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THE ANALYSIS AND DESIGN OF THIN STAINLESS STEEL MEMBRANES SUBJECTED TO UNIFORM PRESSURE LOADING

by

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[redacted]

Geneva, 10th August, 1970

The object of the present paper is to analyse and develop a design guide for thin, stainless steel membranes with clamped edges subjected to uniform pressure such as vacuum system beam windows. The investigation covers the full stress range to failure and includes the following data and graphs to find:

1. Maximum stress in and deflection of the membrane material.

2. Forces on the supporting structure due to the loaded membrane.

Two separate configurations are considered ; infinitely long rectangular membranes and circular ones. (A similar method of solution has previously been applied to the investigation of circular nylon plastic membranes[1)]). Very limited data is available up to the present time to check the predicted values for the rectangular windows. A number of tests have, however, been made so far for the circular membranes and do in fact check reasonably well with the theory.

Glossary of symbols

y = Height of membrane at any point, cm.

y_o = Maximum deflection at center of membrane, cm.

X = Radial distance to any point on the membrane, cm.

a = Radius to the edge of a circular membrane or the half height of a rectangular one, cm. (This should include any small corner radius used at the clamped edge).

t = Material thickness at any point, inches, cm.

t_o = Original material thickness, inches, cm.

t_1 = Minimum material thickness at the center of a strained membrane, inches, cm.

R = Cylindrical radius of curvature for rectangular membrane.

R_m = Meridional radius of curvature, inches, cm, for circular membrane.

R_t = Tangential radius of curvature, inches, cm, for circular membranes .

p = Applied pressure, psi, kg/cm^2.

ϵ = Strain at any point inches/inch, cm/cm or mm/mm.

ϵ_{min} = Minimum meridional strain at edge of membrane, inches/inch, cm/cm or mm/mm.

ϵ_{max} = Maximum meridional and tangential strain at the center of the membrane, inches/inch, cm/cm or mm/mm.

ϵ_t = Strain in the thickness direction at center of the membrane, cm/cm or mm/mm.

E = Modulus of elasticity kg/cm^2.

T = Force in membrane per unit length along the clamped edge, kg/cm

S = Stress, kg/cm^2

S_m, S_t = Meridional and tangential stress, psi, for circular membranes.

ℓ = Meridional arc distance ABC measured along the curve having a varying radius, R_m. See page 5.

CASE I : INFINITELY LONG, RECTANGULAR MEMBRANE

a. Assumptions

1. Material : 300 series stainless steel.
2. Fixed edges - clamped flat before loading.
3. Uniform pressure loading.
4. Curvature assumed to be cylindrical.
5. Linear thickness variation from edge to center assumed.
6. Effects of strain rate and creep are neglected.
7. Poisson's ratio remains ≈ 0.3 throughout the elastic and plastic stress range.*
8. Bending stresses are neglected.

Object

To analyse and develop a design guide for such membranes which will cover the full stress range to ultimate failure and provide convenient data in graphical form giving :

1. Maximum stress in the membrane material.
2. Maximum deflection of membrane under load.
3. Forces on the supporting structure due to the loaded membrane.

b. Procedure

Curve fit typical existing stress-strain data 300 series stainless steel to obtain an equation for stress vs. strain (above the yield point) for purposes of the analysis. See Table I below and Figure I.

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* $\lambda = \dfrac{\text{lateral unit deformation}}{\text{linear unit deformation}} = 0.3$

