_{2N} \\ c_{30} & c_{31} & \dots & c_{3N} \\ c_{40} & c_{41} & \dots & c_{4N} \\ \vdots & \vdots & \vdots & \vdots \\ c_{N0} & c_{N1} & \dots & c_{NN} \end{pmatrix}$$

where the only new elements are c_{00}, c_{33}, and c_{66}. We choose the first to be

$\sum_{i=1}^{N_x} x_i^2 w_i$. The other elements follow from eqs. A.3, A.4 for the linear case. If we consider the other components, the term c_{33} is $\sum_{i=1}^{N_y} y_i^2 w_i$ and the others are associated with the y measurements and c_{66} onwards are associated with the z measurements. Hence the final matrix size is 9×9 for the linear trajectory and 12×12 for the trajectory in the magnetic field.

We now perform the following three operations on the elements of the C matrix in a given sequence as follows:

$$c_{kk}^{new} = -\frac{1}{a_{kk}^{previous}} \qquad k \neq 0$$

$$c_{kr}^{new} = c_{rk}^{new} = \frac{c_{kr}^{previous}}{a_{kk}^{previous}} \qquad r \neq k$$

$$c_{rr'}^{new} = c_{r'r}^{new} = c_{rr'}^{prev.} - \frac{c_{rk}^{prev.} c_{r'k}^{prev.}}{a_{kk}^{prev.}} \qquad r, r' \neq k$$

The sequence one follows is to first change all the elements of the matrix for k=1, then, using the new **C** matrix, repeat the same operation for k=2, and so on until the last operation for k=N.

After these series of reductions take place one can show that

1. c_{00}, c_{33}, c_{66} are the contribution to the total χ^2 for the x, y, z measurements respectively.

2. c_{01} and c_{02} are the solutions to the parameters x_0 and v_{x0}. Similarly c_{34} and c_{35} for y_0 and v_{y0}; c_{67} and c_{68} for z_0 and v_{z0}.

3. The other terms are the elements of the matrix $\mathbf{A}^{-1}$. The elements c_{11}, c_{12}, c_{21}, c_{22} are the terms associated with the x measurements; c_{44}, c_{45}, c_{54}, c_{55} with the y measurements and so on.

In the case of the solution of the trajectory in the magnetic field the C matrix is 12×12 and the associated elements have the same correlation with the χ^2, the solution to the parameters (including β_x, β_y, β_z), and the associated inverse matrix.

A.5. The Calculation of the Momentum Using the Fitted Parameters

In all cases we can use eq. 5 that leads to the relations

$$\vec{v_1} = \vec{v_0} + \frac{q}{c}\frac{L}{P}\vec{v_0} \times \vec{B}$$

$$P = \frac{0.3LBsin(\theta_{v_0,B})}{|\hat{v_1} - \hat{v_0}|}$$

where $\hat{v}$ is a unit vector. In the case of the determination of the momentum using the toroid, $\vec{v_1}$ is the velocity vector after the toroid, and $\vec{v_0}$ is the velocity vector before the toroid. In the case where we use the central tracking region, these vectors are defined at the outer radius of the central tracking and at the collision point.

To a good approximation this leads to the equation

$$P_t = \frac{0.3L_tB}{|\hat{v_1} - \hat{v_0}|}$$

where L_t is the radial length of the magnetic field in each region. Hence we only need to determine the direction cosines of each velocity vector.

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TABLE 1

Resolution $\sigma(1/P_t)\%$
Beam rϕ,z = 20 μ, 1 cm
Silicon Tracker = 15 μ
Straw Tracker = 150 μ
Muon Trackers = 250 μ
Cut at $\pm 4\sigma$

P_t(GeV/c)	θ(deg.)	σ_{inner}	σ_{muon}	σ_{global}
100	10	5.60	14.94	5.52
	15	9.64	13.63	9.57
	20	1.43	14.44	1.43
	25	1.25	17.97	1.25
	35	1.12	15.15	1.12
	45	1.10	15.16	1.10
	55	1.25	17.97	1.25
	65	1.31	16.76	1.31
	75	1.36	19.79	1.36
	85	1.26	20.04	1.26
200	10	11.51	17.97	11.33
	15	21.38	13.79	13.94
	20	2.58	13.98	2.56
	25	2.10	15.41	2.08
	35	2.04	15.44	2.02
	45	2.19	20.30	2.18
	55	2.35	17.32	2.35
	65	2.50	19.16	2.49
	75	2.65	19.84	2.63
	85	2.57	16.71	2.56
300	10	18.47	26.58	18.11
	15	28.59	13.60	13.57
	20	4.01	13.81	4.00
	25	3.09	16.51	3.01
	35	3.12	15.85	3.09
	45	3.12	17.71	3.09
	55	3.42	18.75	3.40
	65	3.53	20.50	3.51
	75	3.84	19.39	3.82
	85	3.76	20.80	3.74

