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Resonance decays,correlations and intermittency in hadronic collisions

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Abstract

We study a very simple model of correlations and intermittency of identical final state pions in hadronic collisions.Final state pions are either products of resonance decays or they are "directly" produced. The "direct" production is simulated by an immediate decay of a resonance. For "direct pions" forming about a half of final state pions and for formation times of resonances less than 0.5fm/c we get density of sources which via Hanbury- Brown and Twiss interference leads to correlations of two identical pions consistent with recent data and shows intermittency patterns for the second factorial moment. The essential ingredient of the scheme is the combination of pions from resonance decays and direct pions. Due to life- times of resonances taken from experiment,pions from resonance decays are responsible for short- range correlations in the longitudinal momentum, whereas directly produced pions, due to their fast production, dominate in the region of longitudinal momentum difference of the order of 100 MeV/c. The combination of both can give an approximate scaling leading to intermittency.

1 Introduction

Analysis of multiparticle production in hadronic collisions in terms of intermittency has been suggested by Bialas and Peschanski [1] about a decade ago. In order to study strong correlations between final state hadrons leading to "spikiness", Bialas and Peschanski proposed to study the dependence of scaled factorial moments on the size of the rapidity bin. To introduce the notation consider rapidity interval $-Y/2 < y < Y/2$ of length Y and divide it into M bins, each bin of length $\delta = Y/M$. Denote by $\rho_1(y_1)$ and $\rho_2(y_1, y_2)$ single and double rapidity distributions

$$\rho_1 = \frac{1}{\sigma_{inel}}\frac{d\sigma}{dy_1}, \qquad \rho_2(y_1, y_2) = \frac{1}{\sigma_{inel}}\frac{d^2\sigma}{dy_1 dy_2} \tag{1}$$

normalized as follows

$$\int \rho_1(y_1) = < n >, \quad \int \rho_2(y_1, y_2) = < n(n-1) > \tag{2}$$

The scaled factorial moment $F_q(\delta)$ corresponding to a given division of the rapidity interval is defined as

$$F_q(\delta) = \frac{< n(n-1)...(n-q-1) >}{< n >^q} \tag{3}$$

where $\delta = Y/M$, $q \geq 2$ and $< ... >$ denotes averaging over bins and events.

Multiparticle distribution is called intermittent if scaled factorial moments obey the power law behaviour for $\delta \to 0$, Δ fixed

$$F_q(\delta) = \left(\frac{\Delta}{\delta}\right)^{f_q} F_q(\Delta) \tag{4}$$

where $f_q > 0$ is called the intermittency exponent. Condition (4) corresponds to a pure scaling behaviour. The analysis of 3- dimensional data by Fialkowski [2] indicates that scaling is not exact and that the second factorial moment behaves rather as

$$F_2(\delta) = 1 + C_L + C_S\left(\frac{Y}{\delta}\right)^{f_2} \tag{5}$$

with C_L being smaller than 1, C_S of the order of 10^{-2} and depending on reaction studied, and f_2 within the range 0.3 - 0.6: all results being within one standard deviation from the average value of $f_2 \approx 0.44$.

Intermittency of multiparticle production has been studied both from experimental and phenomenological points of view, details can be found in review articles [3], the present state of data being summarized by Buschbeck [4] in an excellent review. In early attempts at explaining intermittency cascades with self- similar features have been studied. Later on data [5,6] (see also [3,4]) have shown that short- range correlations between identical particles are significantly stronger than those between non- identical ones. This indicates that a large part of the intermittent behaviour is due to the Hanbury- Brown and Twiss (HBT) correlations [7-11], for reviews on HBT see [12-14].

Scaling behaviour of $F_2(\delta)$ as given by Eq.(4) implies also the scaling behaviour of the correlation function $C_2(y_1, y_2)$ defined as

$$\rho_2(y_1, y_2) = \rho_1(y_1)\rho_1(y_2) + C_2(y_1, y_2) \tag{6}$$

The second factorial moment $F_2(\delta)$ can be expressed as

$$F_2(\delta) = \frac{< n(n-1) >}{< n >^2} = \frac{\int_{-\delta/2}^{\delta/2}\int_{-\delta/2}^{\delta/2} \rho_2(y_1, y_2) dy_1 dy_2}{\left[\int_{-\delta/2}^{\delta/2} \rho_1(y_1) dy_1\right]^2} \tag{7}$$

The term $\rho_1(y_1)\rho_1(y_2)$ when inserted into the numerator of Eq.(7) cannot lead to the behaviour like δ^{-f_2} so this has to be due to $C_2(y_1, y_2)$. For

$$C_2(y_1, y_2) \approx |y_1 - y_2|^{-f_2} \tag{8}$$

we get from Eq.(7) the behaviour of $F_2(\delta)$ required by Eq.(4). For small values of $|y_1 - y_2|$ the intermittent behaviour can be analyzed as well as a function of longitudinal momentum k_L at fixed transverse momentum k_T. Since $y = ln[(E + k_L)/m_T]$, where $m_T = (m^2 + k_T^2)^{1/2}$ we have $dy/dk_L = 1/E$ and the scaling behaviour in Eq.(8) is equivalent to

$$C_2(k_{1L}, k_{2L}) \approx |k_{1L} - k_{2L}|^{-f_2} \tag{9}$$

Most of data on intermittency are however presented as a function of δ related to $y_1 - y_2$. Note also that the behaviour given by Eq.(8) cannot be valid up to $y_1 - y_2 \rightarrow 0$ since $C_2(y_1, y_1) = 1$.

In HBT studies of interferometry of identical particles [12,13,14] the correlation function $C_2(k_1, k_2)$ is expressed as a square of Fourier Transform (FT) of the density of sources

$$C_2(q, K) \equiv C_2(k_1, k_2) = \left| \int \rho(x, K) e^{iq.x} d^4x \right|^2 \tag{10}$$

where four-vectors q, K are defined as

$$q = k_1 - k_2, \qquad K = \frac{k_1 + k_2}{2} \tag{11}$$

and the density of sources (the Wigner distribution) is normalized to 1 by $\int \rho(x, K) d^4x = 1$.

The exact scaling in Eq.(9) for the correlation function $C_2(q)$ can be obtained via Eq.(10), provided that the scaling is built into the density $\rho(x, K)$.To see that it is easiest to make the problem one- dimensional by putting $\vec{q}_T = 0$, $q_0 = 0$. Eq.(10) then reduces to

$$C_2(q) = \left| \int \rho(z, K) e^{iqz} dz \right|^2 \tag{12}$$

Putting now $\rho(z) \equiv \rho(z, K) \approx (z^2)^{-\alpha}$, $\alpha < 1/2$ we get $C_2(q) \approx (q^2)^{2\alpha-1}$. The exact scaling is however not what is seen in the data - and because of normalization $C_2(q = 0) = 1$ it is also impossible. The upper limit on $q = k_1 - k_2$ is given by kinematics and by the region where the assumptions are valid on which the expression in Eq.(10) is based. For this point see e.g. discussion in the introduction to Ref.[15]. The lower limit on q is given by the experimental resolution in measuring momenta of final state particles and by the accuracy with which final state interactions, including the Gamow factor, are known. An estimate of this lower limit is about 20MeV/c.

What is seen in experimental data when studying intermittency or the correlation function $C_2(q)$ is an approximate scaling valid in the region of $|k_{1L} - k_{2L}|$ extending from about 20 MeV/c to a few hundred MeV/c, depending on the kinematics and on the value of K. Such an approximate scaling has been discussed in Refs.[9,16,17]. Bialas [9] has obtained an approximate scaling with the density $\rho(r)$ scaling exactly for $0 < r < L$ and vanishing for $r > L$; in Refs.[16,17] it has been shown that an approximate scaling is obtained in the simple case of bremsstrahlung photons emitted by charged particles which are created in space- time points corresponding to a picture of an inside- outside cascade.

The purpose of the present paper is to point out that an approximate scaling is obtained also from a simple picture of the space- time evolution of hadronic collisions. In this picture about a half of final state pions appear as products of resonance decays. A resonance has a formation time τ_f and a mean life-time τ_d in its rest frame and both these times are Lorentz dilated. Depending on its rapidity, resonance travels some distance before decaying. Two identical pions originated by decays of two different resonances may have close momenta and be produced from two distant sources. This leads via HBT interferometry to an increase of $C_2(q, K)$ for small values of q. We shall show that a superposition of resonances and of directly produced pions gives the two body correlation function $C_2(q)$, which is consistent with data and leads to the second factorial moment $F_2(\delta)$ which shows intermittency patterns.

The model we are studying is admittedly oversimplified, the most drastic assumptions consisting in putting transverse momenta of resonances equal to zero.These simplifications permits us to do most of calculations by hand and keep the discussion as transparent as possible. In our opinion such an approach permits to get an insight into the problem and in this aspect it is complementary to less transparent Monte Carlo computations.

The paper is organized as follows: In the next Sect. we describe our simplified model. Sect.3 deals with the correlation function $C_2(q)$ of two identical pions originated either by resonance decay or produced directly. The calculated $C_2(q)$ is compared with data and lessons following from this comparison are discussed. In Sect.4 we discuss the transverse momentum dependence of two- pion correlation function. In Sect.5 we present the resulting intermittency patterns. Comments and conclusions are contained in Sect.6. Some technicalities are deferred to the Appendix.

2 A simple model of correlations of identical pions originated either by resonance decays or directly produced

A large amount of models of hadronization in e^+e^-, ep and hadron- hadron collisions has been proposed, some of them can be traced back from Refs. [18-21]. In most of these models an intermediate partonic stage is followed by cluster formation and decay. It is not clear whether there are some intermediate "heavy clusters" which decay after some time to known hadronic resonances. Since we wish to have the model as simple as possible we shall not discuss such intermediate stages and we shall only assume that well known hadronic resonances are formed after a common formation time τ_f and after being formed they decay according to schemes known from experiment. The value of the formation time τ_f will be a considered as a free parameter of our model. Studies of resonance production in pp collisions have shown that about a half of final state pions comes from decays of well known hadronic resonances, although there exist also estimates that this fraction is larger. Final state pions which cannot be ascribed to decays of known resonances are reffered to as being "directly" produced. It is possible that a part of these pions is due to decays of rather broad resonances.In our simplified model we describe "directly" produced pions as decay products of a resonance with vanishing life- time. Direct pions are thus produced rather early and not far from the point of the hadronic collision. The influence of resonance production on spectra of their decay products has been studied in detail [23] and literature on the effects of resonance decays on HBT interferometry can be traced back from Ref.[24].