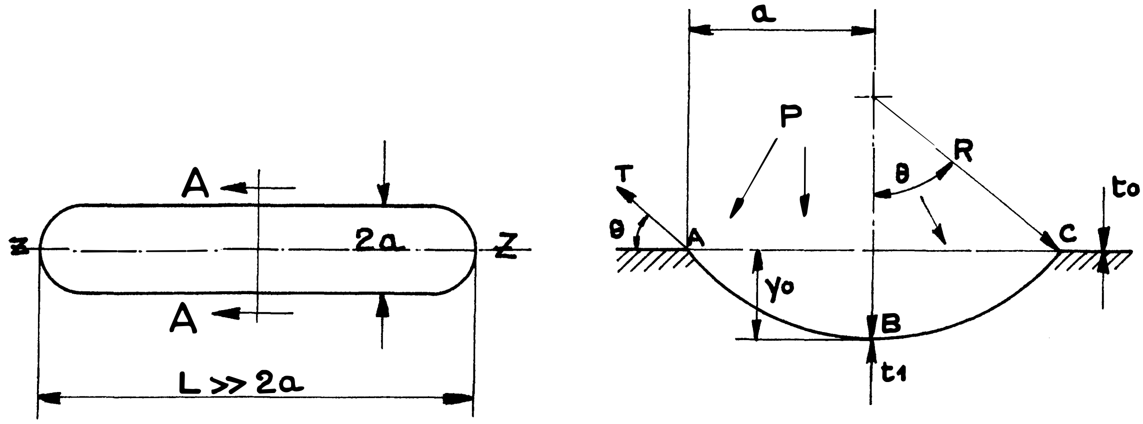
TABLE I

-|-		-|-				
Tensile Load		ε , Strain	* Stress $S = 1.05\left[\varepsilon^{0.178}\right].10^5$	* Stress $S = 7390\varepsilon^{0.178}$	Stress, S,(CERN 18-8 steel)	
Kpsi	kg/cm²	IN./IN or mm/mm	Kpsi	kg/cm²	Kpsi	** kg/cm²
35	2460	.002	34.9	2460	38.4	2700(.2% yield)
50	3515	.015	49.85	3500		
66.6	4690	.075	66.25	4655		
83.3	5850	.234	81.1	5710		
91.2	6410	.467	91.6	6450		
-		.52	93.5	6580	95.2	6700(Rupture)
80	5630	.528	93.8	6600		

* Obtained by curve fitting - use for stress levels > 35000 psi.
<u>Note</u> : The first two columns of data (-|-) were obtained from ASM handbook - 8th Edition for room temperature condition, annealed, 304 stainless steel. (Graph Figure 4, Page 548).

** Check data from M. Bacher - CERN, 17.1.1968.

c. Analysis.



Sect. AA THROUGH WINDOW

Radius of curvature in Z direction along center of membrane is approximately ∞ , therefore the stress, S_z, in that direction is also approximately zero. (For any finite length, however, S_z will have a finite value but it will be diminishingly small as $\frac{L}{2a}$ becomes large).

For the cylindrical shape, the force T in the membrane is constant because of constant radius and the stress is given by $\frac{T}{t}$. Poisson's ratio for the material is assumed = 0.3. It is also assumed that the thickness reduces linearly from edge to center. Figure I.II in the appendix, although for circular membranes, would indicate that this is approximately true.

One can write $\frac{T}{t_1} = S_{max}$ at the center and $\frac{T}{t_o} = S_{min}$ at the clamped edge. Beyond yield, $S = 1.05\epsilon^n 10^5$ from part a above and

$$\frac{S_{max}}{S_{min}} = \frac{t_o}{t_1} = \frac{1.05\ \epsilon^n_{max} \cdot 10^5}{1.05\ \epsilon^n_{min} \cdot 10^5}$$

since there is only one principal stress,

$$t_1 = t_o - .3\epsilon_{max} t_o$$

•

$$\frac{\epsilon^n_{max}}{\epsilon^n_{min}} = \frac{t_o}{t_o(1 - .3\epsilon_{max})} , \quad \epsilon^n_{max}(1 - .3\epsilon_{max}) = \epsilon^n_{min}$$

From the geometry of the figure above, the strained arc length, ABC $= 2R\,\theta = \ell$ where

$$\sin\theta = \frac{a}{R} \quad \text{and} \quad R = \frac{a^2 + y_o^2}{2y_o}$$

Also, for a linear thickness variation,

$$\left[\frac{\epsilon_{max} + \epsilon_{min}}{2}\right]a + a = \ell = \text{Arc ABC}$$

and

$$\epsilon_{max} + \epsilon_{min} = \frac{\ell - 2a}{a} = B, \quad \epsilon_{min} = B - \epsilon_{max} .$$

Substituting from above, $\epsilon^n_{max}(1 - .3\epsilon_{max}) = \epsilon^n_{min} = (B - \epsilon_{max})^n$

which reduces to $\epsilon_{max}(1 + [1 - .3_{max}]^{1/n}) = B$

where n = .178. This equation is solved for ϵ_{max} by trial and error from which S_{max} may be obtaines using the relation

