EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

ISR-BT-TH/69-27

BEATCH - A FORTRAN PROGRAMME FOR THE PARTICLE OPTICS OF BEAM TRANSFER CHANNELS

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[redacted]

Geneva, 30th May, 1969

PS/7222

CONTENTS

Summary

The FORTRAN programme BEATCH deals with the particle motion in BEAM TRANSFER CHANNELS. For a given beam transfer channel it can compute trajectories of particles, beam envelopes along a channel, the matching of phase-space ellipses and momentum compaction, transfer matrices, betatron parameters of a periodic magnetic lattice, and the geometry of the central orbit of the channel. This report describes the fundamental features of the programme and provides the necessary information of its utilisation.

1. Introduction

The design of beam transfer channels for cascade synchrotrons, such as the NAL [1] and the proposed CERN [2] Multi-GeV synchrotrons, or for storage rings [3], presents from the computational point of view some new features with respect to conventional transfer channels. Firstly, because of the greater number of transfers undergone by the beam, a good matching of the beam to the acceptance of the synchrotron is particularly important. Secondly, such channels may consist of a strong focusing periodic magnetic lattice, with sections at either end for matching the input beam to the periodic magnetic lattice of the channel and properly shaping the emittance ellipse of the output beam. Thirdly, machines being linked by a transfer channel may not lie on the same horizontal plane due to the site configuration, and hence three dimensional calculations for the transfer channel are necessary. Finally, a description of the properties of the input and of the output beam in terms of synchrotron parameters is often required.

BEATCH has facilities for computing the geometry of the central orbit of a three dimensional transfer channel in cartesian co-ordinates, the trajectories of particles with respect to the central orbit, the beam envelopes in both horizontal and vertical motion, the horizontal and vertical transfer matrices over sections or the whole of a channel, the horizontal and vertical betatron parameters of any periodic magnetic lattice within a transfer channel, together with a few auxiliary facilities including routines for achieving the matching of betatron phase-space ellipses at two points, and momentum compaction matching at a pre-determined point.

Any desired facility may be called by the use of appropriate control cards.

2. DESCRIPTION OF THE PROGRAMME

A beam transfer may be made up of any sequence of quadrupole lenses, bending magnets and field free sections, which will be referred to as elements. The programme BEATCH can be used for a transfer channel with a non-repetitive sequence of elements, or for a channel with a repetitive sequence of elements constituting a periodic magnetic lattice, or for a combination of both. A data card has to be given for each element of the channel. These data cards, stacked in the sequence of the elements of the channel and preceded by two other data cards (the first carrying a title, and the second the total number of elements in the channel) form a basic stack of cards specifying the channel.

Figure 1 shows the block diagram of BEATCH. The programme is made to execute any or all of its functions by means of control cards following the basic stack, the relative "input" being supplied on data cards following the appropriate control card. The programme consists of a number of sub-routines which perform one of the functions shown in Fig. 1, and which in turn call auxiliary sub-routines. The control cards carry a two character code, which identifies the required computation, followed by a heading which is output at the head of the results for that particular computation. The two character codes used are shown as labels of the corresponding branches of the block diagram ("b" stands for blank).

The particle optics computations which can be carried out and the additional facilities provided are :

i) Tracking of particles relative to the central axis of a channel. The momentum dispersion along a channel is obtained by tracking trajectories of off-momentum particles.

ii) Calculations of the beam envelope along the channel.

iii) Calculation of horizontal and vertical transfer matrices between any two elements of the channel.

iv) Calculation of the betatron parameters of a section of channel having a periodic magnetic lattice.

v) Matching of betatron phase-space ellipses in the horizontal and vertical planes at one or two points along the channel.

vi) Three-dimensional calculation of the geometry of the central trajectory of the channel.

vii) A modification may be made to one or more sections of the basic channel during execution of the programme.

viii) One may obtain a new deck of cards which are labelled and serialized. This deck includes any modification made to the channel and the results of previously executed matching computations.

ix) Comments may be printed, following the output of any computations.

2.1. Reading in the Parameters of a Beam Transfer Channel

The programme has been written for beam transfer channels containing 6 types of elements, each type of element being assigned a code number for identification. Table 1 gives the list of elements and their codes. One may include in the channel up to two sections represented by a 3 x 3 transfer matrix for the horizontal motion and a 3 x 3 transfer matrix for the vertical motion. The two sections are characterized by two different code numbers (6 and 7, see Table 1). In practice 2 x 3 matrices are given, since the last row of these matrices is (0,0,1).

TABLE 1

Elements	Code
Field free section	1
Horizontally focusing quadrupole lens	2
Horizontally defocusing quadrupole lens	3
Horizontal bending magnet with parallel end faces	4
Vertical bending magnet with parallel end faces	5
Transfer matrix	6,7

A data card is punched for each element. It contains an alphanumeric description of the element, the code number for the type of element, and the parameters of the elements, viz., length, bending angle, magnetic field gradient. These cards, stacked in the sequence of the elements in the beam transfer channel, carry all the information relevant to the beam transfer channel. The programme assigns an ordinal number to each of the elements. The maximum allowed number of elements is limited by dimension statements.

The structure of the programme allows for the insertion of additional types of elements. However, because of the 3 x 3 matrix formalism which is used by the programme, no coupling can be introduced between the betatron motion in the horizontal and in the vertical plane.

2.2. Tracking of a Particle Trajectory

The tracking of particles relative to the central trajectory of the beam transfer channel is carried out in the horizontal and vertical planes by matrix multiplication techniques. The programme uses 3 x 3 matrices derived from Penner [4], assuming bending magnets having parallel edges and angles of entry and exit equal to half the bending angle of the magnet.

The horizontal input vector of a particle (x, x', $\Delta p/p$) is multiplied by the matrices of successive elements along the channel to give the vector of the particle at the exit of each element. Similarly for the vertical input vector (z, z', $\Delta p/p$). The symbols x and z denote the horizontal and the vertical displacement with respect to the central trajectory of the channel. Their derivatives with respect to the longitudinal co-ordinate are denoted by x' and z'. The tracking may be forward over the whole beam, forward over a section of the beam, reverse over the whole beam, or reverse over a section of the beam.

Input for the tracking routine requires the ordinal number of the initial and of the final elements of the section to be tracked; the horizontal displacement (x) and slope (x'), the vertical displacement (z) and slope (z') of the particle where it enters the first element stated, and $\Delta p/p$. (if the first number exceeds the second number, the programme carries out a reverse tracking between the elements corresponding to these two numbers). The tracking routine allows for independent scaling of the strengths of one set of focusing quadrupoles and of one set of defocusing quadrupoles. These quadrupoles are indicated by a special control code on the element card. The effect of steering elements in the channel can be included in the tracking.

The tracking routine also has facilities for converting the co-ordinates of the trajectory following the last element of the tracking sequence into normalized co-ordinates [5] (See also the Appendix). These co-ordinates are such that betatron motion is reduced to circular motion in the normalized phase-plane. Therefore, normalized co-ordinates represent a convenient way of describing the motion of particles in a strong focusing lattice.

