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J. Speth[1], R. Tegen[1,2]

[1]Institut für Kernphysik, Forschungszentrum Jülich, D-52425 Jülich, Germany

[2]University of the Witwatersrand, Johannesburg 2050, South Africa

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ωNN Vertex Function and the $\gamma\pi\pi\pi$ Anomaly

J.Speth and R.Tegen*

Institut für Kernphysik, Forschungszentrum Jülich, Postfach 1913, D-52425 Jülich, Germany

Abstract: We show that the empirical ωNN form factor with a cut-off of 1400-1500MeV can be understood as arising from a combination of a quark model form factor with a typical cut-off of 700-800MeV and an anomalous form factor $\sim q^2$ arising from the 3π-intermediate state. The anomaly contributes to the Dirac form factor $F_1(q^2)$ with $F_1(0) = 0$, $-\frac{d}{dq^2} F_1(q^2)\Big|_{q^2=0} > 0$ (and sizable), and to the Pauli form factor $F_2(q^2)$ with $F_2(0) \neq 0$. The resulting tensor coupling $F_2(0)$ is sensitive to the cut-off of the pion momenta in the two-loop integrals and turns out to be small for values around 1 GeV. The quark model ωNN *tensor* coupling $F_2(0)$ vanishes for *point-like* quarks. The anomaly, however, contributes a non-vanishing *tensor* coupling which via the Gordon decomposition can be seen to enhance the *vector* coupling. It remains to be seen if the here discussed *tensor* form factor contribution to the ωNN vertex function has an effect on NN observables similar in magnitude to an enhanced quark model *vector* coupling. This could remove the only remaining discrepancy between empirical ωNN *vector* coupling constants (obtained under the assumption of a near-vanishing tensor coupling) $(g^V_{\omega NN})^2/4\pi$ = 10-20 , necessary to fit the NN phase shifts, and the simplest ($SU(3)_{\text{flavour}}$) quark model prediction, which is $g^{V,q\bar{q}}_{\omega NN} = 3g^{V,q\bar{q}}_{\rho NN}$, $g^{T,q\bar{q}}_{\omega NN} = 0$.

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* On sabbatical leave from: Physics Department, University of the Witwatersrand, Johannesburg 2050, South Africa

1. Introduction

Meson cloud corrections to the nucleon form factors come in two distinct varieties: one-meson loop (Fig.1b-d) and two-meson loop (Fig.1e and Fig.2). The naïve expectation is that all two-meson loop contributions are of higher order and, therefore, small. While this is true for diagrams of the type displayed in Fig.2 it is not a priori so for the contributions in Fig.1e. The reason is that Fig.1e contains the $\gamma\pi\pi\pi$ anomaly (VAAA) which can not be neglected as it is related via Ward identities to the well-known (one-loop) $\pi^0 \rightarrow \gamma\gamma$ anomaly (AVV). The VAAA anomaly has been widely studied in ω(782)-decays as well as in the reaction $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ [1,6,12], but not, to our knowledge, in the ωNN vertex function. This vertex function plays a fundamental role in the understanding of the short-ranged repulsion observed in the NN force [2,13,14]. The presence of this anomaly in nucleon form factors is reflected in the empirical paradox (first noted in ref.[4]) that the isoscalar magnetic moment of the nucleon is small while its isoscalar radius is large. Such anomalous behaviour of nucleon form factors shows also up in the difficulties that chiral quark models have to reproduce *both* the proton *and* the neutron charge form factor. The neglect of contributions like Fig.1e may result in large discrepancies between the empirical and the theoretical $G_E^n(q^2)$ close to $-q^2 = 0$, where the form factor has been accurately measured [21]. The transition $\gamma \rightarrow \pi^+\pi^-\pi^0$ probes the I = 0, G = -1 content of the photon; these are the quantum numbers of the ω(782), ϕ(1020), J/ψ(3097), ... mesons. Therefore, the ωNN coupling is expected to contain a component resulting from the (VAAA) anomaly. We will see below that the anomaly predicts a ωNN vertex function which strongly deviates from the quark model ωNN vertex function for very small 4-momentum transfers $-q^2$ (or: large separation in r-space) while it has a similar q^2-dependence for large $(-q^2)$. It is found empirically that the ωNN (vector) coupling deviates significantly from the simplest $(SU(3)_{flavour})$ quark model prediction, which is $g_{\omega NN}^{V,q\bar{q}} = 3g_{\rho NN}^{V,q\bar{q}}$, $g_{\omega NN}^{T,q\bar{q}} = 0$. Empirically one finds $(g_{\rho NN}^V)^2/4\pi = 0.84 < 1$, and NN phase shift analyses require [2,5,14] $(g_{\omega NN}^V)^2/4\pi$ = 10-20 (and often closer to 20). This is much larger than the quark model prediction which gives less than 9 for this coupling strength. We demonstrate here that the anomaly hardens the quark model vector form factor (i.e. the cut-off is moved from 700-800MeV to close to 1500MeV) and contributes a non-vanishing tensor form factor. The latter effect could be re-interpreted (via the Gordon decomposition of the electromagnetic current) as an enhanced vector coupling. It remains to be seen if a combination of vector and tensor form factors as discussed here has a comparable effect in NN phase shifts as those "hard" and large vector monopole form factors employed in relativistic nucleon-nucleon models [2,5,14]. The paper is organized as follows. In the next Section we present the necessary formalism for the derivation of the ωNN form factors including the VAAA anomaly. In Section 3 our numerical results are presented and their implications for NN models discussed. Technical details of our 2-loop calculation are relegated to three appendices, where we also discuss similarities and differences with a technique developped for scalar two-loop integrals in QCD [15].

2. The Transition $\gamma N \to \pi^+\pi^-\pi^0 N \to N$.

We consider the electromagnetically gauged Lagrangian of the σ-model

$$L^{EM} = L_\sigma - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j_\mu A^\mu \tag{2.1}$$

with $j^\mu = j^\mu_N + j^\mu_{\pi\pi} + j^\mu_{\pi\pi\pi}$, where

$$j^\mu_N(x) = \overline{\psi}(x) e_N \gamma^\mu \psi(x)$$

$$j^\mu_{\pi\pi}(x) = e\varepsilon_{3ab}\phi_a(x) i \overleftrightarrow{\partial^\mu} \phi_b(x)$$

and an "anomalous" contribution related to the Wess-Zumino anomaly term,

$$j^\mu_{\pi\pi\pi}(x) = ieH\varepsilon^{\mu\nu\sigma\tau}\varepsilon_{abc}\partial_\nu\phi_a(x)\partial_\sigma\phi_b(x)\partial_\tau\phi_c(x) \tag{2.2}$$

With this $\mathcal{L}^{EM}$ we calculate now the anomalous contribution (Fig.1e) to the nucleon electromagnetic form factors $F_{1,2}(q^2)$ or, equivalently, the Sachs form factors $G_{E,M}(q^2)$. Note that the $\pi^+\pi^-\pi^0$ intermediate state couples only to the **isoscalar** component of the photon. Under the additional assumption of vector meson dominance (VMD) this process would be dominated by the isoscalar vector mesons (ω(782), ϕ(1020), J/ψ(3097), ...). Without any vector mesons the chiral anomaly (2.2) comes only in one variety (the "contact" term); then [1] $H = H^c = (24\pi^2 f_\pi^3)^{-1}$ in eq.(2.2), leading to $m_N^3 H^c = 4.4$. We note here that H^c is given by a combination of the KSFR relation [8] and the field theoretic connection between the AVV anomaly (as in $\pi^0 \to \gamma\gamma$) and the VAAA anomaly considered here [1,6].
If one includes, however, the vector mesons in (2.1), then the constant H in eq.(2.2) obtains structure due to the now possible processes $\gamma \to \begin{pmatrix} \omega \\ \phi \\ J/\psi \\ Y \end{pmatrix} \to \pi\rho \to \pi^+\pi^-\pi^0$. The ω decays into $\pi^+\pi^-\pi^0$ with a branching ratio of ~ 90%; the corresponding branching ratio for the ϕ (J/ψ) is ~ 3% (1%) [7].
The contribution of Fig.1e to $F^{(I=0)}_{1,2}(q^2)$ can then be decomposed into their vector meson content $F_{1,2}^{(V)}(q^2)$ (V= ω, ϕ, J/ψ , Y , ...).

$$eF^{(I=0)}_{1,2}(q^2) = \sum_{V=\omega,\phi,\dots} \frac{\frac{em_V^2}{g_V}}{m_V^2 - q^2 - im_V\Gamma_V(q^2)} F^{(V)}_{1,2}(q^2) \tag{2.3}$$

In momentum space one has (see Fig.2 for a definition of the momenta)

$$j^\mu_{\pi\pi\pi}(0) = eH(s,t,u)\varepsilon^{\mu\nu\sigma\tau} L^{(+)}_\nu k^{(-)}_\sigma l^{(0)}_\tau \tag{2.2'}$$

where H is in general a Lorentz-scalar vertex function describing the structure of the $\gamma \to 3\pi$ transition (see below), $H = H^c + H^{GSW}$, with [1]

$$H^c = (24\pi^2 f_\pi^3)^{-1} \tag{2.4}$$

and H^{GSW} as in the Gell-Mann-Sharp-Wagner model [9], see below.

We find for the contact term

$$\int d^4x\, e^{-iq\cdot x}\left\langle N_f\left|j^{\mu}_{\pi\pi\pi}(x)\right|N_{in}\right\rangle = ieH^c\varepsilon^{\mu\nu\sigma\tau}g^3_{\pi NN}\int d^4x\, e^{-iq\cdot x}\iiint d^4x_1 d^4x_2 d^4x_3$$
$$\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4l}{(2\pi)^4}\int\frac{d^4L}{(2\pi)^4}(\partial^x_\nu e^{-iL\cdot(x_3-x)})(\partial^x_\sigma e^{-ik\cdot(x-x_1)})(\partial^x_\tau e^{-il\cdot(x_2-x)})$$
$$\frac{i}{k^2-m_\pi^2+i\varepsilon}\frac{i}{l^2-m_\pi^2+i\varepsilon}\frac{i}{L^2-m_\pi^2+i\varepsilon}\int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot(x_3-x_2)}\int\frac{d^4P}{(2\pi)^4}e^{-iP\cdot(x_2-x_1)}e^{iP_f\cdot x_3-iP_{in}\cdot x_1}$$
$$\left\langle N_f\left|\overline{u}(P_f)i\gamma_5\frac{i}{p\!\!\!/-m_N+i\varepsilon}i\gamma_5\frac{i}{P\!\!\!/-m_N+i\varepsilon}i\gamma_5\varepsilon_{abc}\tau_a\tau_b\tau_c u(P_{in})\right|N_{in}\right\rangle$$
$$=(2\pi)^4\delta^{(4)}(q-P_f+P_{in})6(-i)eg^3_{\pi NN}H^c\varepsilon^{\mu\nu\sigma\tau}q_\nu\iint\frac{d^4k\,d^4l}{(2\pi)^8}k_\sigma l_\tau D_k^{-1}D_l^{-1}$$
$$\left\{\overline{u}(P_f)\gamma_5 u(P_{in})(2k\cdot P_{in}-k^2)+l_\alpha k_\beta\overline{u}(P_f)\gamma_5\gamma^\alpha\gamma^\beta u(P_{in})\right\} \tag{2.5}$$

with
$$D_l\equiv(l^2-m_\pi^2+i\varepsilon)((q+k-l)^2-m_\pi^2+i\varepsilon)((P_{in}-k+l)^2-m_N^2+i\varepsilon)$$
$$D_k\equiv(k^2-m_\pi^2+i\varepsilon)((P_{in}-k)^2-m_N^2+i\varepsilon) \tag{2.6}$$

It is easy to show that the $\overline{u}\gamma_5 u$ - term in eq.(2.5) does not contribute; the d^4k - and d^4l - integrations in this case yield terms $g_{\sigma\tau}$, $P^{in}_\sigma P^{in}_\tau$, $P^{in}_\sigma q_\tau$, $q_\sigma P^{in}_\tau$ which all vanish after contraction with $\varepsilon^{\mu\nu\sigma\tau}q_\nu$. The $\overline{u}\gamma_5\gamma_\alpha\gamma_\beta u$ - term in eq.(2.5), however, survives because of the identity

$$i\varepsilon^{\mu\nu\sigma\tau}\gamma_5\gamma_\tau\equiv\gamma^\mu\gamma^\nu\gamma^\sigma-g^{\mu\nu}\gamma^\sigma-g^{\nu\sigma}\gamma^\mu+g^{\mu\sigma}\gamma^\nu \tag{2.7}$$

This leads to the gauge invariant results compiled in table 1.