Our aim in this Section is not to construct a realistic model of the effects of resonance formation and decay on correlations of identical pions in hadronic reactions. Such a model would necessarily include a complicated and not very transparent Monte Carlo computations.What we shall present here is a very simplified and transparent model. In this model we assume that in a hadronic collision:

i) Resonances are formed in a time τ_f after the collision. The value of τ_f is a free parameter of our model.

ii) After being formed a resonance decays with the mean life- time τ_d, taken from experiment.Both τ_f and τ_d are Lorentz dilated by $\gamma = (1-v^2)^{-1/2}$

where v is the velocity of the resonance.

iii) Transverse momentum of resonances vanishes, their velocities have only components along the axis of collision (z-axis). This assumption makes the model somewhat unrealistic, but simplifies substantially calculations and makes the model rather transparent.

iv) A part of pions is produced "directly". The direct production is described as a decay of a resonance with a vanishing mean life- time.

v) We shall work in the cms of hadronic collision and consider only simple kinematical situations in which the momentum $\vec{K} = (\vec{k}_1 + \vec{k}_2)/2$ is small and perpendicular to the axis of the collision (z-axis) and the momentum $\vec{q} = \vec{k}_1 - \vec{k}_2$ is parallel to the z-axis. An example of such a kinematics is shown in Fig.1.This corresponds to $y_{c.m.} \approx 0$ and K_T small.

We shall now study the behaviour of the correlation function $C_2(q, K)$ of two identical pions caused by resonance decays. The two interfering amplitudes are shown in Fig.2. We assume that the two pions have - in the simple situation considered - the same energy, therefore $q_0 = k_{10} - k_{20} = 0$. We shall start with calculating function $\rho(z, K)$ for a particular resonance, then we shall sum over resonance contributions and take the Fourier Transform as shown in Eq.(10).

Width Γ of a resonance of mass M, decaying to two particles of mass m is given in the resonance rest frame as

$$\Gamma = \int |T|^2 \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \delta(\vec{p}_1 + \vec{p}_2)\delta(M - E_1 - E_2) \qquad (13)$$

where the standard and self- explanatory notation has been used. Making use of $E_1 = E_2 = M_T ch(y)$ we can rewrite Eq.(13) for the decay to two equal mass particles as

$$\Gamma = \int |T|^2 \frac{d\phi p_T dp_T dy}{(2\pi)^6 2M\sqrt{M^2 - 4m_T^2}} [\delta(y - y_1) + \delta(y - y_2)] \qquad (14)$$

where $m_T^2 = m^2 + k_T^2$ and

$$y_{1,2} = ln\left((M/2m_T) \pm \sqrt{(M/2m_T)^2 - 1}\right); \quad y_1 = -y_2 \qquad (15)$$

Boosting the resonance to rapidity y_R and normalizing the decay probability to 1, with $|T|^2$ held constant we get

$$\frac{dP}{p_T dp_T d\phi dy} = \frac{1}{\pi}\frac{1}{\sqrt{M^2 - 4m_T^2}}[\delta(y - y_R - y_1) + \delta(y - y_R + y_1)] \tag{16}$$

This probability distribution is normalized as

$$\int \frac{dP}{p_T dp_T d\phi dy} p_T dp_T d\phi dy = 1 \tag{17}$$

Note that in order to keep the calculations simple we are using here and in what follows a "zero width approximation" for distribution of resonance masses.

Eqs.(15) and (16) show that resonance products are shifted in rapidity by $\Delta y = \pm y_1$ with respect to the rapidity of the resonance. The value of this shift may be rather large. For instance for decay of the ρ- meson to two pions with $p_T \approx 0$ we get $\Delta y \equiv y_1 \approx 1.5$. A pion with $y \approx 0$ and $p_T \approx 0$ is thus produced by a ρ with $y_R \approx \pm 1.5$. Such a ρ moves with velocity $v \approx tanh(y_1)$ in the rest frame of the pion.Note that for larger values of p_T of the pion the rapidity difference between the pion and the ρ becomes smaller and for $p_T^2 + m^2 = m_\rho^2/4$ the rapidity difference vanishes.

A ρ with rapidity y_1 needs some time for its formation and some time for its decay. Pion with $y \approx 0$ and $p_T \approx 0$ is thus emitted some distance away from the origin. Two identical pions, both with small y and p_T and originated in decays of two different ρ's come thus from two distant sources as shown in Fig.2.

For resonance decays to two unequal mass particles $M \rightarrow m_1 + m_2$ Eqs.(14)-(17) are somewhat modified.Calculation is sketched in the Appendix, the final result being

$$\frac{dP}{d\phi p_T dp_T dy} = \frac{1}{4\pi\sqrt{k^2 - p_T^2}}[\delta(y - y_R + y_1) + \delta(y - y_R - y_1)] \tag{18}$$

where

$$k \equiv (p_T)_{max} = \frac{1}{2M}[M^2 - (m_1 + m_2)^2]^{1/2}[M^2 - (m_1 - m_2)^2]^{1/2} \tag{19}$$

and, see Eqs.(A.4), (A.5)

$$y_1 = ln(\alpha + \sqrt{\alpha^2 - 1}), \qquad \alpha = \frac{M^2 - (m_2^2 - m_1^2)}{2m_1 M} \tag{20}$$

This equation is valid for vanishing transverse momenta of decay products. For $p_T \neq 0$ masses m_1 and m_2 in Eq.(20) should be replaced by the corresponding transverse masses.as can be seen from the derivation of Eq.(20) in the Appendix.

Expressing the four-vector K in Eq.(11) in terms of y, p_T, ϕ the function $\rho(z, t; K)$ in Eq.(10) can be written as follows

$$\rho(z,t;y,p_T,\phi) = \sum_R \int P(z,t;y_R).\frac{dn_R}{dy_R}.\frac{dP}{p_T dp_T d\phi dy} dy_R \qquad (21)$$

Here dn_R/dy_R is the rapidity density of the resonance R. $dP/p_T dp_T d\phi dy$ is given by Eq.(16) and $P(z,t;y_R)$ is the probability density that resonance R with rapidity y_R decays in the space-time point (z,t).Since we have assumed that resonances move along the z-axis, coordinates x,y of the position of resonance decay vanish.It follows from Eqs.(10) and (21) that the correlation $C_2(q,K)$ is essentially given by the probability distribution $P(z,t;y_R)$. For the case of $y = 0$ which we consider here, the function $P(z,t;y_R)$ is symmetric with respect to $z \rightarrow -z$ and we shall calculate it only for $z \geq 0$. In this case out of two δ- functions in Eq.(16) only the one with $y_R = y_1$ contributes.

The function $P(z,t;y_R)$ is given by the space-time features of formation and decay of resonance R. There are many models of formation of final state hadrons in hadronic collisions. To keep our model as simple as possible we shall select a particularly simple version. We assume that a resonance is formed in its rest frame in time τ_f and in this frame the probability of resonance being already formed at time τ is

$$P_f(\tau) = 1 - exp(-\tau/\tau_f) \qquad (22)$$

In the frame in which resonance R has rapidity y_R its velocity is $v(y_R) = tanh(y_R)$, the formation time is dilated to $t = cosh(y_R)\tau_f$ and the distance travelled by R is $z = v(y_R)t = sinh(y_R)\tau_f$.Probability that resonance R is already formed at the distance z from the origin becomes

$$P_f(z) = 1 - exp(-z/z_f), \qquad z_f = sinh(y_R)\tau_f \qquad (23)$$

The resonance is formed within the interval$(z, z+dz)$ with probability density

$$\rho_f(z) = \frac{dP_f(z)}{dz} = \frac{1}{z_f}e^{-z/z_f} \qquad (24)$$

Assuming a standard exponential decay law, the probability density for decay in the interval $(z, z+dz)$ of resonance produced in z_1 is

$$\rho_d(z) = \frac{1}{z_d} exp[-(z - z_1)/z_d]; \quad z_d = v(y_R)t_d = sinh(y_R)\tau_d$$

where τ_d is the decay time in the rest frame of the resonance. Probability density $P(z, t; y_R)$ in Eq.(21) is then given as (t suppressed)

$$P(z; y_R) = \int_0^z \rho_f(z_1)\rho_d(z - z_1)dz_1 = \frac{1}{z_f - z_d}[e^{-z/z_f} - e^{-z/z_d}] \qquad (25)$$

where

$$z_f = sinh(y_R)\tau_f, \qquad z_d = sinh(y_R)\tau_d$$

It is easy to see that $P(z; y_R)$ satisfies the consistency criteria: (i) Integral from 0 to ∞ of $P(z; y_R)$ is equal to 1,(ii) for $z_f \to 0$ particles are formed immediately and $P(z; y_R)$ approaches $(1/z_d)exp(-z/z_d)$ as expected, (iii) for $z_d \to 0$ particles decay immediately and $P(z; y_R)$ approaches $(1/z_f)exp(-z/z_f)$ as it should.

Function $P(z; y_R)$ for negative z is given as $P(z; y_R) = P(-z; y_R)$. Shapes of $P(z) \equiv P(z; y_R)$ for a few values of z_f, z_d are shown in Fig.3. When summing over contributions of different resonances we shall obtain weighted sums of curves like those in Fig.3. Scaling of correlation function depends on whether such sum of contributions shows an approximately scaling behaviour. A few comments on properties of functions in Eq.(25) are given in the Appendix.

According to Eq.(10) the correlation function is expressed in terms of the Fourier transform of $\rho(z; K)$. As seen from Eq.(21) the z-dependence is given only by $P(z; y_R)$.Note that we consider two pions of equal energy but different longitudinal momenta.In such a situation the time of resonance decay does not enter the results. We shall therefore need the Fourier transform (FT in what follows) $\tilde{P}(q; y_R)$ defined as follows

$$\tilde{P}(q; y_R) = \int_{-\infty}^{\infty} dz e^{iqz} P(z; y_R) \qquad (26)$$

Inserting Eq.(25) into Eq.(26) we get, see Appendix,

$$\tilde{P}(q; y_R) = \frac{1 - z_f z_d q^2}{[1 + (z_f q)^2][1 + (z_d q)^2]} \qquad (27)$$

where $\tilde{P}(q; y_R)$ is normalized by $\tilde{P}(q = 0; y_R) = 1$. The final expression is obtained by Eqs.(10),(21) and (27), inserting branching ratio BR(R) for the decay of resonance R to a pion of given type:

$$|P(q)|^2 \equiv C_2(q, K) = \left| \frac{\sum_R \tilde{P}(q; y_R) w_R(K)}{\sum_R w_R(K)} \right|^2 \tag{28}$$

where $\tilde{P}(q; y_R)$ is given by Eq.(27),$w_R(K)$ is obtained via Eqs.(16) and (21)

$$w_R(K) = \tilde{f}_R(K) . \frac{dn_R}{dy} . BR(R) \tag{29}$$

with y_R given by Eq.(15) for a decay to two pions. Finally $\tilde{f}_R(K)$ comes from Eq.(16) after having normalized $\tilde{f}_R(K) = C(M^2 - 4m_T^2)^{-1/2}$ by the condition

$$\int \tilde{f}_R(K) d\phi p_T dp_T = 1$$

In this way we find

$$\tilde{f}_R(K) = \frac{2}{\pi} \frac{1}{\sqrt{M^2 - 4m^2}} \frac{1}{\sqrt{M^2 - 4m_T^2}} \tag{30}$$

for the equal mass case.