CONTINUE TABLE 1

P_t(GeV/c)	θ (deg.)	σ_{inner}	σ_{muon}	σ_{global}
500	10	28.67	32.51	26.57
	15	34.58	14.00	13.56
	20	6.57	14.08	6.24
	25	5.01	16.22	4.69
	26	6.54	27.15	6.48
	27	6.71	27.31	6.65
	28	6.92	32.56	6.34
	29	5.97	43.79	5.81
	30	6.85	34.49	6.71
	31	6.57	15.45	6.37
	35	6.94	21.64	6.80
	45	5.71	19.14	5.33
	55	6.12	24.04	6.06
	65	6.17	24.68	6.09
	75	6.53	21.21	6.45
	85	6.69	18.44	6.66
750	10	39.41	51.48	35.41
	15	27.03	16.31	16.28
	20	15.42	16.75	14.84
	25	13.21	16.92	13.14
	26	13.22	26.40	12.55
	27	13.41	26.84	13.12
	28	13.40	35.45	11.86
	29	13.57	42.18	12.56
	30	13.47	39.29	11.39
	31	13.07	16.70	12.69
	35	11.67	20.26	11.49
	45	9.85	22.50	9.66
	55	9.83	22.71	9.77
	65	9.24	30.42	9.08
	75	9.70	22.77	9.64
	85	9.03	17.84	8.95
1000	10	44.57	100.66	36.24
	15	33.89	14.86	14.31
	20	18.86	16.23	15.47
	25	14.47	16.96	14.25
	26	14.51	28.86	14.23
	27	14.85	32.18	14.15
	28	14.90	37.41	12.40
	29	15.12	41.95	13.67
	30	14.92	45.36	12.93
	31	14.58	18.01	13.86
	35	14.21	28.80	13.19
	45	13.15	27.91	10.94
	55	13.42	28.24	11.46
	65	13.17	27.54	11.22
	75	13.67	25.83	12.25
	85	12.86	20.80	11.43

CONTINUE TABLE 1

P_t(GeV/c)	θ (deg.)	σ_{inner}	σ_{muon}	σ_{global}
	10	73.12	147.66	40.17
	15	41.82	20.99	18.76
	20	27.25	20.30	18.94
	25	25.91	22.02	20.72
	26	25.74	38.21	20.39
	27	26.12	49.61	19.40
	28	25.91	50.16	18.86
2000	29	26.04	62.19	18.46
	30	25.94	51.98	17.93
	31	25.76	62.48	16.86
	35	26.37	39.43	17.14
	45	25.17	36.81	18.21
	55	25.78	33.22	16.84
	65	24.39	30.11	19.81
	75	24.51	31.17	19.53
	85	23.39	28.44	18.68

TABLE 2

Resolution $\sigma(1/P_t)\%$
Beam rϕ,z = 20 μ, 1 cm
Silicon Tracker = 15 μ
Straw Tracker = 75 μ
Muon Trackers = 250 μ
Cut $\pm 4\sigma$

P_t(GeV/c)	θ(deg.)	σ_{inner}	σ_{muon}	σ_{global}
	10	4.13	14.94	4.13
	15	8.54	13.67	8.49
	20	1.19	14.48	1.19
	25	0.92	16.91	0.92
100	35	0.98	15.09	0.98
	45	0.96	15.16	0.96
	55	1.03	17.97	1.02
	65	1.01	19.19	1.01
	75	1.06	20.75	1.05
	85	1.02	20.12	1.02
	10	8.06	17.97	8.06
	15	17.56	13.79	13.70
	20	2.12	13.98	2.11
	25	1.60	15.58	1.60
200	35	1.68	15.70	1.66
	45	1.61	20.30	1.61
	55	1.74	17.32	1.73
	65	1.99	20.28	1.98
	75	1.91	20.57	1.90
	85	1.89	18.12	1.89
	10	13.15	26.53	13.12
	15	24.19	13.60	13.47
	20	3.23	13.81	3.21
	25	2.48	15.71	2.47
300	35	2.48	17.69	2.45
	45	2.51	17.77	2.50
	55	2.87	18.75	2.85
	65	2.89	20.79	2.88
	75	2.92	19.48	2.90
	85	2.93	20.57	2.91

CONTINUE TABLE 2

P_t(GeV/c)	θ (deg.)	σ_{inner}	σ_{muon}	σ_{global}
	10	20.09	32.29	19.94
	15	34.71	14.03	13.96
	20	5.26	14.09	5.17
	25	4.10	15.36	4.08
500	35	5.21	20.78	5.20
	45	5.17	19.14	4.97
	55	5.64	24.04	5.51
	65	5.86	26.19	5.76
	75	5.17	22.78	5.12
	85	5.29	19.11	5.23
	10	29.27	51.56	26.17
	15	26.14	16.24	16.19
	20	7.93	16.75	7.87
	25	12.20	16.92	12.17
750	35	11.65	20.26	10.59
	45	9.32	20.26	9.16
	55	9.21	22.70	8.80
	65	9.17	27.59	8.25
	75	9.37	27.75	9.16
	85	8.34	19.71	8.22
	10	31.69	100.65	27.54
	15	32.57	14.94	14.90
	20	11.52	16.23	10.24
	25	14.23	16.98	13.77
1000	35	13.34	28.76	12.69
	45	10.94	27.75	10.75
	55	10.65	28.20	9.97
	65	10.83	29.54	9.97
	75	10.54	24.27	9.49
	85	10.17	20.24	9.94

