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Abstract

We investigate the relation between the modified leading log approximation of the perturbative QCD and the sequential binary fragmentation process. We will show that in the absence of inactivation, this process is equivalent to the QCD gluodynamics. The inactivation term yields a precise prescription of how to include the hadronization in the QCD equations.

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The sequential, conservative and off-equilibrium fragmentation process in the fragmentation - inactivation - binary (FIB) model[1,2] is stopped by dissipative effects associated with the inactivation term. We will show that in the limit of 'no dissipation', this very general fragmentation process yields the rate equations of gluodynamics in the modified leading log approximation (MLLA)[3-5] . An extension of the FIB model to treat both quarks and gluons is straightforward and will be discussed elsewhere[6] . The inactivation term of the FIB model yields a unique prescription of how to obtain generalized rate equations of the perturbative QCD (PQCD) in the MLLA including the hadronization and, hence, how to obtain the asymptotic hadron spectrum without invoking the local parton - hadron duality picture.

In the FIB model, one deals with clusters (e.g. partons) characterized by a conservative scalar quantity, that is called the *cluster mass* . The ancestor cluster of mass N is relaxing via an ordered and irreversible sequence of steps. The first step is either a binary fragmentation, or an inactivation. Once inactive, the cluster cannot be reactivated anymore. The fragmentation leads to two clusters, with the mass partition probability $\sim F_{j,N-j}$. In the following steps, the relaxation process continues independently for each descendant cluster until either the low mass cutoff for 'monomers' is reached or all clusters are inactive. Since for any event, the fragmentation and inactivation occur with probabilities per unit of time $\sim F_{j,k-j}$ and $\sim I_k$ respectively, therefore the knowledge of the initial state and the rate-functions $F_{j,k-j}$ and I_k , specifies the fragmenting system and its evolution.

The basic equations of the FIB model, such as master and cascade equations have been given before[1,2] . Below we shall discuss only those features of the FIB model which are relevant for understanding of its relation to the PQCD rate equations. Let us call $P_N[m;t]$ the probability to get a cluster multiplicity m at time t, starting from initial cluster of size N at $t = 0$. The time evolution equation for the multiplicity is given by the following non-linear rate equations :

$$\frac{\partial P_N[m;t]}{\partial t} + \left(I_N + \sum_{j=1}^{N-1} F_{j,N-j} \right) P_N[m;t] =$$

$$= \sum_{j=1}^{N-1} F_{j,N-j} \sum_{m'=1}^{m-1} P_j[m';t] \; P_{N-j}[m-m';t] \; + I_N \delta(m-1) \quad . \tag{1}$$

In terms of the generating function :

$$Z_N(u,t) = \sum_{m=1}^{\infty} P_N[m,t](1+u)^m \quad , \tag{2}$$

one obtains :

$$\frac{\partial Z_N}{\partial t}(u,t) = \sum_{j=1}^{N-1} F_{j,N-j}[Z_j(u,t) \; Z_{N-j}(u,t) \; - \; Z_N(u,t)] +$$

$$+ I_N[1+u-Z_N(u,t)] \quad , \tag{3}$$

with the initial condition (monomer cannot break up) :

$$Z_1(u,t) = 1 + u$$

and the normalization condition :

$$Z_N(u=0,t) = 1 \quad .$$

Note that the partial derivative is taken at a fixed size N. The sum on the right hand side of eq. (3) represents binary fragmentation of the primary cluster N into the daughter clusters of mass j and $N-j$ respectively. The second term on the right hand side is responsible for the inactivation and it is in the essence the dissipative term.

In the following, we shall consider the symmetric form for the fragmentation kernel :

$$F_{j,N-j} = \frac{1}{N} \left(\frac{N^2}{j(N-j)} - 2 + \frac{j(N-j)}{N^2} \right) \quad , \tag{4}$$

which is a superposition of two regular kernels and the singular kernel : $N/(j(N-j))$. We can transform the discrete variable j in (3) into a continuous one : $z = j/N$, which varies from 0 to 1. With this change of variable, the fragmentation kernel

