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Gamow - Teller Strength Function in Spherical Nuclei

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Abstract

The $\sigma t^{(\pm)}$ strength distribution as a function of the excitation energy is investigated in the framework of the Quasiparticle Random Phase Approximation. The results are compared with the available experimental data for ^{54}Fe and with the results of recent shell model calculations. It is demonstrated that the full single-particle space has to be used in order to describe the GT strength function correctly.

1 Introduction

The investigation of the Gamow-Teller (GT) strength function, i.e. the distribution of the strength of the $\sigma_\mu t^{(+)}$ or $\sigma_\mu t^{(-)}$ operators as a function of the excitation energy is of interest not only in nuclear physics, but it has also important applications in astrophysics. Its precise knowledge is requested for nucleosynthesis problems and in the study of supernovae evolution [1]. It also plays an important role in predicting neutrino scattering cross sections used for determining the detector efficiency in solar neutrino flux measurements [2]. In nuclear physics the GT strength function is important in connection with the problem of missing GT strength occuring in the (p, n) charge-exchange reactions at intermediate energies [3], and in connection with the question of a possible renormalization of the axial vector coupling constant of weak interactions in nuclear media, g_A [4]. The GT transitions play a leading role in muon capture, in pion charge-exchange reactions and in other low and medium energy processes in atomic nuclei. In many cases the knowledge of g_A is needed to extract the other parameters of the weak nuclear current, as for example the induced pseudoscalar weak interaction coupling constant, g_P.

The Random Phase Approximation (RPA) as well as multiparticle shell model calculations give a reasonably description of the shape of the GT strength function at low excitation energies and in the giant resonance region. However, agreement with the experimental data is achieved only after application of a quenching factor of roughly 0.8 to the transition amplitude. From the theoretical point of view one is therefore interested in finding a mechanism which is responsible for the transfer of giant GT strength towards higher excitation energies and which simultaneously allows one to reproduce the strength distribution in the giant resonance region and below it. Since now experimental data for both, $\sigma_\mu t^{(+)}$ and $\sigma_\mu t^{(-)}$ are available, it seems important to prove wether the mechanism proposed below is successful in describing both strength functions simultaneously. The purpose of this work is twofold. (a) It is shown within an extension of the RPA that the inclusion of an interaction between $1p-1h$ states (one–phonon states) and $2p-2h$ states (two–phonon states) is not capable of changing the strength distribution in the desired way. (b) It is demonstrated that those parts of the residual interaction coupling single-particle orbitals with different radial quantum numbers are essential in order to achieve a reasonable (at least at a qualitative level) description of the strength distribution. A good overall behaviour of the $\sigma t^{(-)}$ strength function at low energies is then obtained and the position and strength of the giant GT resonance can be reproduced. Simultaneously a large fraction of the $\sigma t^{(-)}$ strength is shifted towards higher excitations, forming new collective states. These collective states are built up from particle-hole (two–quasiparticle) excitations whose single-particle orbitals belong to different major shells. From this it is concluded that it is absolutely necessary to use a full single-particle space in all GT strength calculations. This statement applies in particular also to large shell model calculations. Finally the energy-weighted moments of the strength function are investigated. It is pointed out that in all model calculations where the ground state is not included in the model space (where it serves as a vacuum in the quasiparticle space) the zero and first order energy-weighted moments of the strength function are unchanged by introducing an interaction between

$(1p-1h)$ and $(2p-2h)$ states.

2 The nuclear model

The model Hamiltonian used in the present work consists of separate single particle potential wells of Woods-Saxon shape for neutrons and protons respectively representing the mean field, a superconducting monopole pairing between like particles, and a particle–hole residual interaction in separable form

$$H_{res.} = -2\kappa_1^{01} \sum_{\mu} Q^{\dagger}_{1,\mu} Q_{1,\mu}, \tag{1}$$

$$Q_{1,\mu} = \sum_{j_p, m_p, j_n, m_n} < j_p, m_p | U(r) \sigma_\mu t^{(-)} | j_n, m_n > a^{\dagger}_{j_p, m_p} a_{j_n, m_n}. \tag{2}$$

Here κ_1^{01} is the effective coupling constant of the residual interaction, $a^{\dagger}_{j_p,m_p}$ (a_{j_n,m_n}) is the creation (destruction) operator of a proton (neutron) in the $nljm$-single particle state, $U(r)$ is the radial form factor taken as [5]

$$U(r) = \frac{dW(r)}{dr}$$

and $W(r)$ is the central part of the single-particle shell model potential.