CONTINUE TABLE 2

P_t(GeV/c)	θ (deg.)	σ_{inner}	σ_{muon}	σ_{global}
	10	38.46	133.85	29.16
	15	34.58	21.37	21.20
	20	22.53	20.26	18.37
	25	25.57	22.08	18.91
2000	35	24.17	39.40	15.30
	45	24.76	37.05	16.08
	55	23.74	33.40	15.47
	65	24.61	30.25	16.61
	75	25.61	31.10	17.37
	85	24.97	24.63	15.44

TABLE 3

Resolution $\sigma(1/P_t)\%$
Beam rϕ,z = 20 μ, 1 cm
Silicon Tracker = 15 μ
Straw Tracker = 150 μ
Muon Trackers = 500 μ
Cut $\pm 4\sigma$

P_t(GeV/c)	θ(deg.)	σ_{inner}	σ_{muon}	σ_{global}
100	10	5.60	15.11	5.57
	15	9.57	13.69	9.52
	20	1.43	14.51	1.43
	25	1.25	14.96	1.25
	35	1.10	14.40	1.10
	45	1.10	15.79	1.10
	55	1.25	18.14	1.25
	65	1.31	16.76	1.30
	75	1.36	19.86	1.36
	85	1.30	20.50	1.30
200	10	11.51	18.14	11.48
	15	21.38	13.83	13.83
	20	2.58	14.32	2.57
	25	2.10	15.86	2.09
	35	2.07	17.26	2.06
	45	2.20	17.61	2.19
	55	2.35	17.87	2.34
	65	2.50	19.28	2.48
	75	2.65	19.74	2.64
	85	2.57	18.34	2.54
300	10	18.47	27.94	18.21
	15	28.59	14.00	13.94
	20	4.01	13.94	3.92
	25	3.09	16.08	3.07
	35	3.12	20.30	3.10
	45	3.12	22.68	3.10
	55	3.42	18.94	3.40
	65	3.53	20.74	3.50
	75	3.84	19.60	3.80
	85	3.76	21.12	3.75

CONTINUE TABLE 3

P_t(GeV/c)	θ (deg.)	σ_{inner}	σ_{muon}	σ_{global}
	10	28.67	34.17	27.07
	15	34.58	15.06	15.00
	20	6.57	14.78	6.34
	25	5.07	16.82	5.05
500	35	6.12	25.04	6.09
	45	5.71	26.91	5.56
	55	6.12	24.97	6.04
	65	6.17	25.04	6.14
	75	6.53	24.74	6.45
	85	6.69	19.57	6.62
	10	39.41	53.56	37.20
	15	27.03	18.54	18.21
	20	15.42	17.04	14.87
	25	13.21	17.85	13.04
750	35	11.67	24.87	11.30
	45	9.85	28.97	9.47
	55	9.83	27.13	9.70
	65	9.24	28.76	9.19
	75	9.70	27.61	9.58
	85	9.03	23.76	8.94
	10	44.57	108.43	40.84
	15	33.89	18.48	18.30
	20	18.86	18.74	16.07
	25	14.47	23.46	14.12
1000	35	14.21	38.72	13.74
	45	13.15	31.14	12.76
	55	13.42	32.61	12.71
	65	13.17	34.07	12.70
	75	13.67	32.80	12.92
	85	12.86	29.17	11.46

CONTINUE TABLE 3

P_t(GeV/c)	θ (deg.)	σ_{inner}	σ_{muon}	σ_{global}
	10	73.12	209.89	62.56
	15	41.82	39.38	36.17
	20	27.25	30.51	20.24
	25	25.91	49.53	23.51
2000	35	26.37	60.14	22.21
	45	25.17	57.56	22.94
	55	25.78	59.80	22.56
	65	24.39	60.14	22.04
	75	24.51	58.61	22.37
	85	23.39	57.64	22.41