We define

$$L_{\alpha\tau}(k,q,P_{in})\equiv\int_\Lambda\frac{d^4l}{(2\pi)^4}l_\alpha l_\tau D_l^{-1}(k,q,l,P_{in}) \tag{2.8}$$

where the Λ on the integral signifies that this integral is logarithmically divergent and needs to be regulated. Details have been relegated to Appendix A; the result is

$$L^{\alpha\tau}(k,q,P_{in})=-\frac{i}{8\pi^2}\int_0^1dx\int_0^{1-x}dy\left\{-\frac{1}{4}g^{\alpha\tau}f_1(c,\Lambda_E)+f_2(c,\Lambda_E)(y^2\frac{P^\alpha_{in}P^\tau_{in}}{m_N^2}+\cdots)\right\} \tag{2.9}$$

where the dots replace terms $\sim k^\alpha k^\tau, q^\alpha k^\tau, q^\alpha q^\tau, q^\alpha P^\tau, P^\alpha q^\tau$ which either vanish upon contraction with $\varepsilon^{\mu\nu\sigma\tau}q_\nu$ or vanish after d^4k - integration and contraction with the antisymmetric tensor. The functions $f_{1,2}(c,\Lambda_E)$ are defined in App.A; they depend on the cut-off Λ_E and implicitly (via the polynomial $c(k,q,P_{in};x,y)$ on the momenta k, q, P_{in}. The remaining d^4k - integration can now be done as well; the functions $f_{1,2}(c,\Lambda_E)$ do not contain physical poles so that the same procedure yields (for details see Appendix B)

$$\int_\Lambda\frac{d^4k}{(2\pi)^4}k_\sigma k_\beta L_{\alpha\tau}(k,q,P_{in})D_k^{-1}(k,P_{in})$$
$$=\frac{i}{8\pi^2}\frac{i\pi^2}{4(2\pi)^4}m_N^2\left\{g_{\alpha\tau}(\frac{P_\sigma P_\beta}{m_N^2}a_2^\lambda(q^2)-\frac{1}{4}g_{\sigma\beta}a_1^\lambda(q^2))+g_{\sigma\beta}\frac{P_\alpha P_\tau}{m_N^2}a_3^\lambda(q^2)+\cdots\right\} \tag{2.10}$$

where the dots stand for terms which vanish upon contraction with $\varepsilon^{\mu\nu\sigma\tau} q_\nu \bar{u}_f \gamma_5 \gamma^\alpha \gamma^\beta u_{in}$. The functions $a_{1,2,3}(q^2)$ are given in appendix B. The result of the contraction can be read-off table 1 ($g_{\sigma\beta} P_\alpha P_\tau$ is obtained by permutations from the $g_{\alpha\tau} P_\sigma P_\beta$ result). Finally we obtain

$$\int d^4x\, e^{-iq\cdot x} \langle N_f | j^\mu_{\pi\pi\pi}(x) | N_{in} \rangle =$$

$$= (2\pi)^4 \delta^{(4)}(q - P_f + P_{in}) \frac{3eg^3_{\pi NN} m^3_N H^c}{256\pi^4} \left(-\frac{1}{2} a^\lambda_1(q^2) \Gamma^\mu_{(1)} + (a^\lambda_2(q^2) + a^\lambda_3(q^2)) \Gamma^\mu_{(2)}\right) \qquad (2.11)$$

with $\Gamma^\mu_{(i)}$ as in table 1. This has to be compared with the general form dictated by Lorentz-invariance

$$\int d^4x\, e^{-iq\cdot x} \langle N_f | j^\mu(x) | N_{in} \rangle =$$

$$= (2\pi)^4 \delta^{(4)}(q - P_f + P_{in})\, e\, \bar{u}(P_f) \left[\frac{P^\mu_{in}}{m_N} \Gamma_1(q^2) + \frac{P^\mu_f}{m_N} \Gamma_2(q^2) + \gamma^\mu \Gamma_3(q^2) \right] u(P_{in}) \qquad (2.12)$$

where $\Gamma_1(q^2) = \Gamma_2(q^2)$ due to gauge invariance and the $\Gamma_i(q^2)$ are Lorentz-scalar functions, trivially related to $F_{1,2}(q^2)$ and $G_{E,M}(q^2)$, ($\eta = -q^2/4m_N^2 \geq 0$):

$$\Gamma_1(q^2) \equiv \Gamma_2(q^2) = -\frac{1}{2} F_2(q^2) = \frac{1}{2(1+\eta)} (G_E(q^2) - G_M(q^2))$$
$$\Gamma_3(q^2) = F_1(q^2) + F_2(q^2) = G_M(q^2) \qquad (2.13)$$

A comparison of (2.11) and (2.12) yields

$$\Gamma_1(q^2) \equiv \Gamma_2(q^2) = \Gamma_0\left(-\frac{1}{2} a^\lambda_1(q^2) + a^\lambda_2(q^2) - a^\lambda_3(q^2)\right)$$
$$\Gamma_3(q^2) = \Gamma_0(a^\lambda_1(q^2) - 2(1+\eta)(a^\lambda_2(q^2) + a^\lambda_3(q^2))) \qquad (2.14a)$$

where $\Gamma_0 \equiv \frac{3g^3_{\pi NN} m^3_N H^c}{256\pi^4} = 1.3$. In terms of the Sachs form factors (2.14a) reads

$$G^{3\pi}_E(q^2) = -\Gamma_0 \eta\, a^\lambda_1(q^2) \sim q^2, \qquad G^{3\pi}_E(0) = 0$$
$$G^{3\pi}_M(q^2) = \Gamma_0(\, a^\lambda_1(q^2) - 2(1+\eta)(a^\lambda_2(q^2) + a^\lambda_3(q^2)), \qquad G^{3\pi}_M(0) \neq 0 \qquad (2.14b)$$

and in terms of the Pauli and Dirac form factors

$$F^{3\pi}_1(q^2) = -2\Gamma_0 \eta\, (a^\lambda_2(q^2) + a^\lambda_3(q^2)) \sim q^2, \qquad F^{3\pi}_1(0) = 0$$
$$F^{3\pi}_2(q^2) = 2\Gamma_0\left(\frac{1}{2} a^\lambda_1(q^2) - a^\lambda_2(q^2) - a^\lambda_3(q^2)\right), \qquad F^{3\pi}_2(0) \neq 0 \qquad (2.14c)$$

From this we can extract the ωNN vertex functions

$$F_1^{\omega\to 3\pi\to NN}(q^2) = -2\Gamma_0 g_\omega (1+\eta\frac{4m_N^2}{m_\omega^2})\,\eta\,(a_2^\lambda(q^2)+a_3^\lambda(q^2)) \sim q^2, \qquad F_1^{\omega\to 3\pi\to NN}(0)=0$$

$$F_2^{\omega\to 3\pi\to NN}(q^2) = 2\Gamma_0 g_\omega (1+\eta\frac{4m_N^2}{m_\omega^2})(\frac{1}{2}a_1^\lambda(q^2)-a_2^\lambda(q^2)-a_3^\lambda(q^2)), \quad F_2^{\omega\to 3\pi\to NN}(0)\neq 0$$

where $F^{\omega}_{1(2)}$ refers to the vector (tensor) coupling and g_ω is the γ-ω coupling constant, see section 3. Note that $F_1^{\omega NN}(0)=0,\ \ F_2^{\omega NN}(0)\neq 0$. The vanishing of the vector coupling strength for $q^2 \to 0$ is different from the usual parametrization in terms of a monopole fit with cut-off masses typically Λ_ω=1414 MeV [5]. For completeness we note that the corresponding ρNN monopole fit requires a somewhat different Λ_ρ=1970 MeV, indicative of the different underlying Physics (no anomaly in the ρNN case).

Before we turn to numerical results for the form factors obtained with the contact term we give the corresponding equations for the two-step process γN → πρN → πππN which had first been considered by Gell-Mann, Sharp and Wagner [9]. The integrand in (2.5) is accordingly modified: H_c is replaced under the k- and l-integrals with

$$H(s,t,u) = -H_0(q^2)m_\rho^2\left\{\begin{array}{l}\dfrac{1}{m_\rho^2-(k-l)^2-im_\rho\Gamma_\rho((k-l)^2)}+\dfrac{1}{m_\rho^2-(q-l)^2-im_\rho\Gamma_\rho((q-l)^2)}\\ +\dfrac{1}{m_\rho^2-(q+k)^2-im_\rho\Gamma_\rho((q+k)^2)}\end{array}\right\} \tag{2.15}$$

where

$$H_0(q^2)m_\rho^2 = \frac{-\dfrac{m_\omega^2}{g_\omega}2g_{\omega\pi\rho}g_{\rho\pi\pi}}{m_\omega^2-q^2-im_\omega\Gamma_\omega(q^2)}+\cdots \tag{2.16}$$

and $\Gamma_V(q^2)$ is the momentum-dependent decay width of the vector meson V [10]. The dots in (2.16) stand for other isoscalar vector meson contributions (ϕ, J/ψ, Y, ...) which we disregard here. Instead of eq.(2.5) we obtain now

$$\begin{gathered}(2\pi)^4\delta^{(4)}(q-P_f+P_{in})6ieg_{\pi NN}^3H_0(q^2)\varepsilon^{\mu\nu\sigma\tau}q_\nu(-1)m_\rho^2\\ \iint\frac{d^4k\,d^4l}{(2\pi)^8}k_\sigma l_\tau l_\alpha k_\beta\bar{u}(P_f)\gamma_5\gamma^\alpha\gamma^\beta u(P_{in})D_k^{-1}D_l^{-1}\left\{D_{kl}^{-1}+D_{ql}^{-1}+D_{k,-q}^{-1}\right\}\end{gathered} \tag{2.17}$$

with $D_{ab}\equiv(a-b)^2-m_\rho^2+im_\rho\Gamma_\rho((a-b)^2)$. If we define in analogy to (2.8)