For the unequal mass case (see Appendix) we find in the same way

$$\tilde{f}_R(K) = \frac{1}{2\pi} \frac{1}{k} \frac{1}{\sqrt{k^2 - p_T^2}} \tag{31}$$

where k is given by Eq.(19).Functions $\tilde{f}_R(K)$ are proportional to the probability that a resonance decay leads to a pion with 4-momentum $K = (k_1 + k_2)/2$, see Eq.(11).

Formation time of resonances corresponds to a process in which resonances are - in the statistical average- produced along the boost invariant curve given by

$$\tau_f^2 = t^2 - z^2 \tag{32}$$

In a more realistic model one might think about resonances produced by freeze- out of a thermalized system. The time τ_f in our model mimics the proper time of the freeze- out. but our model does not contain the thermal distribution of resonance momenta within the system at the freeze- out.

Contribution of directly produced pions

In an inside- outside cascade model with hydrodynamical evolution and and with thermalized matter decoupling at (t,z) given by Eq.(32) it is easy to treat directly produced pions and resonances on an equal basis. Both are produced according to Bose- Einstein,or in some approximation, Boltzmann distribution, and after decays of resonances one can calculate the correlation function $C_2(q, K)$.

On the other hand it is not clear whether the hydrodynamical concepts are applicable to a hadronic collision. In our simple model we shall treat direct pions and their contribution to the correlation function in the same way as that of resonances. taking direct pions as products of decay of a resonance with a vanishing life- time. Such a treatment may provide at least some feeling of what may be the effects of directly produced pions.

The life- time of a resonance $\tau_d \approx \hbar/\Gamma$ is approaching zero when Γ is increasing. A resonance with a large width thus corresponds to vanishing τ_d and z_d in Eq.(25). Formation time is taken as equal to that of all other resonances. Taking a large width amounts to integrating over massses of the resonance with a Breit- Wigner distribution. For simplicity we shall take here only a single value of mass. An object with a large width is similar to the behaviour of the S-wave. isospin zero phase shift in $\pi\pi$ scattering. In such a situation we need to add one line to Table 1. Taking the mass of the l=0,I=0 $\pi\pi$ resonance as equal to that of the ρ-meson we get the following parameters

$$y_R = 1.67, \qquad sinh(y_R) = 2.56, \qquad \tau_d = 0, \qquad z_d = 0,$$

$$z_f = 2.56\tau_f, \qquad \hat{f}_R(K_T = 0) = 0.618, \qquad BR(\sigma) = 1$$

Rapidity density of "σ" production is chosen in such a way as to obtain the desired fraction r_{dir} of direct pions.That means that $w_\sigma(K)$ entering Eq.(28) is determined by the condition

$$r_{dir} = \frac{direct\ pions}{all\ pions} = \frac{w_\sigma(K)}{w_\sigma(K) + \sum_R w_R(K)}$$

where the sum over R includes other resonances.The calculation then proceeds as above according to Eq.(28).

3 Correlations of two identical pions

In this Section we shall calculate the correlation function $C_2(q,K)$ for identical pions in our model and compare the results with the data.The calculation contains two free parameters: the formation time τ_f and the ratio r_{dir} of directly produced pions to all pions in the final state.

Calculation of the correlation function proceeds via Eq.(28) where $\tilde{P}(q;y_R)$ is given by Eq.(27) and $w_R(K)$ by Eqs.(29) and (30) or (31) depending on whether resonance decays to two pions or to a pion and another particle.

In π^+p interactions at 16 GeV [25] the authors have identified meson resonances η,ω,ρ^0 and f_2. Relative contributions of different resonances were found to be strongly p_T-dependent: pions from η- and ω-decays populating mostly the low p_T region, those from ρ and f_2 decays dominating at higher p_T. In the low p_T region it seems that

$$\rho^0 : \omega : \eta : f_2 \approx 0.2 : 0.2 : 0.05 : 0.03$$

as ratios of fractions of the total π^- yield.

In pp interactions at 400 GeV/c [26] about a half of pions is estimated to be produced directly (see Table 9 of Ref.[26]). Resonances, most important for pion production in the region $x_F \geq 0.1$ have inclusive cross- sections of the following non- normalized ratios (see Table 6 of Ref.[26]):

$$< \rho >: \omega : f_2 :< K^* >: \Phi \approx 14 : 13 : 3 : 3.5 : 0.6 \qquad (33)$$

where $< \rho >$ denotes averaging over three charged states and $< K^* >$ over four of them.

In pp collisions [27] at CERN- ISR with $\sqrt{s} = 52.5 GeV$, inclusive production of some of vector and tensor mesons has been measured. Results are consistent with extrapolations of data from lower energies and the fraction of pions and kaons due to decays of resonances has been estimated to be larger than 0.55. Refs.[25-27] contain rather complete lists of papers in which resonance production in hadronic collisions has been studied. Patterns of data in different experiments are qualitatively similar and roughly consistent with expectations based on quark- recombination models [28.29] or Lund Fritiof model [30].

We shall now proceed to calculations of the correlation function $C_2(q,K)$ as given by Eqs.(10,11,21,27) and (28). We would like to stress that it is not our aim to get accurate quantitative results. This is hardly possible at least for two reasons: first- our model is rather simplified and second- knowledge of resonance production in hadronic collisions is not complete. We would rather like to gain a qualitative insight into the question of whether a sum of resonance decay contributions and of direct pions can give rise to an approximatively scaling behavior roughly consistent with results of studies of intermittency [2,3]. We would also like to see how the approximate scaling patterns depend on the value of the resonance formation time τ_f and on the ratio r_{dir} of direct to all pions. To start with we have to fix some parameters entering the calculations. We shall take the 4-vector K in Eq.(11) as corresponding to $p_T \approx 0$ and $y \approx 0$ in the c.m.s. of hadronic collision.Rapidity y_R of a resonance of mass M decaying to two pions is then given by Eqs.(15) or (20). where transverse mass reduces to the pion mass. We shall treat three- body decays $\omega \rightarrow 3\pi$ and $\eta \rightarrow 3\pi$ as two- body decays $\omega \rightarrow \pi d$ and $\eta \rightarrow \pi d$ with "d" denoting a "dipion". The mass m_d in the ω-decay is taken as $m_d = m_d(\omega) = 470 MeV$ and $m_d(\eta) = 350 MeV$ what corresponds to symmetric decay kinematics. In this case rapidity of a resonance decaying to a pion with $y = 0$ and small p_T is given by Eqs.(20),and Eqs.(4) and (5) in the Appendix.

$$y_R = ln(\alpha + \sqrt{\alpha^2 - 1}), \qquad \alpha = \frac{M^2 - (m_d^2 - m^2)}{2mM} \tag{34}$$

This expression is valid also for the decay $K^* \rightarrow K\pi$. All parameters entering our calculation of $C_2(q, K)$ via Eq.(28) are given in Table 1, which contains in the last row also parameters concerning directly produced pions. We shall briefly recapitulate symbols in Tab.1 and relations defining them:y_R is the rapidity shift between a resonance and its decay product, see Eqs.(15) and (20) for equal resp. unequal mass cases,$\tau_d = 1/\Gamma$ where Γ is the resonance width, $z_d = sinh(y_R)\tau_d$ is the mean decay distance; $z_f = sinh(y_R)\tau_f$ where τ_f is the formation time of a resonance; $\tilde{f}_R(K)$ is a kinematical factor proportional to the probability density of producing a pion with a given K in the resonance decay. Branching ratio $BR(R)$ is recalculated to an average charge state of the resonance. For instance in the case of the ρ meson we have three charged states.We assume that in the central rapidity region

$$\frac{dn(\rho^+)}{dy} \approx \frac{dn(\rho^0)}{dy} \approx \frac{dn(\rho^-)}{dy} \approx \frac{dn_\rho}{dy} \tag{35}$$

In the sum over ρ^+. ρ^0 and ρ^- decays we shall have $2\pi^- + 2\pi^0 + 2\pi^+$. For $dn_\rho/dy = 1$ we shall thus have two like-sign pions. This factor is included into $BR(\rho)$. In the column Adn_R/dy we give non-normalized ratios of central rapidity density which are guessed from data of Ref.[26]. The symbol dn_R/dy denotes rapidity density averaged over charged states of resonances in the spirit of Eq.(35). The correlation function is then given by Eqs.(28-30).

According to Eqs.(28-31) $C_2(q,k)$ is a weighted sum of contributions of individual resonances. To see that resonances and direct pions give quite different contributions we present in Fig.4 correlation functions corresponding to the assumption that all pions are decay products of a particular resonance - the weight $w_R(K)$ of this resonance is 1 and all other weights vanish. The contribution of direct pions is calculated in the same way and also presented.

In the same Fig.4 we plot also the data of EHS/NA-22 Collaboration given in Fig.5b of Ref.[31]. The data correspond to averaging over transverse momenta $0 < Q_T < 40 MeV/c$ and this narrow interval permits us to compare our calculations done for small transverse momenta with this data.

The interpretation of Figs. 4a, 4b and 4c is rather simple. In Fig.4a corresponding to $\tau_f = 0.2 fm/c$ direct pions are originated by decay of a resonance with formation time of 0.2 fm/c and vanishing mean life- time for the decay. Because of that direct pions are created within a short distance from the collision point and the Fourier Transform of this density of sources is rather broad in q.A typical value of q for directly produced pions is $\hbar/sinh(y_R)\tau_f \approx$0.4GeV/c. For resonances like ρ, Δ, f_2, K^* characteristic time is increased by their decay time, for $\tau_d \approx 1.3$ fm/c, corresponding to a resonance width of about 150MeV, typical longitudinal momentum is $\hbar/sinh(y_R)\tau_d \approx$60MeV/c.

With increasing formation time both resonance contribution and that of direct pions become steeper in q. For τ_f equal to 0.2 or 0.4 fm/c the contribution of direct pions decreases slower than the data so that a cocktail composed of resonance decay products and of direct pions has a chance to describe the data,although at the price of increasing r_{dir} for increasing τ_f.