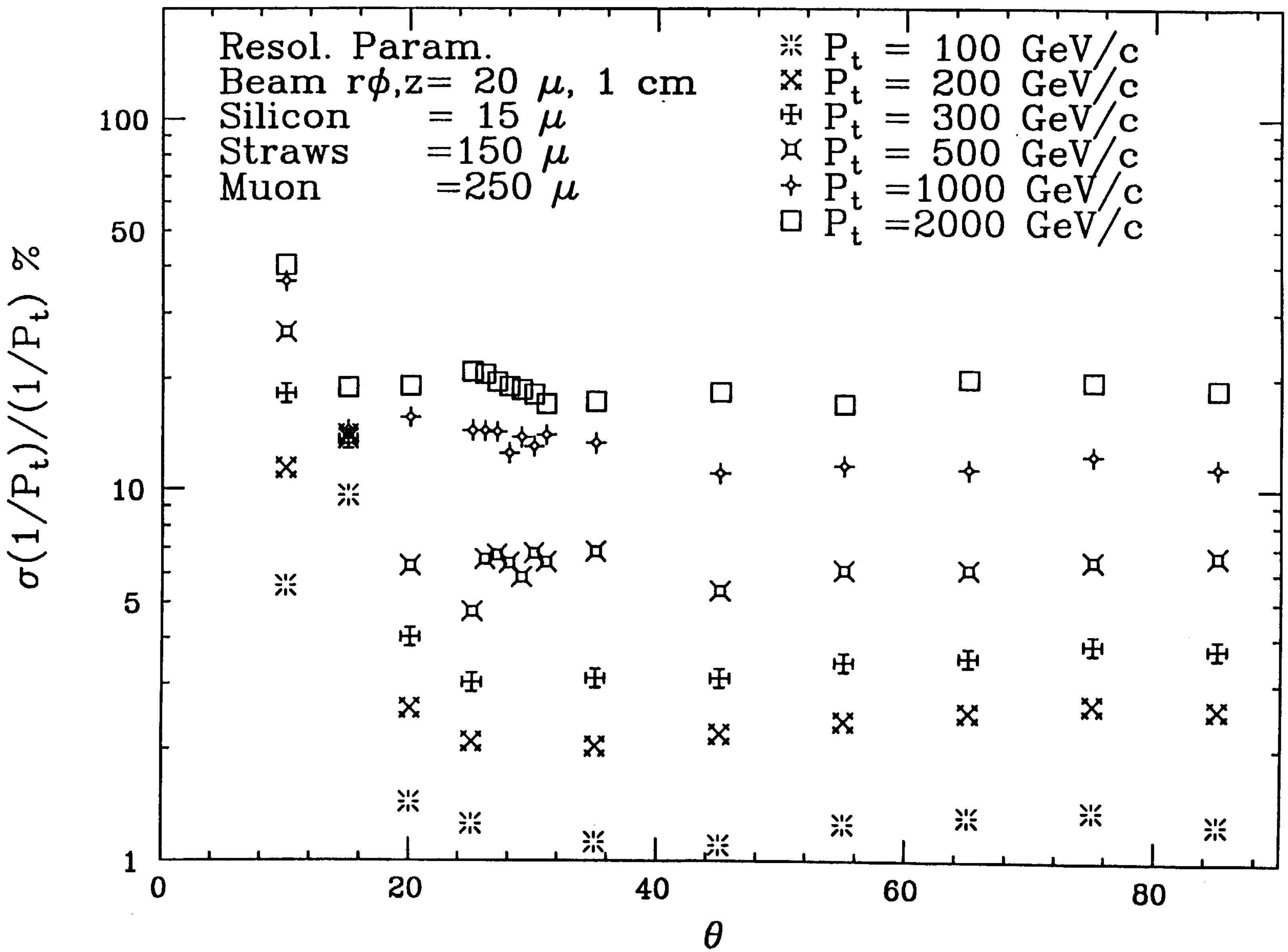

Resol. Param.
Beam rφ,z= 20 μ, 1 cm
Silicon = 15 μ
Straws =150 μ
Muon =250 μ
P_t = 100 GeV/c
P_t = 200 GeV/c
P_t = 300 GeV/c
P_t = 500 GeV/c
P_t =1000 GeV/c
P_t =2000 GeV/c
σ(1/P_t)/(1/P_t) %
θ
100
50
10
5
1
0
20
40
60
80