$$S_{max} = 7390\ \epsilon^{.178}\ \text{kg/cm}^2$$

TABLE II

$\frac{y_o}{a}$	$\frac{\ell}{2a}$	$\varepsilon_{min} + \varepsilon_{max} = B$	ε_{max}	$\frac{t_1}{t_o} = 1 - .3\varepsilon_{max}$	S_{max} $= 1.05\left[\varepsilon_{max}^{0.178}\right].10^5$ psi	S_{max} $7390\varepsilon^{0.178}$ kg/cm^2
.01	1.0001	0.0002	0.0001	0.99997	* 2800	* 197
.02	1.0004	0.0008	0.0004	0.99988	* 19200	* 1350
.05	1.0025	0.005	0.0025	0.99925	36200	2545
.1	1.0067	0.0134	0.00675	0.99798	43100	3030
.2	1.027	0.054	0.0275	0.99175	55400	3890
.3	1.06	0.12	0.063	0.9811	64000	4500
.4	1.103	0.206	0.113	0.9661	71100	5000
.5	1.16	0.320	0.185	0.9445	77800	5460
.6	1.225	0.450	0.275	0.9175	83500	5870
.7	1.300	0.600	0.405	0.8785	89500	6290
.8	1.384	0.768	0.569	0.8293	95100	6700

* Values of stress below the yield point were calculated using the relation $S = 28 \cdot 10^6 \cdot \varepsilon = E\varepsilon$ psi (or $S = 1.97 \cdot 10^6 \cdot \varepsilon$ kg/cm^2).

Here, it is useful to determine the ratio $\frac{pa}{to}$ which is found as follows:

$$R = \frac{a^2 + y_o^2}{2y_o}$$

$$S_{max} = A\varepsilon_{max}^n = \frac{pR}{t_1} = \frac{P}{t_o(1 - .3\varepsilon_{max})}\left[\frac{a^2 + y_o^2}{2y_o}\right]$$

$$2S_{max}\,(1 - .3\epsilon_{max}) = \frac{pa}{t_o}\left[1 + \frac{y_o^2}{a^2}\right]\frac{1}{y_o/a}$$

$$\frac{pa}{t_o} = 2S_{max}\left[1 - .3\epsilon_{max}\right] y_o/a \; \frac{1}{1 + y_o^2/a^2}$$

Both $\frac{pa}{t_o}$ and $\frac{y_o}{a}$ are plotted against maximum stress ; see Figure 2 in Appendix. The tensile force in the membrane per unit length along the clamped edge, T , is found from

$\frac{T}{t_o} = S_{min}$, the stress in the membrane at the clamped edge.

$$T = S_{min}t_o$$

Dividing both side by pa,

$$\frac{T}{pa} = \frac{S_{min}t_o}{pa} = \frac{S_{min}}{pa/t_o}$$

In this form, the term is non-dimensional and need only be multiplied by pa to obtain T directly, in any consistent system of units.

S_{min}, $\frac{pa}{t_o}$ and $\frac{T}{pa}$ are given below in Table III.

TABLE III

$\frac{y_o}{a}$	ϵ_{min} = B- ϵ_{max}	S_{min} = $1.05\left[\epsilon_{min}^{0.178}\right] 10^5$ psi	$\frac{pa}{to}$ psi	$\frac{pa}{to}$ kg/cm^2	$\frac{T}{pa}$ = $\frac{S_{min}}{pa/to}$	* θ DEGREES	$\frac{1}{T/pa}$
.01	0.0001	2800	56	3.94	50	1.146	.02
.02	0.0004	19200	766	54	25	2.293	.04
.05	0.0025	36200	3600	253	10.04	5.71	.0995
.1	0.0665	43000	8510	600	5.05	11.42	.198
.2	0.0265	55100	20950	1475	2.63	22.213	.3805
.3	0.057	63000	33900	2390	1.858	32.615	.539
.4	0.093	68800	46100	3250	1.491	42.1445	.671
.5	0.135	73500	56600	3990	1.295	50.625	.773
.6	0.175	77000	65000	4560	1.186	57.673	.845
.7	0.195	78500	71700	5050	1.093	66.206	.915
.8	0.199	78900	77000	5410	1.022	78.233	.979

$\frac{T}{pa}$ as well as θ are both plotted against $\frac{y_o}{a}$ on a separate graph, Fig. 4, in the appendix.

In order to find the loading at the window support, N_o , refer to Fig. 5 in the appendix. The graph and the data given are shown for the case of a rectangular vacuum box but can equally well be used for circular or elliptically shaped vacuum tubes. Note that N_o, is the total load at the window support including the effects of external pressure on both the vacuum chamber and the window but assuming zero deflection of support. However, when deflection of the window support is taken into account, the force N_o will often be substantially reduced.

The graph of Fig. 6, $\frac{y_o}{a}$ vs $\frac{pa}{to}$, turns out to be reasonably linear and is a useful one for plotting actual data on test windows.

* Note: $T \sin\theta = pa$, $\frac{pa}{T} = \frac{1}{T/pa} = \sin\theta$.