$$L_{\alpha\tau}^{GSW}(k,q,P_{in}) \equiv \sum_{i=1}^{3}L_{\alpha\tau}^{(i)GSW}(k,q,P_{in}) \tag{2.18}$$

where

$$L_{\alpha\tau}^{(1)GSW}(k,q,P_{in}) \equiv m_\rho^2\int_\Lambda\frac{d^4l}{(2\pi)^4}l_\alpha l_\tau D_l^{-1}D_{kl}^{-1}$$

$$L_{\alpha\tau}^{(2)GSW}(k,q,P_{in}) \equiv m_\rho^2\int_\Lambda\frac{d^4l}{(2\pi)^4}l_\alpha l_\tau D_l^{-1}D_{ql}^{-1}$$

$$L_{\alpha\tau}^{(3)GSW}(k,q,P_{in}) = L_{\alpha\tau}(k,q,P_{in})m_\rho^2D_{k,-q}^{-1}$$

one finds (see Appendix C):

$$L_{\alpha\tau}^{(1)GSW}(k,q,P_{in}) = \frac{3i}{8\pi^2}\int_0^1 dx\int_0^{1-x} dy\int_0^{1-x-y} dz\left\{-\frac{1}{4}g_{\alpha\tau}f_3(c_{(1)},\Lambda_E)+f_4(c_{(1)},\Lambda_E)(y^2\frac{P_{in}^\alpha P_{in}^\tau}{m_N^2}+\dots)\right\} \tag{2.19}$$

and

$$\int_\Lambda \frac{d^4k}{(2\pi)^4}k_\sigma k_\beta L_{\alpha\tau}^{(1)GSW}(k,q,P_{in})D_k^{-1}(k,P_{in})$$
$$= \frac{3r_\rho}{32(2\pi)^4}m_N^2\left\{g_{\alpha\tau}(\frac{P_\sigma P_\beta}{m_N^2}\ {}^{(1)}A_2^\lambda(q^2) - \frac{1}{4}g_{\sigma\beta}\ {}^{(1)}A_1^\lambda(q^2)) + g_{\sigma\beta}\frac{P_\alpha P_\tau}{m_N^2}\ {}^{(1)}A_3^\lambda(q^2)\right\} \tag{2.20}$$

Then we get for the first of the three contributions

$$\Gamma_1^{(1)GSW}(q^2) \equiv \Gamma_2^{(1)GSW}(q^2) = \Gamma_0^{(1)}(q^2)(\frac{1}{2}\ {}^{(1)}A_1^\lambda(q^2) - {}^{(1)}A_2^\lambda(q^2) - {}^{(1)}A_3^\lambda(q^2))$$
$$\Gamma_3^{(1)GSW}(q^2) = \Gamma_0^{(1)}(q^2)(-\ {}^{(1)}A_1^\lambda(q^2) + 2(1+\eta)({}^{(1)}A_2^\lambda(q^2) + {}^{(1)}A_3^\lambda(q^2))) \tag{2.21a}$$

with $\Gamma_0^{(1)}(q^2) = \frac{9g_{\pi NN}^3 m_N^3}{256\pi^4}r_\rho H_0(q^2) \equiv 3r_\rho\frac{H_0(q^2)}{H^c}\Gamma_0$. The functions ${}^{(1)}A_i^\lambda(q^2)$ are given in App.C. The second contribution is also of the form (2.21a) with the superscript (1) replaced with (2) on all A-functions; the resulting ${}^{(2)}A_i^\lambda(q^2)$ have the same functional form as ${}^{(1)}A_i^\lambda(q^2)$; only $c_{(1)}$ needs to be replaced with $c_{(2)}$; see App.C.
The third contribution is different

$$\Gamma_1^{(3)GSW}(q^2) = \frac{2}{3}\Gamma_0^{(1)}(q^2)(\frac{1}{2}\ {}^{(3)}A_1^\lambda(q^2) - {}^{(3)}A_2^\lambda(q^2) - {}^{(3)}A_3^\lambda(q^2))$$
$$\Gamma_3^{(3)GSW}(q^2) = \frac{2}{3}\Gamma_0^{(1)}(q^2)(-\ {}^{(3)}A_1^\lambda(q^2) + 2(1+\eta)({}^{(3)}A_2^\lambda(q^2) + {}^{(3)}A_3^\lambda(q^2))) \tag{2.21c}$$

where ${}^{(3)}A_i^\lambda(q^2)$ is now distinct from ${}^{(j)}A_i^\lambda(q^2)$ $(j=1,2)$ see Appendix C.The 5-fold integration in the ${}^{(j)}A_i^\lambda(q^2)$ can only partly be done by analytical integration. The c-integration leads to di-logarithms (or: Spence integrals) typical for regularized loop-integrations. The remaining 4-Feynman parameter integrals have to be done numerically.

3. Results and Discussion

Combining eq.(2.14a) and (2.21a+c) we obtain as final result

$$F_{1,an}^{\omega NN}(q^2) = F_1^{\omega\to 3\pi\to NN}(q^2) + F_1^{\omega\to\pi\rho\to 3\pi\to NN}(q^2)$$
$$= 2\Gamma_0 g_\omega (1+\eta\frac{4m_N^2}{m_\omega^2})\,\eta\left[-(a_2^\lambda(q^2)+a_3^\lambda(q^2)) + 3r_\rho\frac{H_0(q^2)}{H^c}(A_2^{GSW,\lambda}(q^2)+A_3^{GSW,\lambda}(q^2))\right]$$
$$F_{2,an}^{\omega NN}(q^2) = F_2^{\omega\to 3\pi\to NN}(q^2) + F_2^{\omega\to\pi\rho\to 3\pi\to NN}(q^2) \qquad (3.1)$$
$$= 2\Gamma_0 g_\omega (1+\eta\frac{4m_N^2}{m_\omega^2})\left[-(-\frac{1}{2}a_1^\lambda(q^2)+a_2^\lambda(q^2)+a_3^\lambda(q^2)) + 3r_\rho\frac{H_0(q^2)}{H^c}(-\frac{1}{2}A_1^{GSW,\lambda}(q^2)+A_2^{GSW,\lambda}(q^2) + A_3^{GSW,\lambda}(q^2))\right]$$

where $A_i^{GSW,\lambda} = {}^{(1)}A_i^\lambda + {}^{(2)}A_i^\lambda + \frac{2}{3}\,{}^{(3)}A_i^\lambda$; $(i = 1,2,3)$. A similar form is obtained for the Sachs form factors $G_E = F_1 - \eta F_2$, $G_M = F_1 + F_2$. Note that $F_{1,2,an}^{\omega NN}(q^2)$ and $G_{E,M,an}^{\omega NN}(q^2)$ are completely determined by the 2-loop integrals $a_i(q^2), A_i^{GSW}(q^2)$ once the coupling constants $f_\pi, g_{\pi NN}, g_\omega, g_{\omega\pi\rho}, g_{\rho\pi\pi}$ and masses $m_N, m_\omega, m_\rho, m_\pi$ are known. In Fig.4 we display the 2-loop integrals for three pion momentum cut-offs Λ =1.2,1.0 and 0.8 GeV between q^2=0 and $-q^2$=1.75 GeV^2. Not surprisingly the contact term produces much flatter 2-loop integrals $a_i(q^2)$ than the GSW process, $A_i(q^2)$. Because $g_{\omega\pi\rho} < 0$, while all other coupling constants are positive, we observe for $-q^2 > 0$ a cancellation of contributions from the contact term and from the GSW process. This is manifest in a zero of $F_{1,an}^{\omega NN}(q^2)$ for $\eta \geq 0.5$ and of $F_{2,an}^{\omega NN}(q^2)$ for $\eta \leq 0.5$, dependent on the cut-off, see Fig.5a. We have used $(m_N, m_\omega, m_\rho, m_\pi)$ = (940,782,770,140) MeV and $(f_\pi, g_{\pi NN}, g_\omega, g_{\omega\pi\rho}, g_{\rho\pi\pi})$ = (92.4 MeV, 13.45, 16, -19 GeV^{-1}, 6) [17]. These are typical values with some freedom of variation in g_ω *and* $g_{\omega\pi\rho}$. While our numerical results depend very little on g_ω they do depend almost linearly on $g_{\omega\pi\rho}$ due to the smallness of the contact term relative to the GSW process. If heavier vector mesons are included these values would have to change accordingly.

A striking feature of our result, eq.(3.1) is that $F_{1,an}^{\omega NN}(q^2) \sim \eta$; the slope at $\eta = 0$ increases with increasing Λ. A *larger* slope of the form factor $F_{1,an}^{\omega NN}(q^2)$ at zero momentum-transfers could only be accommodated by a *larger* $/g_{\omega\pi\rho}/$, if the cut-off remains unchanged. $F_{2,an}^{\omega NN}(q^2)$ depends sensitively on Λ and changes sign (from positive to negative) for $\Lambda > 1.0$ GeV. The same trend is displayed by $G_{E,M,an}^{\omega NN}(q^2)$, see Fig.5b. In Fig.6 we show our total result

$F_1^{\omega NN}(q^2) = F_{1,an}^{\omega NN}(q^2) + F_1^{\omega\to q\bar{q}}(q^2)$, $F_2^{\omega NN}(q^2) = F_{2,an}^{\omega NN}(q^2)$

where

$$F_1^{\omega\to q\bar{q}}(q^2) = \frac{g_{\omega NN}^{V,q\bar{q}}}{1-\frac{q^2}{\Lambda_Q^2}} \qquad (3.2)$$

is a reasonable parametrization of the quark model form factor with $g_{\omega NN}^{V,q\bar{q}} = 3g_{\rho NN}^{V,q\bar{q}} = 10$ and Λ_Q= 800 MeV [16], and compare with the empirical monopole form

$$F_1^{\omega NN}(q^2) = \frac{g^V_{\omega NN}}{1 - \frac{q^2}{\Lambda^2_{mon}}} \tag{3.3}$$