Tracking output lists, for each element, its description and parameters, the horizontal displacement (x) and slope (x'), the vertical displacement (z) and slope (z') at the exit (in the direction of the tracking) of the element together with the magnitude of any horizontal and or vertical abrupt deflection (or "kick") introduced at the entrance of that element. If required the normalized co-ordinates and the normalized amplitudes of oscillation following the last element are also computed, for both the horizontal and the vertical motion.

2.3. Beam Envelope Calculation

The beam envelope dimensions and the slope of the beam emittance ellipses are computed at the exit of each element. The beam emittance ellipse is expressed by complex numbers [6] (Z-values) and transformed through each element by matrix multiplication using the transfer matrix of that element. The beam dimensions are computed for given horizontal and vertical emittances. The routine may also be used for investigating the effects of gradient errors in the quadrupole elements on the beam envelope.

Input for the routine requires the horizontal and vertical Z-values at the entrance of the initial element and the horizontal and vertical beam emittances.

Alternative to using these unnormalized Z-values as input, the ellipses may be specified using horizontal and vertical β and α values [7] and giving the normalized Z-values [5]. Inside the routine, only unnormalized Z-values are used for the computations. Elements other than the first and last one may be specified to start and respectively end the computation. A given number of gradient errors may be introduced.

Output lists the initial half-width and half-height and for each element, its description and parameters, the half beam-width, half beam-height, horizontal and vertical unnormalized Z-values, and the length of the beam from the beginning of the channel up to the exit of the element. If β and α values have been fed in, then the output lists the normalized Z-values (horizontal and vertical) and their inverses, at the exit from the last element of the computation, together with the β and α values.

2.4. Transfer Matrices

The 3 x 3 horizontal and vertical transfer matrices between two elements within the transfer channel are computed by this routine. Input requires the ordinal number of the element from the beginning of which the transfer matrix is required, and the ordinal number of the element at the end of which the transfer matrix is to finish. Output lists the ordinal number of these elements and the matrices.

2.5. Betatron Parameters of a Periodic Magnetic Lattice

For a section of the beam transfer channel which has a periodic lattice structure, this routine first calculates the horizontal and vertical transfer matrices from the beginning of one period to the beginning of the next. Then from each of these matrices according to Courant and Snyder[7] it calculates the betatron amplitude function β, $\alpha = -(d\beta/ds)/2$ and the betatron phase advance μ per period. The Z-values for the acceptance ellipses are also computed from the values of β and α. Input requires the ordinal number of the first and last elements in the period. Output lists the ordinal number of these elements, the transfer matrices, the β-values, the α-values, the phase advances and the Z-values for both the horizontal and vertical motion.

2.6. Betatron and Momentum Compaction Matching

This subsidiary programme uses the strength and longitudinal postion of a number of independently variable quadrupole magnets, designated matching quadrupoles, to match a) the transformed betatron phase-space ellipse of an input beam to acceptance ellipses at up to two matching points in the channel, and b) the momentum dispersion at one point to a required momentum compaction at that point. For matching in betatron phase-space, the input and the acceptance ellipses are either specified in complex numbers or computed by the programme using a procedure similar to that described in Section 2.5. A transfer matrix is required between matching quadrupoles. This matrix can either be computed by the programme as in Section 2.4. or be read in directly.

Momentum compaction matching is achieved in conjunction with the tracking programme. The tracking programme gives the normalized amplitudes of oscillation in the horizontal and vertical planes (with respect to a specified equilibrium orbit) of an off-momentum particle at a predetermined point. Ideally, these values should be zero and the matching process uses the strengths of matching quadrupoles to find a minimum in these normalized amplitudes of oscillations.

The matching programme develops a function, F, which is a combination of horizontal and vertical betatron mismatch factors for both matching points and horizontal and vertical momentum compaction mismatch factors. For both the horizontal and the vertical motion, the programme takes as betatron mismatch factors the amplitude of beat oscillation of the beam size with respect to the matched beam size; as momentum compaction mismatch factor, it takes the final, normalized amplitudes of oscillations of the trajectory which is tracked. Weighting factors are given to each of these mistatch factors in order to determine the prominent factor to be matched. A weighting factor or zero for any of the mismatch factors eliminates these from the matching process thus reducing the number of requirements to be matched.

The function F is determined for initial parameters, i.e. position and strengths, of the matching quadrupoles. This function is minimized using the general purpose minimization programme MINUIT [8]. This programme incorporates three different minimization methods, each of which may be used alone or in combination with the others depending on the behaviour of the function. These methods are outlined below. For more details the reader is referred to Ref. [8].

i) A Monte Carlo search of the minimum. The function F is computed for random values of the parameters, chosen according to gaussian distributions centred at the initial values of the parameters. Although this method is very slow, it may be used to determine the starting points for subsequent minimizations, in particular when the function is expected to have several minima.

ii) A minimization using the Rosenbrock method [9]. This method is based on a search of the minimum in each of the orthogonal directions of the parameter space, followed by the definition of new orthogonal directions until the improvement on the minimum is smaller than a preset value. This method is reasonably fast even when far from tne minimum.

iii) A minimization based on a variable matrix method by Davidon [10], which proceeds toward the mimimum by making successive approximations to the covariance matrix. It then converges simultaneously toward the minimum and toward the true covariance matrix. This method is extremely fast close to the minimum, but is slow and unreliable, for badly behaved functions, far from the minimum.

The maximum number of matching parameters is fixed by dimension statements. Strengths and displacements of different quadrupoles may be denoted by the same parameter, so that they remain equal throughout the matching. Minimum and maximum values may be assigned to the strengths, as well as to the longitudinal displacements of the quadrupoles. Also the matching process may be requested to determine the maximum beam sizes at the matching quadrupoles for each set of parameters set by the minimization programme. If these exceed specified upper limits of the beam size, the mismatch factor is artificially increased, so that different sets of parameters are sought by the programme. Following the matching programme, the beam envelope through the sequence of matching quadrupoles is obtained for a beam of specified horizontal and vertical emittances. Computations following a matching programme take the matching quadrupole parameters computed by the matching programme, if so specified.

The matching need not be used with a complete beam data stack, i.e. separate matching computations can be carried out by reading in data appropriate to matching only. However, momentum compaction matching can only be carried out if the matching data has been preceded by appropriate tracking data. Out of all the routines, the matching routine is usually by far the most time consuming.

2.7. Beam Geometry Computation

The geometry of the central orbit of the beam transfer channel is computed in the cartesian co-ordinates, X, Y, Z, used for geodetic measurements, where Z is the altitude. This computation may be carried out forward over the whole beam, forward over a section of the beam, reverse over the whole beam or reverse over a section of the beam.