(4) defines a new splitting function $\Phi_{z,1-z} = NF_{j,N-j}$ which is identical to the kernel of gluodynamics[3] . The time t appearing in (3), arises within the fragmentation and inactivation kernels , which themselves are probabilities per unit of t. We define then the time as : $t = T \ln Y$, where T is a constant, $Y = \ln(N\Theta/Q_0)$, $Q_0 = const$ and Θ plays the role of time, ordering the sequence of events. Assuming now that all physical quantities depend only on the variable Y and not on N and Θ separately, we transform (3) into :

$$\frac{\partial Z}{\partial Y}(Y,u) = \int_0^1 {\gamma_0}^2(Y)\Phi_{z,1-z}[Z(Y+\log z,u)Z(Y+\log(1-z),u) -$$

$$- Z(Y,u)]dz + R(Y,u) \qquad , \tag{5}$$

where :

$$R(Y,u) = \mathcal{I}(Y)[1 + u - Z(Y,u)] \quad . \tag{6}$$

The initial and normalization conditions are :

$$Z(0,u) = 1 + u \quad , \qquad Z(Y,0) = 1 \quad ,$$

and :

$${\gamma_0}^2(Y) \equiv \frac{2N_C\pi}{\alpha_s(Y)} \quad ,$$

where $\alpha_s(Y)$ is the QCD running coupling constant. $\mathcal{I}(Y)$ in (6), is the inactivation function written in the new variables. Eq. (5) is analogous to the gluodynamics equations in the MLLA if N is the initial momentum, Θ is the angular width of the gluon jet considered and T is chosen such that $T = 12N_C/(11N_C - 2N_f)$. The gluodynamics equations correspond to neglecting the 'dissipation term' $R(Y,u)$ in (5). Notice, that the precise form of this term follows from the identification of the 'dissipation mechanism' and hence of the hadronization with the 'inactivation mechanism' of the conservative FIB process.

Coming back to the FIB-equation, one can take the derivative of eq. (3) with respect to u and taking $u = 0$, to obtain *exact* linear recurrence equations for the multiplicity average :

$$\frac{\partial < m >_N}{\partial t} = \sum_{j=1}^{N-1} F_{j,N-j}(2 < m >_j - < m >_N) + I_N(1- < m >_N)$$

$$< m >_1 \; = 1 \quad , \tag{7}$$

which are easy to solve exactly for finite values of the size N. The normalized factorial $\hat{F}_q$ and cumulant $\hat{K}_q$ moments of the $P_N[m]$ distribution, which describe respectively full and genuine q-cluster correlations, can be also easily calculated :

$$\hat{F}_q = \frac{1}{< m >^q} \frac{d^q Z(u)}{du^q}|_{u=0}$$

$$\hat{K}_q = \frac{1}{< m >^q} \frac{d^q \ln Z(u)}{du^q}|_{u=0} \tag{8}$$

For finite N and $t \to \infty$, values of $\hat{F}_q$ can be found from the *exact* FIB recurrent equations[6] :

$$\hat{F}_q(N)[(q-1)(I_N + \sum_{j=1}^{N-1} F_{j,N-j}) - \frac{q}{< m >_N}(I_N + 2\sum_{j=1}^{N-1} F_{j,N-j} < m >_j)]$$

$$= \sum_{j=1}^{N-1} F_{j,N-j} \sum_{l=0}^{q} \begin{pmatrix} q \\ l \end{pmatrix} \frac{< m >_j^l < m >_{N-j}^{q-l}}{< m >_N^q} \hat{F}_l(j)\hat{F}_{q-l}(N-j) \tag{9}$$

with $\hat{F}_0 = \hat{F}_1 = 1$, which are obtained by taking successive derivatives of eq.(3) . It is important for practical applications that one is able to solve easily exact recurrent FIB equations (9) for the moments of the generating function (up to $N \sim 10^4$), to give a natural framework for the Monte-Carlo simulations for bigger sizes up to $N \sim 10^7$, and then to find leading behaviour for asymptotically large N [8] .

To illustrate this method, we can first notice that different behaviours in the FIB model can be separated into three classes according to the feature of the inactivation probability of a k-cluster :

$$p_I(k) = 1/(1 + \sum_{j=1}^{k-1} F_{j,k-j}/I_k) \tag{10}$$

If this quantity is an increasing function of the size k then the system belongs to the so-called ∞-*cluster* phase, where there is in average one 'macroscopic' cluster and some small-size clusters. On the contrary, if p_I is decreasing, the system is in the *shattering* phase, and almost all the system breaks up into small clusters. These two phases are separated by the *transition line* characterized by the independence of the probabilities p_I and p_F $(\equiv 1 - p_I)$ on the cluster size (the scale-invariant branching process) at any stage of the process[7] . At the transition line , the system is critical[1,2] and the composed particle first moment $N_C \equiv 1- < m >_1 /N$, where $< m >_1$ is the average number of monomers, is an order parameter.

