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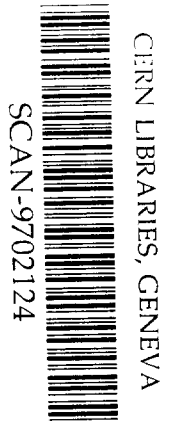
# **THE ORIGIN OF MILLISECOND PULSAR VELOCITIES**

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# The Origin of Millisecond Pulsar Velocities

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**Abstract.** To help determine the origin of millisecond pulsars, we compute the space velocities predicted by various models of their formation. It is difficult to produce a millisecond pulsar velocity greater than  $270 \text{ km s}^{-1}$  with any model, unless the formation of the neutron star is accompanied by some form of asymmetric kick. If the accretion-induced collapse of a white dwarf is the sole formation mechanism, then kicks are essential to reproduce the high velocities of some of the millisecond pulsars observed. Our computations show that, in general, we expect those systems with shorter orbital periods to have larger velocities than those with longer periods, but any relation between the final orbital period and space velocity is fairly weak, especially if asymmetries are involved. We demonstrate that binary millisecond pulsars probably evolved through a common envelope before the formation of the neutron star, and that the energy used to expell the envelope is more than that available from the gravitational potential.

More millisecond pulsar proper motions and distances will help to constrain the various formation models, but early indications are that low-velocity millisecond pulsars are over-represented in the sample, especially if the most recent estimates of kick velocities are used.

**Key words:** binaries: evolution, mass-loss, compact stars – stars: neutron – pulsars: general, formation

## 1. Introduction

Recently, a large number of millisecond pulsars have been reported: Camilo, Nice & Taylor (1993), Nice, Taylor & Fruchter (1993), Foster, Wolszczan & Camilo (1993), Johnston et al. (1993), Bailes et al. (1994), and Lorimer et al. (1995a). Most of these pulsars appear in binary systems with orbital periods between 0.4 and 147 d and spin

periods between 1.6 and 16 ms. The high incidence of binaries among millisecond pulsars, in combination with rapid spin and relatively weak magnetic fields, strongly suggest that a large fraction of them (possibly all) are old neutron stars that have been “recycled” by accreting mass and angular momentum from a companion star in a binary system (e.g. Alpar et al. 1982; van den Heuvel 1984). In the only other viable formation model, binary millisecond pulsars are formed immediately following the accretion-induced collapse (AIC) of a white dwarf – see e.g. Nomoto & Kondo (1991); Canal, Isern & Labay (1990) and references therein. This model has some attractive features because it eliminates the need for a long spin-up phase which leads to a birthrate problem between the millisecond pulsars and the low-mass X-ray binaries (Kulkarni & Narayan 1988).

Binary systems in which supernovae occur recoil as the result of sudden mass loss from the system (Blaauw 1961). Most radio pulsars are thought to receive a kick at birth as a result of the asymmetry of the explosion which forms the pulsar (Dewey & Cordes 1987; Bailes 1989). Recent studies of the proper motion of single pulsars by Lyne & Lorimer (1994) indicate that these kicks could be typically  $\sim 450 \text{ km s}^{-1}$ . Kicks imparted to the neutron star tend to increase the runaway velocity of any resulting binary (Cordes & Wasserman 1984). It is possible that such kicks could also affect neutron stars created by AIC. The amount of mass lost in a type Ib/c supernova explosion is considerably greater than that due to AIC, and hence the velocities of millisecond pulsars may provide some clue to their likely origin (Bailes 1989).

Monte Carlo simulations on a large ensemble of binary systems enables one to examine the expected characteristics of the binary millisecond pulsar population and the physics behind the interactions during their evolution. Only a few general simulation studies are known in the literature (e.g. Dewey & Cordes 1987; Lipunov et al. 1994; Pols & Marinus 1994). Dewey & Cordes used simulations to determine whether or not the velocities of pulsars could arise from the break-up of binary systems, whereas

Lipunov et al. used them primarily to calculate the number of binary pulsars with black hole companions. Pols & Marinus used simulations to study the evolution of young open clusters and the formation of blue stragglers.

Here we use Monte Carlo simulations to determine the range of potential runaway velocities predicted by the various models for the formation of millisecond pulsars. In particular we examine any potential relationship between the orbital period of a binary millisecond pulsar and its runaway velocity. We highlight the importance of the physical assumptions used in the binary evolution model especially during the common envelope and spiral-in phase. The assumptions governing the orbital evolution are outlined in Section 2, and we present the results of our simulations in Section 3. These are compared with the observational data in Section 4. Finally, we present our conclusions in Section 5.

## 2. A model of binary evolution

We have developed a computer code to follow the evolution of a binary system from the zero-age main sequence to its “final” state, at all stages keeping careful track of the mass and orbital separation of the two stars. A large number of outcomes is possible from massive binary evolution (see Dewey & Cordes 1987) ranging from systems which are disrupted at the time of the supernova explosion, to binary pulsars with white dwarf, neutron star or black hole companions. In this paper we restrict our attention to systems which are likely to form millisecond pulsars.

In Fig. 1 we depict the standard model of millisecond pulsar formation in which the neutron star is formed from a type Ib/c supernova explosion. Type Ib,c supernovae like type II are found among early-type stars and young stellar populations, but the absence of hydrogen in their spectra indicate that these exploding He-stars originate from closely interacting binaries, whereas single early-type stars are thought to be progenitors of type II supernovae – see e.g. van den Heuvel (1994). Binary millisecond pulsars (hereafter BMSPs) are thought to originate from close binaries which survive the type Ib/c supernova of the He-star (the progenitor of the neutron star) and continue to evolve through a mass-exchanging X-ray phase. This causes the neutron star to spin-up to very high rotation frequencies. Since the mass accretion rate is, at most, the Eddington accretion limit  $\sim 1.5 \times 10^{-8} M_{\odot} \text{ yr}^{-1}$  (for spherical accretion), recycled binary millisecond pulsars are likely to originate from low-mass X-ray binaries (LMXBs) which evolve on long ( $\gtrsim 10^8$  yr) timescales. Thus binary millisecond pulsars should possess white dwarf companions. For an excellent review on the formation of binary and millisecond radio pulsars see Bhattacharya & van den Heuvel (1991).

To simulate the formation of binary millisecond pulsars we assume that the initial binary system consists of two

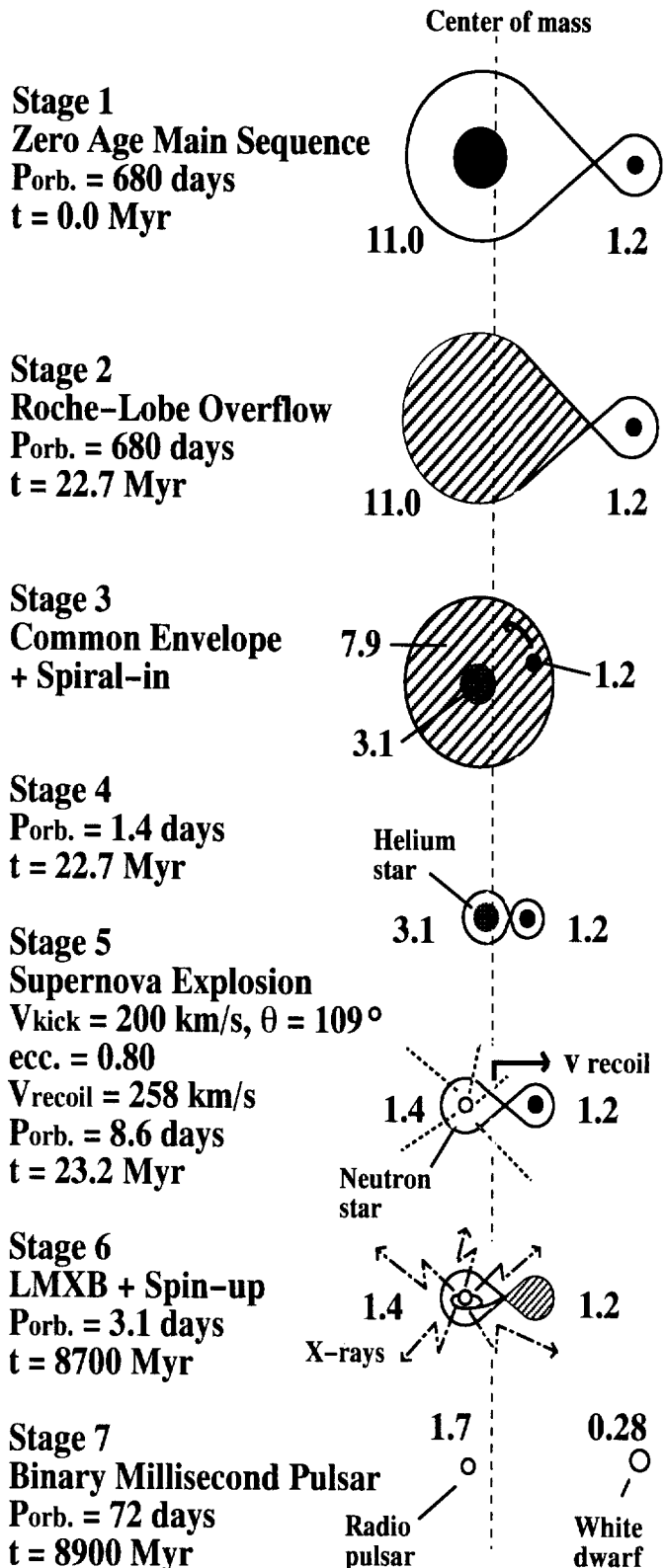


Fig. 1. Illustration of the formation of a binary millisecond pulsar according to the standard evolutionary model – see text. The stellar masses are in units of  $M_{\odot}$ .