Input requires initial co-ordinates, "gisement" (defined as the angle formed by the central trajectory with the Y-axis, measured in clockwise direction in grades, where 100 grades = 90^{o}) the intital vertical angle (the slope measured in radians with respect to the X, Y plane) and the ordinal numbers of the inital and of the final element in the channel between which the geometric computation is required. The required co-ordinates and gisement for the end of the beam can also be fed in so that the accuracy of the position of the end of the beam can be noted. If the first ordinal number stated exceeds the second, a reverse geometry computation is carried out.

The output gives the initial co-ordiantes and slopes including the initial horizontal angle (the slope measured in an anti-clockwise direction in radians with respect to the X,Z plane) and the required final co-ordinates and slopes. Then output lists for each element, its description, length, horizontal or vertical bending angle, the X-co-ordinate, Y-co-ordinate, altitude, gisement, horizontal angle, horizontal length (i.e. horizontal projection of the distance from the entrance of the initial element) and the beam length (longitudinal distance from the entrance of the first element) at the exit of that element.

2.8. Modification to the Beam Transfer Channel

Modifications to the original beam transfer channel may be introduced without altering the basic stack of cards. The first card following the control card specifies the number of groups of successive elements to be changed. This is followed by the appropriate number of groups of data cards. Each group consists of a data card giving the ordinal number of the first and last elements to be changed, followed by the data cards, in sequence, of the new elements to be inserted. The length of the beam may be increased.

2.9. Summary Print-Out

A summary of the most significant information relating to a beam transfer channel may be obtained on one page of print-out following calculations involving all the three sub-routines for tracking, beam envelope, and geometry. Output lists the relevant initial conditions and for each element its description and parameters, the x, x', z and z' co-ordinates of the last tracked trajectory, the half-width and half-height of the beam envelope, the X and Y co-ordinates and altitude Z of the central trajectory, and the beam length. Also listed is a classified summary of the elements used in the channel, quoting the total number of field free sections, of horizontally focusing quadrupoles, of horizontally defocusing quadrupoles, of horizontal bending magnets, of vertical bending magnets, and of tr nsfer matrix elements.

3. PROGRAMME UTILISATION

The programme requires a control data card for each desired operation to be carried out, each control card being followed by the appropriate data cards before the next control card is inserted. The first two columns of a control card contain two control characters which indicate the operation to be carried out (Fig. 1). The last 78 columns contain a heading or a sub-heading, as explained below.

A control card having blank control characters defines the start of a basic stack of data cards representing a beam transfer channel, and any number of control and associated data cards may follow such a basic stack. Furthermore any number of beam transfer channels may be included by following the data for computations on a preceding channel by a new blank character control card and the stack of the data cards for the new beam transfer channel. A typical stack of data cards would be :

bb Control Card for channel data
Element Data Cards
MO Control Card for a modification of the channel
Modification Data Cards
TR Control Card for tracking of a trajectory
Input Data Cards for tracking
BE Control Card for beam envelope computation
Input Data Cards for beam envelope computation

bb Control Card for new channel data
¦ Element Data Cards
TR Control Card for tracking of a trajectory
¦ Input Data Cards for tracking
MH Control Card for betatron and momentum compaction matching
¦ Input Data Cards for matching
BE Control Card for beam envelope computation
¦ Input Data Cards for beam envelope computation
GE Control Card for Geometry computation
¦ Input Data Cards for geometry
SU Control Card for a Summary Print-Out
ST Control Card to stop execution

The last 78 columns of the bb control card contain in alphanumeric format, a heading which precedes the output of any subsidiary programme. The last 78 columns of control cards for subsidiary programmes, for example, the tracking programme, contain a sub-heading, which is printed after the heading. The heading is always printed at the top of a new page. Table 2 at the end of this section gives a summary of data cards.

3.1. Reading in the Heading and the Parameters of a Beam Transfer Channel

The last 78 columns of the blank character control card contain in alphanumeric format a heading which will be printed out on any output concerning the channel. Data cards for describing a beam transfer channel are read in the following order :

a) FORMAT I5

A card with N, number of elements of the channel. The maximum value for the standard version of the programme is 400. It is assigned by DIMENSION statements.

b) FORMAT A8, I2, F10.4, F10.7, F10.5, I10

N cards describing the elements of the transfer channel. These cards must be introduced in the order in which the elements are traversed by the beam. Each element is given an ordinal number I=1....N by the programme. Each card describes one element and contains in sequence :

- ELEMENT — the name of the element
- CODE — the code number describing the type of element. The code number of each type of element is given in Table 1
- L — the effective length of the element, expressed in metres
- ANGLE — the bending angle (horizontal if CODE=4, vertical if CODE=5) given to the central trajectory of the channel expressed in radians. A scaling of these deflections may be successively applied (see point e below)
- K — the quadrupole field $K = |dB_z/dx|/(B_z r)$, expressed in m^{-2} where dB_z/dx is the magnetic field gradient of the element and $B_z r$ the magnetic rigidity of the particles. A scaling of these quadrupole fields may be successively applied (see point e below)
- M — an integer number the use of which is explained at point e below.

c) FORMAT 10X, 6F10.5

If there is an element having CODE=6 (see Section 2.1.) one must provide two cards containing the elements (from left to right and from top to bottom) of the horizontal 2x3 transfer matrix of this element and the elements of its vertical 2x3 transfer matrix, respectively.

d) Similarly for the element having CODE=7.

e) FORMAT 4F10.5

A card with the quantities FH, FV, FF and FD. The role of these quantities is the following : for bending angles of the magnets where M=0, the values read in on the data cards are multiplied by FH for horizontal bending magnets and by FV for vertical bending magnets. If M≠0, the values read in remain unchanged. Similarly, if M=0, the K-values of the horizontally focusing or defocusing quadrupoles lenses read in on the cards are multiplied by FF or FD respectively. If M≠ 0 the values remain unchanged.

3.2. Tracking of a Trajectory

The last 78 columns of the TR control card contain a sub-heading which is printed immediately below the heading of the transfer channel in the tracking output. The programme computes the transfer matrices of the elements of the transfer channel and carries out the tracking by matrix multiplication methods. Directly after the TR control card the following cards are read in :

a) FORMAT 7F10.5, 3I3, I1

A card containing in sequence the quantities :

- X, X' the initial horizontal displacement and slope of the trajectory, expressed in mm and mrad respectively
- DP the particle momentum deviation $\Delta p/p$ (expressed in o/oo) with respect to the central trajectory (DP=0) of the transfer channel.

- Z, Z' the initial vertical displacement and slope of the trajectory, expressed in mm and mrad respectively

- FF,FD (See Section 3.1.) to be used for the tracking. These values of FF and FD will remain unchanged for successive calculations for example when working out beam envelope or matching computations. If the data card has a zero or blank in the field for FF the previous value assigned to FF will remain unchanged. Similarly for FD.