In the limit of 'no inactivation', the evolution is cut sharply at the low-mass cutoff Q_0 (the monomer) and one assumes usually that the cutoff scale is of the order of the hadron masses[9] . This is the usual procedure of solving the PQCD rate equations in the MLLA. One then relates parton and hadron spectra assuming the validity of the local parton - hadron duality picture. This *ad hoc* assumption is surprisingly successful[10] although it is conceptually unsatisfactory because the evolution equations do not yield the asymptotic $(t \rightarrow \infty)$ spectra. The inclusion of the dissipative effects $(\mathcal{I}(Y) \neq 0)$ is associated with one additional free function : $\mathcal{I}(Y)$, which has to be determined by fitting the data. In the shattering phase, for example, the equations (7) can be asymptotically (for $N \rightarrow \infty$) solved, and the result is that the anomalous multiplicity dimension :

$$\gamma = \frac{d \ln(< m >)_N}{d \ln(N)} \tag{11}$$

behaves like $\gamma_0 \sim 1/\sqrt{\ln(N)}$ if the system is stopped at a fixed time (sharp low-mass cutoff), and like a constant when the system is allowed to evolve till its final state. The dependence of γ on energy ('initial parton size') has been studied in various approximations for PQCD gluodynamics and for $\lambda\phi_6^3$ theory $(F_{j,N-j} = 6j(N-j))$ and many results are now available[3–5,10] . These results are obviously identical to the fixed time limiting results of the FIB model. For $\mathcal{I}(Y) \neq 0$, various classes of the multiplicity distributions have been found for multiplicative fragmentation kernels : $F_{j,N-j} = (j(N-j))^{\alpha}$ at the transition line[8] . For $p_I > 1/2$ and for any value of the homogeneity index α, the cluster multiplicity is asymptotically a constant independent of N. This is the 'Cayley domain' of the multiplicity distribution, where the FIB process is analogous to the invasion percolation on the Cayley tree[11] . For $p_I < 1/2$ and $\alpha > -1$, the cluster multiplicity is : $< m >_N \sim aN^{\tau-1}$ $(1 \leq \tau \leq 2)$ and $\gamma(N) = \tau - 1$. This is the 'Brand-Schentzle (BS) domain', where the multiplicity distribution is a special solution of the nonlinear stochastic equation with multiplicative fluctuations[12] . The solution of this equation includes many analytic functions used to describe multiplicity distributions in pp and e^+e^- collisions[13] . For $p_I < 1/2$ and $\alpha < -1$ (the 'evaporative domain') , the cluster multiplicity is approximately N - independent. In the transition region between the BS and evaporative domains ($\alpha = -1,\ 0 < p_I < 1/2$), one finds[8] :

$$< m >_N \sim (\ln N)^{\frac{1-2p_I}{p_I}} \tag{12}$$

and, hence, $\gamma(N) \sim (\ln N)^{-1}$. In gluodynamics[3] , the vector nature of massless gluons leads to the kernel which is a superposition of regular multiplicative kernels with $\alpha = 0, 1$ in the BS domain and the singular kernel with $\alpha = -1$ in the transition region $\alpha = -1,\ 0 < p_I < 1/2$. It turns out however, that the multiplicity distribution in this case is dominated by the singular kernel $\alpha = -1$, and the leading term of the inactivation function is $I_k \sim \ln k$. Details of the multiplicity distributions can be found in Ref. 8 . Let us only remind that the multiplicity probability distribution in the BS domain and that in the transition region $\alpha = -1,\ 0 < p_I < 1/2$ satisfy the KNO scaling[14] *exactly.* The KNO scaling is a special property of the critical fragmentation process and is absent in both ∞-cluster and shattering phases[6] .

Present experimental data for e^+e^- reactions shows that (i) the KNO scaling is approximately satisfied and (ii) the mean multiplicity :

$$< m >_N \sim a_0 + a_1 \ln N + a_2 \ln^2 N \quad . \tag{13}$$

The first observation (i) allows to locate the fragmentation domain close to the transitional region $\alpha = -1,\ 0 < p_I < 1/2$ between BS and evaporative domains. To fit the experimental dependence of the mean multiplicity on energy[15] for e^+e^- reactions at $\sqrt{s} \leq 100 GeV$, the inactivation function should be :

$$I_k = c\,(\ln k)\exp[-d\left(\frac{k}{N}\right)^2] \quad . \tag{14}$$

The two parameters : $c = 3/2$ and $d = 10$ have been used as free parameters to fit the experimental KNO-scaling function[15] (see Fig. 2). For large clusters, $p_I(k)$ (eq. (10)) corresponding to (14) decreases fast with increase of the size k , i.e. at the beginning of the process, the fragmenting system is found in the shattering phase. With decreasing cluster size, the fragmenting system approaches the region $\alpha = -1$, $0 < p_I < 1/2$ of the transition line and the inactivation probability becomes close to : $p_I(k) \rightarrow 1/(1 + 2/c) = 3/7$. The influence of big clusters and hence of the evolution in the shattering phase on the cluster multiplicity distribution becomes negligible for very large initial cluster size N . Hence, small deviations from the KNO scaling seen in the data for $\sqrt{s} \leq 100 GeV$ [15] should be interpreted as the transition phenomenon related to the small size of the initial gluon jet. Also the dependence of $< m >_N$ on $\ln N$, as seen in the data[15] and in the Fig. 1, is a pre-asymptotic feature which is going to be replaced by (12), with $p_I = 3/7$, when $N \rightarrow \infty$. Experimental finding of this transition to the regime of the critical fragmentation at which the asymptotic ($N \rightarrow \infty$) conditions dominate, would allow to fix the inactivation probability and therefore one could learn about the details of the hadronization phase.