zero-age main sequence (ZAMS) stars, and chose primary masses in the interval  $10\text{--}25 M_{\odot}$ , and initial separations between  $5\text{--}2000 R_{\odot}$ . In our code we used a flat logarithmic initial separation distribution ( $\Gamma(a) \propto a^{-1}$ ) and assumed a Salpeter initial mass function for the ZAMS primary stellar masses of  $N(m) \propto m^{-2.35}$  combined with a mass-ratio function:  $f(q) = 2/(1+q)^2$  (Kuiper 1935). We adopt the term “primary” to refer to the *initially* more massive star, regardless of the effects of mass transfer or loss as the system evolves. The results presented in this paper are based on a typical simulation of  $10^5$  ZAMS binaries.

We assume that all the initial binary systems consist of two ZAMS stars in a circular orbit. The mass ratio of the binary is denoted by  $q = M_2/M_1$ , its total mass by  $M = M_1 + M_2$ , its orbital period by  $P_{\text{orb}}$ , the semi-major axis of the relative orbit by  $a$ , and the eccentricity by  $e$ .

### 2.1. Critical radius for Roche-lobe overflow

The effective gravitational potential in a binary is determined by the masses of the two stars, and the centrifugal force arising from the motion of the stars around one another. The equipotential surfaces from the two stars meet in the critical first Lagrangian point,  $L_1$ , and defines the “pear shaped” Roche-lobe, cf. Fig. 1. If the primary star evolves to fill its Roche-lobe, the unbalanced pressure at  $L_1$  will cause mass transfer (Roche-lobe overflow, RLO) from the primary to the secondary star. The critical radius of the primary  $R_L$ , defined as the radius of a sphere which has the same volume as the Roche-lobe, is given by Eggleton (1983):

$$\frac{R_L}{a} = \frac{0.49}{0.6 + q^{2/3} \ln(1 + q^{-1/3})} \quad (1)$$

A ZAMS star born in a close binary system with a radius smaller than that of its Roche-lobe may, because of expansion of its envelope at a later evolutionary stage, begin RLO. Kippenhahn & Weigert (1967) define three types of close binary interaction: cases *A*, *B* and *C*. In case *A*, the system is so close that the primary star fills its Roche-lobe while burning core-hydrogen on the main sequence. In case *B*, its Roche-lobe is filled after the end of core-hydrogen burning but before helium ignition, and in case *C* the Roche-lobe is filled during helium shell burning before carbon ignition. Wide binaries where the primary never fills its Roche-lobe will not be considered here, since such systems will remain very wide and do not evolve into a BMSP.

A very close binary can also be forced into RLO if the binary shrinks as a result of orbital angular momentum losses (Section 2.7.2).

In our simulations we used interpolations of the evolutionary grids of Maeder & Meynet (1988, 1989) for initial masses below  $12 M_{\odot}$  and those of Maeder (1990) for masses in the range  $15$  to  $25 M_{\odot}$ . These grids take into account stellar-wind mass loss in massive stars and

giant-branch stars, and also include a moderate amount of overshooting from the convective core. For helium-stars, we used the calculations of Paczynski (1971). From these models of stellar evolution we calculate the radius and age of the stars at the onset of RLO. Using other models with different chemical composition and convective overshooting may have changed these values slightly, but would not have had any significant effect on the parameters of the final population of BMSPs.

### 2.2. Mass loss from a stellar wind

For early-type stars a significant amount of mass is lost from the system in the form of a stellar wind during main-sequence evolution. The change in orbital separation depends crucially on the specific angular momentum of the wind matter – which is rather poorly known. Assuming that the average angular momentum (per unit mass) carried away by a spherically symmetric stellar wind at high velocity is the same as the average orbital angular momentum of the mass-losing star, then the orbital separation will increase simply as:

$$\frac{a_f}{a_i} = \frac{M_i}{M_f} \quad (2)$$

where the subscripts *i* and *f* denote the initial and final parameters, respectively.

### 2.3. Roche-lobe overflow (RLO)

Once the RLO has started it continues until the primary no longer fills its Roche-lobe because of the orbit’s expansion. This will happen if the star has become the lighter of the two stars (because of the mass transfer), or if matter is lost from the system in the form of a stellar wind. The orbital angular momentum of a circular binary is given by:

$$J_{\text{orb}} = \frac{M_1 M_2 \Omega a^2}{M_1 + M_2} \quad (3)$$

where  $\Omega$  is the orbital angular velocity,  $\Omega = \sqrt{GM/a^3}$ . A simple differentiation of eq.(3) then yields the rate of change in orbital separation, which in the general case can be written as:

$$\frac{\dot{a}}{a} = -2 \left[ 1 + (\beta - 1) \frac{M_1}{M_2} - \frac{\beta}{2} \frac{M_1}{M} \right] \frac{\dot{M}_1}{M_1} + 2 \frac{\dot{J}_{\text{orb}}}{J_{\text{orb}}} \quad (4)$$

where  $\beta$  is the fraction of the exchanged matter that leaves the system – *i.e.*  $\dot{M}_2 = -(1 - \beta) \dot{M}_1$  and  $\dot{M}_1 < 0$ . Non-conservative interactions arise in situations where  $\beta \neq 0$  and/or  $\dot{J}_{\text{orb}} \neq 0$ .

### 2.3.1. Conservative RLO

If no mass or orbital angular momentum is lost from the system during the mass exchange, one then has  $\beta = \Delta J_{\text{orb}} = 0$ , and from eq.(4) we see that mass transfer from the primary to the secondary will result in a decrease of the orbital separation if  $M_1 > M_2$ , and a widening of the orbit if  $M_1 < M_2$ . Thus the orbit can initially shrink, then reach a minimum separation when the masses of the two stars are equal and thereafter expand until the mass transfer ceases. Using eq.(4), and assuming that  $\beta = \Delta J_{\text{orb}} = 0$ , gives the ratio of the final and initial separation:

$$\frac{a_f}{a_i} = \left( \frac{M_{1i} M_{2i}}{M_{1f} M_{2f}} \right)^2 \quad (5)$$

In our simulations we assumed that mass transfer before the neutron star formed was conservative if  $q > 0.4$ , which we adopt as a mass-ratio threshold value,  $q_{\text{ce}}$ , for the formation of a common envelope (see below).

### 2.4. Common envelope and spiral-in

A very important stage of the evolution of close binaries is the formation of a common envelope (CE). The onset of a CE-phase and break-down of the conservative mass transfer assumptions depend on:

1. the depth of the donor convective envelope,
2. the mass-ratio,  $q$ , and
3. the evolutionary state of the accreting star.

Generally, if a star in a binary fills its Roche-lobe (either because nuclear fuel at the center has been exhausted and its envelope swells up or from loss of orbital angular momentum) there will be an attempt to transfer mass. However, if the thermal re-adjustment timescale of the accreting star is larger than the mass-transfer timescale, then the accreted layer piles up above the companion, heats up and expands so that the accreting star will also fill its Roche-lobe. The result is that the transferred matter will form a CE embedding both stars (Paczynski 1976; Ostriker 1976), cf. stage 3 in Fig. 1. Furthermore, in some cases the creation of a drag-force, arising from the motion of a star through the envelope, will lead to dissipation of orbital angular momentum (spiral-in) and deposit of orbital energy in the envelope. Hence, the global outcome of a CE-phase is:

1. reduced binary separation
2. often ejection of the CE.

For a review see Iben & Livio (1993). They showed that the change of the Roche-lobe radius,  $R_L$  of the star losing mass is given by:

$$d \ln R_L \approx 2 \left| \frac{d M_1}{M_1} \right| \left[ 0.78 - \left( \frac{f/q + 1}{1 + q} \right) \right] + 2 d \ln J_{\text{orb}} \quad (6)$$

where  $f = 1 - \beta$  is the fraction of the transferred matter that remains in the system. Thus for  $q < 1.28$  a runaway mass transfer can occur. In order to have an effective spiral-in effect, we assumed a CE-phase, only if  $q_{\text{ce}} < 0.4$ , i.e. if the donor star is at least more than twice as heavy as the accretor (cf. Bhattacharya & van den Heuvel 1991).