The case of a strong attractive particle-particle interaction which has been shown to be especially important for low energy $\sigma t^{(+)}$ transitions [6] should be discussed separately. Here we would like to note only that there is a definite contradiction between the description of the β^+ decay of the proton rich spherical nuclei and the β^- decay of neutron rich nuclei when using the particle-hole and particle-particle interactions simultaneously [7].

The diagonalization of the model Hamiltonian is done in two steps. First, we make the transition to the quasiparticle operators by means of the Bogoliubov transformation (for protons and for neutrons separately):

$$\alpha_{j,m} = v_j a_{j,m} + (-1)^{j-m} u_j a^{\dagger}_{j,-m} \ . \tag{3}$$

In the next step the charge-exchange $1^{(+)}$ phonons are introduced

$$\Omega_{\mu,i} = \sum_{j_p, j_n} \{ \psi^i_{j_p,j_n} [\alpha_{j_p} \alpha_{j_n}]_{1,\mu} + (-1)^{\mu} \phi^i_{j_p,j_n} [\alpha_{j_p} \alpha_{j_n}]^{\dagger}_{1,-\mu} \} \ , \tag{4}$$

where

$$[\alpha_{j_p}\alpha_{j_n}]_{\lambda,\mu} \equiv \sum_{m_p,m_n} < j_p m_p j_n m_n | \lambda\mu > \alpha_{j_p,m_p}\alpha_{j_n,m_n}$$

and $< j_p m_p j_n m_n | \lambda\mu >$ is a Clebsch-Gordan coefficient. The normalization condition for the phonon amplitudes is

$$\sum_{j_p,j_n} \{\psi^i_{j_p,j_n}\psi^{i'}_{j_p,j_n} - \phi^i_{j_p,j_n}\phi^{i'}_{j_p,j_n}\} = \delta_{i,i'} \ .$$

The equations defining the phonon amplitudes and the excitation energy ω_i of one–phonon $1^{(+)}_i$ state over the ground $0^{(+)}$ state are

$$R^+_{q,q'}g^i_{q'} - \omega_i w^i_q = 0 \quad (5)$$

$$-\omega_i g^i_q + R^-_{q,q'}w^i_{q'} = 0 \ , \quad (6)$$

where

$$g^i_q \equiv \psi^i_{j_p,j_n} + \phi^i_{j_p,j_n} \ , \quad w^i_q \equiv \psi^i_{j_p,j_n} - \phi^i_{j_p,j_n} \ ,$$
$$R^{\pm}_{q,q'} \equiv \epsilon_q\delta_{q,q'} - (2/3)\kappa^{01}_1 h_q u^{\pm}_q h_{q'} u^{\pm}_{q'} \ ,$$
$$\epsilon_q \equiv \epsilon_{j_p} + \epsilon_{j_n} \ , \quad u^{\pm}_q \equiv u_{j_p}v_{j_n} \pm v_{j_p}u_{j_n} \ ,$$
$$h_q \equiv h(j_p, j_n) \equiv < j_p \parallel U(r)\sigma t^{(-)} \parallel j_n > \ .$$

The amplitudes of the transitions from the ground state to the $1^{(+)}$ one–phonon excited states are given by

$$b^+_\mu(1^{(+)}, i) = \frac{1}{\sqrt{3}} \sum_{j_p,j_n} < j_p \parallel \sigma t^{(+)} \parallel j_n > (v_{j_p}u_{j_n}\psi^i_{j_p,j_n} + u_{j_p}v_{j_n}\phi^i_{j_p,j_n}) \ , \quad (7)$$

$$b^-_\mu(1^{(+)}, i) = \frac{1}{\sqrt{3}} \sum_{j_p,j_n} < j_p \parallel \sigma t^{(-)} \parallel j_n > (u_{j_p}v_{j_n}\psi^i_{j_p,j_n} + v_{j_p}u_{j_n}\phi^i_{j_p,j_n}) \ . \quad (8)$$