and Λ_{mon} = 800 and 1400 MeV. Between $\eta = 0$ and $\eta = 0.3$ our result $F_1^{\omega NN}(q^2)$ is very close to the monopole form (3.3) with Λ_{mon} = 1400 MeV. The form factor $F_2^{\omega NN}(q^2)$ is (for the same pion cut-off $\Lambda = 1.0$ GeV) essentially flat between $\eta = 0$ and $\eta = 0.2$. Our result $F_2^{\omega NN}(0) \approx 1 \approx \frac{1}{10} F_1^{\omega NN}(0)$ confirms the empirical finding from fits to the NN phase shifts that $g^T_{\omega NN}$ usually comes out much smaller than $g^V_{\omega NN}$. Note that the relevant range for nuclear physics applications is $0 \le \eta \le 0.1$, corresponding to $0 \le -q^2 \le (600\text{MeV/c})^2$. In the usually used Breit frame one has $-q^2 = \vec{q}^{\,2}$. It will be interesting to see if our $F_2^{\omega NN}(q^2)$, Fig.5a, could produce effects similar to those arising from an unnaturally high $g^V_{\omega NN} \cong 16$ in eq.(3.3)[1]. The ωNN vertex function corresponds to an *isoscalar* vector field probing the nucleon and, thus, gives rise to a size parameter for the target nucleon which is necessarily isoscalar in nature. The peculiar q^2-dependence of the anomaly contribution near $q^2=0$ will lead to a *decrease* of the size parameter originating from the quark-antiquark sector. If added to the usual quark model contribution one sees that the $\omega \to 3\pi$ anomaly *stabilizes* the isoscalar radius of the nucleon, a well-known effect observed also in Skyrme-type models [3] ; here this effect is manifest in a **shrinking** of the nucleon isoscalar radius due to the presence of the $\omega \to 3\pi$ anomaly which adds a *stabilizing* mesonic cloud to the nucleon. We recall here the presence of $\log(m_\pi)$ and m_π^{-1} terms in the iso**vector** radii $\langle r_{1,2}^2 \rangle^{I=1}$ due to the presence of the ρ (2π) cloud [18], which tend to '*destabilize*' the nucleon, in the sense that these radii become infinite in the chiral limit. The iso**scalar** radii (here including the anomaly) contain only terms $O(m_\pi^2 \log m_\pi)$ and $O(m_\pi^2)$ which are finite in the chiral limit ; there is, therefore, no obvious connection between isoscalar radii and the pion Compton wave length. The contribution of heavier isoscalar vector mesons (φ(1020), J/ψ(3097), ...) in (2.3) is suppressed because the time-like pole they induce via (2.3) moves further and further away from the space-like region considered here; we have neglected them here for the purpose of our qualitative discussion. We note also that the Euclidean cut-off procedure used here for simplicity does not interfere with the gauge invariance of the contributions considered here. This is not necessarily so, see the discussion in [19]. Finally we want to mention that there is an interesting relation between the Euclidean cut-off (which happens to respect the gauge invariance here) and the Pauli-Villars cut-off procedure (which is known to respect gauge invariance), see [19].

In summary, we have shown that the empirical ωNN form factor with a cut-off of 1400-1500MeV can be understood as arising from a combination of a quark model form factor with a typical cut-off of 700-800MeV and an anomalous form factor $\sim q^2$ arising from the 3π-intermediate state. The anomaly contributes to the Dirac form factor $F_1(q^2)$ with $F_1(0) = 0$, $-\frac{d}{dq^2} F_1(q^2)\Big|_{q^2=0} > 0$ (and sizeable), and to the Pauli form factor $F_2(q^2)$ with $F_2(0) \neq 0$. The resulting tensor coupling $F_2(0)$ is sensitive to the cut-off of the pion momenta in the two-loop integrals and turns out to be small for values around 1 GeV. While the quark model ωNN *tensor* coupling $F_2(0)$ vanishes for *point-like* quarks this is not so for the anomaly contribution which is a non-negligible *tensor* coupling. This can be seen (via the Gordon decomposition of

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[1] Two different parametrisations are being used in the literature, eq.(3.3) and $g(\Lambda^2 - m_\omega^2)/(\Lambda^2 - q^2)$ [2,14]. The coupling constants differ by a factor $(1 - m_\omega^2/\Lambda^2)^{-1} \approx 1.45$ which could explain the bulk part of the discrepancy, with the balance possibly provided by the tensor coupling.

the electromagnetic current) to enhance the *vector* coupling. It remains to be seen if the here discussed *tensor* form factor contribution to the ωNN vertex function has an effect on NN observables similar in magnitude to an enhanced quark model *vector* coupling. This could remove the only remaining discrepancy between empirical ωNN *vector* coupling constants (obtained under the assumption of a near-vanishing tensor coupling) $(g^{V}_{\omega NN})^2/4\pi$ = 10-20 , necessary to fit the NN phase shifts, and the simplest $(SU(3)_{flavour})$ quark model prediction, which is $g^{V,q\bar{q}}_{\omega NN} = 3g^{V,q\bar{q}}_{\rho NN}$, $g^{T,q\bar{q}}_{\omega NN} = 0$. Our result confirms the findings in [5] which model the ωNN coupling in NN scattering via correlated πρ exchange, leaving much less room for explicit quark-gluon effects than previously assumed.

Table 1

$\Gamma^{(i)}_{\alpha\beta\sigma\tau}$	$F^{\mu}_{(i)}/(-im_N)$
$g_{\alpha\tau}\dfrac{P^{in}_{\sigma}P^{in}_{\beta}}{m_N^2}$	$\Gamma^{\mu}_{(1)}$
$g_{\alpha\tau}\,g_{\sigma\beta}$	$2\Gamma^{\mu}_{(2)}$

with the definition $F^{\mu}_{(i)} \equiv \varepsilon^{\mu\nu\sigma\tau}q_{\nu}\bar{u}(P_f)\gamma_5\gamma^{\alpha}\gamma^{\beta}u(P_{in})\Gamma^{(i)}_{\alpha\beta\sigma\tau}$ and the gauge invariant combinations ($q_{\mu}\Gamma^{\mu}_{(i)} \equiv 0$),

$$\Gamma^{\mu}_{(1)} \equiv \bar{u}(P_f)\left[\frac{1}{m_N}(P^{\mu}_f + P^{\mu}_{in}) - 2(1+\eta)\gamma^{\mu}\right]u(P_{in}); \quad \Gamma^{\mu}_{(2)} \equiv \bar{u}(P_f)\left[\frac{1}{m_N}(P^{\mu}_f + P^{\mu}_{in}) - 2\gamma^{\mu}\right]u(P_{in})$$

Appendix A

We use Feynman's parametrization to treat the pole-structure of D_l^{-1} in $L^{\alpha\tau}(k,q,P_{in})$

$$L^{\alpha\tau}(k,q,P_{in}) = \frac{2}{(2\pi)^4}\int_{-\infty}^{+\infty}dz_1\int_{-\infty}^{+\infty}dz_2\int_{-\infty}^{+\infty}dz_3\delta(1-\sum_{i=1}^{3}z_i)\prod_{i=1}^{3}\theta(z_i)\int d^4l\frac{l^\alpha l^\tau}{\left[\tilde{l}^2 - m_N^2 c + i\varepsilon\right]^3} \quad (A.1)$$

with $\tilde{l} \equiv l + z_3 P_{in} - z_2 q - (z_2+z_3)k$ and

$$c = -(z_2+z_3)(1-z_2-z_3)\left(\frac{k}{m_N} - \frac{z_3}{z_2+z_3}\frac{P_{in}}{m_N} + \frac{z_2}{z_2+z_3}\frac{q}{m_N}\right)^2 + \frac{z_3^2}{z_2+z_3} + \frac{m_\pi^2}{m_N^2}(1-z_3) \quad (A.2)$$

Note that $c(z_2=1, z_3=0) = \frac{m_\pi^2}{m_N^2} \neq 0$. Therefore, one can not neglect the m_π^2-term compared to the m_N^2-term ! The origin can be shifted $l \to \tilde{l}$ because of the logarithmic divergence. The $d^4\tilde{l}$ - integration can be done via a Wick-rotation to Euclidean 4-space:

$$L^{\alpha\tau}(k,q,P_{in}) = -\frac{i}{8\pi^2}\int_0^1 dz_2\int_0^{1-z_2}dz_3\left\{-\frac{1}{4}g^{\alpha\tau}f_1(c,\Lambda_E) + \right.$$

$$\left. f_2(c,\Lambda_E)\left(-z_3\frac{P_{in}}{m_N} + z_2\frac{q}{m_N} + (z_2+z_3)\frac{k}{m_N}\right)^\alpha\left(-z_3\frac{P_{in}}{m_N} + z_2\frac{q}{m_N} + (z_2+z_3)\frac{k}{m_N}\right)^\tau\right\} \quad (A.3)$$

with

$$f_1(c,\Lambda_E) = \int_0^{\Lambda_E^2}dx\frac{x^2}{(x+m_N^2 c)^3} = \ln\frac{|\lambda^2+c|}{|c|} - \frac{3}{2}\frac{\lambda^2}{\lambda^2+c} + \frac{c}{2}\frac{\lambda^2}{(\lambda^2+c)^2} \quad (A.4)$$

and

$$f_2(c,\Lambda_E) = m_N^2\int_0^{\Lambda_E^2}dx\frac{x}{(x+m_N^2 c)^3} = \frac{1}{2c}\frac{\lambda^4}{(\lambda^2+c)^2} \quad (A.5)$$

where $\lambda \equiv \Lambda_E / m_N$ is the cut-off on the Euclidean 4-momenta.

Appendix B

With eq.(2.6) and (2.9) we obtain

$$\int_\Lambda \frac{d^4k}{(2\pi)^4} k_\sigma k_\beta L_{\alpha\tau}(k,q,P_{in}) D_k^{-1}(k,P_{in}) = \int_0^1 dt \int_\Lambda \frac{d^4k}{(2\pi)^4} \frac{k_\sigma k_\beta L_{\alpha\tau}}{\left[\tilde{k}^2 - m_N^2 R + i\varepsilon\right]^2} \qquad (B.1)$$

with $\tilde{k} \equiv k - t\,P_{in}$ and $R \equiv t^2 + (1-t) r_\pi$; $(r_\pi \equiv m_\pi^2 / m_N^2)$. The shift of origin can be done because $L_{\alpha\tau}(k) \sim \frac{1}{k^2}$, which follows from $f_{1,2}(c) \sim \frac{1}{c}$ for $|k^2| \to \infty$, so that (B.1) is only logarithmically divergent (hence no surface terms). Then (B.1) reads

$$\int_0^1 dt \int_\Lambda \frac{d^4\tilde{k}}{(2\pi)^4} \frac{\tilde{k}_\sigma \tilde{k}_\beta + t^2 P_\sigma^{in} P_\beta^{in} + t(\tilde{k}_\sigma P_\beta^{in} + \tilde{k}_\beta P_\sigma^{in})}{\left[\tilde{k}^2 - m_N^2 R + i\varepsilon\right]^2} L_{\alpha\tau}((\tilde{k}+Q)^2) \qquad (B.2)$$

where $L_{\alpha\tau}$ depends (via c) on $(\tilde{k}+Q)^2$ with

$$Q \equiv (t - \frac{z_3}{z_2+z_3}) P_{in} + \frac{z_2}{z_2+z_3} q, \quad c = -(z_2+z_3)(1-z_2-z_3)(\frac{\tilde{k}+Q}{m_N})^2 + \frac{z_3^2}{z_2+z_3} + r_\pi(1-z_3) \qquad (B.3)$$