CASE II : CIRCULAR STAINLESS STEEL MEMBRANES

a. Assumptions

The assumptions here are the same as those made for the rectangular membranes in part I-a above except for the shape of the membrane (part I-a-4). Here, the shape is assumed to be the so-called Hencky curve which is approximated by the relationship

$$\frac{Y}{y_o} = \frac{1-0.9\ x^2}{a^2} - \frac{0.1\ x^5}{a^5}$$ see Fig. 7

This curve, according to H.H. Stevens[3)], is supported by considerable experimental data on plastic membranes streched both elastically and plastically.

b. Analysis

1. Determination of maximum stress and maximum strain.

Referring to Fig. 8, the equation of equilibrium for any thin-walled vessel having the form of a surface of revolution and subjected to a uniform internal pressure is, from Timoshenko[(4)],

$$\frac{S_m}{R_m} + \frac{S_t}{R_t} = \frac{p}{t}$$

To solve this equation, first find R_m , the meridional radius of curvature.

$$R \text{ for any curve in general} = \frac{\left[1 + (dy/dx)^2\right]}{\frac{d^2y}{dx^2}}$$

$$y/y_o = 1 - \frac{0.9\ x^2}{a^2} - \frac{0.1\ x^5}{a^5} \quad \text{Hencky Eq.}$$

$$\frac{dy}{dx} = - \frac{1.8y_o x}{a^2} - \frac{.5y_o x^4}{a^5}$$

$$\frac{dy^2}{dx^2} = - \frac{1.8y_o}{a^2} - \frac{2y_o x^3}{a^5}$$

Note: Minus signs above merely indicate that the curve is concave downward.

$$\text{then } R_m = \frac{\left[1 + \left(\frac{1.8y_o x}{a^2} + \frac{0.5y_o x^4}{a^5}\right)^2\right]^{3/2}}{\frac{1.8y_o}{a^2} + \frac{2y_o x^3}{a^5}}$$

at $x = 0$, $S_m = S_t$ and $R_m = R_t = \frac{a^2}{1.8y_o}$ at the center of the membrane. ($R_m \neq R_t$ at any other value of x.)

Both maximum stress, S_{max}, and maximum strain, ε_{max}, occur at the center of the membrane.

$S_{max} = A\varepsilon^n_{max}$, (see Table I above) $\frac{S_{max}}{R_m} = \frac{p}{2t_1} = \frac{A\varepsilon^n_{max}}{a^2}(1.8y_o)$

$$\varepsilon^n_{max} = \frac{pa^2}{3.6Ay_o t_1} \text{ and } S_{max} = \frac{pa^2}{3.6y_o t_1}$$

In order to get the minimum thickness t_1 at the center in terms of the original material thickness, t_o, consider ε_t, strain in the thickness direction = 2 (μ ε_{max}) where μ is Poisson's ratio assumed = 0.3. Thus $\varepsilon_t = 2(0.3)\varepsilon_{max}$ (the factor of 2 is needed because there are two equal principal stresses at $x = 0$.). Further,

$$(t_o - 0.6\ \varepsilon_{max}\ t_o) = t_1$$

$$\text{and } S_{max} = \frac{pa^2}{3.6 y_o t_o} \left(\frac{1}{1 \,.\, 0.6\, \varepsilon_{max}} \right)$$

2. Referring again to Fig. 8 the average strain can be expressed in terms of the arc length

as arc $AB = \frac{\ell}{2} = a + \left(\frac{\varepsilon_{min} + \varepsilon_{max}}{2} \right) a$ assuming a linear increase in strain from edge to center.

and $\varepsilon_{min} + \varepsilon_{max} = \left(\frac{\ell}{2} - a \right) \frac{2}{a}$ The sum, $\varepsilon_{min} + \varepsilon_{max}$, will be evaluated by finding the strained arc lengths for various values of y_o/a. Integration of $d\ell$ for the Hencky Equation is too difficult. A glance at Fig. 7, however, shows that reasonable values can be obtained by averaging $\ell/2$ for a circle and $\ell/2$ found by integrating $d\ell$ for the simplified equation,

$$y/y_o = 1 - \frac{x^2}{a^2} \,.$$ This was done as follows:

$$\ell/2 \text{ for circle} = R\theta \text{ where } R = \frac{a^2 + y_o^2}{2y_o} \text{ and } \sin\theta = \frac{a}{r}$$

$$\text{for } y/y_o = 1 - x^2/a^2 \quad ; \quad dy = \frac{2xy_o dx}{a^2}$$

$$d\ell = \sqrt{dx^2 + dy^2} = \sqrt{dx^2 + \frac{4x^2 y_o^2 dx^2}{a^4}}$$

$$\text{the arc length, } \frac{\ell}{2} = \int_{x=0}^{x=a} d\ell = \int_0^a dx \sqrt{1 + \frac{4x^2 y_o^2}{a^4}}$$