- I1 tracking starts at the point where the trajectory enters the element I1. If on the data card I1=0 or blank, the programme will put I1=1

- I2 the tracking ends when the trajectory comes out of the element I2. If on the data card I2=0 or blank, the programme will put I2=N. In the case where I2 < I1 the programme carries out a backward tracking from the entrance (for particles going backwards) of the element I2, to the exit (for particle going backwards) of the element I1

- NKICK at the entrance (with respect to the direction of the tracking) of some elements, one can give to the trajectory horizontal and vertical kicks. NKICK gives the number of points where the kicks are given (see point b below)

- INORM in the case where the channel transfers a beam into a strong focusing lattice (for example into a synchrotron), it may be useful to express the coordinates of the trajectory at the end of the transfer channel in normalized machine units (see Appendix). This is done when INORM=1. If this is not required one should put INORM =0 or blank.

b) FORMAT I5, 2F10.5

NKICK cards, each of which contains the ordinal number (I) of an element at the entrance of which a kick must be given, the horizontal kick and the vertical kick to be given at this point (expressed in mrad).

c) FORMAT 8F10.5

If INORM=1 there must follow a card containing in sequence :

β_n, β, α for horizontal motion (BETAHN, BETAH, ALPHAH),

β_n, β, α for vertical motion (BETAVN, BETAV, ALPHAV), the horizontal momentum compaction function $\alpha_p = \Delta x/(\Delta p/p)$ and its azimuthal derivative $\alpha' \equiv d\alpha_p/ds$ (ALPHAP, DALPHAP). The functions α_p and α'_p give the displacement Δx and the slope $\Delta x'$ of the equilibrium orbit of particles having $\Delta p/p=1$. The quantities β_n, β and α_p are given in metres, whereas α and α'_p are given in radians. The quantities listed above must be given at the exit from element I2.

The output produced by the tracking routine consists of :

i) heading

ii) sub-heading i.e. the last 78 columns of the TR control card

iii) the matrices CODE=6 and CODE=7, if these have been given

iv) the values of FH, FV, FF and FD

v) DP and the initial co-ordinates X, X', Z and Z' of the trajectory

vi) for each element the output gives I, ELEMENT, CODE, the actual *) K, M, the actual *) bending angle (ANGLE)

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*) i.e. the value on the element data card multiplied by FF, FD FH and FV according to the type of the element.

the co-ordinates of the trajectory at the exit from the I-th element (X, X', Z and Z'), the horizontal (XKICK) and the vertical (ZKICK) kicks given to the trajectory at the entrance of the I-th element and the longitudinal distance (BEAM LENGTH) from the entrance of the first element of the transfer channel to the end of the I-th element. BEAM LENGTH is measured along the central trajectory of the transfer channel and is given in metres

vii) in the case where INORM=1, at the exit from the element I2 the output gives the normalized co-ordinates (XN, X'N, ZN, Z'N) and the normalized amplitudes of oscillations (AXN = $\sqrt{XN^2 + X'N^2}$ and AZN = $\sqrt{ZN^2 + Z'N^2}$) of the trajectory with respect to the trajectory having horizontal displacement ALPHAP * DP (expressed in mm), horizontal slope DALPHAP * DP (expressed in mrad) and zero vertical displacement and slope. The values of BETAHN, BETAH, ALPHAH, ALPHAP, DALPHAP, BETAVN, BETAV and ALPHAV are also printed. The above displacements, slopes and amplitudes are expressed in mm.

3.3. Beam Envelope Calculation

The programme computes the shape of the beam emittance ellipses and the beam dimensions at the exit of each element of the channel. The shape of emittance ellipses is described by complex numbers ZH and ZV (for horizontal and vertical motion respectively) as defined in Hereward's paper [6]. The computations are carried out with the values of FF and FD previously stored in tracking or in reading in the channel. Directly after the BE control card there are the following data :

a) FORMAT 8F10·5

For the horizontal and vertical Z-values at the entrance of the first element (ZIH and ZIV) there is a card which contains RIH, XIH, RIV, XIV, BETAH, ALPHAH, BETAV and ALPHAV.
RIH and XIH are respectively the real and imaginary parts of ZIH. In the case where the horizontal normalized Z-value (i.e. Z-value evaluated in a normalized phase-plane) at the exit from element I2 (see below) is required, the horizontal β and α values at this location (BETAH, ALPHAH) must be given. If not, then these fields must be left blank. Similarly for vertical motion.

b) FORMAT 2F10·5, 3I5

A second card contains the quantities :

- EH/PI,EV/PI	the horizontal and vertical beam emittances divided by π, expressed in mm mrad
- I1	the ordinal number of the element at the entrance of which computations start
- I2	the ordinal number of the element at the exit of which the computation has to be stopped. If I1 or I2 are not specified, I1=1 and I2=N
- NMOD	the gradients of NMOD elements are modified with respect to the gradients assumed in the preceding calculation (see point c below) After the beam envelope calculations, the gradients are reset to their original value.

c) FORMAT I5, F10.5

NMOD cards each containing the ordinal number of a quadrupole lens and its gradient modification DK/K expressed as a fraction of the nominal gradient K.

The following output is produced :

i) heading

ii) sub-heading, i.e. the content of the last 78 columns of the BE control card

iii) the matrices CODE=6 and CODE=7, if they have been given

iv) FH, FV, FF and FD

v) ZIH, EH/PI, ZIV and EV/PI

vi) for each element the output gives I, ELEMENT, the nominal K, DK/K, ANGLE, the half betatron beam width WH, the half betatron beam height WV, ZH and ZV at the exit from the element and BEAM LENGTH

vii) in the case where β and α-values have been read in, the output gives the normalized Z-values (ZHN and ZVN) and their inverses (YHN=1/ZHN and YVN=1/ZVN) at the exit from the element I2, as well as the β and α-values at this point.

3.4. Transfer Matrices

The MX control card is followed by a card (FORMAT 2I5) containing two integers I1 and I2. The routine gives the horizontal and vertical transfer matrices AH and AV from the entrance of the element I1 to the exit of the element I2 ≥ I1.

3.5. Betatron Parameters

This routine computes the betatron parameters of a periodic lattice. The LA control card is followed by a card (FORMAT 2I5) containing the numbers I1 and I2 ≥ I1 of the first and of the last element in the period. The routine works out the horizontal and vertical transfer matrices from the entrance of I1 to the exit of I2. From these matrices it deduces the betatron functions β and α (7), the betatron phase advance μ and the Z-values for both the horizontal and the vertical betatron motion (6).

3.6. Matching

3.6.1. General Description

Let us consider three locations 1, 2 and 3 situated in this azimuthal sequence in the channel and another loaction 4 situated anywhere downstream from 1.
At 1 the beam has assigned horizontal and vertical emittance ellipses and dispersion according to particle momentum.
The routine determines the strength and the location (called "matching parameters") of a number of quadrupoles ("matching quadrupoles") such that the second of the following conditions and optionally all of the others are satisfied :

i) at 2, the emittance ellipses have an assigned shape (betatron matching)

ii) at 3, the emittance ellipses have an assigned shape (betatron matching)

iii) at 4, the beam has an assigned dispersion according to particle momentum (momentum compaction matching).