The parameters of the single-particle potentials and the monopole pairing constants are taken from [8]. Only one parameter will be varied during the calculations – the effective coupling constant of the residual interaction κ^{01}_1. The calculated GT strength function is given by the running sum

$$S^{(\pm)}(E) = \sum_{i:E_i \leq E} \sum_\mu |b^{\pm}_\mu(1^{(+)}, i)|^2 \ .$$

3 Results and Discussion

We present here the results of our calculations of the GT strength function for ^{54}Fe and compare them with detailed experimantal studies of the $\sigma t^{(+)}$ and $\sigma t^{(-)}$ strength functions by means of the (p,n) [9, 10] and (n,p) [9] reactions on ^{54}Fe. These experimental results together with our theoretical evaluation are shown in figs. 1 and 2 and 3. A detailed discussion is given in the following sections.

3.1 The $\sigma t^{(+)}$ strength in ^{54}Fe

In this section we discuss the theoretical results of the calculations for $\sigma t^{(+)}$ transitions. Fig. 1 shows the experimental running sum $S^{+}(E)$ from the $^{54}Fe(n,p)^{54}Mn$ reaction[9] in comparison with the theoretical results calculated for different values of κ_1^{01} ($-0.23/A$, $-0.43/A$, $-0.63/A$ and $-0.83/A$). The distribution of the $\sigma t^{(+)}$ strength at low excitation energies is determined by the positions of the two–quasiparticle states. There is one collective state which absorbs the main part of the transition strength. With increasing absolute value of the effective interaction constant $|\kappa_1^{01}|$ the collective state is shifted towards higher excitation energies and its strength decreases. Simultaneously the total transition strength becomes smaller. The calculated and measured strength distributions [9] are in qualitative agreement. It should be mentioned that in the low excitation energy region a much richer experimental spectrum is observed than calculations show. The total $\sigma t^{(+)}$ strength measured in the (n,p) reaction up to excitation energies of 10 MeV is equal to 3.1 ± 0.6 (all energies are measured with respect to the ground state of the residual nuclei). The calculated QRPA strength for this energy range lies between 4.2 and 6.5 depending on the value of $|\kappa_1^{01}|$. A recent shell model calculation [11] needs a quenching factor of 0.77 for the σt operators in order to reproduce the experimentally observed transition strength. The work of [11] contains also a review of previous calculations of $S^{(\pm)}$ in $^{54,56}Fe$. Various shell model calculations and the QRPA with different residual interactions give usually higher values for $S^{(+)}$ than those obtained in the present work. An exception is the QRPA case with the particle-particle interaction of [6] which gives $S^{(+)} = 4.2$. The relative low value $S^{(+)}$ in our present calculation is due to the used residual interaction. The radial form factor $U(r)$ varies rapidly with radius and differs from zero only in the surface region. Therefore the matrix elements between single–particle states with different radial quantum numbers contribute appreciably. In this way a mixing not only between the usual spin–orbit partners of the valence shell occurs (as should be the case in shell model calculations), but is present also between all the other single particle states. The influence of this mixing becomes especially important in the case of $\sigma t^{(-)}$ transitions, when the Gamow-Teller resonance can be excited.

3.2 The $\sigma t^{(-)}$ strength in ^{54}Fe

The results of our calculations of $S^{(-)}(E)$ for the same set of values of κ_1^{01} are shown in fig. 2 together with the GT strength function measured in the $^{54}Fe(p,n)^{54}Mn$ reaction [9, 10] (data are taken from tables I and III of [10]). The results of [9] are close to those of [10] and have larger error bars. Fig. 3 shows a comparison between the experimental and theoretical ($\kappa_1^{01} = -\frac{0.43}{A}$) strength functions plotted as a function of excitation energy. The following observations can be made

(i) a collective GT state located above the conventional GT giant resonance appears above a certain value of the effective interaction constant. This state is formed on the basis of two–quasiparticle states in which neutron and proton quasiparticles occupy levels with different radial quantum numbers;

(ii) as $|\kappa_1^{01}|$ increases, this collective state absorbs a steadily-increasing part of the total $\sigma t^{(-)}$ strength and is shifted gradually up to higher excitation energies;

(iii) accordingly, the conventional giant resonance around 10 MeV looses part of its transition strength and is shifted only slightly towards higher excitation energies. The reason for this positional stability are additional two-quasiparticle poles appearing in the QRPA secular equation located above those forming the giant GT resonance. It should be mentioned that the standard way of obtaining the value of the effective coupling constant from the position of the giant resonance meets difficulties in this case.