For $\tau_f \approx$1fm/c even the contribution of direct pions decreases faster than the data and any cocktail composed of resonance decay products and direct pions is bound to fail.

In Fig.5 we show the dependence of $C_2(q)$ on both τ_f and r_{dir}.As can be seen in Fig.5a a reasonable qualitative agreement with data is obtained for $\tau_f \approx$0.2fm/c and $r_{dir} \approx$0.50 - 0.6. For $\tau_f \approx$0.4fm/c the agreement can be reached with $r_{dir} \approx 0.7$ which seems to be excluded by data on resonance production preferring a lower fraction of directly produced pions.

In Figs.4 and 5 our curves are somewhat below the point with the lowest value of q.This might be due to the fact that we have not included the contribution of η.For the study of this point one would need to take into the account also details of the binnig procedure.

In our simplified model transverse momenta of resonances are put equal to zero.With transverse momenta of resonances included we expect that formation times τ_f required by the data will slightly increase, since resonances moving not exactly along the z-axis would need more time to decay in the region with the same longitudinal dimension.

4 Transverse momentum dependence of the correlation function $C_2(q, K)$

So far we have assumed that $K = (k_1+k_2)/2$ has been small. calculations have been performed for $K_T = 0$, $K_L = 0$. It is easy to see, that the correlation function $C_2(q, K)$ is very sensitive to K_T, even if we restrict ourselves to vanishing K_L. Value of K_T enters results via two factors. The weight $w_R(K)$ as given by Eq.(29) contains the factor $\tilde{f}_R(K)$ given by Eqs.(30) and (31). Functions $\tilde{f}_R(K)$ increase with increasing K_T and their ratios are changing. More important is the dependence of y_R on m_T. As seen from Eqs.(15) and (20) values of y_R decrease with increasing K_T which in turn decreases values of z_f in Eqs.(23) and (24) and in the same way values of z_d in Eq.(25). From Eq.(20) it follows that

$$sinh(y_R) = \sqrt{\frac{M^2}{(2m_T)^2} - 1} \tag{36}$$

for resonance decay to two pions.The value of $sinh(y_R)$ is thus decreasing with K_T. As follows from expressions of z_f and z_d in Eq.(25), they are both proportional to $sinh(y_R)$. For small values of q Eq.(27) gives

$$\tilde{P}(q; y_R) \approx 1 - q^2[sinh^2(y_R)(\tau_f^2 + \tau_f\tau_d + \tau_d^2)] \tag{37}$$

Writing $\tilde{P}(q, y_R) \approx 1 - q^2 a_R^2$ where a_R is a typical length,analogous to R_L ,we can see that the contribution of resonance R to a_R^2 decreases with p_T in the same way as $sinh^2(y_R)$ as given by Eq.(36).

To illustrate the effect we give in Table 2 the dependence of y_R on K_T. The rapidities y_R were calculated by Eqs.(15) and (16) for fixed values of resonance masses.Blank places in the table indicate that resonance decay for the given K_T is kinematically forbidden.

The calculation of correlation function $C_2(q,K)$ for a given value of K_T would proceed exactly as above via Eqs.(28),(30) and (31).In a realistic model resonance contributions are averaged over their mass- and transverse momentum distributions. The simplified model shows where are the reasons for the decrease of R_L with K_T of pions.We shall not discuss this effect in more detail here.

5 Intermittency patterns due to resonance decays

The second factorial moment as defined by Eqs.(6),(7) and (10) can be written as

$$F_2(\delta) = 1 + \frac{1}{\delta^2}\int_{-\delta/2}^{\delta/2} dy_1 \int_{-\delta/2}^{\delta/2} dy_2 C_2(m_T sinh y_1 - m_T sinh y_2) \qquad (38)$$

where we have suppressed the K_T and expressed the longitudinal momentum difference in terms of rapidities

$$q \equiv k_{1L} - k_{2L} = m_T sinh y_1 - m_T sinh y_2 \qquad (39)$$

The function $C_2(q, K_T)$ in Eq.(38) is calculated by Eq.(28) and presented in Fig.4. We shall study here the second factorial moment $F_2(\delta)$ for τ_f = 0.2fm/c with r_{dir}=0.5 and τ_f=0.4fm/c with r_{dir} =0.7 which give a reasonable description of the correlation function $C_2(q)$.We shall include the same resonances and the same weights as in Fig.4.

In Fig.6a we plot $F_2(\delta)$ as a function of $2/\delta$, both in the logarithmic scale for τ_f=0.2fm/c and r_{dir}=0.5.The fit of results by the Fialkowski curve Eq.(5) is also given in Fig.6a.The values of the coefficients obtained are

$$C_L = 0.11 \pm 0.20, \quad C_S = 0.22 \pm 0.17, \quad f_2 = 0.31 \pm 0.13$$

In Fig.6b we present $F_2(\delta)$ for τ_f=0.4fm/c and r_{dir}=0.7.The values of coefficients are

$$C_L = 0.16 \pm 0.3, \quad C_S = 0.22 \pm 0.25, \quad f_2 = 0.33 \pm 0.2$$

Rather large "errors" are probably due to the fact that the coefficients are strongly correlated.The value of the intermittency exponent f_2 is larger than values found from data integrated over rather large interval of p_T [32-35]. This is not quite surprising since the EHS/NA-22 Collaboration [32] have found a rather strong dependence of the intermittency index on the size of the p_T bin over which one integrates.In particular the increse of $F_2(\delta)$ with $1/\delta$ has been found [32] stronger for $0 \leq p_T \leq 0.15$ GeV/c than for $0 \leq p_T \leq 0.15$ GeV/c and for $0.15 \leq p_T \leq 0.30$ GeV/c.

The interval $0.0 \leq p_T \leq 0.04$GeV/c studied by the NA-22 Collaboration in Ref.[31] is more narrow than those studied previously and it is not surprising that the intermittency index f_2 is larger.

The f_2 found in our analysis is somewhat smaller than values found in three- dimensional analyses by Fialkowski [2] what is also not surprising if the correlation is of a short range in the momentum space. It will be very interesting to see the intermittency analyses of the data of Ref.[31] in the bin $0 \leq p_T \leq 0.04$ GeV/c.Our model indicates that values of f_2 should be about 0.3±0.15.

6 Comments and Conclusions

We have described above a very simplified model of effects caused by resonance formation and decay on Bose- Einstein correlations of identical pions in hadronic collisions.Due to the simplicity of the model our results should be rather considered as hints to what one can expect in more detailed calculations.The results can be summarized as follows:

- The correlation function $C_2(q, K)$ for $\vec{K} \approx 0$ as measured by the EHS/NA-22 Collaboration [31] in π^+ interactions at 250GeV/c can be understood as due to an interplay of resonance decays and of directly produced pions provided that the fraction of directly produced pions $r_{dir} \approx 0.5$ and the formation time of resonances and direct pions is rather short $\tau_f \approx 0.2$fm/c. For the formation time of $\tau_f \approx 0.4$fm/c the fraction r_{dir} increases to about 0.7 and for $\tau_f \approx 1$fm/c consistency with data cannot be achieved.

Note that our estimate of the fraction of directly produced pions is larger than results obtained by Lednický and Progulova [24].

-The second factorial moment $F_2(\delta)$ calculated within the model shows intermittent patterns.When described by the Fialkowski's formula the value of intermittency exponent is $f_2 \approx 0.3\pm0.15$ for small K.This value is larger than those obtained from data averaged over larger regions of p_T [32-35] and smaller than values found in three- dimensional analyses [2].It would be most interesting to see the intermittency analysis of EHS/NA-22 data for 0¡p_T¡0.04GeV/c.

-Our simple model shows in a very transparent way a strong dependence of the correlation function $C_2(q.K)$ on the value of K=$(k_1 + k_2)/2$ and in particular on the average transverse momentum K_T of the two identical pions. We shall discuss this point in more detail in a separate work.

- In our model resonance formation and decay plays an important role and as a consequence of that the correlation function $C_2(q)$ is quite different from a Gaussian.This indicates that the data on correlations in hadronic collisions should be rather fitted by functions which correspond to a sum of directly produced pions and one or two resonances.When taking only one resonance one should probably take parameters of the ρ to take into account resonances of width comparable to that of the ρ and when taking also a second resonance one could take parameters of the ω to take into account also objects with a longer life- time.

Models analyzing effects of resonance formation and decay on correlations of identical particles have been studied earlier by numerous authors [36- 46]. As far as we know the intermittency patterns in this type of models have not been studied in a quantitative way as we have tried to do in the present work. Conclusions about resonance formation times and average life- times have been made by Lednický and Progulova [24] who have considered a model containing ρ- mesons and direct pions, by Csörgõ et al. [40] who have evaluated analytically the average formation time of resonances as 0.77±0.1fm/c and mean life- time of resonances as 2.88 fm/c and used then the Monte Carlo program SPACER to analyze data on Si+Au collisions at 14.5GeV per nucleon and O+Au interactions at 200 GeV/nucleon.

Padula and Gyulassy [42- 44] have analyzed pp and $\bar{p}$p data at CERN ISR energies and in particular the sensibility of data to the abundance of resonances. They have found that the data are inconsistent with the full resonance fractions as predicted by the Lund model.Their results are consistent with those of Kulka and Lörstad [46] and with our results at lower energies as shown in Fig.4 above. The reason of this result is due to to the fact that resonances tend to increase R_L whereas direct pions work in the opposite direction.

In most of analyses the presence of resonances leads to marked deviations from Gaussian shapes of the correlation function $C_2(q)$,reasons for that being simply visible in our model.

It would be most interesting to have data on correlation functions for pp, pA, and AB collisions at the same energy which would permit to study differences of correlation functions as a function of the atomic number of colliding particles and search for the onset of collective expansion, which should be visible via long time delays [47- 50].Unfortunately the increase of $\ll z^2 \gg$ may be due both to an increase of the time delay and to the increase of the abundance of resonances and these two mechanisms should be disentangled before firm conclusions could be done.A step in this direction has been recently performed by Wiedemann [51] in an interesting analysis which combines hydrodynamics in heavy-ion collisions with effects of resonance decays.

There is a lot of most interesting aspects of data which we have not discussed in the present paper.Apart of the p_T- dependence of correlation function these include at least: multiparticle correlations and higher factorial moments, correlations of un- like pions which appear naturally in models based on resonance decays, and the rapidity dependence of correlation functions.We have also limited ourselves to a simple situation with two identical pions having the same energy and have studied only the dependence of the correlation function on the difference of the longitudinal momenta Q_L. The model can be generalized also to other types of variables upon which the correlation function depends and we hope to return to these issues in the near future.