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Figure captions

Fig. 1

Shapes of the multiplicity distributions in KNO variables adjusted to fit the e^+e^- data[15].

Fig. 2

Dependence of the mean multiplicity on energy fitting experimental data in e^+e^- reactions[15].

Fig. 1

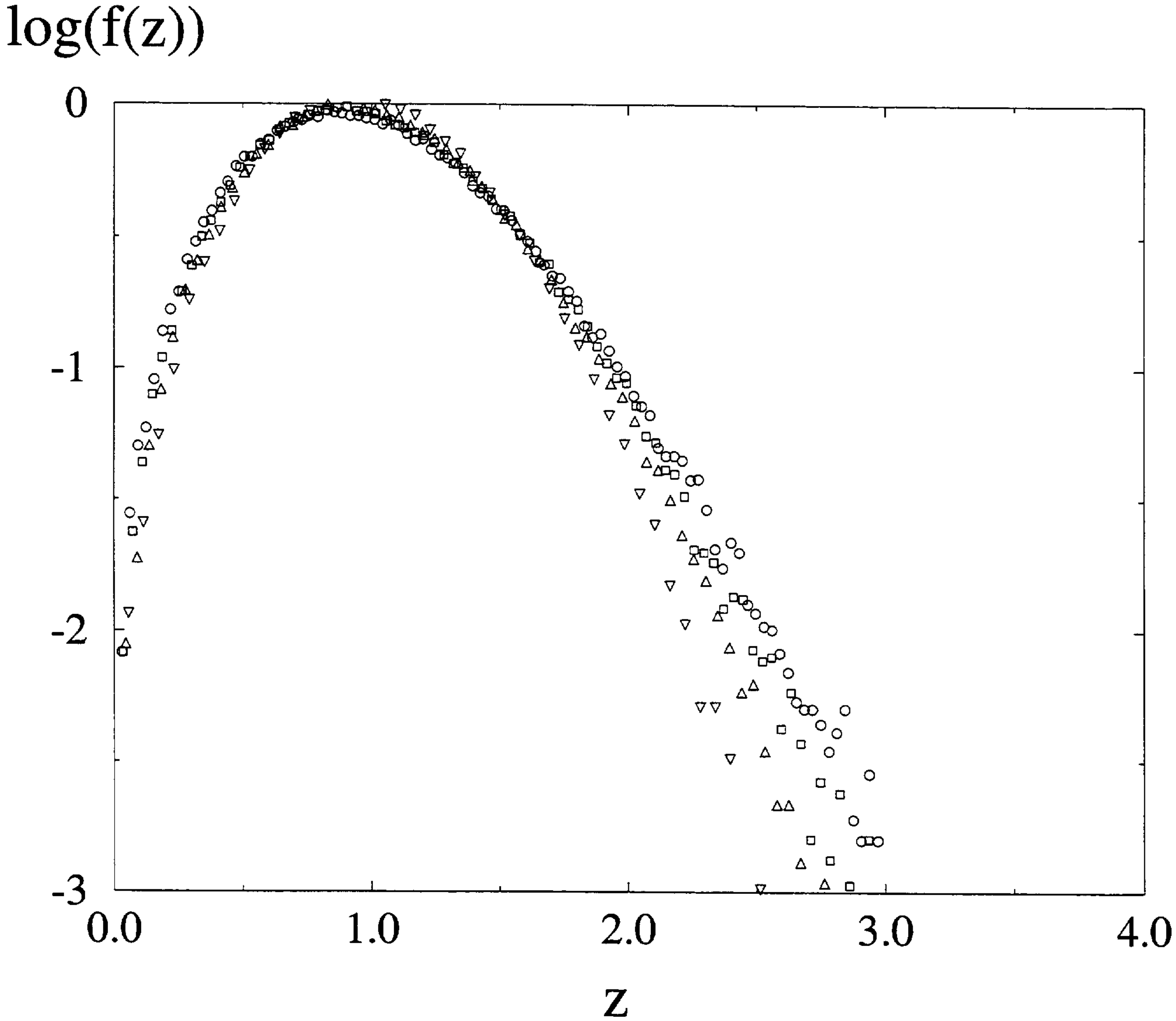

Fig. 2