Commensurate with our drastic cut-off on the Euclidean 4-momenta, we make another simplifying assumption

$$L_{\alpha\tau}((\tilde{k}+Q)^2) \equiv L_{\alpha\tau}(\tilde{k}^2 + Q^2) + \tilde{k}^\mu R_{\alpha\tau\mu}(\tilde{k}^2) \quad (B.4)$$

where $R_{\alpha\tau\mu}$ contains derivatives of c with respect to $\tilde{k}$ *at* $Q^\mu = 0$. In other words : we assume that the curvature of $L_{\alpha\tau}$ in $\tilde{k}$ - space is much smaller than it's gradient at $Q^\mu = 0$. This approximation is exact at $q^2=0$ and at large $q^2 \to \infty$. Eq.(B.4) has the effect that c in $f_{1,2}(c,\Lambda_E)$ in (A.4) is replaced with the scalar (in $\tilde{k}$) function

$$c(s,\eta,z_2,z_3,t) = (z_2+z_3)(1-z_2-z_3)(s-t^2) + z_3^2 + r_\pi(1-z_3) + 4\eta z_2(1-z_2-z_3)(1-t)$$
$$+ 2tz_3(1-z_2-z_3) \qquad (B.5)$$

with $s = -\tilde{k}^2 / m_N^2$ the normalized Euclidean 4-momentum squared. Note that c does not contain mixed terms $(\tilde{k}\cdot Q)$ anymore which vanish after symmetric integration. Now the $\tilde{k}$ -integral in (B.2) can be done using Wick-rotation. Then c becomes positive as a function of $s \geq 0$ except for the negative t^2-term which introduces a zero into c and a manageable singularity into the $(\ln|c|)$ - term in $f_1(c)$. Inserting (B.4) and (B.5) in (B.2) yields

$$(B.2) = \frac{m_N^2}{8(2\pi)^4}\left[-\frac{1}{4} g_{\alpha\tau} \{ -\frac{1}{4} g_{\sigma\beta} \int_0^1 dt \int_0^{\lambda^2} ds \frac{s^2 F_1(s)}{(s+R)^2} + \right.$$
$$\left. + \frac{P_\sigma^{in} P_\beta^{in}}{m_N^2} \int_0^1 dt t^2 \int_0^{\lambda^2} ds \frac{s\, F_1(s)}{(s+R)^2} \} - \frac{1}{4} g_{\sigma\beta} \frac{P_\alpha^{in} P_\tau^{in}}{m_N^2} \int_0^1 dt \int_0^{\lambda^2} ds \frac{s^2 F_2(s)}{(s+R)^2} + \cdots \right] \qquad (B.6)$$

where the dots stand for terms

$$P_\alpha P_\tau P_\sigma P_\beta, (2g_{\sigma\beta} P_\alpha P_\tau + g_{\beta\tau} P_\alpha P_\sigma + g_{\sigma\alpha} P_\tau P_\beta), (g_{\sigma\beta} P_\alpha P_\tau + g_{\beta\tau} P_\alpha P_\sigma + g_{\sigma\alpha} P_\tau P_\beta + g_{\alpha\tau} P_\sigma P_\beta) ,$$

which all vanish upon contraction with $\varepsilon^{\mu\nu\sigma\tau}$ q_ν $\bar{u}_f\gamma_5\gamma^\alpha\gamma^\beta u_{in}$. The functions $F_{1,2}$ are integrals over the functions $f_{1,2}(c)$:

$$F_1(s) = \int_0^1 dx \int_0^{1-x} dy f_1(c(s,\eta,x,y,t))$$

$$F_2(s) = \int_0^1 dx \int_0^{1-x} dy y^2 f_2(c(s,\eta,x,y,t)) \qquad (B.7)$$

with $f_{1,2}(c)$ as in (A.4+5) and

$$c(s,\eta,x,y,t) = (x+y)(1-x-y)(s-t^2) + y^2 + r_\pi(1-y) + 4\eta x(1-x-y)(1-t) + 2ty(1-x-y)$$

Then

$$(B.6) = -\frac{m_N^2}{32(2\pi)^4}\left[g_{\alpha\tau}(-\frac{1}{4}g_{\sigma\beta}a_1^\lambda(q^2) + \frac{P_\sigma^{in}P_\beta^{in}}{m_N^2}a_2^\lambda(q^2)) + g_{\sigma\beta}\frac{P_\alpha^{in}P_\tau^{in}}{m_N^2}a_3^\lambda(q^2)\right] \qquad (B.8)$$

where the new functions $a_i(q^2)$ $(i=1,2,3)$ are defined as follows

$$a^\lambda_{\binom{1}{3}}(q^2) = \int_0^1 dt \int_0^{\lambda^2} ds \frac{s^2}{(s+R)^2} \int_0^1 dx \int_0^{1-x} dy \binom{1}{y^2} f_{\binom{1}{2}}(c(s,\eta,x,y,t))$$

$$a_2^\lambda(q^2) = \int_0^1 dt t^2 \int_0^{\lambda^2} ds \frac{s}{(s+R)^2} \int_0^1 dx \int_0^{1-x} dy f_1(c(s,\eta,x,y,t)) \qquad (B.9)$$

Eq.(B.8) is identical with eq.(2.10) in the main text. In order to isolate the singularity (for $c\to 0$ the $(\ln|c|)$ - term diverges) we transform the s-integration first, writing $c(s,\eta,x,y,t)= a(x,y)(s-t^2)+b(x,y,\eta,t)$ with $a(x,y)=(x+y)(1-x-y) \geq 0$ $(1/4 \geq a(x,y) \geq 0)$ and $b(x,y,\eta,t)= y^2+r_\pi(1-y)+4\eta x(1-x-y)(1-t)+2yt(1-x-y)$ ($r_\pi \leq b(x,y,\eta,t) \leq 1$) and $R(t)$ as defined below (B.1) with $R(t)\geq r_\pi > 0$, we transform (first for $a(x,y) \neq 0$):

$$\int_0^{\lambda^2} ds \frac{s^2}{(s+R)^2} f_{\binom{1}{2}}(c(s)) = \frac{1}{a}\int_d^{d+a\lambda^2} dc \frac{(c-d)^2}{(c-d+Ra)^2} f_{\binom{1}{2}}(c) \qquad (B.10)$$

with $d=b-at^2$. Then the c-integral becomes a principal-value integral for $d(x,y,\eta,t) <0$. For $d(x,y,\eta,t) >0$ the integral is easily done without singularities of the integrand. The case $a(x,y)=0$ in (B.9) requires special care. For $a(x,y)=0$ we have $c(s)=b$ independent of s, then $f_1(c)=f_1(b)$ and

$$\int_0^{\lambda^2} ds \frac{s^2}{(s+R)^2} = \lambda^2 - 2R\ln\frac{\lambda^2+R}{R} + \frac{R\lambda^2}{\lambda^2+R}$$

so that we finally obtain

$$a^\lambda_{\binom{1}{3}}(q^2) = \int_0^1 dt \int_0^1 dx \int_0^{1-x} dy \binom{1}{y^2} \begin{cases} \frac{1}{a}\int_d^{d+a\lambda^2} dc \frac{(c-d)^2}{(c-d+Ra)^2} f_{\binom{1}{2}}(c) & \text{for } a(x,y)\neq 0 \\ f_{\binom{1}{2}}(b)(\lambda^2 - 2R\ln\frac{\lambda^2+R}{R} + \frac{R\lambda^2}{\lambda^2+R}) & \text{for } a(x,y)=0 \end{cases} \qquad (B.11)$$

and, similarly,

$$a_2^{\lambda}(q^2) = \int_0^1 dtt^2 \int_0^1 dx \int_0^{1-x} dy \int_d^{d+a\lambda^2} dc \frac{c-d}{(c-d+Ra)^2} f_1(c)$$

Note that for a(x,y)=0 the c-integral has been done explicitly; for a(x,y) $\neq$ 0 we can express the c-integral in terms of dilogarithms (which are related to the Spence function [11])

$$Li_2(z) = \int_z^0 dt \frac{\ln(1-t)}{t}$$

The remaining t-, x-, y-integrations have to be done numerically. Finally, we would like to compare our procedure (layed out in Appendices A,B,C) with a new technique (generalized Wick rotation) for the evaluation of certain scalar two-loop integrals appearing in QCD diagrams [15]. The method of ref.[15] is elegant and efficient; it has been tested extensively. Unfortunately our two-loop integrals are tensorial in nature and not directly applicable to this method. If the method of [15] could be generalized to include tensor 2-loop contributions to 3-point functions (vertex functions like ours), the approximation, eq.(B.4) would become unnecessary. None of our conclusions, however, depend sensitively on this approximation. There is certainly room for future improvements of our procedure, should the need arise from applications in NN physics.

Appendix C

We calculate here $L^{(i)GSW}_{\alpha\tau}(k,q,P_{in})$ as defined after eq.(2.18). First for (i=1):

$$L^{(1)GSW}_{\alpha\tau}(k,q,P_{in}) = \frac{6m_\rho^2}{(2\pi)^4}\int_{-\infty}^{+\infty}dz_1\int_{-\infty}^{+\infty}dz_2\int_{-\infty}^{+\infty}dz_3\int_{-\infty}^{+\infty}dz_4\,\delta(1-\sum_{i=1}^{4}z_i)\prod_{i=1}^{4}\theta(z_i)\int d^4l\frac{l_\alpha l_\tau}{\left[\tilde{l}^2 - m_N^2 c_{(1)} + i\varepsilon\right]^4} \quad (C.1)$$

with $\tilde{l} \equiv l + z_3P_{in} - z_2q - (1-z_1)k$ and

$$c_{(1)} = -z_1(1-z_1)\left(\frac{k}{m_N} - \frac{z_3}{1-z_1}\frac{P_{in}}{m_N} + \frac{z_2}{1-z_1}\frac{q}{m_N}\right)^2 + \frac{z_3^2}{1-z_1} + 4\eta\frac{z_2z_4}{1-z_1} + r_\pi(z_1+z_2) + r_\rho(1-z_1-z_2-z_3)$$

Note that $c_{(1)}(z_1=1, z_2=0, z_3=0) = \frac{m_\pi^2}{m_N^2} \neq 0$. The origin can be shifted $l \to \tilde{l}$ due to convergence. The $d^4\tilde{l}$ - integration can be done via a Wick-rotation to Euclidean 4-space:

$$L^{(1)GSW}_{\alpha\tau}(k,q,P_{in}) = \frac{3ir_\rho}{8\pi^2}\int_0^1 dz_2\int_0^{1-z_2}dz_3\int_0^{1-z_2-z_3}dz_4\left\{-\frac{1}{4}g_{\alpha\tau}f_3(c_{(1)},\Lambda_E) + \right.$$

$$\left. f_4(c_{(1)},\Lambda_E)\left(-z_3\frac{P_{in}}{m_N} + z_2\frac{q}{m_N} + (z_2+z_3+z_4)\frac{k}{m_N}\right)_\alpha\left(-z_3\frac{P_{in}}{m_N} + z_2\frac{q}{m_N} + (z_2+z_3+z_4)\frac{k}{m_N}\right)_\tau\right\}$$

$$(C.2)$$

with

$$f_3(c) = \int_0^{\lambda^2} ds\frac{s^2}{(s+c)^4} = \frac{1}{3c}\frac{\lambda^2}{(\lambda^2+c)} - \frac{2}{3}\frac{\lambda^2}{(\lambda^2+c)^2} + \frac{1}{3}\frac{c\lambda^2}{(\lambda^2+c)^3}$$

and

$$f_4(c) = \int_0^{\lambda^2} ds\frac{s}{(s+c)^4} = \frac{1}{6c^2} + \frac{1}{3}\frac{c}{(\lambda^2+c)^3} - \frac{1}{2}\frac{1}{(\lambda^2+c)^2}$$

Note that $f_{3,4}(c)$ are finite for $\lambda\to\infty$. Up to terms which vanish upon contraction (indicated as dots) we obtain

$$L^{(1)GSW}_{\alpha\tau}(k,q,P_{in}) = \frac{3ir_\rho}{8\pi^2}\int_0^1 dz_2\int_0^{1-z_2}dz_3\int_0^{1-z_2-z_3}dz_4\left\{-\frac{1}{4}g_{\alpha\tau}f_3(c_{(1)},\Lambda_E) + z_3^2\frac{P_\sigma^{in}P_\tau^{in}}{m_N^2}f_4(c_{(1)},\Lambda_E) + \cdots\right\} \quad (C.3)$$