FIG. 1

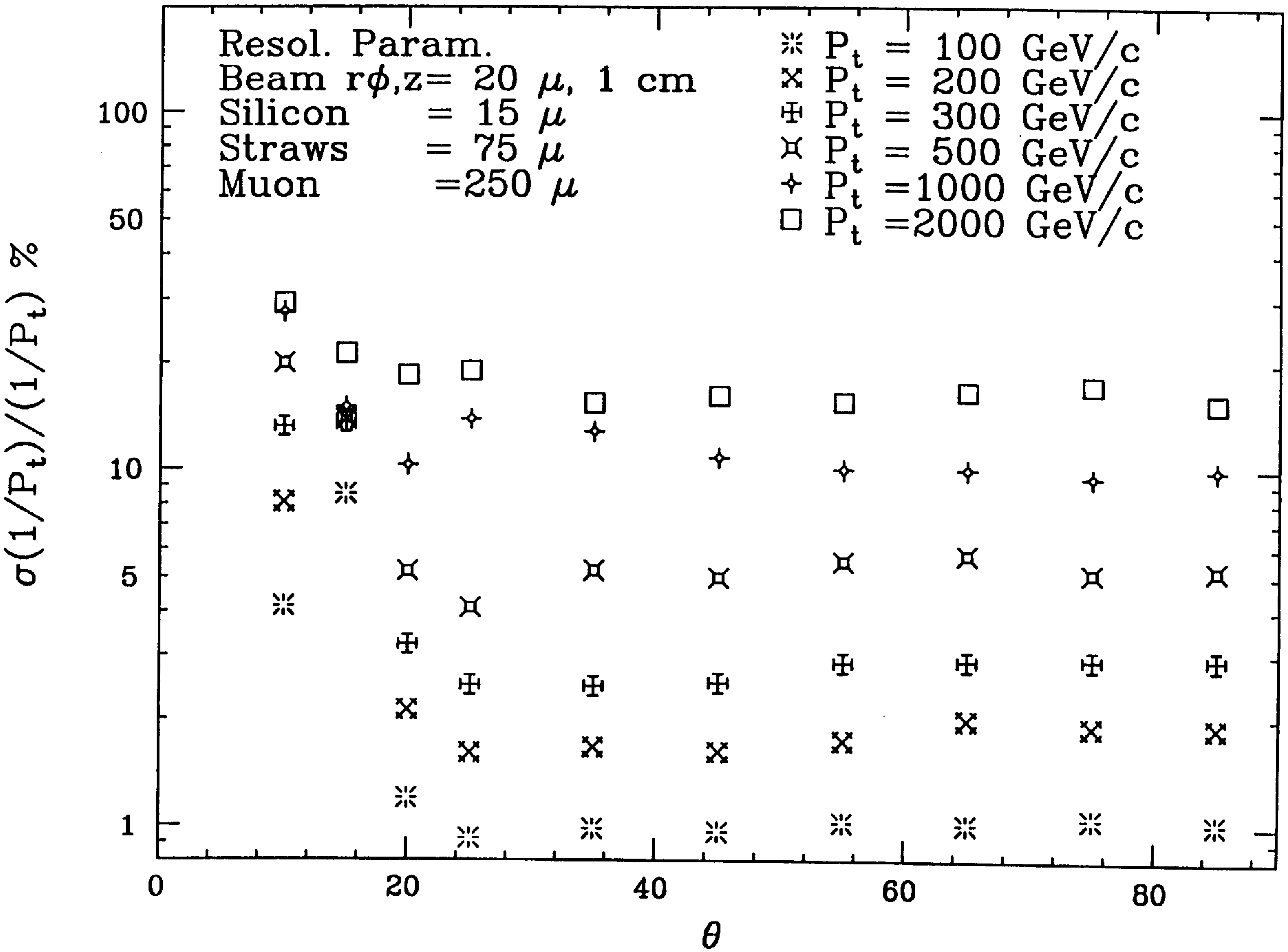

Resol. Param.
Beam rφ,z= 20 μ, 1 cm
Silicon = 15 μ
Straws = 75 μ
Muon =250 μ
P_t = 100 GeV/c
P_t = 200 GeV/c
P_t = 300 GeV/c
P_t = 500 GeV/c
P_t =1000 GeV/c
P_t =2000 GeV/c
σ(1/P_t)/(1/P_t) %
100
50
10
5
1
0
20
40
60
80
θ