Since each of these conditions implies four constraints, in order to be satisfied it requires at least four available matching parameters (i.e. strengths or locations of matching quadrupoles) upstream to the corresponding point. The computer time required increases rapidly when increasing the number of parameters. Hence one should handle the minimum number of parameters at the same time : whenever possible one should group the matching quadrupoles in different groups such that each group is used in connection with only some of the points i) to iii). Matching is then carried out using each of these groups separately. Let us consider, for example, the case where 4 coincides with 3, where all the conditions i), ii) and iii) have to be satisfied and where in the section from 1 to 2 there is a sufficient number of parameters to satisfy condition i) and in the section 2 to 3 there is a sufficient number of parameters to satisfy conditions ii) and iii). In this case it is convenient to use the quadrupoles from 1 to 2 only for matching at 2 and to use the quadrupoles from 2 to 3 for the other matchings.

The matching is carried out by means of an iterative procedure. At each step the programme MINUIT [8] calls a sub-routine FCN (belonging to BEATCH), which gives back the value of a function F to be minimized, corresponding to the current parameters. The function F is defined as the sum of six mismtach factors (AH, AV, AHM, AVM, AXN and AZN) described below, each of them multiplied by a weighting factor (FAH, FAV, FAHM, FAVM, FAXN and FAZN).

The mismatch factors are defined as follows :

i) If at least one of the two weighting factors FAHM and FAVM is different from zero, at point 2 the subroutine FCN compares the emittance ellipses, obtained with the current parameters, to the required emittance ellipses. Then it works out the betatron mismatch factor on the horizontal plane (AHM) and on the vertical plane (AVM). Following Hereward [6] these mismatch factors are defined as the radius of the circle circumscribed to the normalized emittance ellipse of area π. This factor, called A in Hereward's paper, gives the maximum beam size in the periodic lattice.

ii) At point 3 the betatron mismatch factors in the horizontal plane (AH) and in the vertical plane (AV) are computed in a similar way.

iii) The momentum compaction matching at point 4 is obtained as follows. At each step the routine repeats with a new set of parameters the last tracking executed by the programme (Section 3.2.). The initial co-ördinates specifiedin the last tracking must be such that the trajectory has zero amplitude of betatron oscillation with respect to a given trajectory (equilibrium orbit in the case where I1 is the first element of a channel which transfers the beam out of a circular accelerator). At the exit of I2 (point 4) the sub-routine works out the normalized amplitudes of oscillations AXN and AZN with respect to the trajectory having horizontal displacement and slope ALPHAP * DP and DALPHAP * DP and zero vertical displacement and slope. A perfect momentum compaction matching is achieved if AXN and AZN are zero. The computations of AXN and AZN are carried out if at least one of the two weighting factors FAXN and FAZN is different from zero.

One may assign maximum allowed values WHMAX and WVMAX for the half betatron width and for the half betatron height, respectively, at the matching quadrupoles, in which case the horizontal and vertical beam emittances divided by π (EH/PI and EV/PI) have to be given and the beam size is computed at each step. If WHMAX and WVMAX are set equal to zero of left blank, the beam size is computed only when matching is finished. In order to save computer time, whenever possible FAHM, FAVM, FAXN, FAZN, WHMAX and WVMAX should be zero.

The section of the channel which is involved in the matching is called the "matching section" (it could also consist of all the channel). The matching section is made up of "matching quadrupoles" and "interquadrupole sections" as specified below. In azimuthal sequence one has location 1, an interquadrupole section, NQUAD matching quadrupoles each followed by an interquadrupole section and location 3. Location 2 is described on data cards as a quadrupole having CODE=0 (all the other quantities on the quadrupole data card are in this case irrelevant and the length L is set equal to zero). The position of location 4 is specified by the tracking routine. The total number of parameters (NPAR) and of quadrupoles (NQUAD) are given. Their maximum values are fixed by dimension statements and are equal to 15 in the standard version of the programme.

Each matching quadrupole has two variable parameters, namely the gradient K and the displacement S (taken positive is forward directed along the beam) from a position which is defined by initially assigning the interquadrupole sections. Each matching quadrupole is characterized by two parameter indices : the gradient index KP > 0 and the displacement index LP ≥ 0. The actual quadrupole gradient and displacement are represented by the KP-th parameter and by the LP-th parameter, respectively, which are varied in order to minimize the function F. The initial values of the parameters are part of the data required by MINUIT. Quadrupoles having the same KP are given the same strength and quadrupoles having the same LP are given the same displacement. One can in this way, for example, displace rigidly a doublet of quadrupoles in order to achieve matching. Quadrupoles having LP=0 are kept in a fixed position. The minimizing programme incorporates facilities which allows lower and upper limits to the values to be assumed by the parameters.

If one gives the ordinal number IQ of the quadrupole, then after matching the IQ-th element of the transfer channel is given a gradient equal to the computed gradient "K(OUT)" for that quadrupole, is azimuthally displaced by the computed displacement "S(OUT)" and at the same time its M-value is set equal to 1, such that the gradient is not modified by FF and FD.

The interquadrupole sections are made up of elements of fixed locations and strenghts. In the computations they are represented by transfer matrices. If one gives the ordinal number I1 and I2 at the first and last elements of an interquadrupole section, the sub-routine computes the transfer matrices for zero momentum deviation, and if momentum compaction matching is also required, for momentum deviation DP. Alternatively, instead of specifying I1 and I2, one can give directly the elements of these matrices, to be used for both the values of momentum deviation.

For the description of elements of the matching section one can therefore refer to a transfer channel previously introduced, or one can describe them completely in the matching routine. In the first case the computed values for the parameters can be used for further computations on the transfer channel. In the second case the matching is completely separated from the other computations which are performed.

3.6.2. Input Data

The following data must be provided :

a) data cards required by MINUIT (see Ref. 8)

- FORMAT (8A10) Title Card - Any BCD characters, serving as a title for the print-out

- FORMAT (I10,A10,4F10.5) Parameter Cards

Col. 1-10	Parameter Number as referenced in FCN	
Col. 11-20	Name for the parameter	
Col. 21-30	Starting value	
Col. 31-40	Approximate error or step size (if zero, parameter is constant)	
Col. 41-50	Lower bound on parameter	if both blank not bounded
Col. 51-60	Upper bound on parameter	

- One blank card signals end of parameters cards - Note : parameter cards must be in order of increasing parameter number, but they need not include all parameters numbers if some numbers are not used by FCN.
  Points b) to h) refer to data required by FCN.

b) FORMAT 8F10.5
a card which reads in FAH, FAV, FAXN, FAZN, FAHM, FAVM, WHMAX, WVMAX

c) FORMAT 8F10.5

a card which contains RIH, XIH, RIV, XIV, BETAIH, ALPHAIH, BETAIV and ALPHAIV.
During the matching computations, the programme makes use of unnormalized Z-values. There are three ways of feeding in the horizontal and vertical unnormalized Z-values at location 1, namely ZIH=RIH+i·XIH and ZIV=RIV +i·ZIV :