If RLO is *always* conservative, then this implies that all BMSPs must have evolved through the phase of a common envelope and spiral-in. Conservative RLO would make the companion star heavy enough to explode in a supernova forming another neutron star, not a white dwarf. This follows from the fact that, to avoid a CE-phase, the secondary must have an initial mass  $M_2 \geq 3-4 M_{\odot}$  (assuming a threshold mass of 8-10  $M_{\odot}$  for producing a neutron star) and therefore, after the RLO, it will also have a mass above the critical mass limit for undergoing a supernova explosion, leaving a double neutron star system if the orbit stays bound. Thus in our model, we assume all BMSPs to have evolved through a CE-phase before the supernova explosion that created the neutron star.

We used a simple approach (Webbink 1984) to describe the CE-evolution (see e.g. Bhattacharya & van den Heuvel 1991; Iben & Livio 1993). Let  $\eta_{\text{ce}}$  describe the efficiency of ejecting the envelope, i.e. of converting orbital energy into the kinetic energy that provides the outward motion of the envelope:  $\Delta E_{\text{bind}} \equiv \eta_{\text{ce}} \Delta E_{\text{orb}}$  or

$$\frac{GM_1 M_{1\text{env}}}{\lambda a_i r_L} = \eta_{\text{ce}} \left[ \frac{GM_{1\text{core}} M_2}{2a_f} - \frac{GM_1 M_2}{2a_i} \right] \quad (7)$$

which yields:

$$\frac{a_f}{a_i} = \frac{M_{1\text{core}} M_2}{M_1} \frac{1}{M_2 + 2M_{1\text{env}} / (\eta_{\text{ce}} \lambda r_L)} \quad (8)$$

where  $r_L = R_L / a_i$  is the dimensionless Roche-lobe radius of the donor star (eq. 1),  $\lambda$  is a weighting factor ( $\leq 1.0$ ) for the binding energy of the core and envelope of the donor star and, finally, where  $M_{1\text{core}}$ ,  $M_{1\text{env}}$ ,  $a_i$  and  $a_f$  are the mass of the helium core and hydrogen-rich envelope of the evolved star, and the initial and final separation, respectively.

The choice of  $\eta_{\text{ce}}$  and the survival constraint for a binary undergoing a CE + spiral-in phase is of great importance for the kinematics of the surviving binaries – a discussion follows later.

### 2.5. The supernova explosion, SN

We assumed that the primary lost all of its envelope in the RLO (or CE-phase) leaving a helium star (Wolf-Rayet star) which exploded leaving a neutron star (stage 4 and 5, respectively in Fig. 1), or became a white dwarf, depending on whether or not its mass was above a critical limit. We chose a critical mass limit for the helium star of  $2.8 M_{\odot}$ . This value corresponds roughly to an initial mass of  $10 M_{\odot}$  (8-9  $M_{\odot}$  for case C and 10-12  $M_{\odot}$  for case B

RLO) or  $8M_{\odot}$  for very wide binaries and single stars.

To calculate the mass of the He-core of the primary stars, we used the expressions below which we found to agree well with the grids used in our code:

$$M_{\text{core}}^{\text{caseB}} = \frac{1}{80}(M_{\text{zams}} - 1)^2 + \frac{1}{8}(M_{\text{zams}} - 1) + 0.2 \quad (9)$$

$$M_{\text{core}}^{\text{caseC}} = \frac{1}{100}(M_{\text{zams}} - 1)^2 + \frac{1}{4}(M_{\text{zams}} - 1) + 0.4$$

for Roche-lobe overflow of case *B* and case *C*, respectively. In our model we have not taken into account for the possibility of a second RLO, prior to the SN, of helium stars with masses  $M_{\text{He}} \lesssim 3.5M_{\odot}$  (Habets 1985).

If the helium star forms a supernova it is a good approximation that the collapse of the helium star is instantaneous compared with the binary's orbital period. We assumed all neutron stars to be born with the canonical mass of  $1.4M_{\odot}$ . If the companion star is not affected significantly by the impact of the supernova shell, and if the pre-supernova orbit is circular, then systems which eject at least half of their total mass will be disrupted. However, if a random orientated kick velocity is added to the newborn neutron star, due to a slight asymmetry in the SN, then binaries can, in some cases, be stabilized and survive losing more than half of the total mass. For a review on the consequences of sudden mass loss in a binary, see Hills (1983).

The orbital energy of a binary is given by:

$$E_{\text{orb}} = -\frac{GM_1M_2}{2a} = -\frac{GM_1M_2}{r} + \frac{1}{2}\mu v_{\text{rel}}^2 \quad (10)$$

where  $a$  is the semi-major axis,  $r$  is the separation between the stars at the moment of explosion and  $\mu$  is the reduced mass of the system. Assuming a pre-SN circular orbit, the relative velocity of the two stars is simply:  $v_{\text{rel}} = \sqrt{G(M_1 + M_2)}/r$  and one can show that the change of the semi-major axis, as a result of the SN, is given by (Flannery & van den Heuvel 1975):

$$\frac{a_f}{a_i} = \left[ \frac{1 - (\Delta M/M)}{1 - 2(\Delta M/M) - (v_{\text{kick}}/v_c)^2 - 2\cos\theta v_{\text{kick}}/v_c} \right] \quad (11)$$

where  $a_i=r$  and  $a_f$  are the initial and final semi-major axis, respectively,  $\Delta M$  is the amount of matter lost,  $v_c$  is the pre-SN velocity of the helium star in a reference frame fixed on the companion star,  $v_{\text{kick}}$  is the magnitude of the kick velocity and  $\theta$  gives the direction of the kick relative to the orientation of the pre-SN velocity. There exists a critical angle,  $\theta_{\text{crit}}$  for which all SNe with  $\theta < \theta_{\text{crit}}$  will result in disruption of the orbit. Thus the probability for a binary to survive the SN in which the newborn neutron star receives a kick,  $v_{\text{kick}}$  with a random orientation,  $\theta$  is given by: (*i.e.*  $P(\theta > \theta_{\text{crit}})$ )

$$P_{\text{bound}} = \frac{1}{2} \left\{ 1 + \left[ \frac{1 - 2\Delta M/M - (v_{\text{kick}}/v_c)^2}{2v_{\text{kick}}/v_c} \right] \right\} \quad (12)$$

In case the system stays bound (e.g. PSR B1259-63, Johnston et al. 1992 and PSR J0045-7319, Kaspi et al. 1994) the eccentricity,  $e$  of the post-SN orbit, containing a pulsar and an unevolved star, is calculated from:

$$e = \sqrt{1 + \frac{2E_{\text{orb}}L_{\text{orb}}^2}{\mu G^2 M_1^2 M_2^2}} \quad (13)$$

where  $\mu$  is the reduced mass,  $E_{\text{orb}}$  is given by eq.(10) and the orbital angular momentum can be derived from:

$$L_{\text{orb}} = r\mu \sqrt{(v_c + v_{\text{kick}} \cos\theta)^2 + (v_{\text{kick}} \sin\theta \sin\phi)^2} \quad (14)$$

Prior to the SN, the momentum of the primary star is equal (and opposite) to that of the secondary. Because the primary loses mass in the SN, the system will receive a recoil velocity,  $v_{\text{recoil}}$ . In the general case of an asymmetric SN where a kick velocity is added, the recoil velocity of the system is given by:

$$v_{\text{recoil}} = \sqrt{\Delta P_x^2 + \Delta P_y^2 + \Delta P_z^2} / (M_{\text{NS}} + M_2) \quad (15)$$

where the change in momentum is:

$$\begin{aligned} \Delta P_x &= M_{\text{NS}}(v_{\text{He}} + v_{\text{kick}} \cos\theta) - M_{\text{He}}v_{\text{He}} \\ \Delta P_y &= M_{\text{NS}} v_{\text{kick}} \sin\theta \cos\phi \\ \Delta P_z &= M_{\text{NS}} v_{\text{kick}} \sin\theta \sin\phi \end{aligned} \quad (16)$$

and  $v_{\text{He}}$  is the pre-SN velocity of the primary (He-star) in the center of mass reference frame and  $\phi$  is the angle between the projection of the kick velocity onto a plane perpendicular to the velocity vector of the He-star and the orbital plane (*i.e.*  $v_{\text{kick},y} = v_{\text{kick}} \sin\theta \cos\phi$ ).

We assumed binaries to coalesce if the neutron star is shot into the envelope of its companion due to an asymmetric kick.