Note that $f_3 \sim \frac{1}{c_{(1)}}$ *for* $c_{(1)} \to 0$ *and* $f_4 \sim \frac{1}{c_{(1)}^2}$ *for* $c_{(1)} \to 0$. The further evaluation follows the steps given in App.B,

$$\int_\Lambda \frac{d^4k}{(2\pi)^4}k_\sigma k_\beta L^{(1)GSW}_{\alpha\tau}(k,q,P_{in})D_k^{-1}(k,P_{in}) =$$

$$\int_0^1 dt\int_\Lambda\frac{d^4\tilde{k}}{(2\pi)^4}\frac{\tilde{k}_\sigma\tilde{k}_\beta + t^2P_\sigma^{in}P_\beta^{in} + t(\tilde{k}_\sigma P_\beta^{in} + \tilde{k}_\beta P_\sigma^{in})}{\left[\tilde{k}^2 - m_N^2R + i\varepsilon\right]^2}L^{(1)GSW}_{\alpha\tau}$$

$$= \frac{3m_N^2r_\rho}{32(2\pi)^4}\left[g_{\alpha\tau}\left(-\frac{1}{4}g_{\sigma\beta}\,{}^{(1)}A_1^\lambda(q^2) + \frac{P_\sigma^{in}P_\beta^{in}}{m_N^2}\,{}^{(1)}A_2^\lambda(q^2)\right) + g_{\sigma\beta}\frac{P_\alpha^{in}P_\tau^{in}}{m_N^2}\,{}^{(1)}A_3^\lambda(q^2)\right] \quad (C.4)$$

with

$$ {}^{(1)}A^{\lambda}_{\binom{1}{3}}(q^2)=\int_0^1 dt\int_0^1 dx\int_0^{1-x} dy\binom{1}{y^2}\int_0^{1-x-y} dz\begin{cases}\dfrac{1}{a_{(1)}}\displaystyle\int_{d_{(1)}}^{d_{(1)}+a_{(1)}\lambda^2} dc\,\frac{(c-d_{(1)})^2}{(c-d_{(1)}+Ra_{(1)})^2}f_{\binom{3}{4}}(c) & for\ a_{(1)}(x,y,z)\neq 0\\ f_{\binom{3}{4}}(b_{(1)})(\lambda^2-2R\ln\dfrac{\lambda^2+R}{R}+\dfrac{R\lambda^2}{\lambda^2+R}) & for\ a_{(1)}(x,y,z)=0\end{cases} \tag{C.5} $$

and

$$ {}^{(1)}A^{\lambda}_{2}(q^2)=\int_0^1 dtt^2\int_0^1 dx\int_0^{1-x} dy\int_0^{1-x-y} dz\int_{d_{(1)}}^{d_{(1)}+a_{(1)}\lambda^2} dc\,\frac{c-d_{(1)}}{(c-d_{(1)}+Ra_{(1)})^2}f_3(c) $$

where

$c_{(1)}=a_{(1)}s+d_{(1)}$, $a_{(1)}=(x+y+z)(1-x-y-z)$, $d_{(1)}(x,y,z,t)=b_{(1)}(x,y,z,t)-a_{(1)}(x,y,z)t^2$,
$b_{(1)}(x,y,z,t)=y^2+r_\pi(1-y-z)+r_\rho z+2ty(1-x-y-z)+4\eta x(z+(1-t)(1-x-y-z))$

With $L^{(2)GSW}_{\alpha\tau}$ we obtain the same form (C.4) with ${}^{(1)}A^{\lambda}_{(i)}\to{}^{(2)}A^{\lambda}_{(i)}$ and $(c_{(1)},a_{(1)},b_{(1)})\to(c_{(2)},a_{(2)},b_{(2)})$ with $c_{(2)}=a_{(2)}s+d_{(2)}$, $a_{(2)}(x,y,z)=(x+y)(1-x-y)$, $d_{(2)}=b_{(2)}-a_{(2)}t^2$, $b_{(2)}(x,y,z,t)=y^2+r_\pi(1-y-z)+r_\rho z+2ty(1-x-y)+4\eta[(1-x-y-z)(x(1-t)+z(1+t(x+y)))-tz(x-z(x+y))]$.

For $L^{(3)GSW}_{\alpha\tau}$ in (2.18) we can go straight to the k-integration:

$$ \int_\Lambda\frac{d^4k}{(2\pi)^4}k_\sigma k_\beta L_{\alpha\tau}(k,q,P_{in})D_k^{-1}D_{k,-q}^{-1}=2\int_{-\infty}^{+\infty}dt_1\int_{-\infty}^{+\infty}dt_2\int_{-\infty}^{+\infty}dt_3\delta(1-\sum_{i=1}^{3}t_i)\prod_{i=1}^{3}\theta(t_i)\int_\Lambda\frac{d^4k}{(2\pi)^4}\frac{k_\sigma k_\beta L_{\alpha\tau}}{\left[\tilde{k}^2-m_N^2R_{(3)}+i\varepsilon\right]^3} $$

with $R_{(3)}=t_2^2+r_\pi(1-t_2-t_3)+t_3r_\rho+4\eta t_3(1-t_2-t_3)$. The usual shift of origin can be done $k\to\tilde{k}=k-t_2P_{in}+t_3q$ with a result of the form (2.21c) with

$$ {}^{(3)}A^{\lambda}_{\binom{1}{3}}(q^2)=\int_0^1 dt_2\int_0^{1-t_2} dt_3\int_0^1 dx\int_0^{1-x} dy\binom{1}{y^2}\int_{d_{(3)}}^{d_{(3)}+a_{(3)}\lambda^2} dc\,\frac{(c-d_{(3)})^2}{(c-d_{(3)}+R_{(3)}a_{(3)})^3}f_{\binom{1}{2}}(c) $$

and (C.6)

$$ {}^{(3)}A^{\lambda}_{2}(q^2)=\int_0^1 dt_2t_2^2\int_0^{1-t_2} dt_3\int_0^1 dx\int_0^{1-x} dy\,a_{(3)}(x,y)\int_{d_{(3)}}^{d_{(3)}+a_{(3)}\lambda^2} dc\,\frac{c-d_{(3)}}{(c-d_{(3)}+R_{(3)}a_{(3)})^3}f_1(c) $$

and $c_{(3)}=a_{(3)}s+d_{(3)}$, $a_{(3)}(x,y)=(x+y)(1-x-y)$, $d_{(3)}=b_{(3)}-a_{(3)}(t_2^2-4\eta t_3(t_2+t_3))$
$b_{(3)}(x,y,z,t)=y^2+r_\pi(1-y)+2y(1-x-y)(t_2-2t_3\eta)+4\eta x(1-x-y)(1-t_2-2t_3)$.

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[17] We take f_π from ref.[20], $g_{\rho\pi\pi}$ can be obtained from the KSFR relation [8] $g_{\rho\pi\pi} = m_\rho / \sqrt{2} f_\pi = 5.89$ or directly from the decay width $g_{\rho\pi\pi} = 4\sqrt{3\pi\, \Gamma_{\rho\to\pi\pi} / m_\rho} / (1 - 4m_\pi^2 / m_\rho^2)^{3/4} = 6.04$ with very similar results; the coupling constant $g_{\omega\pi\rho}$ can be derived via a low-energy theorem [6]

$g_{\omega\pi\rho} = -3m_\rho^2 / (16\pi^2 f_\pi^3) = -14.3\,GeV^{-1}$ or directly from the ω→πγ decay, $|g_{\omega\pi\rho}| = 2(g_{\rho\pi\pi}/m_\omega)\sqrt{6(\Gamma_{\omega\to\pi\gamma}/m_\omega)/(\alpha(1-m_\pi^2/m_\omega^2))} = 14.0\,GeV^{-1}$. Note that $g_{\omega\pi\rho}$ comes out even more negative from the GSW treatment of the reaction $e^+e^- \to \pi^+\pi^-\pi^0$ [9,12] in the energy region between the ω-mass and the φ-mass. We use here a typical value from the latter source, $|g_{\omega\pi\rho}|/g_\omega \cong 1.2\,GeV^{-1}$ and $g_\omega = 16$, because this reaction is closer to our application than the ω→πγ decay, for example. The spread of empirical values for $g_{\omega\pi\rho}$ hints at a siz able off-shell dependence of this vertex function.

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Figure Captions

Fig.1 : Coupling of a virtual photon to the nucleon via (a) direct coupling to the nucleon charge , (b) coupling to the isovector pion cloud, (c) vertex correction (both isovector and isoscalar), (d) self-energy corrections to the nucleon mass, and (e) the isoscalar $\pi^+\pi^-\pi^0$ state; the solid dot in (e) represents a contact interaction as well as the two-step process via the πρ intermediate state, see text.

Fig.2 : A two-pion - loop contribution of higher order.

Fig.3 : kinematics associated with Fig.1e.

Fig.4 : (a) q^2-dependence (note : $\eta = -q^2/4m_N^2 \geq 0$) of the contact two-loop integral a_1, eq.(B.11), (dashed curves) and of the GSW two-loop-integral A_1 , eq.(3.1), (solid curves) for three cut-off values around 1 GeV.

(b) same notation as in (a) but now for the combinations $(a_2 + a_3)$, eq.B.11, and (A_2+A_3), eq.(3.1).

Fig.5: (a) ωNN vertex functions $F_{1,an}^{\omega NN}(q^2)$ (solid curves) and $F_{2,an}^{\omega NN}(q^2)$ (dashed curves) versus η,eq. 3.1, for three cut-offs around 1 GeV.

(b) ωNN vertex functions $G_{E,an}^{\omega NN}(q^2)$ (solid curves) and $G_{M,an}^{\omega NN}(q^2)$ (dashed curves) versus η, for three cut-offs around 1 GeV.

Fig.6: ωNN vertex function $F_1^{\omega NN}(q^2)$. Long-dashed curve: anomaly contribution, eq.(3.1) with pion cut-off Λ= 1 GeV; short-dashed curves: monopole form factor with Λ_{mon}=1.4 GeV (empirical fit to NN-data) and Λ_Q=0.8 GeV (quark contribution, eq.(3.2)) and $g_{\omega NN}^{V} = 10$ for both, see eq.(3.3); solid curve: sum of anomaly and quark contributions.