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Appendix

A Rapidity shifts in a two body decay

For the sake of completeness we shall give here some simple formulas for rapidity difference between the decaying particle and its decay products and we shall derive Eq.(16).Consider a particle with mass M decaying in its rest frame to two particles with masses μ_1 and μ_2. Decay products are characterized by their transverse momenta $\vec{p}_{T1}$ = -$\vec{p}_{T2} \equiv \vec{p}_T$ and by their rapidities y_1 and y_2. Denoting transverse masses as $m_1 = \sqrt{\mu_1^2 + p_T^2}$, $\mu_2 = \sqrt{\mu_2^2 + p_T^2}$ the energy and momentum conservation implies

$$M = m_1 cosh(y_1) + m_2 cosh(y_2) \tag{1}$$

$$0 = m_1 sinh(y_1) + m_2 sinh(y_2) \tag{2}$$

Expressing $sinh(y_2)$ from (A.2) and using $cosh^2(y_2) = 1 + sinh^2(y_2)$ we obtain from (A.1)

$$M = m_1 cosh(y_1) + m_2 \sqrt{1 + \left(\frac{m_1}{m_2} sinh(y_1)\right)^2} \tag{3}$$

From Eq.(A.3) we find

$$cosh(y_1) \equiv \alpha = \frac{M^2 - (m_2^2 - m_1^2)}{2 m_1 M} \tag{4}$$

Via quadratic equation for $x = exp(y_1)$ we get

$$y_1 = \pm ln(\alpha + \sqrt{\alpha^2 - 1}) \tag{5}$$

and

$$sinh(y_1) = \pm \left(\frac{M^4 - 2(m_1^2 + m_2^2)M^2 + (m_2^2 - m_1^2)^2}{4 m_1^2 M^2}\right)^{1/2} \tag{6}$$

For the equal mass case $m_1 = m_2$ and formulas simplify to

$$cosh(y_1) = \frac{M}{2m_1} \equiv \alpha$$

$$y_1 = \pm ln\left((M/2m_1)^2 + \sqrt{(M/2m_1)^2 - 1}\right)$$

$$sinh(y_1) = \pm\sqrt{(M/2m_1)^2 - 1}$$

Starting with Eq.(13).putting $|T| = 1$ and integrating over $\vec{p}_2$ we get for the equal mass case

$$(2\pi)^6\Gamma = \int \frac{d^3p_1}{2E_1 2E_2}\delta(E - E_1 - E_2) = \int \frac{d^3p_1}{2E_1 2E_2}\delta(M - 2E_1) \qquad (7)$$

writing $d^3p_1/E_1 = 2\pi p_T dp_T dy_1$. $E_1 = m_1 cosh(y_1)$ where m_1 is the transverse mass, and using $2E_1 = 2E_2 = M$ we find

$$(2\pi)^6\Gamma = \int \frac{2\pi p_T dp_T dy}{2M}\delta(M - 2m_1 cosh(y_1)) \qquad (8)$$

$$(2\pi)^6\Gamma = \int \frac{2\pi p_T dp_T dy}{2M}\sum_{y_1}\frac{1}{2m_1|sinh(y_1)|}\delta(y - y_1) \qquad (9)$$

where the sum goes over two possible values of y_1.Expressing y_1 in terms of M and m_1 we find

$$(2\pi)^6\Gamma = \int \frac{2\pi p_T dp_T dy}{2M\sqrt{M^2 - 4m_1^2}}(\delta(y - y_1) + \delta(y + y_1)) \qquad (10)$$

what corresponds to Eq.(14) in the text.For the unequal mass case we have instead of Eq.(A.7)

$$(2\pi)^6\Gamma = \int \frac{2\pi p_T dp_T dy_1}{2E_1 2E_2}\sum \frac{1}{|m_1 sinh(y_1) + m_2 sinh(y_2)(dy_2/dy_1)|}\delta(y - y_1) \qquad (11)$$

where the sum goes over the two solutions in Eq.(A.5).For the expression in the denominator we use Eq.(A.2) to get

$$(2\pi)^6\Gamma = \int \frac{2\pi p_T dp_T dy_1}{4E_2}\frac{1}{|m_1 sinh(y_1)(1 + m_1 cosh(y_1)/m_2 cosh(y_2)|}[\delta(y - y_1) + \delta(y + y_1)] \qquad (12)$$

By using Eqs.(A.4) and (A.5) we find

$$(2\pi)^6\Gamma = \int \frac{2\pi p_T dp_T dy_1}{2[M^4 - 2(m_1^2 + m_2^2)M^2 + (m_2^2 - m_1^2)^2]^{1/2}}[\delta(y - y_1) + \delta(y + y_1)] \qquad (13)$$

For the equal mass case $m_1 = m_2$ we come back to Eq.(A.10).

The maximal p_T ,$(p_T)_{max} = k$ is given by the condition

$$M = \sqrt{\mu_1^2 + k^2} + \sqrt{\mu_2^2 + k^2} \tag{14}$$

leading to

$$k = \frac{1}{2M}\sqrt{[M^2 - (\mu_1 + \mu_2)^2][M^2 - (\mu_1 - \mu_2)^2]} \tag{15}$$

In terms of k we can rewrite Eq.(A.14) as

$$(2\pi)^6 = \int \frac{2\pi p_T dp_T dy_1}{4M\sqrt{k^2 - p_T^2}}[\delta(y - y_1) + \delta(y + y_1)] \tag{16}$$

The normalized probability distribution for the resonance decay then becomes

$$\frac{dP}{d\phi p_T dp_T dy} = \frac{1}{4\pi k\sqrt{k^2 - p_t^2}}[\delta(y - y_1) + \delta(y + y_1)] \tag{17}$$

For the equal mass case and for $p_T = 0$ Eq.(A.18) leads to Eq.(16) in the main text.

Contribution of a resonance to P(z) and to its Fourier transform

For $z > 0$ the contribution of a single resonance to $P(z) \equiv P(z; y_R)$ is given by Eq.(21).Putting $f \equiv z_f$ and $d \equiv z_d$ we have

$$P(z) = \frac{1}{f - d}\left(e^{-z/f} - e^{-z/d}\right); \qquad z > 0 \tag{18}$$

$$P(z) = P(-z); \qquad z < 0$$

$P(z)$ vanishes for $z = 0$ then increases and reaches a maximum at

$$z_m = \frac{lnf - lnd}{f - d} fd \tag{19}$$

It is easy to show that for $d < f$ it holds $d < z_m < f$ and that as expected z_m increases with both f and d. The Fourier transform of $P(z)$ is

$$\tilde{P}(q) = \int_{-\infty}^{+\infty} P(z)e^{iqz}dz = \frac{1 - fdq^2}{[1 + (fq)^2][1 + (dq)^2]} \tag{20}$$

Figure Captions

Fig.1 Simple kinematical situation for study of HBT interferometry in the c.m.s. of hadronic collision.

Fig.2 Two interfering amplitudes for production of identical pions with momenta $\vec{k}_1$ and $\vec{k}_2$.

Fig.3 The shape of function P(z) as given by Eq.(21) for a set of values z_f and z_d: a) $z_f = z_d = 1$; b)$z_f = 1$, $z_d = 3$; c) $z_f = 3$, $z_d = 5$.

Fig.4 Contributions of individual resonances and of directly produced pions to the correlation function $C_2(q)$. Data points taken from Fig.5b of Ref.[31]. Contributions are plotted for three values of the formation time: a) τ_f= 0.2fm/c; τ_f = 0.4fm/c; c) τ_f = 1fm/c.

Fig.5 Comparison of data [31] on $C_2(q)$ for $0 < p_T < 0.04$GeV/c with coctails of resonances and directly produced pions: a) τ_f = 0.2fm/c; b)τ_f = 0.4fm/c.

Fig.6 Second factorial moment $F_2(\delta)$ as calculated in our model: a)τ_f = 0.2fm/c, r_{dir} = 0.5; b) τ_f = 0.4fm/c, r_{dir} = 0.7. The lines are fits by the Fialkowski's formula.Parameters are described in the text.

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Table 1. Basic parameters for calculation of correlations of identical pions (lengths in GeV^{-1})

Res.	y_R	$sh(y_r)$	τ_d	z_d	z_f	$f_R(K)$	$BR(R)$	$A\frac{dn_r}{dy}$	$w(r)$
ρ	1.67	2.56	6.66	16.96	$2.56\tau_f$	0.618	2	0.31	0.38
ω	1.257	1.615	118.6	191.6	$1.615\tau_f$	1.615	0.89	0.31	0.45
f_2	2.35	5.2	5.41	28.1	$5.2\tau_f$	0.21	0.57	0.07	0.01
K^*	1.47	2.06	41.2	84.9	$2.06\tau_f$	0.96	1.33	0.08	0.1
Δ	1.26	1.62	8.3	13.5	$1.62\tau_f$	1.57	1.33	0.11	0.23

Table 2. Dependence $y_R = y_R(K_T)$ for a selected set of resonances

$K_T[GeV/c]$ Resonance	*0.00*	*0.05*	*0.10*	*0.15*	*0.20*	*0.25*	*0.30*	*0.35*	*0.40*
ρ	1.67	1.62	1.45	1.24	1.03	0.80	0.56	0.205	
f_2	2.19	2.14	1.98	1.80	1.61	1.43	1.27	1.11	0.96
K^*	1.47	1.41	1.23	1.01	0.77	0.34			
Δ	1.25	1.19	0.99	0.74	0.40				
ω	1.24	1.18	0.98	0.73	0.38				
η	0.76	0.67	0.34						