Introducing the residual interaction in its present form allows to describe the main features of the $\sigma t^{(\pm)}$ strength distribution and to reproduce the experimentally observed GT transition strength without any additional effective charges [12] or quenching factors [11].

4 Sum rules analysis

For all values of κ_1^{01} the difference between the total $\sigma t^{(-)}$ and $\sigma t^{(+)}$ strength is

$$S^{(-)} - S^{(+)} = 5.61 \ .$$

This value is somewhat less than $S^{(-)} - S^{(+)} = 6.0$ predicted by the well known Ikeda sum rule [13]. The origin of this difference is mainly the small nonorthogonality of the neutron and proton single–particle wave functions due to the difference in the single–particle potential wells. It can easily be proved that this sum rule is conserved in the QRPA, i.e. the difference $S^{(-)} - S^{(+)}$ calculated in the QRPA does not depend on the interaction between the two–quasiparticle states and is determined by the included two–quasiparticle space only [14].

We are turning to a discussion of the mechanisms which govern the distribution of strength in QRPA and which make a satisfactory description possible. First we observe that the interaction between $1p-1h$ and $2p-2h$ (or between one– and two–phonon states), responsible for the width of the giant resonance, causes some redistribution of the transition strength over the excitation energies [15, 16]. It is far from obvious why this interaction changes the strength function in such a way that almost half of the total strength of the giant resonance is shifted to very high excitation energies. Therefore a description of the general features of the GT strength distribution in the framework of the QRPA seems to be important. To be more precise let us consider the energy-weighted moments of the strength function

$$S^k = \sum_\nu E_\nu^k | < \Psi_\nu | T | 0 > |^2 \ , \tag{9}$$

where $< \Psi_\nu | T | 0 >$ is the transition amplitude from the ground state $|0>$ to the excited state Ψ_ν with excitation energy E_ν under the influence of a transition operator T, k are non-negative integers. The sum is to be taken over all final states Ψ_ν. Let us suppose that we have a certain given model wave function for the ground state $|0>$ and that we solve the Schrödinger equation for the excited state wave functions only. It is possible to decompose the wave function of the excited states in some basis spanning the relevant space

$$\Psi_\nu = \sum_m c_{\nu,m} \Phi_m + \sum_n \tilde{c}_{\nu,n} \tilde{\Phi}_n \tag{10}$$

In this expression the set of basis vectors has been divided into two groups according to the values of the matrix elements of the transition operator T

$$< \Phi_m|T|0 > \neq 0 \ , \qquad < \tilde{\Phi}_n|T|0 > = 0 \ .$$

For example, if one had used the Hartree-Fock ground state and the space of excited states spanned by $1p-1h$ and $2p-2h$ basis vectors, particle-hole components would belong to the first group and two-particle–two-hole vectors would fall into the second group for every one-body transition operator.

It was proved in [17], that in such cases the zero and first energy-weighted moments (S^0 and S^1) are determined by the Φ-subspace of simple excited states only, and do not depend on the interaction between Φ– and more complicated $\tilde{\Phi}$–states and on the interactions acting inside the $\tilde{\Phi}$ subspace alone. This is a direct consequence of freezing the ground state.

In connection with the problem of missing GT strength this means, that by adding $2p-2h$ components into excited state wave functions only, the total transition strength and energy centroid of the strength function (equal to S^1/S^0) must be necessarily the same as in the $1p-1h$ (or QRPA) calculation! The aforementioned separation was done practically in all calculations investigating the influence of complicated states on the GT strength function (see, for example, [18]). The simultaneous conservation of the total transition strength and of the energy centroid has the following consequence: if a large part of the strength from the giant resonance could be shifted by the interaction with more complicated states to higher excitation energies, then some strength would be shifted to the very low excitation energies too. As a result the strength function in the giant resonance region and below changes completely. This effect was noticed in the shell model calculation of [19]. Our arguments are valid in the shell model case too. Several theoretical calculations [18] showed that the high energy tails of the GT strength function contained a large fraction of the total GT strength and simultaneously the low energy part of the strength function was reproduced correctly. Therefore there is a definite contradiction between our statement on the conservation of S^0 and S^1 and the results of several numerical calculations. Probably, the changes in the $1p-1h$ and $2p-2h$ propagators (inclusion of an energy shift) made in these papers to imitate some of the omitted terms are responsible for this sum rule violations.