- They can be read in directly on the data cards, in which case BETAIH or respectively BETAIV must be zero.
- They can be read in as normalized values and converted by the programme into unnormalized values by using (see Appendix) the specified values of the radial and vertical betatron functions at point 1 (BETAIH, ALPHAIH, BETAIV and ALPHAIV).
- The third way is described at point d below.

d) FORMAT 2I5

If the previous card is blank then one reads in a second card with the quantities I1 and I2 ≥ I1, which are the ordinal numbers of the first and last elements of a period of the magnetic lattice. The Z-values are computed by means of these two numbers, a procedure similar to that described in Section 3.5.

e) FORMAT 8F10·5
FORMAT 2I5

If FAMH or FAMV are different from zero one reads in 1 card or 2 cards, as described at points c) and d) above.

f) FORMAT 8F10·5
FORMAT 2I5

The data for the Z-values ZOH and ZOV at location 3 are read in as described at points c) and d) above, even when FAH or FAV are zero.

g) FORMAT 2F10·5, I5

The horizontal and vertical beam emittance EH/PI and EV/PI (expressed in mm mrad) and the number of matching quadrupoles NQUAD (including location 2) are read in. If the interquadrupole sections and the matching quadrupoles are identical and in the same order as after a previous matching (including quadrupole displacements given by the matching routine) where the elements have been defined, it is sufficient to put NQUAD=0. No further cards are in this case required by the matching routine, except those required at point i) below.

h) A sequence of cards describing, an interquadrupole section, a matching quadrupole (or location 2), an interquadrupole section, etc. There are a total of NQUAD matching quadrupoles and NQUAD + 1 interquadrupole sections. The sequence ends with an interquadrupole section.

- Interquadrupole Section, FORMAT 2I5, 6F10.5

For an interquadrupole section one reads in a card containing I1, I2 and the elements (from left to right and from top to bottom) of the matrix AH. If I1=0 or I2=0, AH represents the horizontal transfer matrix of the interquadrupole section; the vertical transfer matrix must be given on a subsequent card, with the same format. If I1≠0 and I2≠0, the transfer matrix is computed by the programme, as described in Section 3.4 ; no further card is required to describe the interquadrupole section.

- Matching quadrupole, FORMAT A8, I2, 5F10.5, 3I5
For a matching quadrupole one reads on a card the name of the quadrupole, CODE, L, SMIN, SMAX, KMIN, KMAX, KP, LP and IQ. The physical boundaries for the parameters K and L are actually set in the MINUIT cards. The values SMIN, SMAX, KMIN and KMAX are simply output in the print-out produced by the routine FCN (see below).

i) MINUIT "command" cards (see Ref. 8) which determine the method of minimization.

3.6.3. Output

The output of the matching routine can be divided into two parts which give :

a) MINUIT output, giving information (such as the total no. of steps required etc.), concerning the minimization process. If the minimization was unsuccessful no further output is given.

b) FCN output. Starting on a new page one finds :

- The description of the matching section, preceded by the heading and sub-heading.
- Mismatch factors, weighting factors, maximum allowed beam size and beam emittances.
- Matching section after matching. This part of the output gives the computed gradients and displacements, the horizontal and vertical half betatron beam size (WH and WV) through the matching section as well as the Z-values (ZH and ZV) and the betatron functions (BETAH, ALPHAH, BETAV and ALPHAV) which describe the emittance ellipses.

3.7. Computation of the Geometry of the Channel

This sub-routine calculates the co-ordinates and the slopes of the central trajectory of the channel with respect to the cartesian co-ordinate system X, Y, Z used for geodetic measurements. The co-ordinate Z will also be called ALTITUDE. After the GE control card two further data cards are required :

a) FORMAT 5F10·5, 2I5

This first data card gives :

- The initial X, Y, Z co-ordinates.
- The initial "GISEMENT", defined as the angle formed by the central trajectory with the Y-axis, measured in the clockwise direction in grades (100 grades = 90°).
- The initial vertical angle (VERT. ANGLE), which is the slope with respect to the X, Y plane, measured in radians.
- The ordinal numbers (I1 and I2) of the first and last elements for the geometric calculation. If on the data card I1=0 or blank, the programme sets I1=1. If on the data card I2=0 or blank, the programme sets I2=N. If I2<I1, the programme computes the geometry in a reverse direction along the beam line (N.B. In this case the initial gisement must still be given as for the forward direction of the beam).

b) FORMAT 5F10·5

The second data card gives the required final X, Y, Z co-ordinates and the required final gisement and vertical angle. These are not used by the programme but simply output in order to permit a quick estimate of the accuracy of the geometry of the channel.

The output consists of

i) heading

ii) sub-heading, i.e. last 78 columns of the GE control card

iii) FH and FV

iv) the initial and the required final X, Y, ALTITUDE, GISEMENT, HOR. ANGLE and VERT. ANGLE. The HOR. ANGLE is the angle (in radians) formed by the central trajectory of the channel with the X-axis, measured in the anti-clockwise direction.

v) I, ELEMENT, L, ANGLE, X, Y, ALTITUDE, HOR. LENGTH (horizontal projection of the distance from the entrance of the I1-th element) HOR. ANGLE, GISEMENT and BEAM LENGTH at the exit from each element.

3.8. Modification of the Transfer Channel

The last 78 columns of the MO control data card contain the heading for the modified transfer channel. Directly after this MO card must follow :

a) FORMAT I5

A card containing NMOD, the number of groups of elements to be modified. Each of these groups consists of elements having consecutive ordinal numbers. In order to modify only the factors FH,FV, FF and FD, NMOD=0.

b) FORMAT 2I5, FORMAT A8, I2, F10.4, F10.7, F10.5, I10

NMOD sets of data cards. Each of these sets describes a group of elements to be modified and consists of

- a card (FORMAT 2I5), containing I1 and I2, which are respectively the ordinal numbers of the first and last elements of the group. If I2=I1 or I2=0 (in which case the programme puts I2=I1) the group consists of the element I1. In the case I2>N, the total number of elements, N, is increased to I2.

- (I2-I1+1) cards of the type described in Section 3.1. describing the elements of the group to be modified. These cards must be in the order in which the elements are traversed by the beam.

c) If, among the modified elements, there is a card having CODE=6 its horizontal and vertical transfer matrices must be given as described in Section 3.1.

d) Similarly if there is a card having CODE=7.

e) A card containing FH, FV, FF and FD as described in Section 3.1.