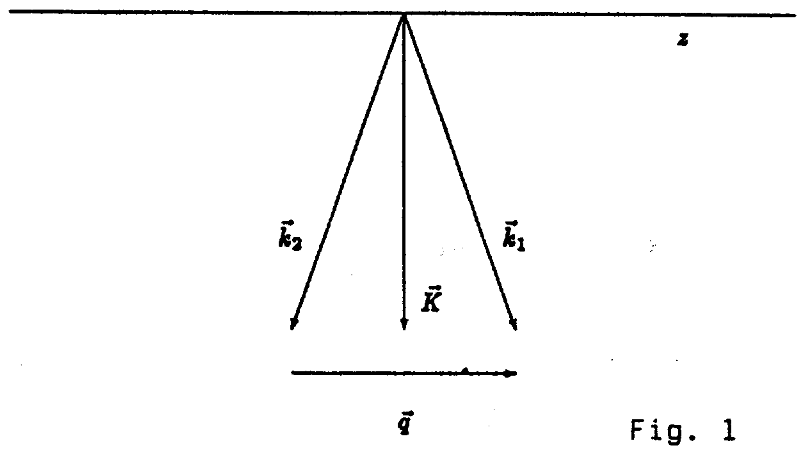


Fig. 1

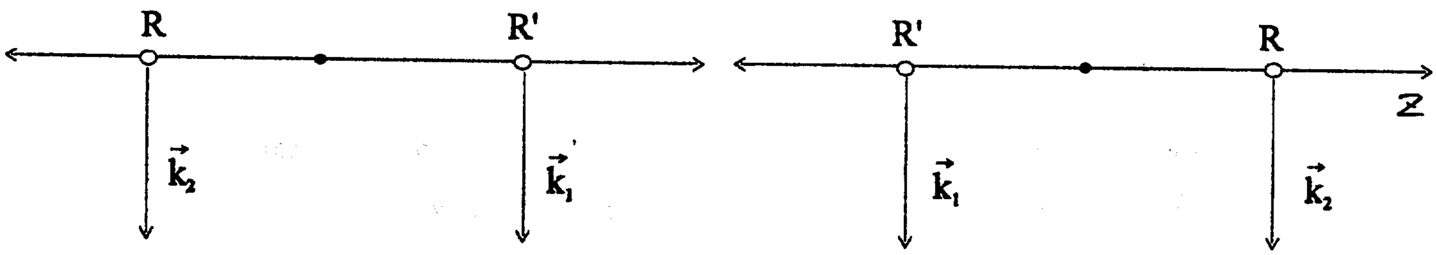


Fig. 2

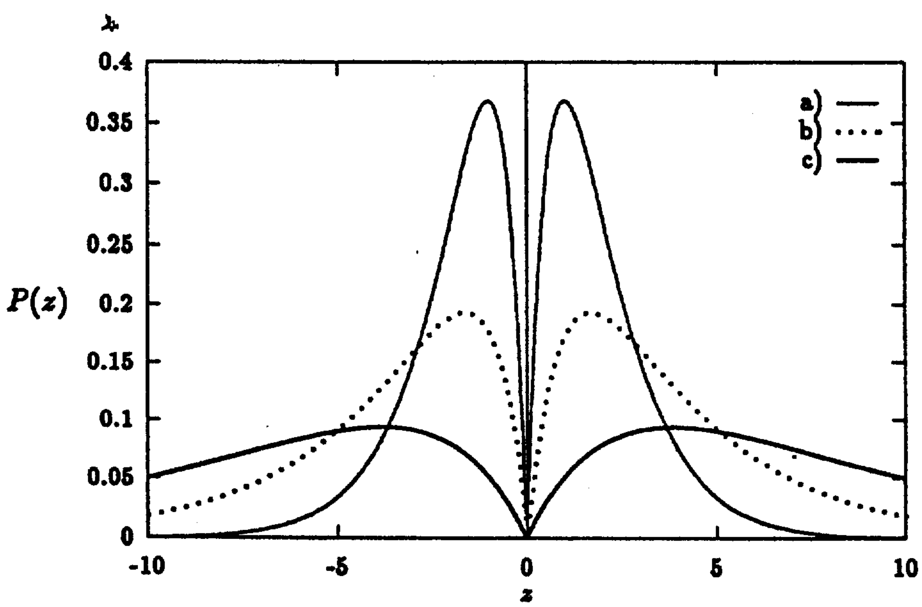
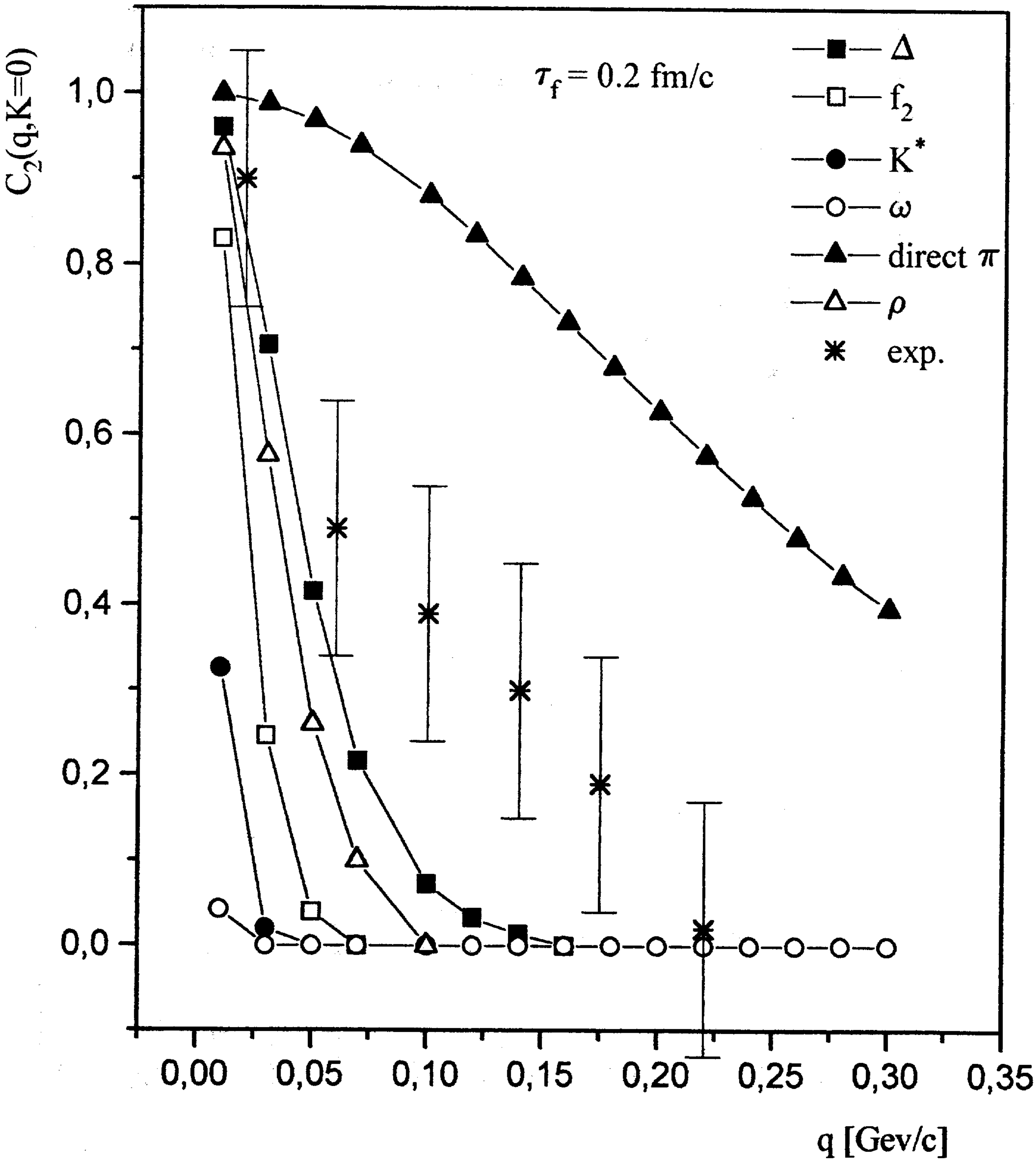
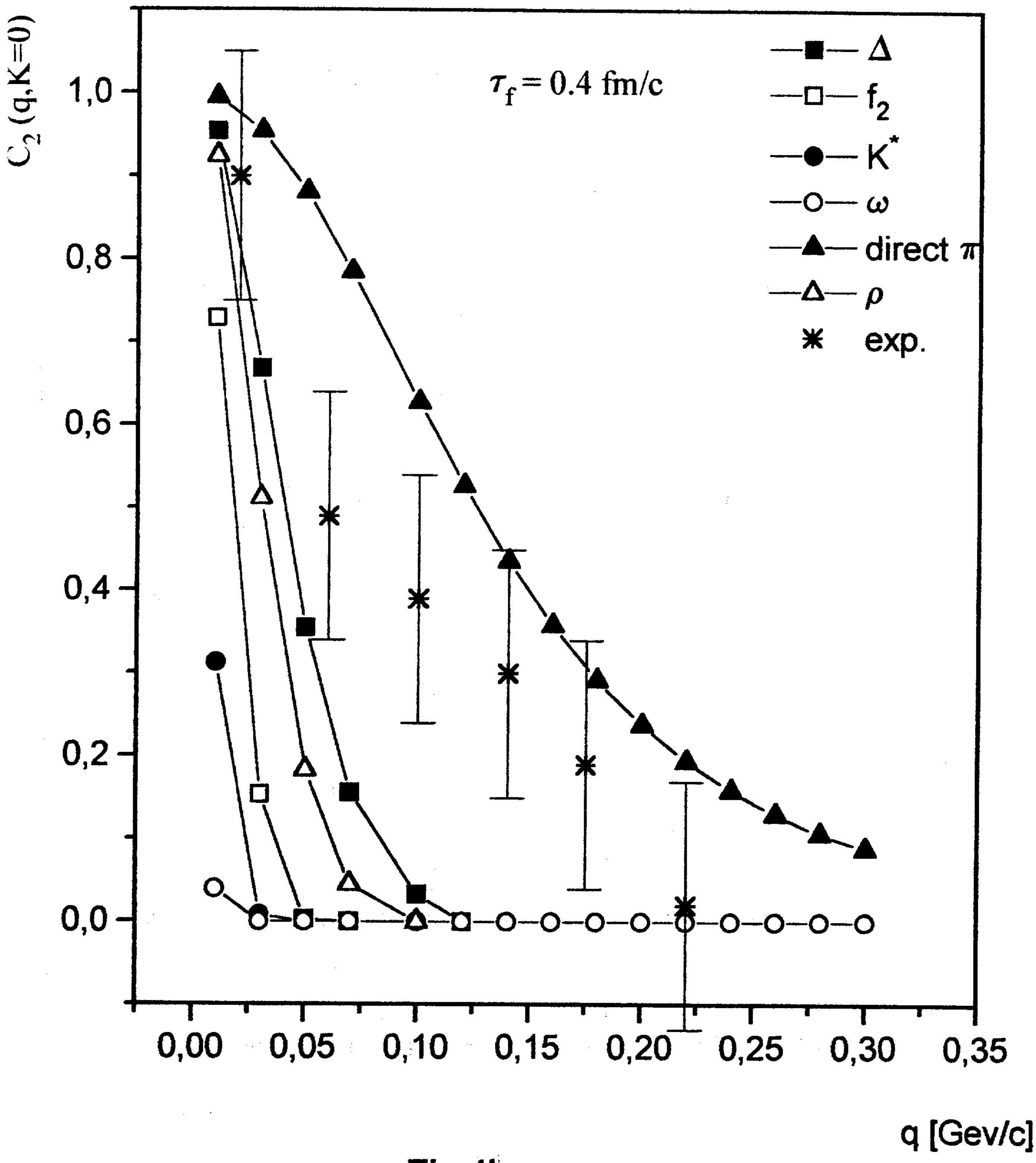