The limitation arising from S^0 and S^1 conservation can be removed if one considers the ground state to be lying in the same space as the excited states.

Calculations with explicit inclusion of two-phonon correlations in the ground state (see, for example [20, 21]) have shown that the strength distribution cannot be changed very much and an opening of the model space is therefore necessary.

5 Conclusion

The present work leads to the following conclusions.

(i) It is possible to describe in the QRPA simultaneously the strength functions of $\sigma t^{(\pm)}$ transitions in ^{54}Fe. The use of a separable nonlocal residual interaction allows a qualitative description of the $\sigma t^{(+)}$ strength function. The calculated $S^{(+)}$ strength below 10 MeV is between 4.3 and 5.4 and to be compared with 3.1 ± 0.6 obtained experimentally in the (n,p) reaction [9]. The calculated $\sigma t^{(-)}$ strength function agrees with the strength function obtained from the (p,n) reaction in the low excitation energy region and in the giant resonance region without any quenching factors. The rest of the GT strength is absorbed by high-lying collective $1^{(-)}$ states formed around two-quasiparticle states having different number of nodes in the radial part of their single-particle wave functions, i.e. belonging to different major shells. This is basically the reason why it is important to use the full single-particle space in the calculation of the $\sigma t^{(-)}$ strength function.

(ii) Some arguments were presented why it is impossible to shift a large part of $\sigma t^{(-)}$ strength to the higher excitation energies and simultaneously describe the low-energy part of the GT strength function by using $2p-2h$ admixtures in the nuclear wave functions only.

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Figure caption

Fig. 1 Running sum $S^{(+)}(E)$ for the $\sigma t^{(+)}$ transition operator as a function of the excitation energy of the residual nuclei. The shaded area represents the experimental strength function for the $^{54}Fe(n,p)^{54}Mn$ reaction [9]. a), b), c) and d) represent QRPA calculation for various values of κ_1^{01} (a: $-0.23/A$, b: $-0.43/A$, c: $-0.63/A$ and d: $-0.83/A$).

Fig. 2 Running sum $S^{(-)}(E)$ for the $\sigma t^{(-)}$ transition operator as a function of the excitation energy of the residual nuclei. The notation is the same as in Fig. 1. The experimental data for the $^{54}Fe(p,n)^{54}Co$ reaction are from ref.[10].

Fig. 3 $\sigma_\mu t^-$ strength function as a function of excitation energy. Experimental data (shaded area) for the $^{54}Fe(p,n)^{54}Co$ reaction are from [10]. The theoretical calculation for $\kappa_1^{01} = -0.43/A$ corresponds to curve b) of Fig. 2.