## 2.6. Tidal circularization

If the post-SN orbit is not too wide the binary will, on a timescale depending on separation and mass of the companion star, become an X-ray source. Close binaries will also circularize their orbit because of tidal interactions, resulting in a reduced separation between the stars (Sutantyo 1974). In our model we assumed that all binaries with post-SN  $P_{\text{orb}} \lesssim 12^{\text{d}}$  are affected significantly by tidal interaction and reduced the semi-major axis by a factor of  $(1 - e^2)$  in order to conserve the orbital angular momentum. In our model we assumed the rest of the bound orbits to become circular with the new radius equal to the old semi-major axis<sup>1</sup>.

<sup>1</sup> However, see Verbunt & Phinney (1995) for the possibility of circularizing wider orbits in the companion's giant phase.

## 2.7. The evolution of X-ray binaries

If the primary exploded to form a neutron star and the binary stayed bound in a relatively close orbit, the system will proceed into an X-ray phase – stage 6 in Fig. 1.

### 2.7.1. High-mass X-ray binaries (HMXBs)

If the companion mass is of the order  $8 M_{\odot}$  or above, the HMXB (or Be/X-ray binary for wider orbits) evolves on a short nuclear timescale of  $\sim 10^7$  yr. During this period X-rays are emitted from the accreting neutron star because of stellar wind mass loss from the companion. Once the companion star enters its (sub)giant stage and swells up it will fill its Roche-lobe and commence mass transfer on a short time scale ( $\sim 10^4$  yr). During this phase the system is very likely to evolve into a second CE, because of the large mass-ratio between the massive early-type star and the neutron star. If a second supernova explosion follows the HMXB the result is either a bound double-neutron star system (e.g. PSR 1913+16) or two runaway pulsars – see Bailes (1989). In case the secondary star does not explode in the SN, a system with a heavy ( $\sim 1M_{\odot}$ ) white dwarf is left behind – cf. PSR 0655+64.

If the spiral-in is complete, a Thorne-Żytkow object may form (such objects might be progenitors of single millisecond pulsars).

### 2.7.2. Low- and intermediate-mass X-ray binaries

The evolution of binaries with lower masses, the likely progenitors of BMSPs, is more complicated. For a review see e.g. Verbunt (1990).

If  $P_{\text{orb}} < 2^d$  ( $a \lesssim 5R_{\odot}$ ) then loss of angular momentum drives the two stars together (via a magnetic stellar wind, gravitational wave radiation or both) until the low-mass star fills its Roche-lobe and starts transferring mass. In this case we have an X-ray binary with a main-sequence mass donor. To simplify further calculations we assumed that the final outcome of such a system is a neutron star – white dwarf (NS+WD) binary with a randomly generated period between 0 and  $\frac{1}{2}(P_{\text{orb}}^i/\text{days})^2$  days.

If  $P_{\text{orb}} > 2^d$  ( $a \gtrsim 10R_{\odot}$ ) the mass transfer is driven by the interior nuclear evolution of the companion star. In this case we get an X-ray binary with a (sub)giant donor. We model the orbital evolution by assuming that matter flows over from the companion star onto the neutron star in a conservative way, before it is ejected isotropically with the specific angular momentum of the neutron star. In this situation the change in orbital separation is given by (Tauris 1996):

$$\frac{a_f}{a_i} = \left( \frac{q_i(1-\beta) + 1}{q_f(1-\beta) + 1} \right)^{\frac{3\beta-5}{1-\beta}} \left( \frac{q_i + 1}{q_f + 1} \right) \left( \frac{q_i}{q_f} \right)^2 \quad (17)$$

in terms of the mass-ratios ( $q = M_{\text{donor}}/M_{\text{accretor}}$ ) before and after the mass transfer. The final outcome (stage 7

in Fig. 1) is an NS+WD system with a period between a few hours and  $10^3$  days, depending on the mass of the companion star and  $\beta$ .

To determine the final mass of the white dwarf we used the calculations of Joss et al. (1987) for  $M_2 < 2.3M_{\odot}$  and assumed  $M_{\text{WD}} = 0.07 M_2 + 0.5$  for  $2.3 < M_2/M_{\odot} < 10$  in order to produce CO and O-Ne-Mg white dwarfs with masses  $0.65 < M_{\text{WD}}/M_{\odot} < 1.2$ . If the mass of the donor is too low ( $< 0.9M_{\odot}$ ), the star will not evolve within the Hubble time.

The possibility that some relatively heavy low- or intermediate-mass X-ray binaries ( $1.5 \lesssim M_2 \lesssim 6M_{\odot}$ ) evolve into a second CE and spiral-in phase is very important for the formation of short-orbital period BMSPs and could help explain the lack of observed binary pulsars with orbital periods between 12 and 56 d (Tauris 1996). However, since the final orbital period evolution during the X-ray phase does not affect the runaway velocity of the binary systems, we have not included the possibility of a spiral-in of the neutron star in the calculations presented in this paper – for a discussion of this issue, the parameter  $\beta$  and an analysis of a possible  $M_{\text{WD}} - P_{\text{orb}}$  relation of wide-orbit low-mass binary pulsars, see Tauris (1996).

### 2.8. Spin-up of the neutron star

Most millisecond pulsars are members of binary systems which indicate that they are old neutron stars (dead pulsars from the “graveyard”) that have been recycled by accreting mass and angular momentum from the companion star. The theory of the spin-up process (see e.g. Lamb, Pethick & Pines 1973) predicts a lower limit for the final spin period of the pulsar:

$$P_{\text{eq}} \approx (2.4\text{ms}) B_9^{6/7} M_{\text{NS}}^{-5/7} \left( \frac{\dot{M}}{M_{\text{Edd}}} \right)^{-3/7} R_6^{18/7} \quad (18)$$

where  $B_9$  is the strength of the surface dipole magnetic field in units of  $10^9$  Gauss,  $R_6$  is the radius of the neutron star in units of 10 km,  $M_{\text{NS}}$  is its mass in units of  $M_{\odot}$  and  $\dot{M}$  and  $M_{\text{Edd}}$  are the accretion rate and the Eddington accretion limit ( $\sim 1.5 \times 10^{-8} M_{\odot} \text{yr}^{-1}$ ), respectively, of the neutron star.

### 2.9. Evaporation of companion star

The possibility of ablating the companion star, from either the X-ray flux during accretion and/or the wind of relativistic particles after the pulsar turns on, has interesting effects on close binary evolution and the formation of pulsars with ultra light companions and planets (PSR B1257+12, Wolszczan 1994) as well as single millisecond pulsars. We have not yet implemented such calculations in our code since they do not affect the velocities. For a review of radiation-driven evolution in close LMXBs – see Tavani (1992).

### 3. Results

Monte Carlo simulations are a powerful tool for investigating complicated interactions which depend on many parameters. Often the difficult task is not to produce a reasonable code but to analyse the results, pinpoint the important physical parameters behind the major trends in a simulated population and try to understand these results in terms of relatively simple physics.

Thus we have concentrated on the influence of  $\eta_{ce}$  and the amount of asymmetry in the SN before the pulsars form. These two parameters, together with the poorly known specific angular momentum of matter which leaves a binary system during mass loss, are the three most important parameters for the final appearance of a BMSP.

#### 3.1. A relation between $v_{recoil}$ and $P_{orb}$ ?

As the final velocity of a BMSP is dependent upon the separation of the two components at the time of the supernova explosion, we might expect there to be a relationship between the final orbital period and the runaway velocity of the binary. In Fig. 2a the distribution of BMSPs in the  $(v_{recoil}, P_{orb})$ -diagram is shown, using an evolutionary model with  $\eta_{ce} = 2.0$ , symmetric SN and stable RLO mass transfer from the evolved companion onto the neutron star assuming  $\beta = 0.7$ . The width of the interval of recoil velocities is entirely determined by the outcome of the CE-evolution (assuming a wide range of initial separations and masses). According to eq.(15), the post-SN  $v_{recoil}$  is proportional to the pre-SN orbital velocity of the exploding helium star. The minimum values of  $v_{recoil}^{min} \approx 25 \text{ km s}^{-1}$  are because the secondary main sequence star spiralled into the envelope of the giant as the result of drag forces, and thus the primary helium star obtained a large orbital velocity ( $> 70 \text{ km s}^{-1}$ ) before the supernova explosion. Similarly, the maximum values in this figure of  $v_{recoil}^{max} \approx 270 \text{ km s}^{-1}$  arise because binary systems coalesce if the separation between the components after spiral-in is shorter than the sum of the stellar radii. This is quite a soft survival constraint and thus allows for the existence of very tight, and thereby fast, pre-SN orbital velocities. Changing this survival constraint to a stricter one so that all binary components, where the secondary main sequence star fills its Roche-lobe after spiral-in, are assumed to coalesce, thus prevents the existence of very close (*i.e.* high-velocity) pre-SN orbits and yields  $v_{recoil}^{max} \approx 180 \text{ km s}^{-1}$ .