Fig. 3



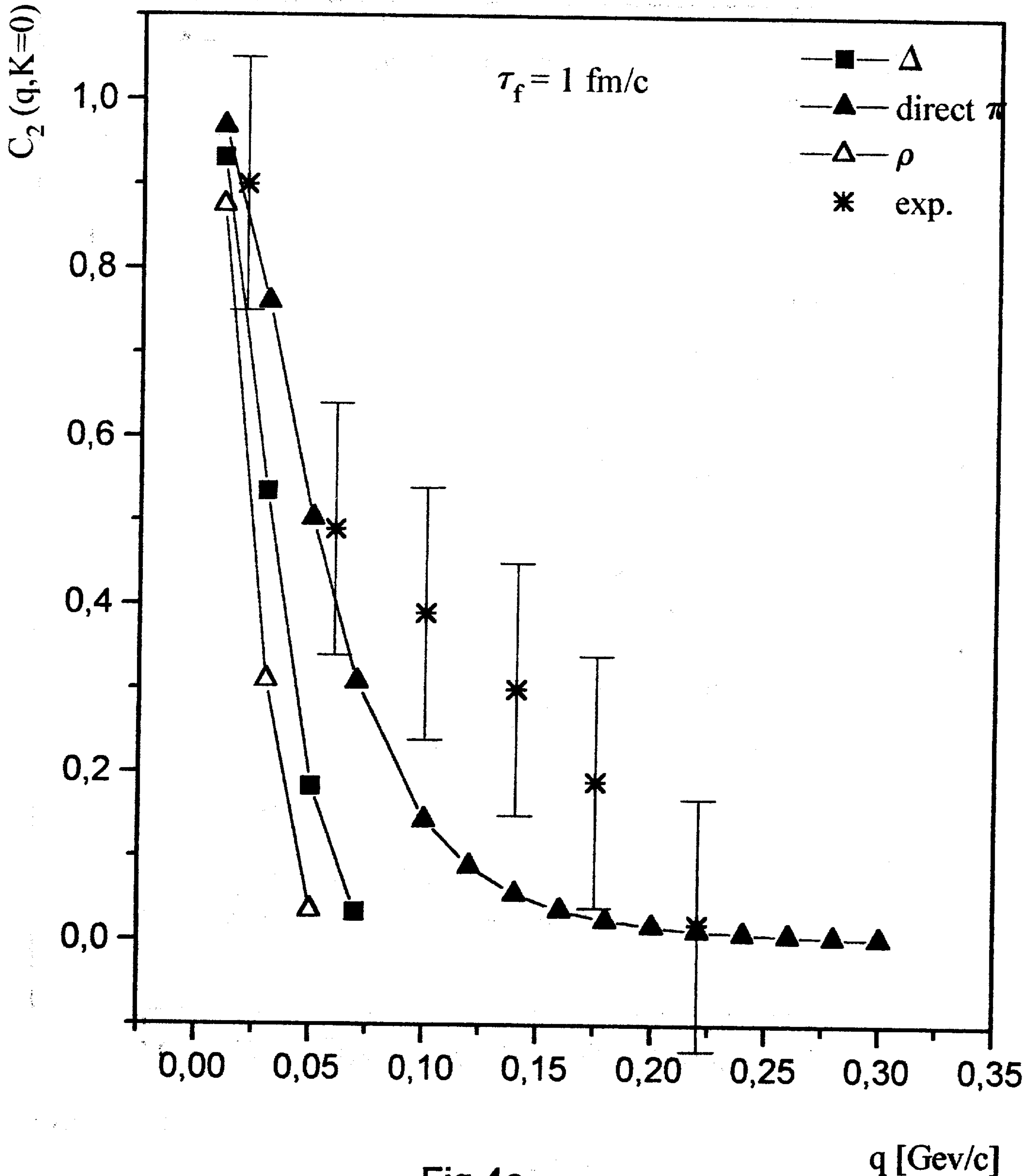
τ_f = 0.2 fm/c
Δ
f_2
K^*
ω
direct π
ρ
exp.
$C_2(q,K=0)$
1,0
0,8
0,6
0,4
0,2
0,0
0,00
0,05
0,10
0,15
0,20
0,25
0,30
0,35
q [Gev/c]

Fig.4a



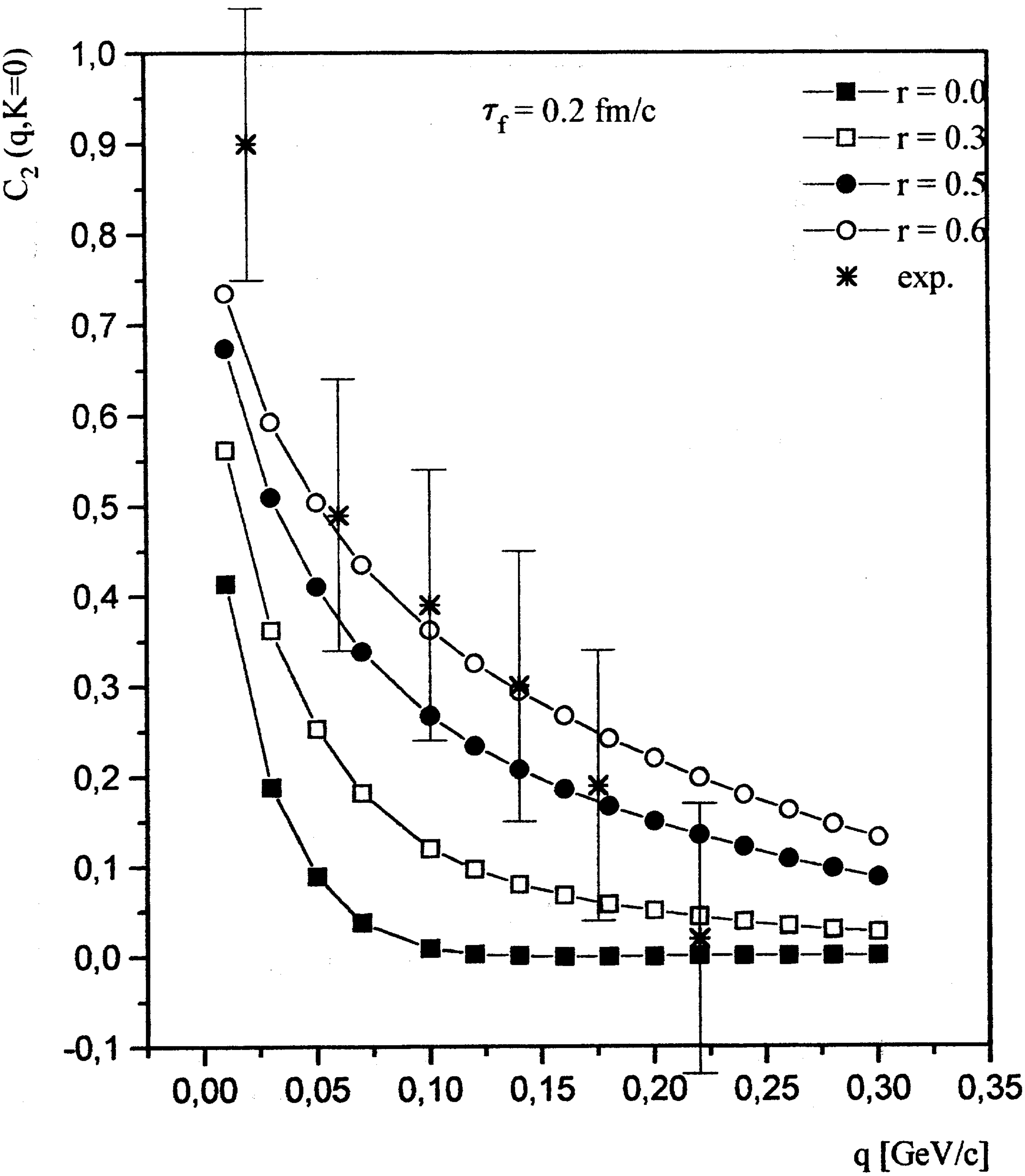
C_2 (q,K=0)
τ_f = 0.4 fm/c
Δ
f_2
K^*
ω
direct π
ρ
exp.
1,0
0,8
0,6
0,4
0,2
0,0
0,00
0,05
0,10
0,15
0,20
0,25
0,30
0,35
q [Gev/c]