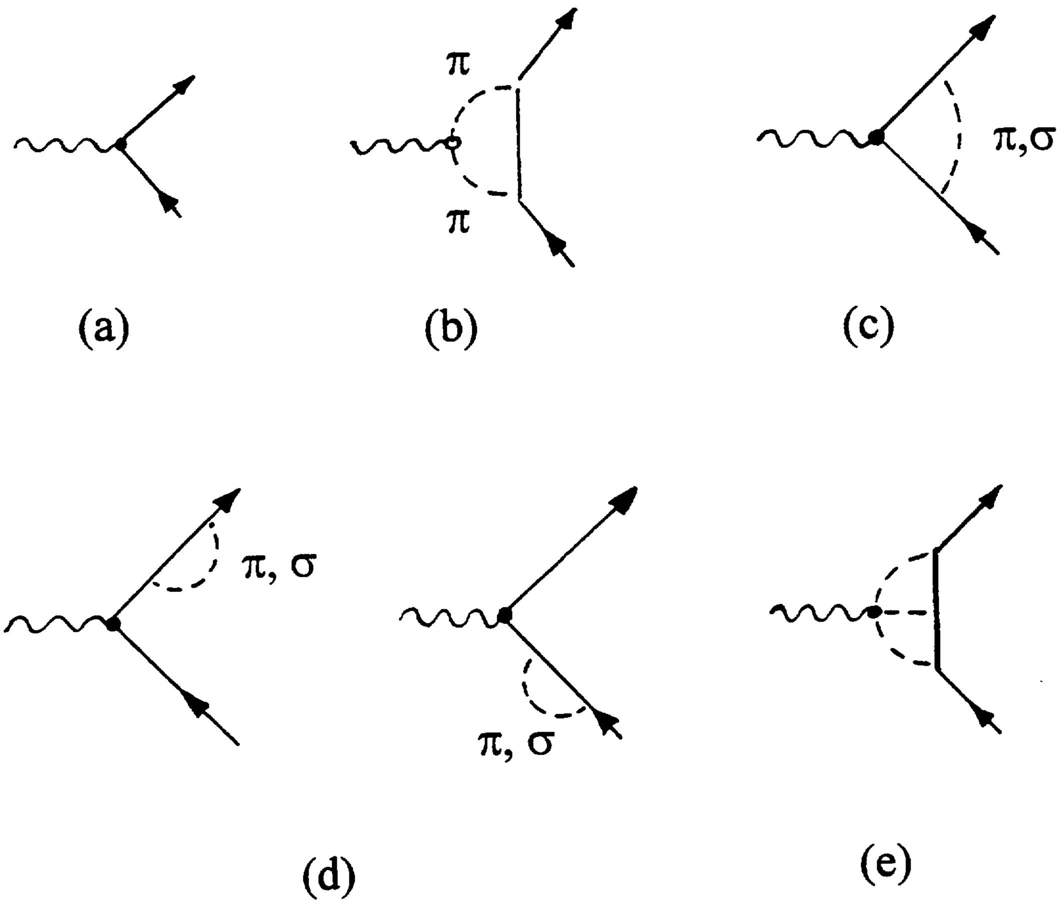


Fig. 1

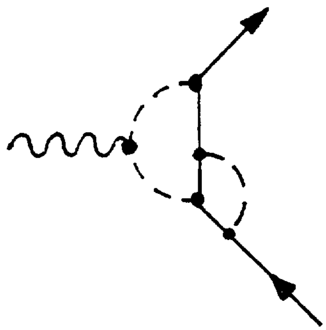


Fig. 2

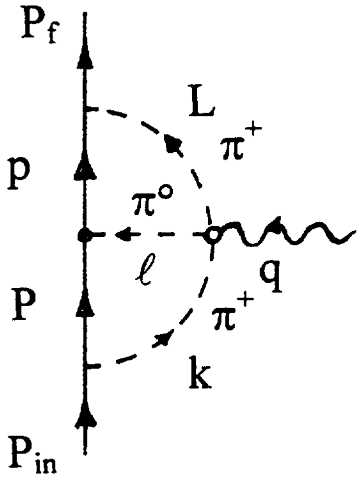


Fig. 3

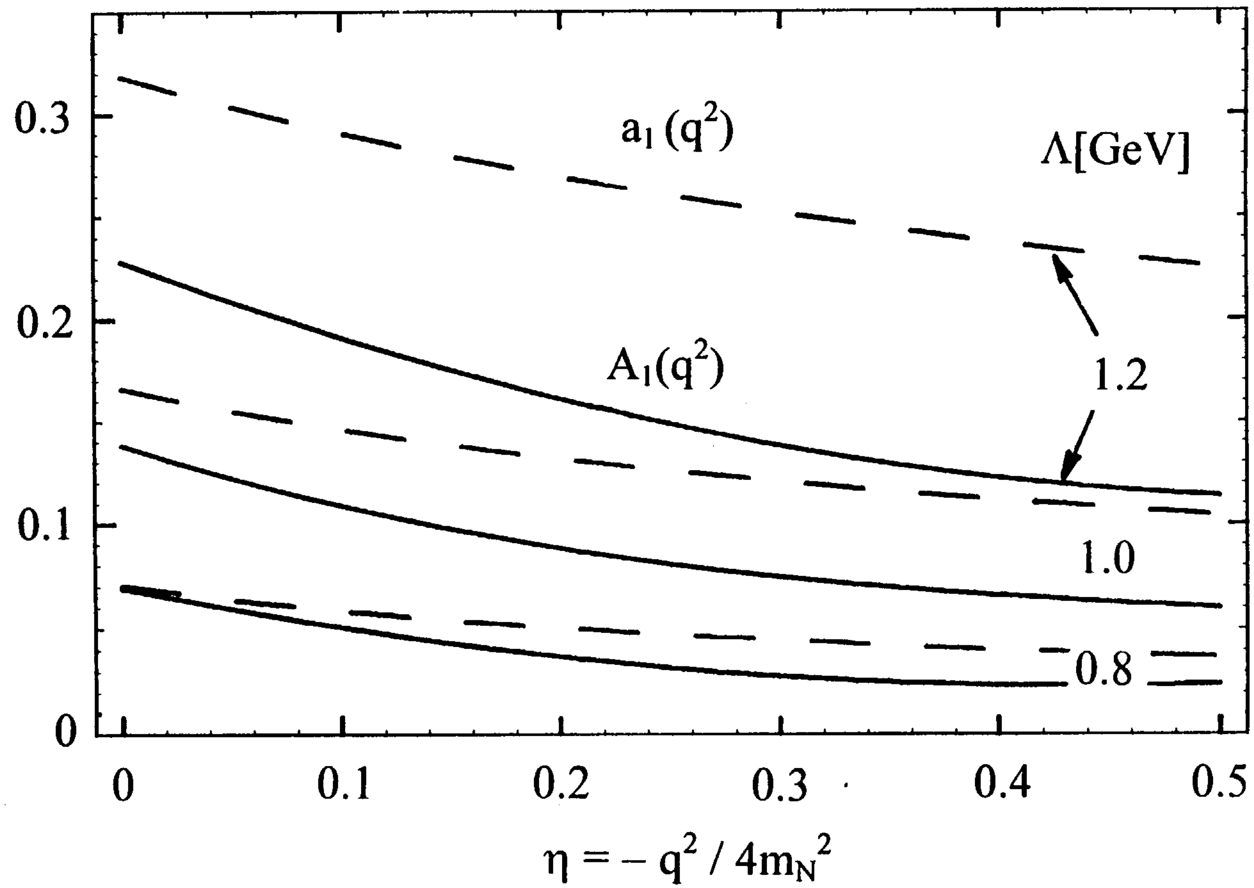
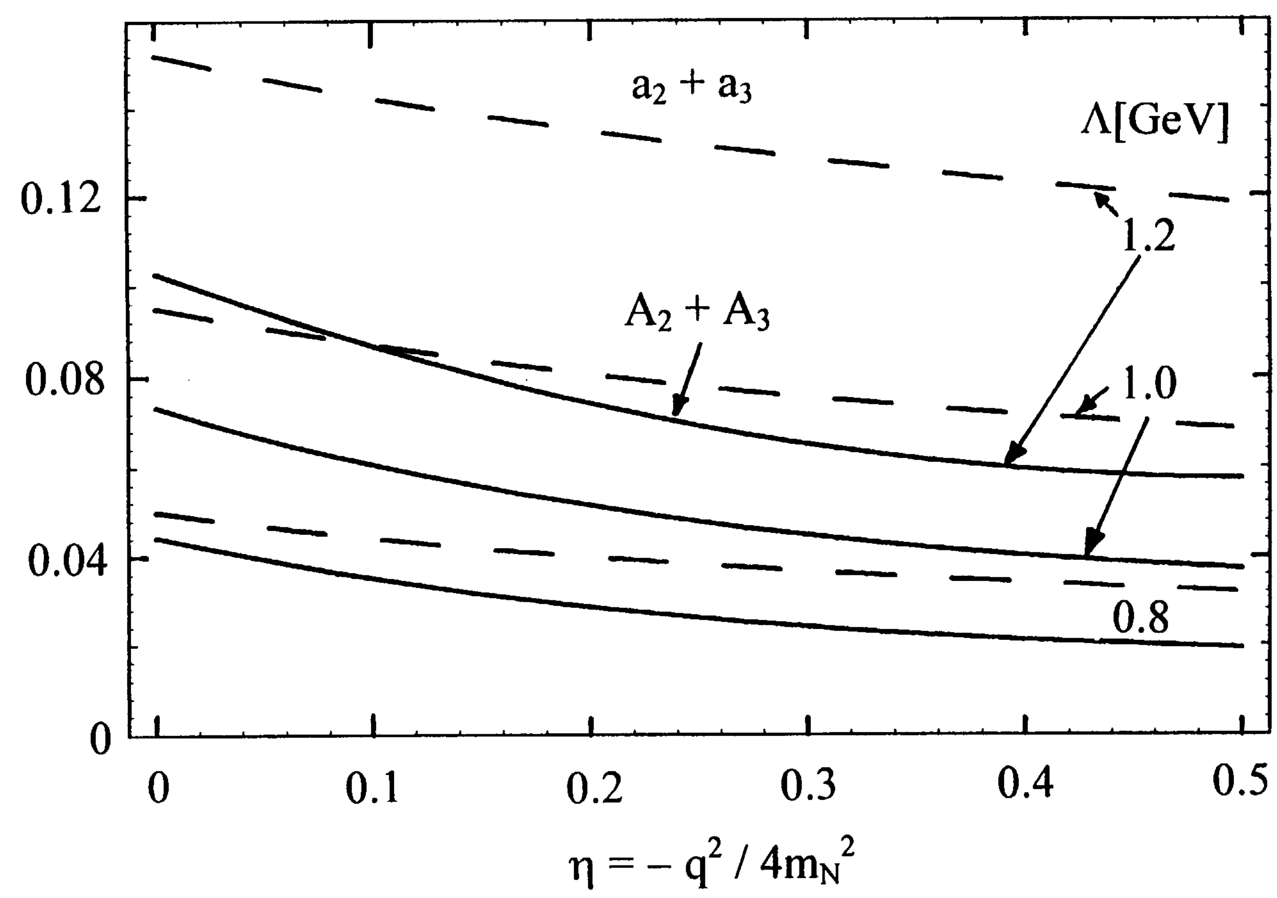
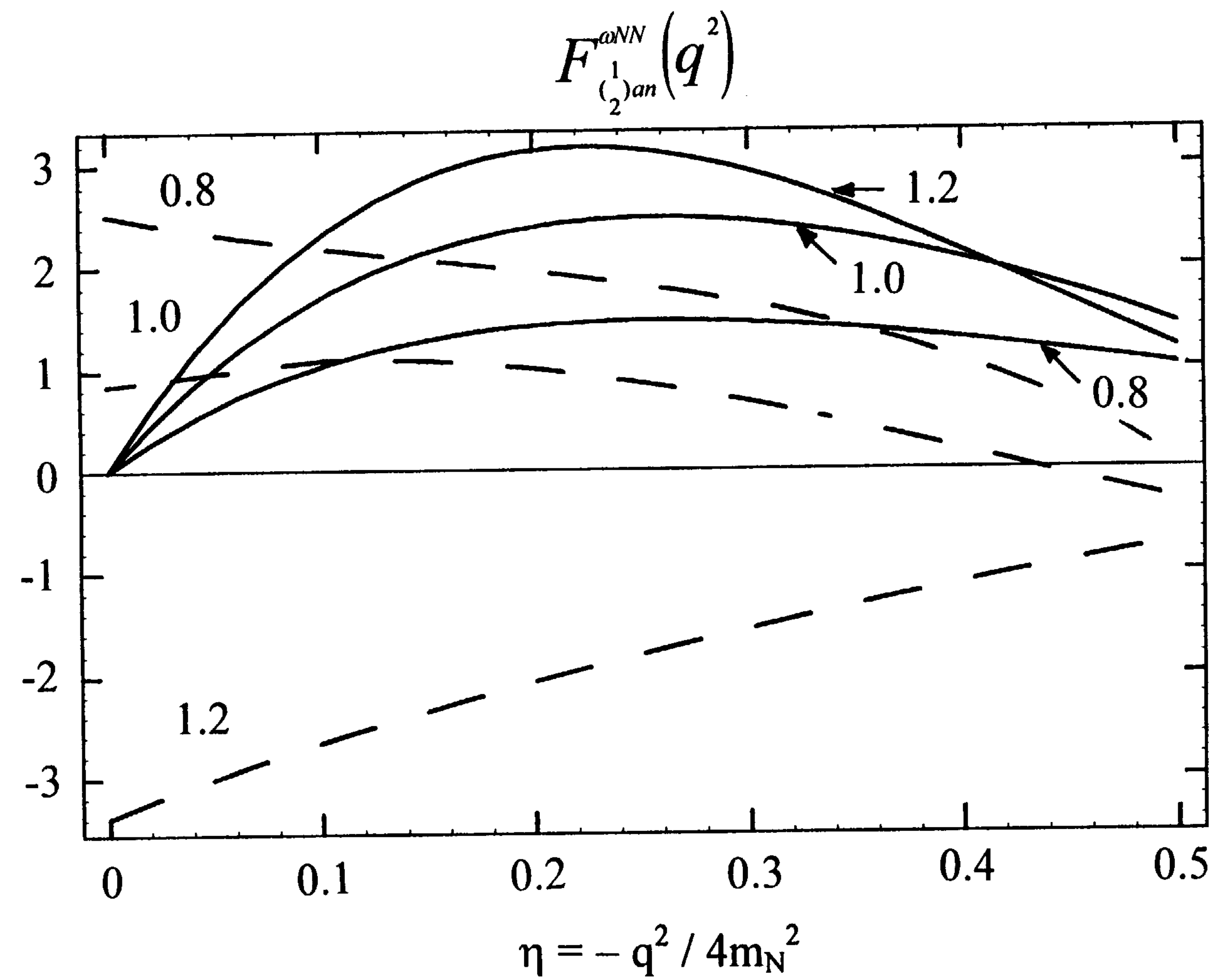