The scatter in the diagram is due to the initial spread in separation and masses (see Section 3.2) and is also a function of some of the parameters (e.g.  $\eta_{ce}$ ,  $q_{ce}$  and  $\beta$ ) governing the physics of binary interactions. Also, the differential tidal circularization as a function of the post-SN orbital period contributes to the scatter but does not change the velocity distribution.

Fig. 2a indicates a  $v_{recoil}$ - $P_{orb}$  anti-correlation though

some scattering is apparent. Binaries with high recoil velocities ( $> 100 \text{ km s}^{-1}$ ) and  $P_{orb} > 10^d$  arise from systems that almost became disrupted in the SN (*i.e.* binaries which lost almost half of their total mass in the explosion). Thus, in general if  $\eta_{ce} = 2.0$ , BMSPs with low recoil velocities should have large orbital periods and we expect high velocity systems to have tighter orbits. By reducing  $\eta_{ce}$  to 0.5 the  $v_{recoil}$ - $P_{orb}$  anti-correlation is destroyed, as illustrated in Fig. 2b. Note that the few surviving binaries with  $10^d < P_{orb} < 200^d$ , which did not coalesce in the CE-phase, are expected to have rather high velocities of  $\sim 200 \text{ km s}^{-1}$ . However, we are inclined to believe that  $\eta_{ce}$  should be greater than unity – see Section 4.1.

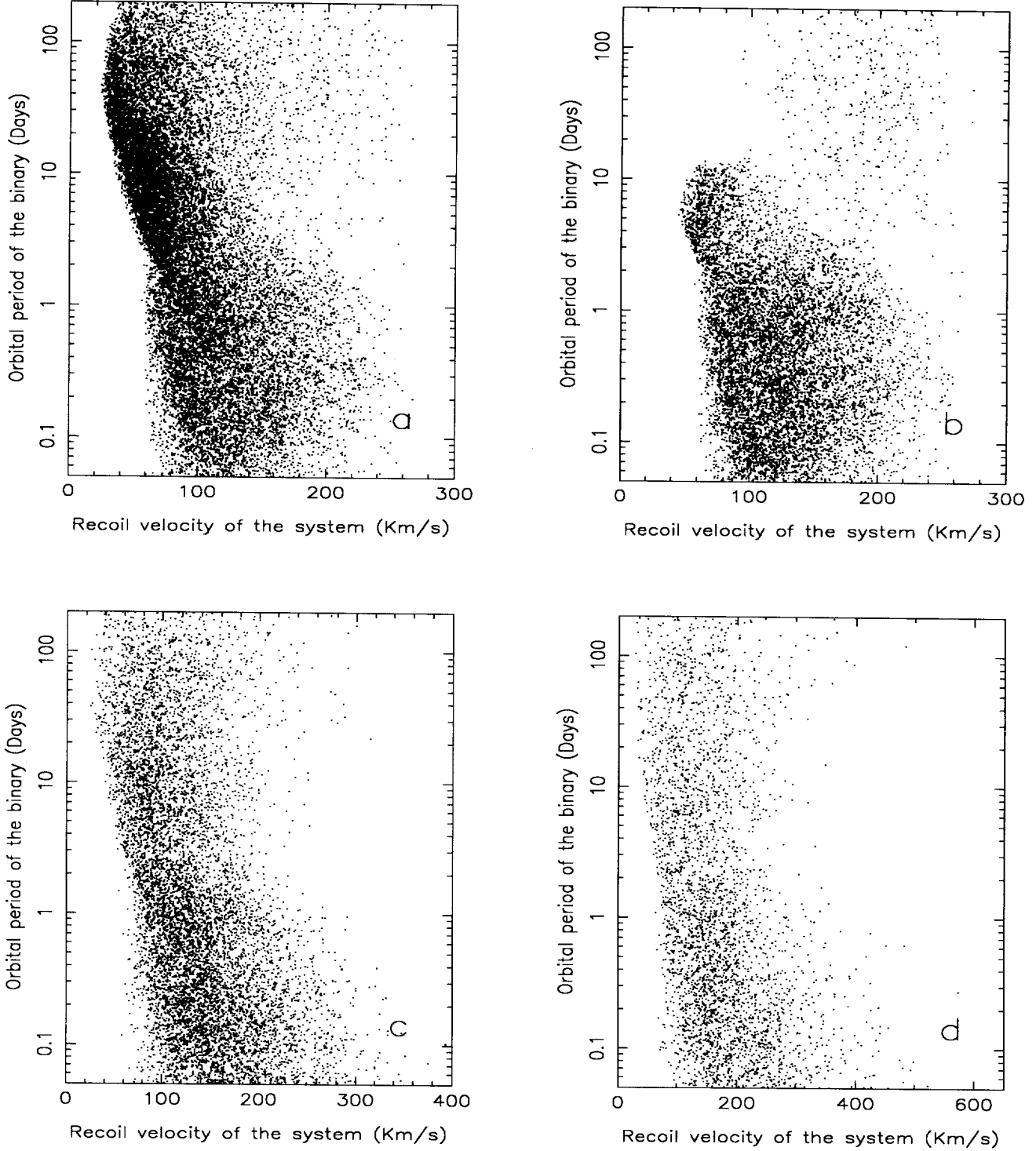
#### 3.1.1. Asymmetric SNe

In Figs 2c,d we assumed an asymmetric SN and added a randomly orientated kick velocity with a mean of 200 and 450  $\text{km s}^{-1}$ , respectively. The kick velocity distribution was taken to be Gaussian with standard deviation  $\sigma$  of 100 and 200  $\text{km s}^{-1}$  respectively. This increases the final recoil velocity of the systems (compare Figs 3a,c and d), but tends to weaken the  $v_{recoil}$ - $P_{orb}$  correlation, since most of the pre-SN wide systems are disrupted and the velocity scattering of those that survive is large. Furthermore, tight binaries which almost disrupt in the SN will have large  $P_{orb}$  (and large  $v_{recoil}$ ) – cf. Fig. 1, where  $e = 0.80$ , assuming  $\phi = 0^\circ$ . In the simulations used for Figs 2c,d we used  $\eta_{ce} = 2.0$ . However, when including a kick velocity of  $\sim 200 \text{ km s}^{-1}$  or more, the resulting velocity distribution of the surviving NS+WD binaries is only slightly changed by using other (lower) values of  $\eta_{ce}$ .

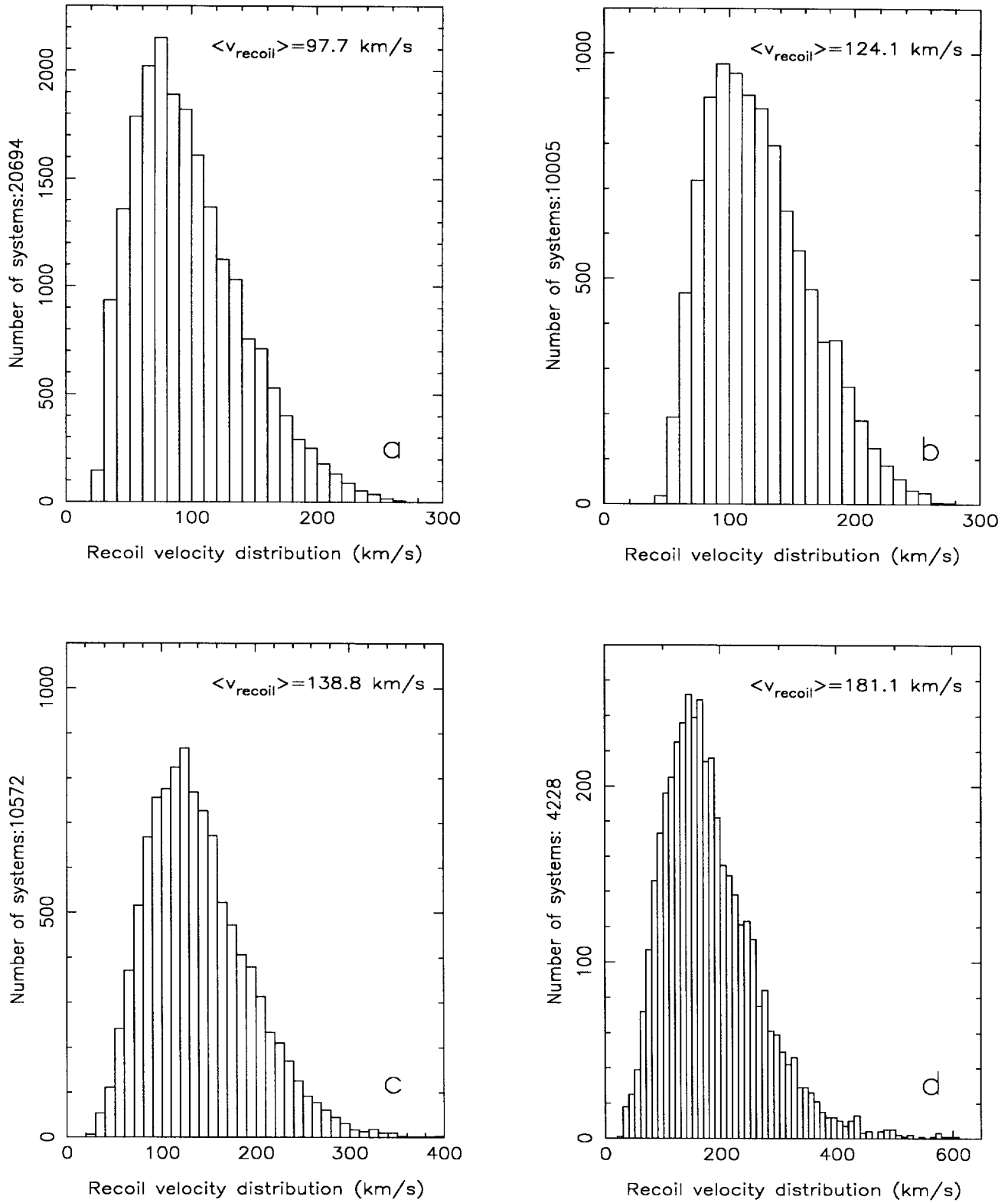
It should be noticed in the case of an asymmetric SN, that short separations ( $a \lesssim 3R_{comp.}$ ) before the explosion also increase the possibility of penetrating the envelope of the companion star with the newborn neutron star. A significant fraction of such systems might therefore coalesce (Leonard, Hills & Dewey 1994) if one assumes only a soft survival constraint (see above) which allows for very tight orbits after CE and spiral-in evolution.