FIG. 2

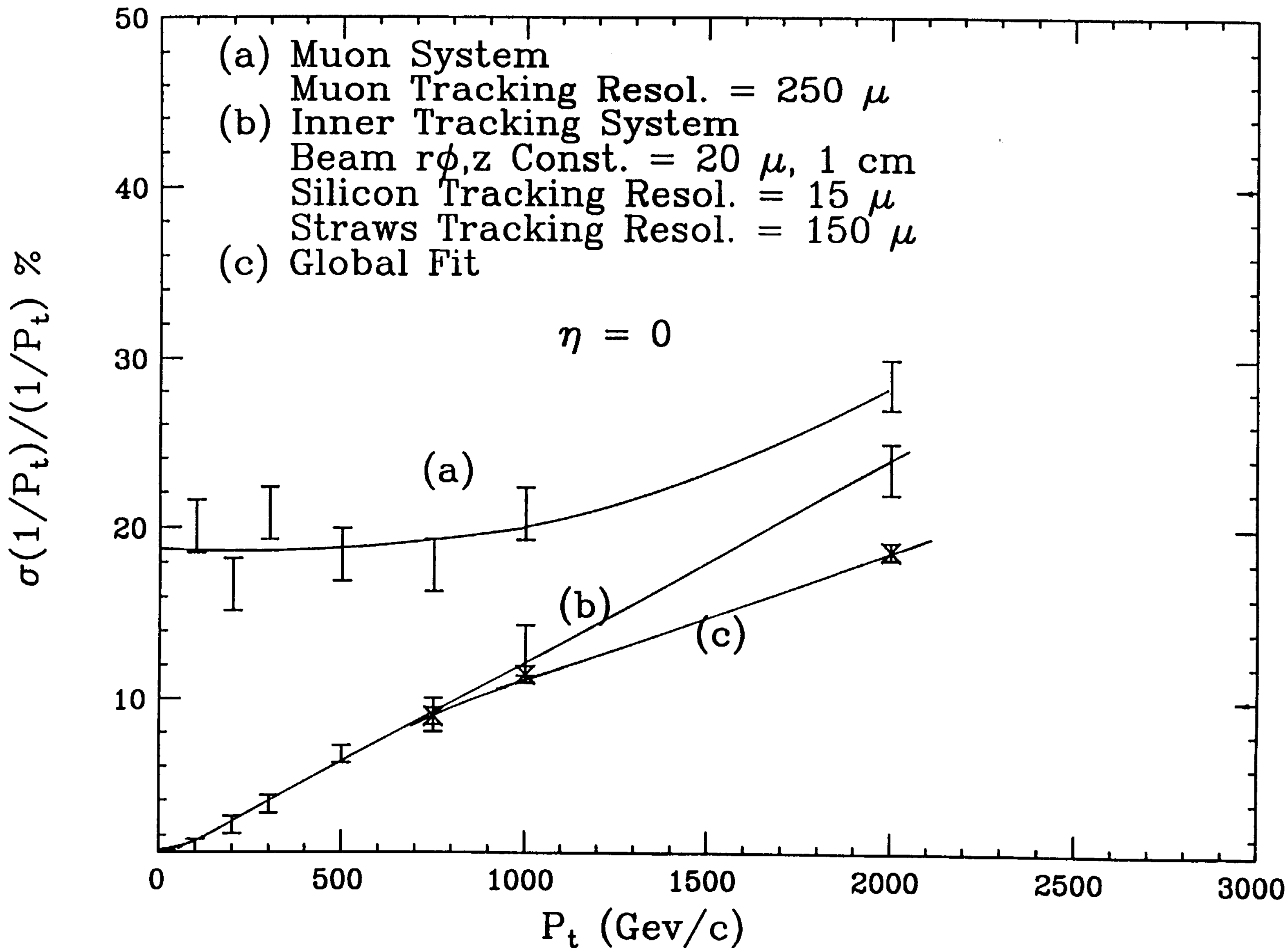
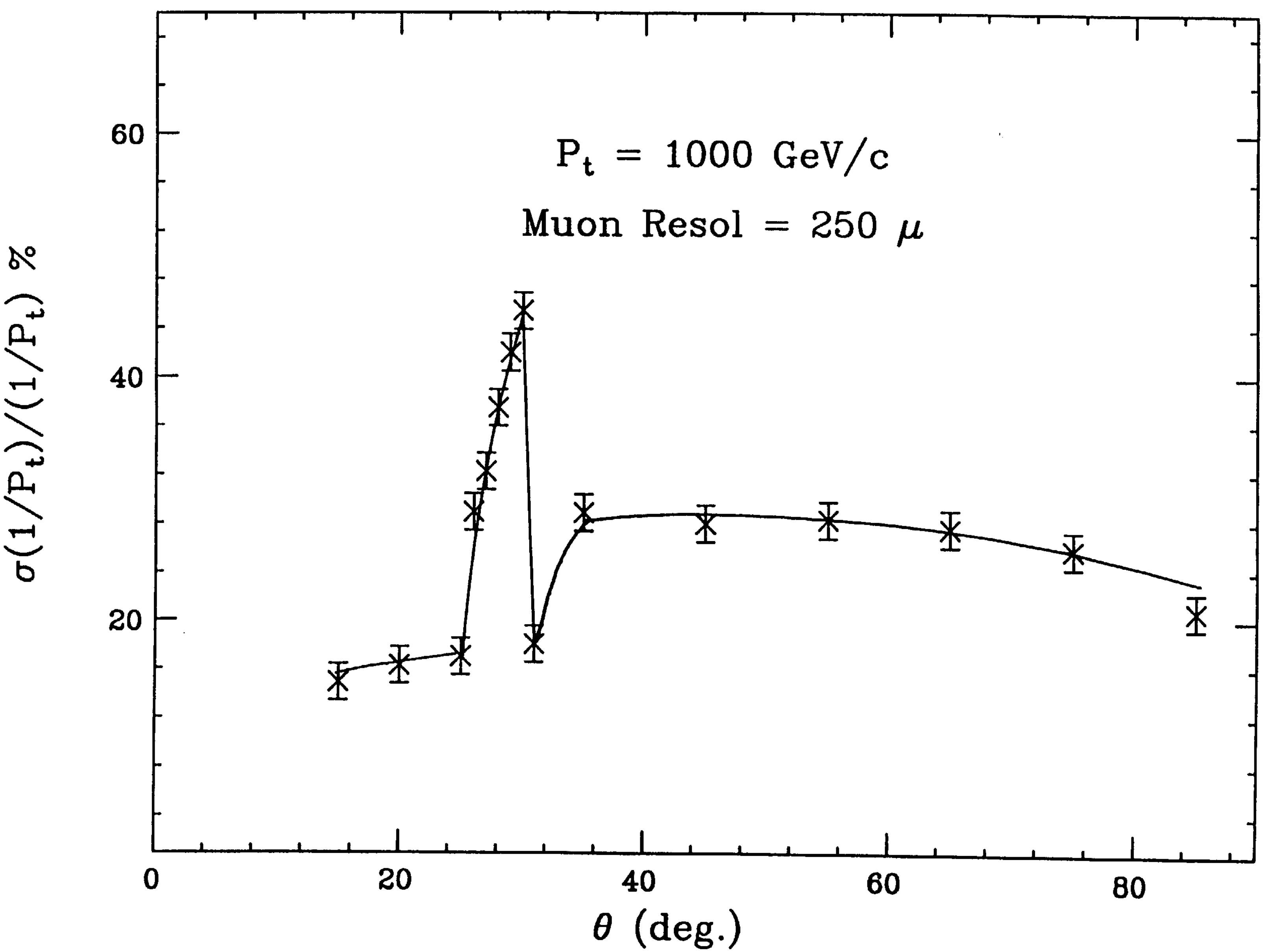