Fig. 4a



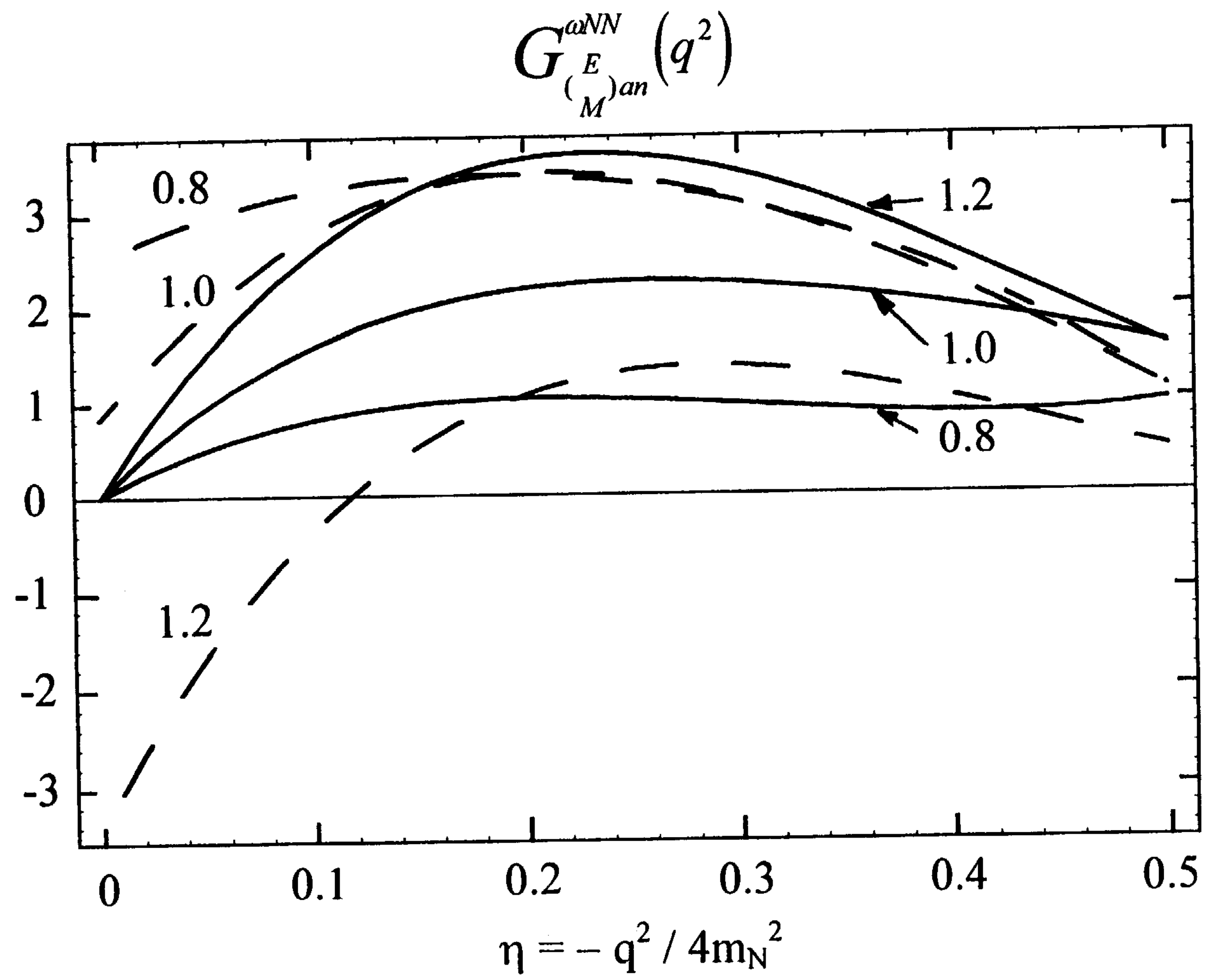
$a_2 + a_3$
Λ[GeV]
1.2
1.0
0.8
$A_2 + A_3$
0.12
0.08
0.04
0
0.1
0.2
0.3
0.4
0.5
$\eta = -q^2 / 4m_N^2$

Fig. 4b



$F^{\omega NN}_{(\frac{1}{2})an}\left(q^2\right)$
3
2
1
0
-1
-2
-3
0.8
1.0
1.2
1.2
1.0
0.8
0
0.1
0.2
0.3
0.4
0.5
$\eta = -q^2 / 4m_N^2$

Fig. 5a



$G^{\omega NN}_{(^{E}_{M})an}(q^2)$
0.8
1.0
1.2
1.2
1.0
0.8
3
2
1
0
-1
-2
-3
0
0.1
0.2
0.3
0.4
0.5
$\eta = -q^2 / 4m_N^2$

Fig. 5b

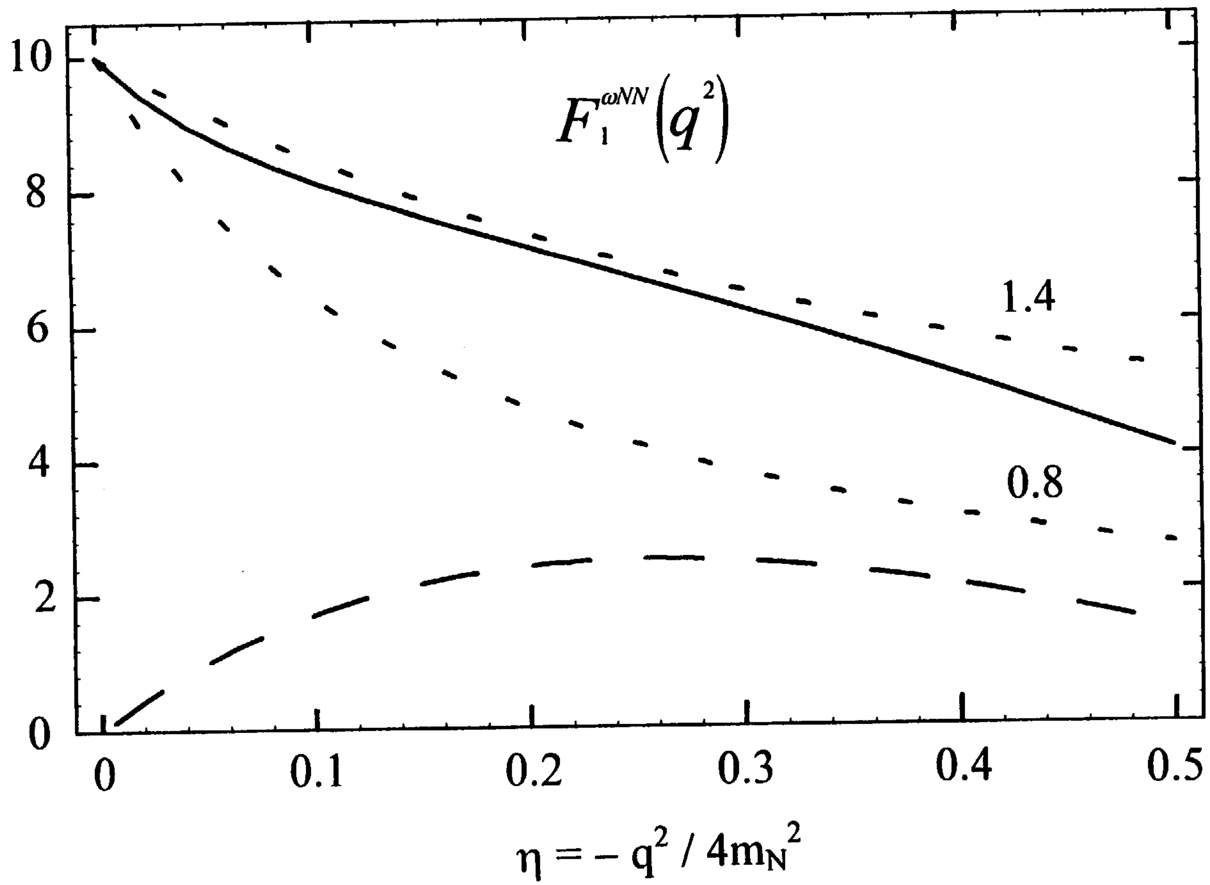


Fig. 6