3.9. Summary Print-Out

The sub-routine prints out parts of the output obtained from the last Tracking, Beam and Geometry calculations, as described below.

i) heading of the transfer channel

ii) elements of the horizontal and vertical transfer matrices of the elements having CODE=6 and CODE=7, if such elements exist

iii) DP and initial co-ordinates X, X', Z and Z' of the last trajectory which has been tracked

iv) ZIH, EP/PI, ZIV and EV/PI

v) I, ELEMENT, L, K, ANGLE, X, X', Z, Z' (co-ordinates of the last tracked trajectory with respect to the central trajectory of the channel) WH, WV (horizontal and vertical betatron half beam size), X, Y, ALTITUDE (co-ordinates of the central trajectory of the channel with respect to the geodetic reference system) and BEAM LENGTH

vi) the total number of field free sections (SS), of horizontally focusing and defocusing quadrupoles (QHF and QHD), of horizontally and vertically bending magnets (HBM and VBM), and of transfer matrix elements (MAT).

3.10. Serializing Labelling and Punching of a Deck

This routine is used to punch a labelled and serialized deck of cards describing the last transfer channel which has been read in. One can assign new values of the scaling factors, FH, FV, FF and FD. In this case the programme modifies bending angles and quadrupole gradients punched on data cards, such that when multiplied (if M=0) by the new scaling factors they give the same actual gradients as the previous deck having different gradients and scaling factors punced on the cards.

The columns 61 to 70 of the SE control card contain the label, which is punched on columns 61 to 70 of the cards describing the elements of the transfer channel. Columns 71 to 80 of the punched cards contain the ordinal number I. After the SE control card one must have a card (FORMAT 4F10·5) containing the required values of FH, FV, FF and FD. The punched element cards have the same format as when reading in a channel.

3.11. Comment Cards

The contents of the last 78 columns of the CO control card are printed immediately after having skipped a line.

4. Conclusion

The programme BEATCH has been extensively used, for the design of the transfer channels which lead the proton beam delivered by the CERN Protron Synchrotron to each of the CERN Intersecting Storage Rings or to the new Experimental Hall. The programme has been currently run on the CDC 6400 and 6600 Computers and is included in the CERN Programme Library.

APPENDIX

Normalized Systems of Units

Denoting by s the distance along the equilibrium orbit and by x either the radial or the vertical component of the displacement from the equilibrium orbit, the general expression of the betatron oscillation in an alternating gradient synchrotron is :

$$x(s) = a\sqrt{\beta(s)}\ \cos\left[\Psi(s) + \delta\right] , \qquad (A1)$$

where $\beta(s)$ is the betatron amplitude function, $\Psi(s) = \int(ds/\beta)$ is the betatron phase function and a and δ are arbitrary constants. In terms of the "normalized" displacement

$$\bar{x} = \sqrt{\beta_n/\beta}\ x \qquad (A2)$$

and of the longitudinal co-ordinate Ψ, the betatron oscillation is reduced to a harmonic oscillation. β_n is a constant which has the same dimensions as β, such that $\bar{x}$ has the same dimensions as x. β_n also acts as a scaling factor.

According to Eq. (A2), at any particular azimuth the phase-plane $(x,\ x' \equiv dx/ds)$ linearly transforms into the "normalized" phase plane $(\bar{x},\ \bar{x}' \equiv d\bar{x}/d\psi)$. In matrix notation :

$$\begin{pmatrix} \bar{x} \\ \bar{x}' \end{pmatrix} = N \begin{pmatrix} x \\ x' \end{pmatrix} \qquad (A3)$$

The matrix N is expressed by :

$$N = \sqrt{\frac{\beta_n}{\beta}} \begin{pmatrix} 1 & 0 \\ \alpha & \beta \end{pmatrix} \qquad (A4)$$

where $\alpha = -(d\beta/ds)/2$. The determinant of N is β_n. The inverse matrix of N is :

$$N^{-1} = \frac{1}{\sqrt{\beta\beta_n}} \begin{pmatrix} \beta & 0 \\ -\alpha & 1 \end{pmatrix} \qquad (A5)$$

The matrix for the transformation of the phase-plane $(\bar{x}, \bar{x}')$ from an azimuth s_1 to an azimuth s_2 is expressed as :

$$\bar{M}_{12} = \begin{pmatrix} \cos\Psi & \sin\Psi \\ -\sin\Psi & \cos\Psi \end{pmatrix} \qquad (A6)$$

where $\Psi = \Psi(s_2) - \Psi(s_1)$. This shows that on the normalized phase-plane particles rotate on a circle centred on their equilibrium orbit.

The transformation of the complex numbers $Y = 1/Z$ which describe the emittance ellipses (6), from the un-normalized phase-plane (Y) to the normalized phase-plane $(\bar{Y})$ is :

$$\bar{Y} = \beta\, Y - i\, \alpha \qquad (A7)$$

where β and α are the betatron functions (7). In the normalized phase-plane an emittance ellipse which is matched to the betatron functions of the lattice is represented by a circle. Therefore it is characterized by $\bar{\bar{Y}} = 1$.

References

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TABLE 2. DATA CARDS

DATA FOR READING IN A CHANNEL (SECTION 3.1)

		FORMAT
CONTROL CARD	bb, HEADING	A2,A8,7A10
	N	I5
N CARDS	ELEMENT,CODE,L, ANGLE, K, M	A8,I2,F10.4,F10.7,F10.5,I10
ONLY IF AMONG PREVIOUS CARDS AN	2x3 HORIZONTAL MATRIX	10X,6F10.5
ELEMENT HAS CODE=6 OR 7	2x3 VERTICAL MATRIX	10X,6F10.5
	FH, FV, FF, FD	4F10.5

TRACKING (SECTION 3.2)

CONTROL CARD	TR, SUB-HEADING	A2,A8,7A10
	X,X',DP,Z,Z',FF,FD,I1,I2,NKICK,INORM	7F10.5,3I3,I1
NKICK CARDS	I,HOR.KICK,VERT.KICK	I5,2F10.5
ONLY IF INORM=1	BETAHN,BETAH,ALPHAH,BETAVN,BETAV,ALPHAV,ALPHAP,DALPHAP	8F10.5

BEAM ENVELOPE CALCULATIONS (SECTION 3.3)

CONTROL CARD	BE, SUB-HEADING	A2,A8,7A10
	RIH,XIH,RIV,XIV,BETAH,ALPHAH,BETAV,ALPHAV	8F10.5
	EH/PI,EV/PI,I1,I2,NMOD	2F10.5,3I5
NMOD CARDS	I,DK/K	I5,F10.5

TRANSFER MATRICES (SECTION 3.4)

CONTROL CARD	MX, SUB-HEADING	A2,A8,7A10
	I1, I2	2I5

BETATRON PARAMETERS (SECTION 3.5)

CONTROL CARD	LA, SUB-HEADING	A2,A8,7A10
	I1, I2	2I5

TABLE 2 (Continued)

MATCHING (SECTION 3.6)