(a) Muon System
Muon Tracking Resol. = 250 μ
(b) Inner Tracking System
Beam rφ,z Const. = 20 μ, 1 cm
Silicon Tracking Resol. = 15 μ
Straws Tracking Resol. = 150 μ
(c) Global Fit
η = 0
(a)
(b)
(c)
σ(1/P_t)/(1/P_t) %
P_t (Gev/c)
0
10
20
30
40
50
500
1000
1500
2000
2500
3000

FIG. 3



P_t = 1000 GeV/c
Muon Resol = 250 μ
$\sigma(1/P_t)/(1/P_t)$ %
θ (deg.)
0
20
40
60
80

FIG 4

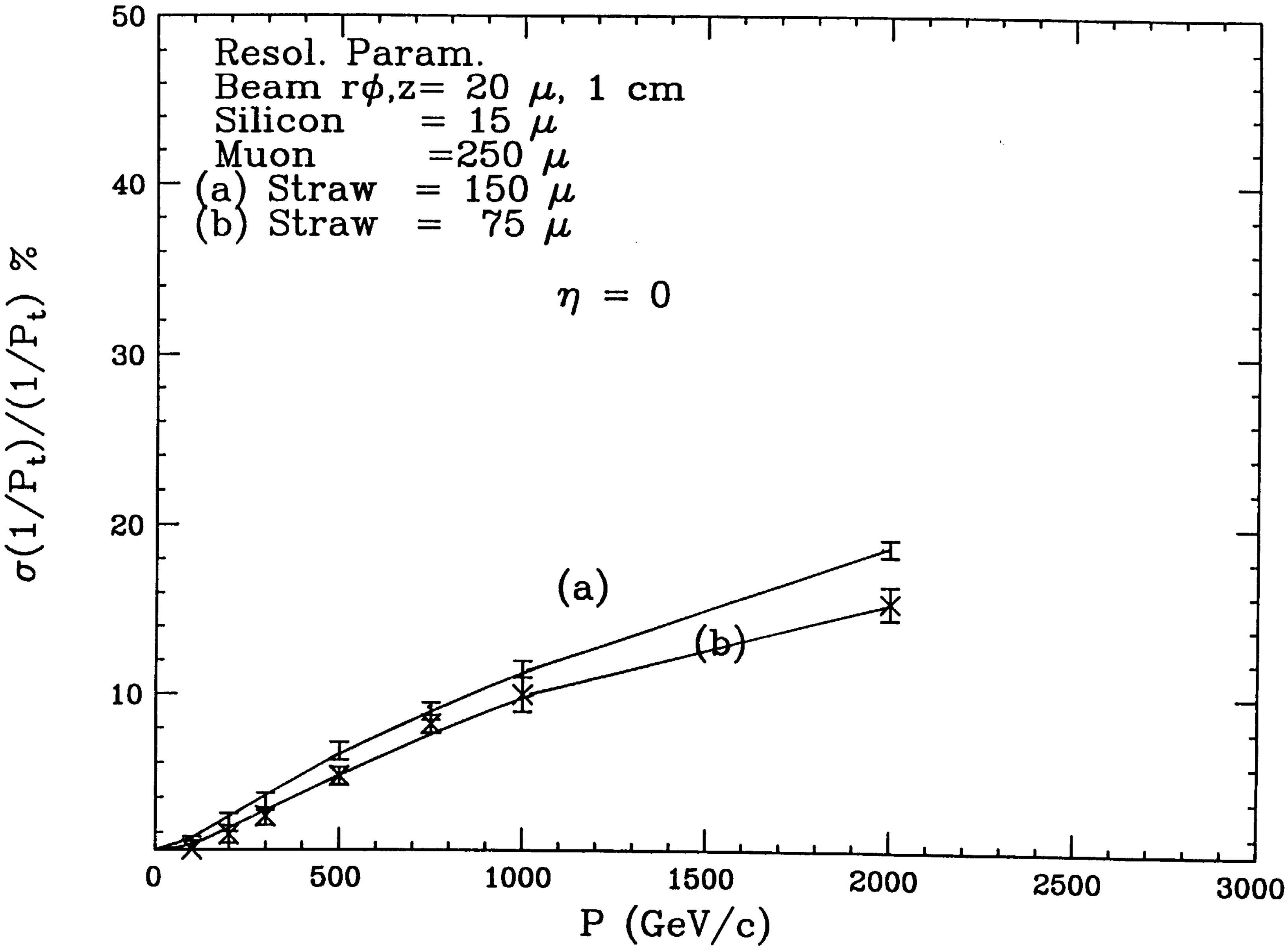

Resol. Param.
Beam rφ,z= 20 μ, 1 cm
Silicon = 15 μ
Muon =250 μ
(a) Straw = 150 μ
(b) Straw = 75 μ
η = 0
(a)
(b)
σ(1/P_t)/(1/P_t) %
P (GeV/c)
0
500
1000
1500
2000
2500
3000
10
20
30
40
50