Fig.4b



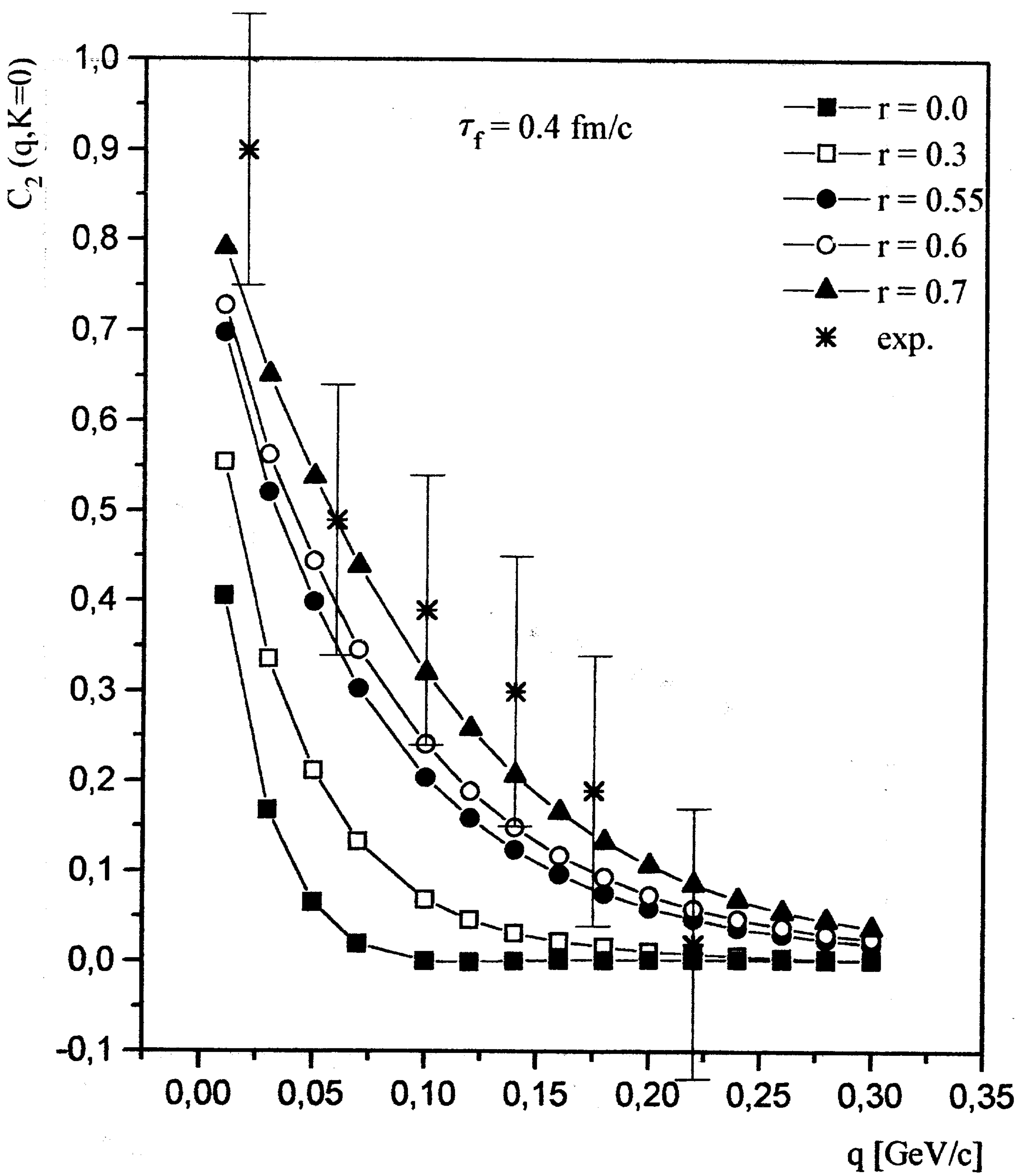
C_2 (q,K=0)
τ_f = 1 fm/c
Δ
direct π
ρ
exp.
1,0
0,8
0,6
0,4
0,2
0,0
0,00
0,05
0,10
0,15
0,20
0,25
0,30
0,35
q [Gev/c]

Fig.4c



C2 (q,K=0)
1,0
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0,0
-0,1
τf = 0.2 fm/c
r = 0.0
r = 0.3
r = 0.5
r = 0.6
exp.
0,00
0,05
0,10
0,15
0,20
0,25
0,30
0,35
q [GeV/c]

Fig.5a



C2 (q,K=0)
τf = 0.4 fm/c
r = 0.0
r = 0.3
r = 0.55
r = 0.6
r = 0.7
exp.
1,0
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0,0
-0,1
0,00
0,05
0,10
0,15
0,20
0,25
0,30
0,35
q [GeV/c]

Fig.5b

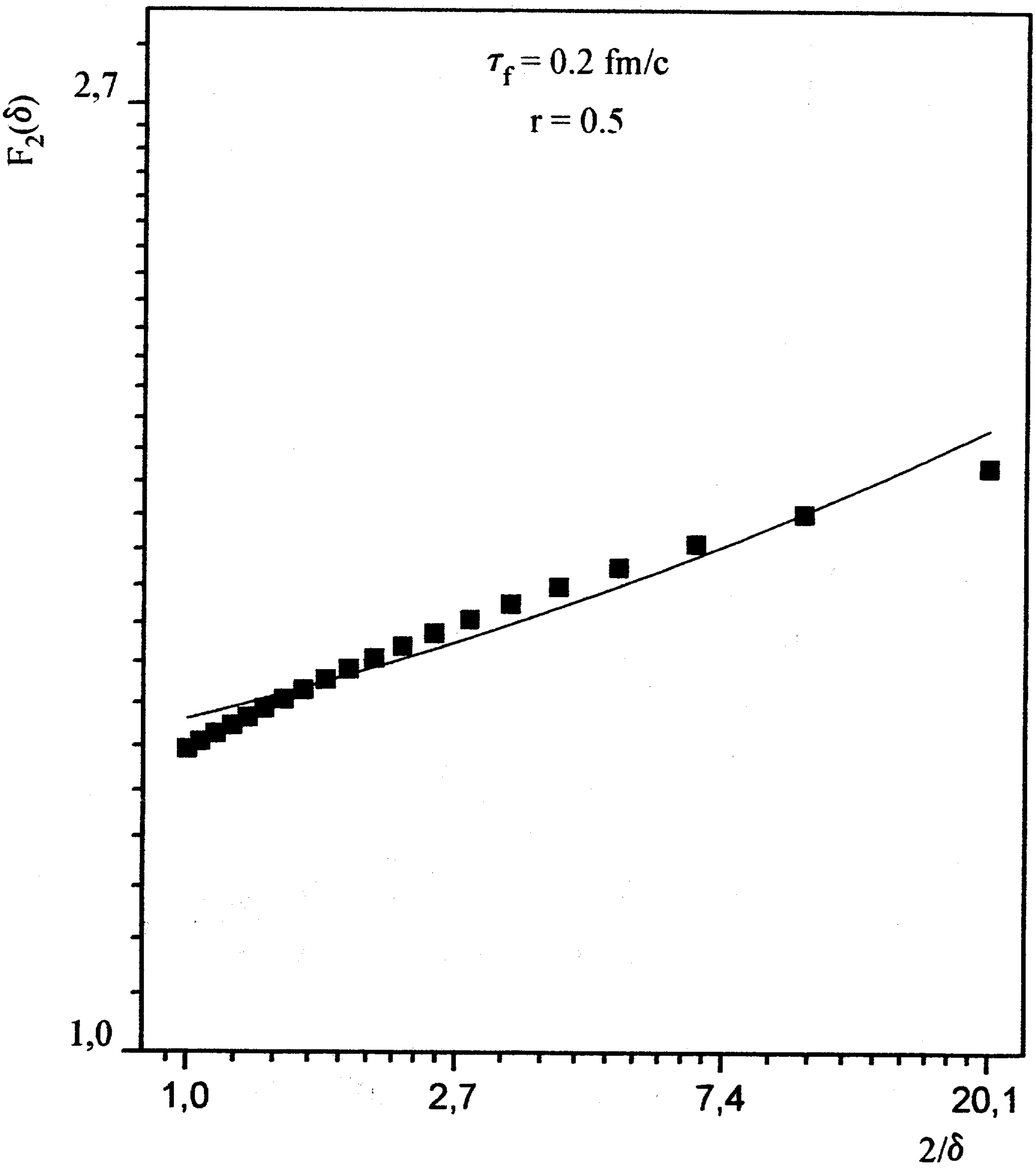


Fig.6a

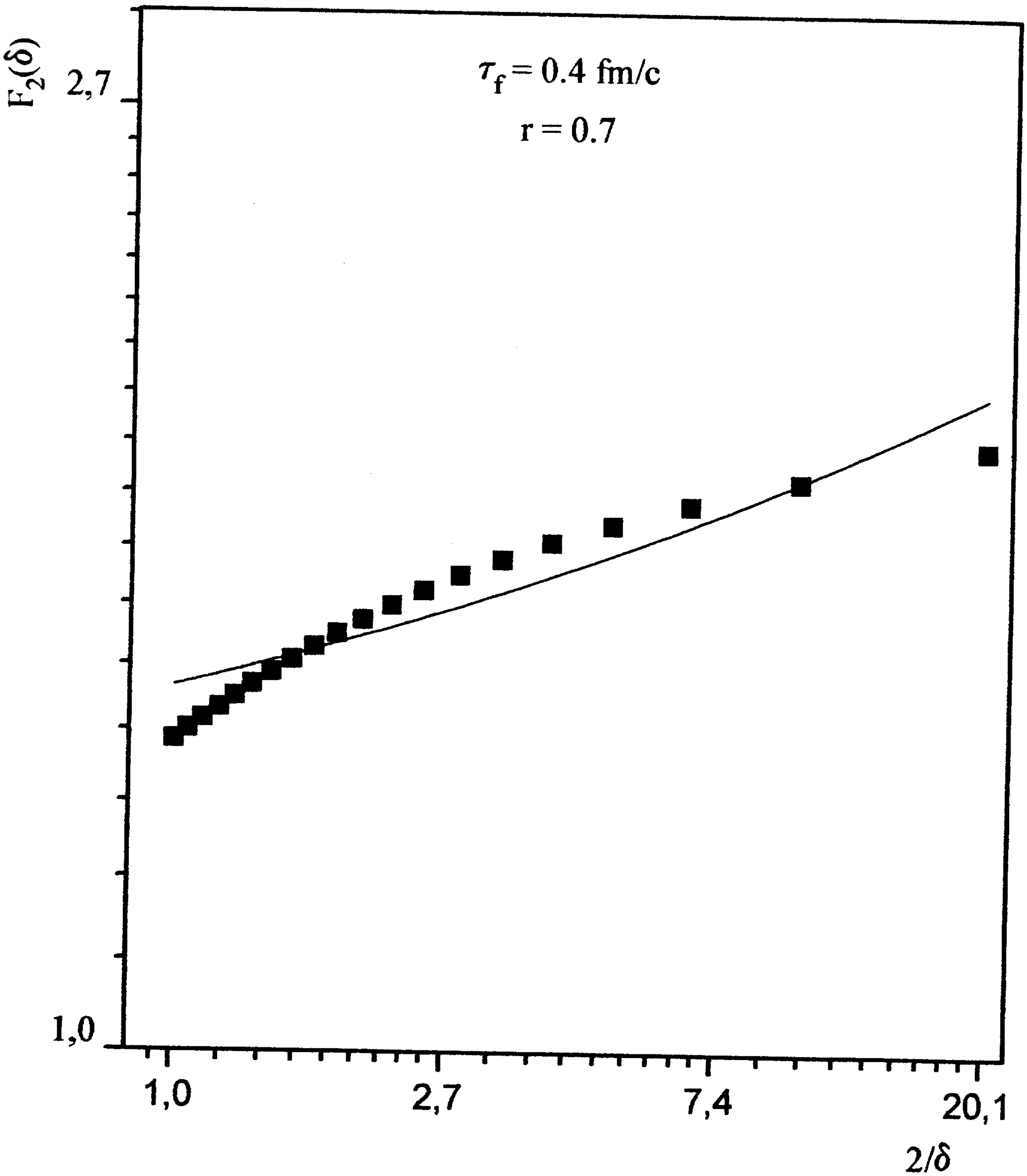


Fig.6b