Fig. 3 shows the recoil velocity distribution of the simulations presented in Fig. 2. To produce the above figures we evolved 100 000 binaries with initial conditions as indicated in Section 2. Of these binaries  $\sim 1/5$  evolve into BMSPs (or at least binaries containing a neutron star and a white dwarf) in the case of a symmetric SN and  $\eta_{ce} = 2.0$ . The remaining binaries either coalesced, became disrupted in the SN, evolved into bound or unbound double neutron star systems or produced the neutron star after the white dwarf. Decreasing  $\eta_{ce}$ , *i.e.* increasing the effect of the spiral-in, reduces the number of surviving binaries as expected. If a random orientated Gaussian kick of  $\langle v_{kick} \rangle = 200$  or 450  $\text{km s}^{-1}$  ( $\sigma = 100$  and 200  $\text{km s}^{-1}$ ) is added to the newborn neutron star in the SN, then the number of surviving BMSPs is further reduced by a factor of  $\sim 2$  and 5, respectively. It should be noticed that

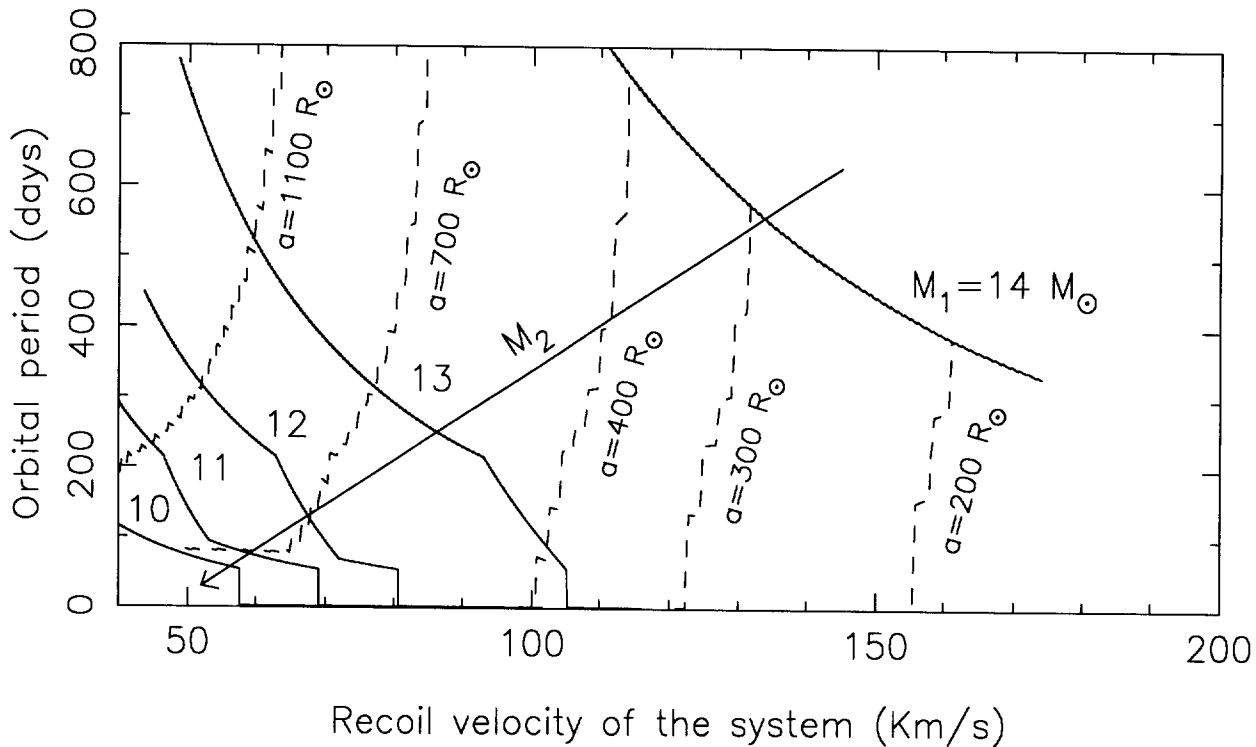




**Fig. 2.** Distribution of NS+WD binaries in the  $(v_{\text{recoil}}, P_{\text{orb}})$ -diagram. In Figs 2a,b we assumed a symmetric SNe for the formation of the neutron star using  $\eta_{\text{ce}} = 2.0$  and  $\eta_{\text{ce}} = 0.5$ , respectively. Figs 2c,d present results assuming that  $\eta_{\text{ce}} = 2.0$  and show the effect of including asymmetry in the SN using  $\langle v_{\text{kick}} \rangle = 200$  or  $450 \text{ km s}^{-1}$  ( $\sigma = 100$  and  $200 \text{ km s}^{-1}$ ), respectively. In the above simulations we used a soft survival constraint (see Section 3.1) after CE and spiral-in evolution, and assumed  $\beta = 0.7$  for the final mass transfer. Approximately 5~15 % of the binaries (decreasing with  $\eta_{\text{ce}}$  and  $\langle v_{\text{kick}} \rangle$ ) have  $P_{\text{orb}} > 200^{\text{d}}$  and another 5~15 % (increasing with  $\eta_{\text{ce}}$  and  $\langle v_{\text{kick}} \rangle$ ) have  $P_{\text{orb}} < 0.05^{\text{d}}$ .



**Fig. 3.** Velocity distribution of the binaries in Figs 2a-d. See text for further details.



**Fig. 4.** The dependence of the initial ZAMS-binary parameters on the location of the final NS+WD system in the  $(v_{\text{recoil}}, P_{\text{orb}})$ -diagram assuming  $\eta_{\text{ce}} = 2.0$  and  $M_2 = 1.2M_{\odot}$ . The arrow indicates the direction in the diagram of increasing initial secondary masses. Thus for a given observed binary pulsar it is possible to put constraints on the progenitor ZAMS-binary if one assumes that a *symmetric* SN created the neutron star.

large kicks of  $\sim 450 \text{ km s}^{-1}$  only increase  $\langle v_{\text{recoil}} \rangle$  with  $\sim 80\%$  compared to symmetric SN. Fig. 3c seems to fit observations reasonably well (see Section 4.3). The value of  $\beta$  has no effect on the velocity distribution.

### 3.2. Constraints on parameters of BMSP progenitors

If a *symmetric* SN created the neutron star, then a one-to-one correspondence between initial and final orbital parameters exists. We illustrate the dependence of the BMSPs on the assumed parameters of their ZAMS progenitors in Fig. 4. The results shown in Fig. 4 assume, as in Fig. 2, that systems only coalesced after a common envelope and spiral-in evolution if the separation between the two stars was less than the sum of their radii. The figure demonstrates the correspondence between the location of the BMSP in the  $(v_{\text{recoil}}, P_{\text{orb}})$ -diagram and the initial ZAMS parameters of separation,  $a$  primary mass,  $M_1$  and secondary mass,  $M_2$ . The plot shows the dependence of initial separation for five different values of the primary mass. The interpretation of this figure is straightforward and in accordance with the equations of binary evolution outlined in Section 2. The figure illustrates a clear trend of an inverse dependence of  $v_{\text{recoil}}$  on the initial separation,  $a$ . The lower values of  $a$  give rise to higher values of the pre-SN orbital velocity,  $v_{\text{He}}$ , of the exploding He-star and

hence of  $v_{\text{recoil}}$ . It is also seen that larger  $a$  results in larger final  $P_{\text{orb}}$ . Furthermore, a more massive primary star will lead to higher values of  $v_{\text{recoil}}$ . This is partly because a higher mass leads to a higher  $v_{\text{He}}$  and partly because  $v_{\text{recoil}}$  is an increasing function of the amount of matter lost in the SN,  $\Delta M = M_{\text{He}} - M_{\text{NS}}$  (see eqs 15,16). That increasing primary masses would lead to wider final orbits might, at first, appear peculiar. However, eq.(11) shows that increasing values of  $\Delta M/M$  also results in larger values of  $P_{\text{orb}}$ .