			FORMAT
CONTROL CARD		MH, SUB-HEADING	A2, A8, 7A10
* * * INSERT FIRST LOT OF DATA CARDS FOR MINUIT * * * SEE REFERENCE 8			
Z-VALUES, ZIH, ZIV AT LOCATION 1		FAH, FAV, FAXN, FAZN, FAHM, FAVM, WHMAX, WVMAX	8F10.5
CARD 1 (Z'S GIVEN)		RIH, XIH, RIV, XIV, BETAIH, ALPHAIH, BETAIV, ALPHAIV	8F10.5
IF CARD 1 IS BLANK THEN READ			
CARD 2 (Z'S COMPUTED)		I1, I2	2I5
Z-VALUES, ZMH, ZMV AT LOCATION 2			
ONLY IF FAMH OR FAMV ≠ 0			
CARD 1			8F10.5
CARD 2			2I5
Z-VALUES, ZOH, ZOV AT LOCATION 3			
CARD 1			8F10.5
CARD 2			2I5
		EH/PI, EV/PI, NQUAD	2F10.5, I5
CARDS FOR INTERQUADRUPOLE SECT.	(1)	I1, I2, AH MATRIX,	2I5, 6F10.5
		AV MATRIX,	10X, 6F10.5
CARD FOR MATCHING QUADRUPOLE	(1)	NAME, CODE, L, SMIN, SMAX, KMIN, KMAX, KP, LP, IQ	A8, I2, 5F10.5, 3I5
INTERQUADRUPOLE SECTION	(2)		
MATCHING QUADRUPOLE	(2)		
⋮	⋮		
MATCHING QUADRUPOLE	(NQUAD)		
INTERQUADRUPOLE SECTION	(NQUAD+1)		
* * * INSERT MINUIT 'COMMAND' DATA CARDS * * * SEE REFERENCE 8			

GEOMETRY (SECTION 3.7)

CONTROL CARD	GE, SUB-HEADING	A2, A8, 7A10
	X, Y, Z, GISEMENT, VERTICAL ANGLE (INITIAL), I1, I2	5F10.5, 2I5
	X, Y, Z, GISEMENT, VERTICAL ANGLE, (FINAL)	5F10.5

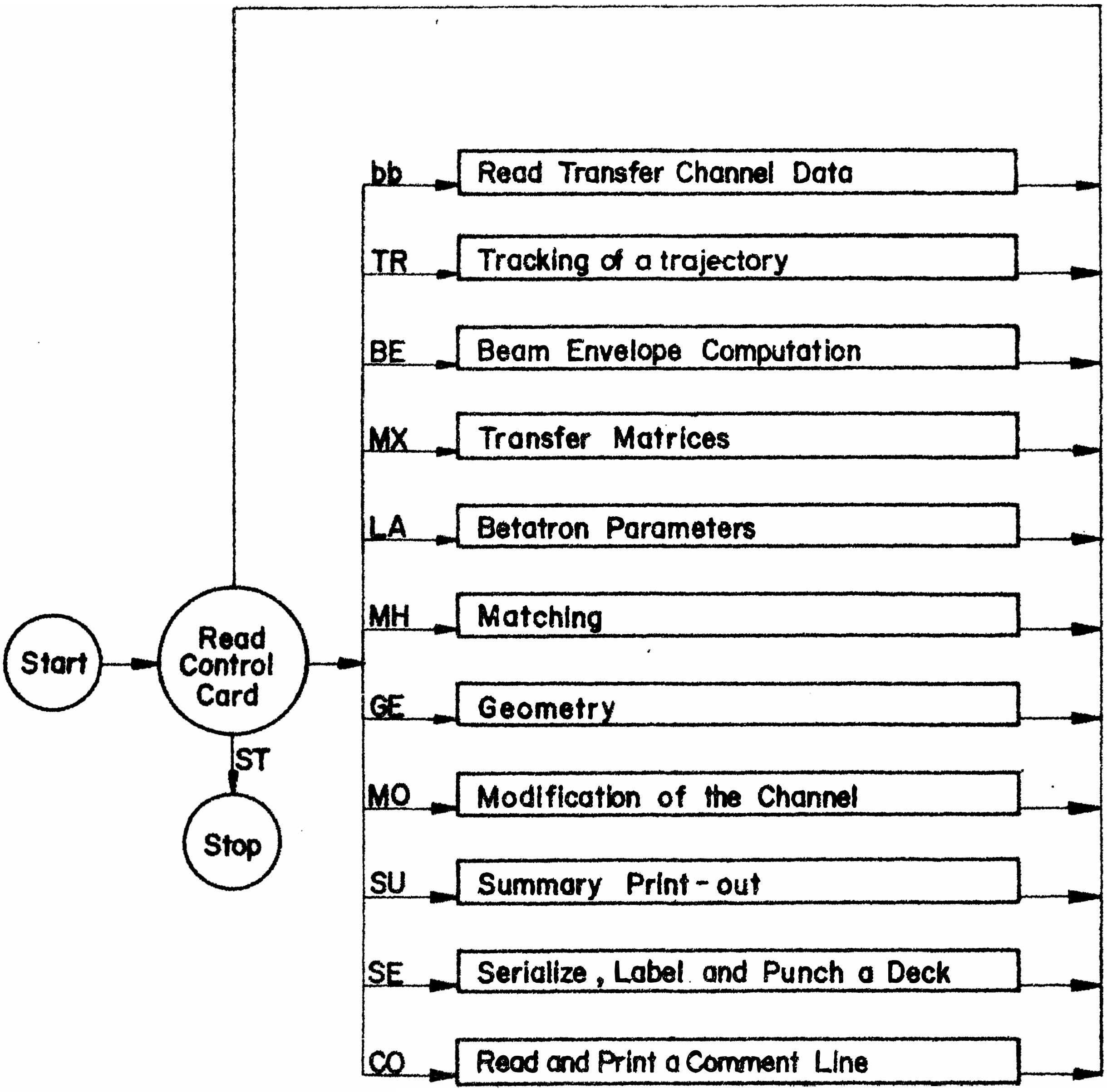

Fig. 1: **Block Diagram**

TABLE 2 (Continued)

			FORMAT
MODIFICATION OF CHANNEL (SECTION 3.8)			
CONTROL CARD	MØ, SUB-HEADING		A2, A8,7A10
	NMOD		I5
THEN FOLLOW NMOD SETS OF DATA			
	I1, I2	1 SET OF DATA	2I5
(I2-I1+1) CARDS	ELEMENT, CODE, L, ANGLE, K, M		A8,I2,F10.4,F10.7,F10.5 I10
ONLY IF AMONG PREVIOUS CARDS AN ELEMENT HAS CODE=6 OR 7	2x3 HORIZONTAL MATRIX		10X, 6F10.5
	2x3 VERTICAL MATRIX		10X, 6F10.5
	FH, FV, FF, FD		4F10.5
SUMMARY PRINT-OUT (SECTION 3.9)			
CONTROL CARD	SU		A2,A8,7A10
SERIALIZING A DECK (SECTION 3.10)			
CONTROL CARD	SE (LABEL IS IN COLUMNS 61-70)		A2,A8,7A10
	FH, FV, FF, FD		4F10.5
COMMENT (SECTION 3.11)			
CONTROL CARD	C¢, COMMENT		A2, A8, 7A10