FIG. 5

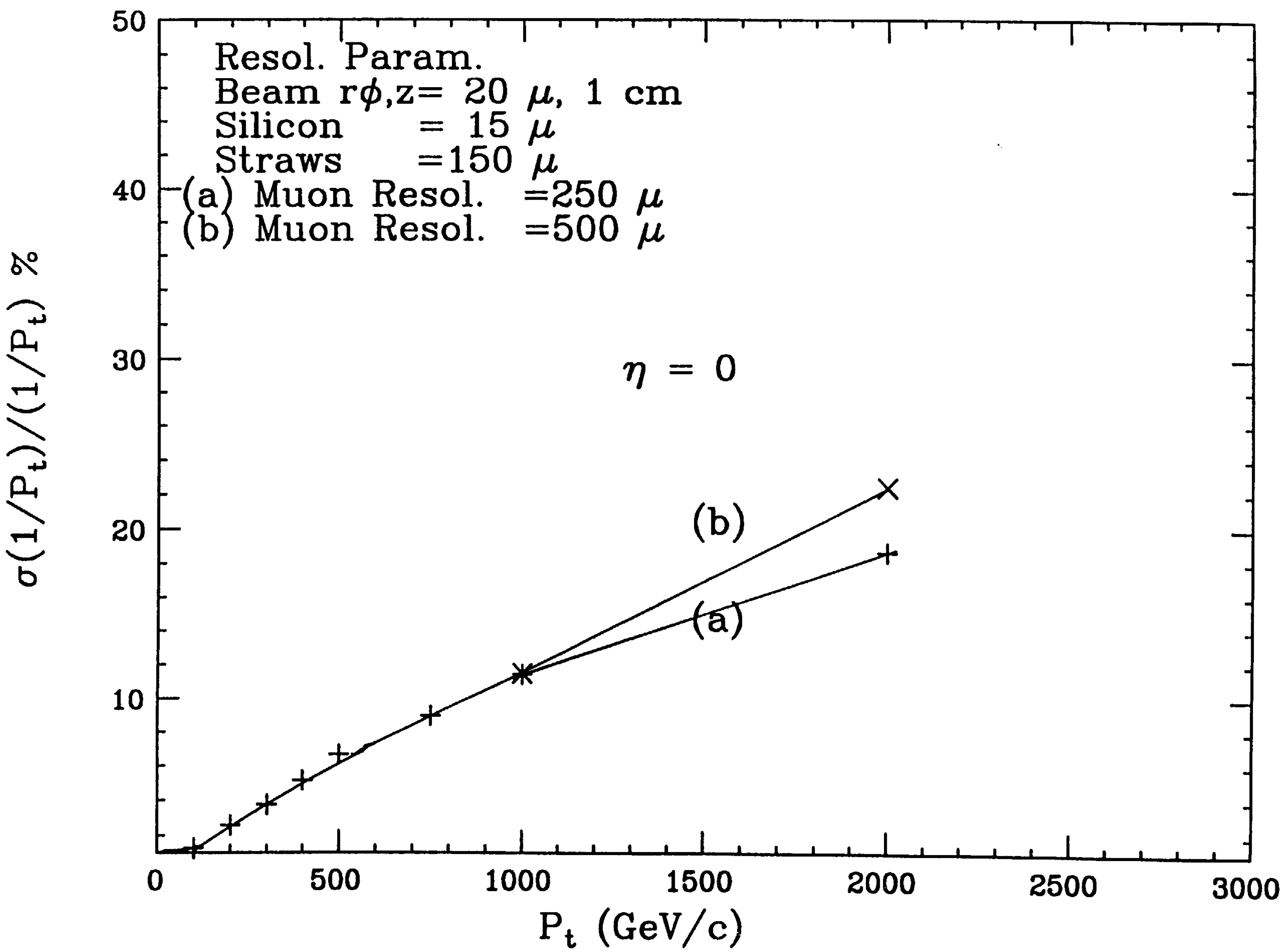

Resol. Param.
Beam rϕ,z= 20 μ, 1 cm
Silicon = 15 μ
Straws =150 μ
(a) Muon Resol. =250 μ
(b) Muon Resol. =500 μ
η = 0
(b)
(a)
σ(1/P_t)/(1/P_t) %
P_t (GeV/c)
0
500
1000
1500
2000
2500
3000
10
20
30
40
50

FIG. 6