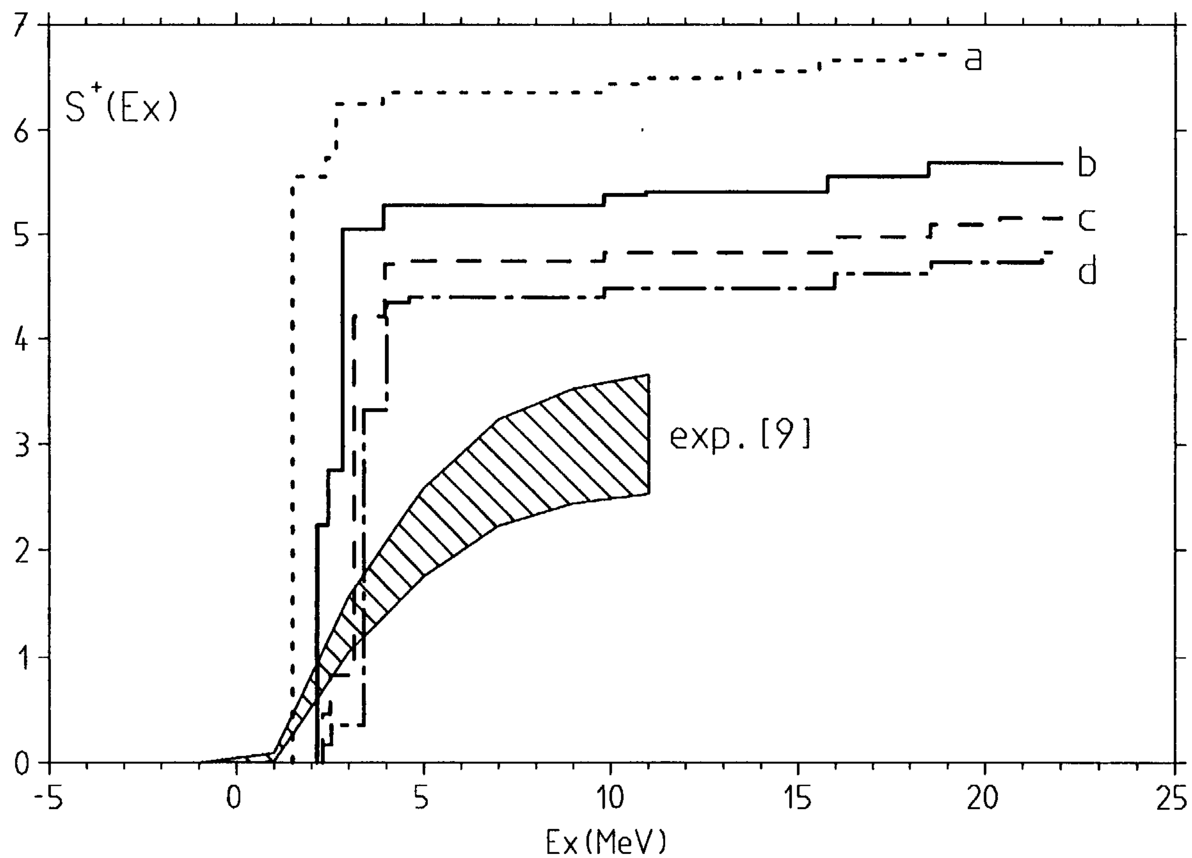
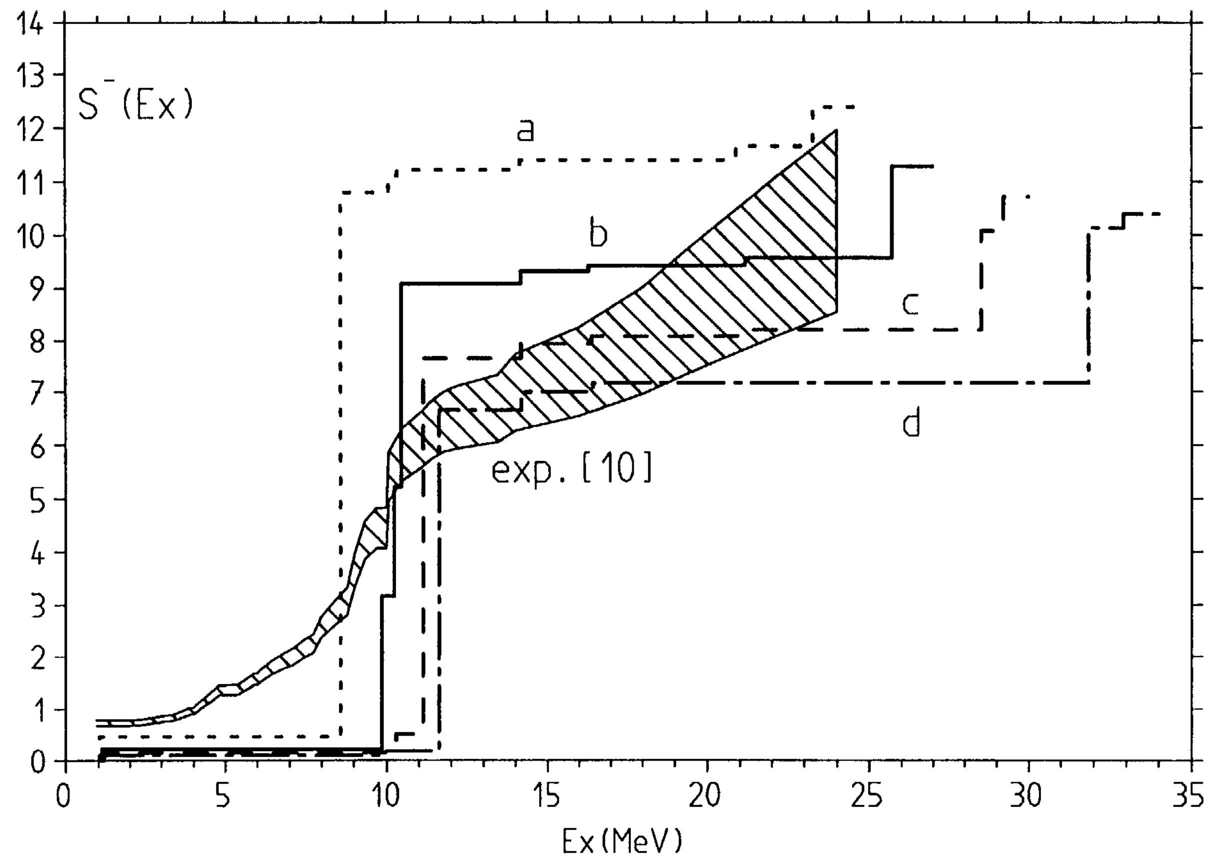