It is important to notice that these trends are also valid for other choices of  $\eta_{\text{ce}}$  though the shape of the lines in the  $(v_{\text{recoil}}, P_{\text{orb}})$ -diagram would be different.

## 4. Discussion

### 4.1. The Efficiency Parameter, $\eta_{\text{ce}}$

If orbital energy is all that is available to expel the envelope of the giant star during CE-evolution then  $\eta_{\text{ce}} \leq 1$ . However, sources other than the drag force may also contribute to this process – e.g. recombination energy, enhanced nuclear burning in shells or pulsations (cf. Iben & Livio 1993). Thus it is quite possible that the efficiency parameter,  $\eta_{\text{ce}}$ , could be greater than unity.

Using the simple fact that the initial separation between the two ZAMS-stars in a given binary is limited illustrates how one could put constraints on  $\eta_{ce}$ . On the one hand, the initial separation between the stars must not be too large, otherwise the system would never come into contact. On the other, the system must contain enough orbital energy to avoid coalescence during the CE-phase. The change in orbital separation during CE-evolution can be estimated from eq.(8). For BMSP progenitor systems we simplify this equation (using  $\lambda \sim 0.5$ ) to:

$$\frac{a_f}{a_i} \approx \frac{q_i}{20} \eta_{ce} \quad (19)$$

which is a good approximation for a large range of pre-CE mass-ratios,  $q_i \lesssim 0.4$  assuming a simple relation between  $M_{1core}$  and  $M_1$  (e.g.  $M_{1core} = 0.28 M_1$ ), and  $M_2 \ll (2M_{1env})/(\eta_{ce} \lambda r_L)$  which are both reasonable approximations. Thus the orbital period after the CE-phase (which in our code is also equal to the pre-SN orbital period) is proportional to  $\eta_{ce}^{3/2}$ . Therefore we also expect binary periods after the SN to be proportional to the same factor, and so using  $\eta_{ce} = 0.5$  should produce BMSPs with orbital periods that are shorter by a factor of 8 compared to a CE-evolution using  $\eta_{ce} = 2.0$ . Hence to produce a reasonable amount of BMSPs with  $P_{orb} > 100^d$  ( $\sim 15\%$  of the observed systems) we are inclined to believe that  $\eta_{ce} > 1.0$ .

Assuming a typical CE survival constraint of  $a_f \geq 2.0 R_\odot$  yields a minimum separation before the CE of  $a_i > 50\text{--}200 R_\odot$  – depending on  $q_i$  – in the case of  $\eta_{ce} = 2.0$ , whereas  $a_i > 200\text{--}800 R_\odot$  in the case of  $\eta_{ce} = 0.5$ . Similarly, a given BMSP with a large  $P_{orb}$  and assuming a fixed relative change in the orbital separation (independent of  $\eta_{ce}$ ) during the SN could rule out any CE-evolution with  $\eta_{ce}$  below a certain value – depending on the assumed maximum radii the giant stars can attain.

The conclusion, at least in some cases, that  $\eta_{ce} > 1.0$  is supported by observations of double nuclei of planetary nebulae (De Kool 1990; Yungelson, Tutukov and Livio 1993).

We shall discuss below the much more crucial effect of including a kick velocity in the SN.

#### 4.2. Symmetry vs. Asymmetry

In the case of spherically symmetric supernova explosions, we have demonstrated that, depending on  $\eta_{ce}$ , a correspondence exists between the recoil velocity and orbital period, as a function of initial masses and separations, of binaries containing a neutron star and a white dwarf. In case of asymmetric SN the relation between the recoil velocity and orbital period becomes quite weak partly as a result of the random orientation of the kick and its magnitude<sup>2</sup>.

<sup>2</sup> Throughout this paper we have assumed that the amount of asymmetry in the SN is independent of the orbital parameters.

However, there is evidence that some relatively newborn pulsars have enormous velocities ( $\sim 1000 \text{ km s}^{-1}$ , cf. Cordes, Romani & Lundgren 1993; Frail & Kulkarni 1991), which in combination with the recent velocity measurement of PSR B1257+12 (see below) suggests that at least some (*i.e.* possibly *all*) pulsars formed in a type II or type Ib/c SN receive a kick velocity. This idea is also supported by the work of Lyne & Lorimer (1994) which indicate pulsars are born with a large kick velocity of average  $\sim 450 \text{ km s}^{-1}$ .

#### 4.3. Comparison with Observations

Only a very few proper motions of binary pulsars have been measured. These are presented in Table 1. The magnitude of the transverse velocities observed are consistent with those 3-D velocities predicted in the simulations ( $\bar{v}_{recoil} = 4/\pi \times \bar{v}_\perp$ ) – cf. Fig. 3. PSRs B1257+12 and B1957+20 have high recoil velocities of  $\gtrsim 280 \text{ km s}^{-1}$  and  $\gtrsim 220 \text{ km s}^{-1}$ , respectively, if we assume the dispersion-measure distances to these objects to be correct. For the former, we have shown that such a high velocity can only be obtained when the supernova explosion is asymmetric.

**Table 1.** Transverse velocities of BMSPs

PSR-name	$\mu$ mas. yr <sup>-1</sup>	dist. kpc	$v_\perp$ km s <sup>-1</sup>	$P_{orb}$ days
J0437–4715	135	0.14	91 <sup>a</sup>	5.7
J1012+5307	<33	0.52	<75 <sup>b</sup>	0.60
B1257+12	95.0	0.62	280 <sup>c</sup>	planets
J1455–3330	-	0.73	60 <sup>d</sup>	76.2
J1713+0747	6.3	0.89	27 <sup>c</sup>	67.8
B1855+09	6.2	0.70	21 <sup>c</sup>	12.2
B1957+20	30.4	1.53	220 <sup>c</sup>	0.38
J2019+2425	23.5	0.91	101 <sup>c</sup>	76.5
J2145–0750	-	0.50	31 <sup>d</sup>	6.8
J2317+1439	7.6	1.9	70 <sup>e</sup>	2.5

<sup>a</sup> Bell et al. 1995

<sup>b</sup> Lorimer et al. 1995b

<sup>c</sup> Nice & Taylor 1995 (and references therein).

<sup>d</sup> Nicastro & Johnston 1995

<sup>e</sup> Camilo et al. 1996

However, the expected recoil velocities of the rest (8 out of 10) of these binary millisecond pulsars seem to be  $\lesssim 100 \text{ km s}^{-1}$ , depending on the unknown line-of-sight velocity. It has been pointed out by Nice & Taylor (1995) that such small space velocities are comparable with velocities originating from gravitational interactions with massive interstellar cloud complexes and that therefore the initial intrinsic velocities of these old pulsars could even be much lower, approximately  $\sim$  a few 10s of  $\text{km s}^{-1}$ . If this is true,

it seems to challenge the assumption that the magnitude of the average kick velocity, imparted to neutron stars formed via type Ib/c SN, is  $\sim 450 \text{ km s}^{-1}$ .

One interpretation of the small observed velocities is that it may be possible that kick velocities originating from type Ib/c SN are in general smaller than those of type II SN, which dominate the Lyne & Lorimer (1994) value of  $\sim 450 \text{ km s}^{-1}$  obtained from analyzing observations of single pulsars. However, we must bear in mind the selection effects at work here, and it is possible that millisecond pulsars with large velocities are under-represented in the sample because of the large  $z$ -heights they will obtain from their velocity. Pulsars with large  $z$ -heights will be a long way from the Sun and harder to detect than those with small  $z$ -heights. It should also be noted that very close binaries (which have higher than average recoil velocities) dominate the velocity distribution of our simulations and such systems are usually harder to detect because of the rapidly changing apparent period and enhanced occurrence of eclipsing phenomena from ablation. Quantifying the magnitude of the selection effects is very important.

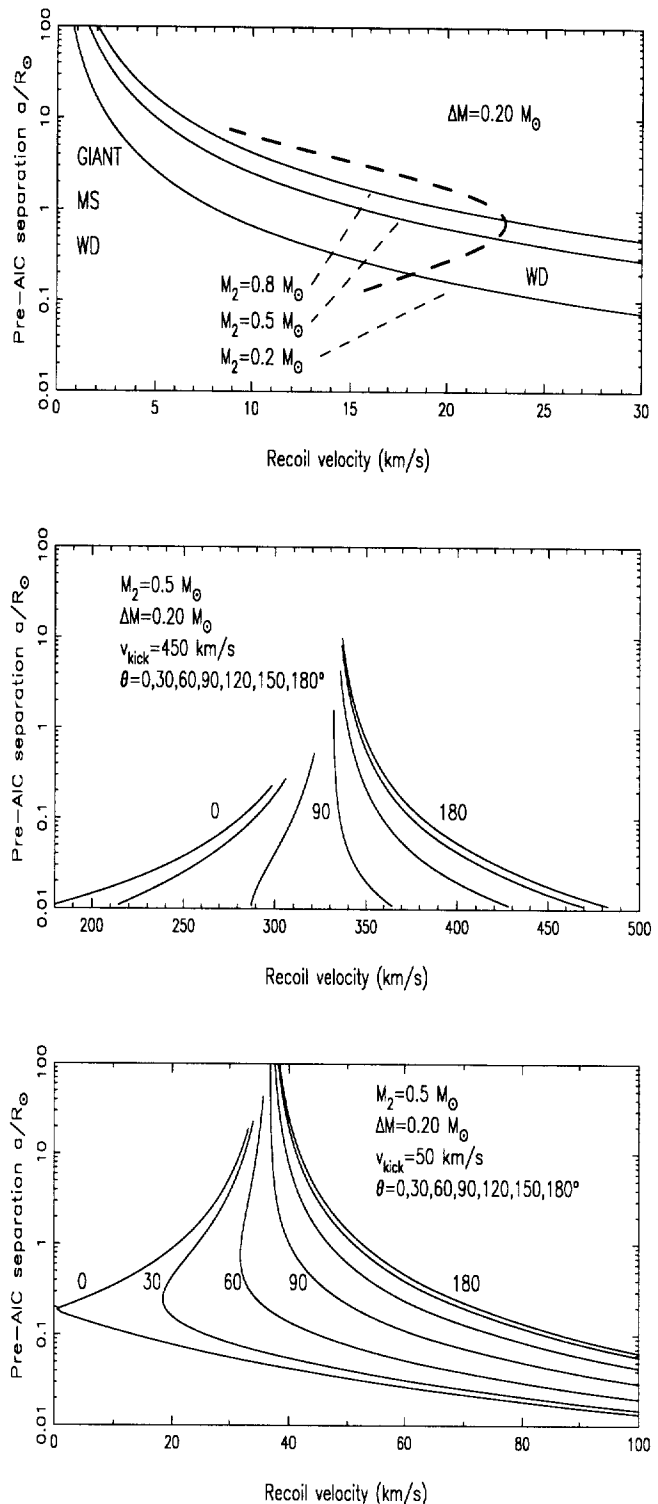
Only a large ensemble of BMSP proper motions together with careful analysis of observational selection effects will yield reliable information about the kick velocities imparted to the newborn neutron stars.

Note, that the velocity distributions presented in Fig. 3 also apply to LMXBs as well as single millisecond pulsars, if these origin from binaries where the companion star has been evaporated. Furthermore, compared to the recoil velocities of the BMSPs, we expect the average recoil velocities of the single millisecond pulsars to be slightly larger as a result of the weak  $v_{\text{recoil}}-P_{\text{orb}}$  anti-correlation and increased efficiency of evaporation with shorter separations<sup>3</sup>.

#### 4.4. Accretion Induced Collapse (AIC)

We now investigate the velocities we expect if millisecond pulsars are the product of the accretion-induced collapse (AIC) of a massive white dwarf. The formation of a binary millisecond pulsar via AIC of a massive accreting O-Ne-Mg white dwarf is expected to yield different kinematic properties of the surviving binary than those BMSPs formed by a type Ib/c SN because of the small amount of mass ejected following an AIC. In Fig. 5 we show the recoil velocities of binaries surviving the AIC with and without kicks. We have assumed that  $0.20M_{\odot}$  is ejected instantaneously,  $M_{\text{NS}} = 1.4M_{\odot}$  and that the mass of the companion, at the time of implosion, is in the range  $0.2-0.8M_{\odot}$ . Also indicated in Fig. 5a (top) is the nature of the donor star as a function of pre-AIC separation.

<sup>3</sup> However, if single millisecond pulsars origin from Thorne-Żytkow objects (Section 2.7.1) then they are expected to have lower recoil velocities as a result of their former relatively heavy companion star.



**Fig. 5.** Recoil velocities of BMSPs formed via symmetric AIC (top) and asymmetric AIC (center, bottom), as a function of pre-AIC separation. In the case of asymmetric AIC we assumed a kick of 450 and 50  $\text{km s}^{-1}$ , respectively. Only bound systems are shown. The kick angle,  $\theta$ , is defined in Section 2.5. See text for further details.

In the case of a symmetric AIC the recoil velocities are limited because of the size of the companion star in the pre-AIC binary. The pre-AIC velocity of the collapsing star, in the center of mass reference frame, is given by:  $v = M_2((M_{\text{NS}} + \Delta M + M_2)a/G)^{-1/2}$ , which in combination with eq.(15) and assuming a minimum separation of  $a_{\text{min}}/R_{\odot} \approx M_2/M_{\odot}$ , between the massive accreting white dwarf and a non-degenerate low-mass companion, yields  $v_{\text{recoil}}^{\text{max}} \lesssim 20 \text{ km s}^{-1}$  (the area to the left of the dashed line in Fig. 5a). Thus if the neutron stars in BMSPs are formed via AIC the implosion must be asymmetric in order to explain their observed velocities.

On the other hand, if AICs are asymmetric (e.g. Fig. 5b) we conclude that one is unlikely to be able to distinguish between BMSPs formed via AIC and those formed via an asymmetric SN of type Ib/c, if the magnitude and spread of the kick velocities are the same in both cases.

In Fig. 5c we have demonstrated that even a slight asymmetry ( $v_{\text{kick}} \sim 50 \text{ km s}^{-1}$ ) in the AIC will have important consequences for LMXBs and BMSPs formed in globular clusters. If the donor star in the pre-AIC mass transfer phase is non-degenerate (i.e.  $a \gtrsim 1R_{\odot}$ ), then all BMSPs produced via AIC will leave any cluster with  $v_{\text{esc}} \lesssim 25 \text{ km s}^{-1}$ . Thus if the observed BMSPs in globular clusters were indeed created via AIC, we have demonstrated that the implosion must have been almost symmetric. It is possible that the magnitude of the kick velocity is related to the amount of mass ejected in the SN, which is very low in AIC compared to SNe of type Ib/c or II.

## 5. Conclusions

1. It is possible to determine the recoil velocity and orbital period of a NS+WD binary from semi-analytical calculations of binary stellar evolution, *only* if the initial separation and masses are known and supernova explosions of type Ib/c are symmetric. Furthermore there is an anti-correlation between  $v_{\text{recoil}}$  and  $P_{\text{orb}}$  depending on the initial masses and separations as well as the physics governing the CE-evolution. However, we have presented evidence that SNe cannot be expected to be symmetric and if kicks in general are large ( $\sim 450 \text{ km s}^{-1}$ ) then we do not expect to see any clear correlation between  $P_{\text{orb}}$  and  $v_{\text{recoil}}$  (or the transverse velocity which is the actual observed parameter) emerging from future observations of binary millisecond pulsars.
2. Whilst the observed sample is still small, there seem to be more low-velocity pulsars than the simulations with large kicks would suggest. Analysis of the selection effects is very important for determining the magnitude of the kick velocity received by neutron stars at birth, from future observations of millisecond pulsar velocities.

3. It is not possible to explain large velocities ( $\sim 300 \text{ km s}^{-1}$ ) of pulsars such as PSR B1257+12 without a kick velocity.
4. If the AIC of massive white dwarfs leads to BMSPs then the ejection of mass must be asymmetric to explain the observed velocities of BMSPs. It is difficult to distinguish between models where AIC leads to the production of the neutron star and the “standard” model on kinematic grounds if the production of the neutron star is always accompanied by an asymmetric kick.
5. We have argued that if the AIC of a massive white dwarf with a main sequence or (sub)giant donor star is the progenitor of BMSPs in globular clusters, then the implosion must be symmetric (or  $v_{\text{kick}} \lesssim 50 \text{ km s}^{-1}$ ) for the system to be retained in the cluster.
6. Binary millisecond pulsars have probably evolved through a CE-phase before the formation of the neutron star if the alternative of a RLO is expected to be (almost) conservative. The required efficiency parameter  $\eta_{\text{ce}}$  is, in general, larger than unity in order to explain the wide orbits among observed binaries containing a neutron star and a white dwarf.

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