Fig. 1



S⁻(Ex)
a
b
c
d
exp. [10]
Ex (MeV)
0
5
10
15
20
25
30
35
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0

Fig. 2

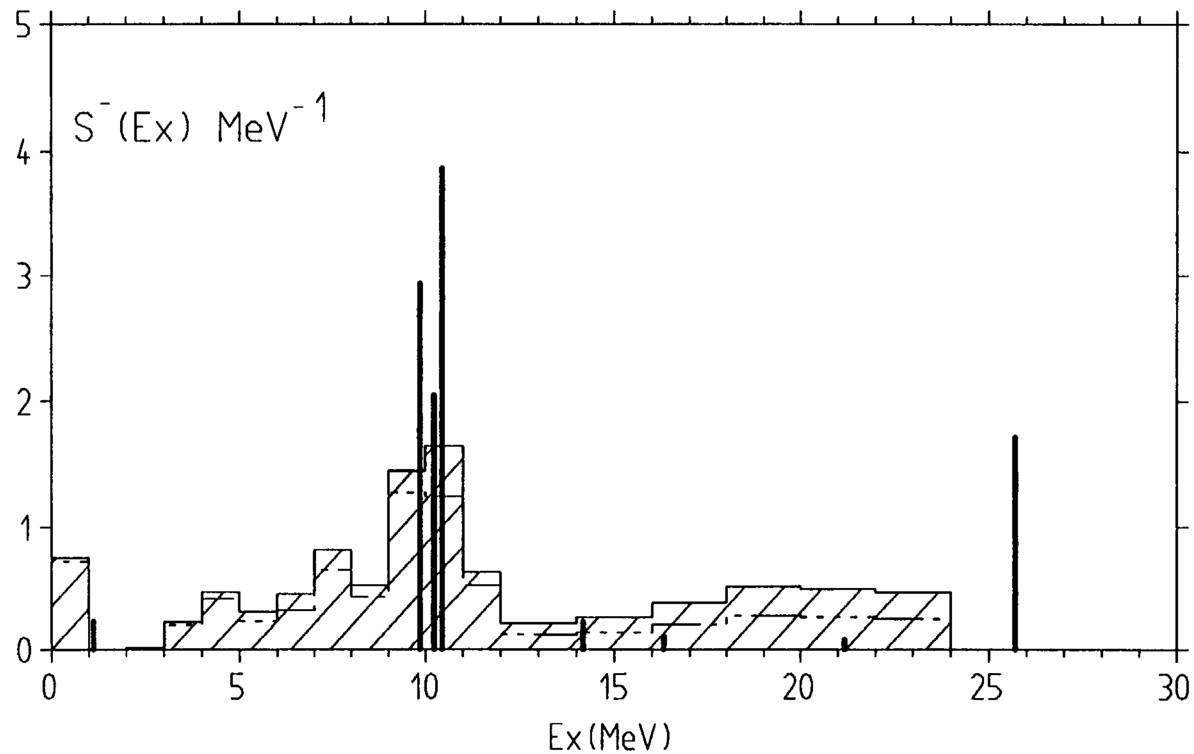


Fig. 3