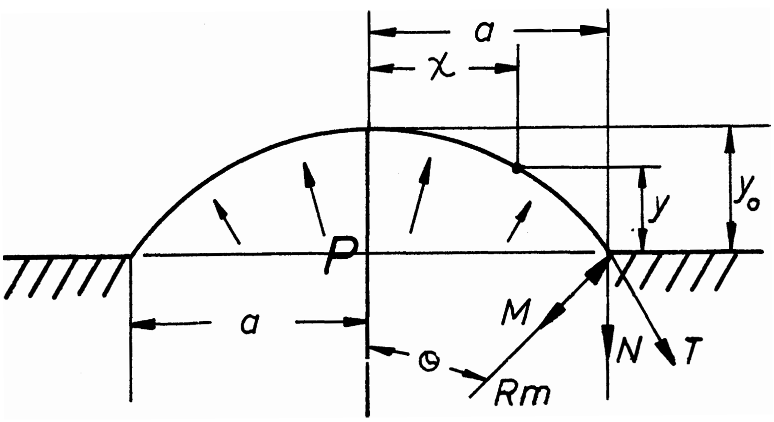
$$\text{and } \frac{\ell}{2} = \frac{a}{2} \sqrt{1 + \frac{4y_o^2}{a^2}} + \frac{a^2}{4y_o} \sinh^{-1}\left[\frac{2y_o}{a}\right] .$$

TABLE IV

STRAINED ARC LENGTHS

y_o/a	$\ell/2a$ Circle	$\ell/2a$ $y/y_o = 1-x^2/a^2$	$\ell/2a$ Average	$\varepsilon_{min} + \varepsilon_{max} = \left[(\ell/2-a)\frac{2}{a}\right]$
.01	1.0001	1.0001	1.0001	0.0002
.02	1.0004	1.0004	1.0004	0.0008
.05	1.0025	1.0025	1.0025	0.005
.1	1.0067	1.0067	1.0067	0.0134
.2	1.027	1.026	1.0265	0.053
.3	1.06	1.057	1.059	0.118
.4	1.103	1.0983	1.101	0.202
.5	1.16	1.1477	1.154	0.308
.6	1.225	1.2043	1.215	0.430
.7	1.300	1.2667	1.283	0.566

3. In order to eliminate ε_{min} and get everything in terms of ε_{max}, next find min value of meridional strain, ε_{min}, at x=a



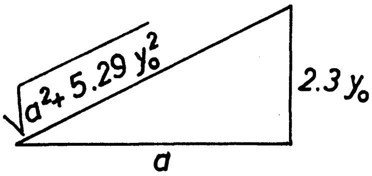
$N = p\pi a^2$ = Total load around circumference.

Summing vertical and horizontal components,

$$M\cos\theta + T\ \mathrm{Sin}\theta = N = p\pi a^2$$

$$M\sin\theta = T\cos\theta$$

from above, $\tan\theta = \frac{dy}{dx} = \frac{1.8y_o a}{a^2} + \frac{.5y_o a^4}{a^5} = \frac{2.3y_o}{a}$



$$\mathrm{Sin}\theta = \frac{2.3y_o}{\sqrt{a + 5.29y_o^2}} = \frac{2.3y_o}{a\sqrt{1 + 5.29y_y^2/a}}$$

$$\mathrm{Cos}\theta = \frac{a}{\sqrt{a^2 + 5.29y_o^2}} = \frac{1}{\sqrt{1 + 5.29y_o^2/a^2}}$$

$$M = \frac{T\cos\theta}{\sin\theta} = \frac{T}{\tan\theta} = \frac{Ta}{2.3y_o}$$

and $M\ \cos\theta + T\sin\theta = \frac{Ta}{2.3y_o} \cdot \frac{1}{\sqrt{1 + 5.29y_o^2/a^2}} + \frac{T(2.3y_o)}{\sqrt{a}\ \ 1+5.29y_o^2/a^2}$

Now the minimum value of meridional stress, S_{ma}, occurs at x=a

and, $S_{ma} = \frac{T}{2\pi a t_o}$ $\qquad T = S_{ma} 2\ (\pi a\ t_o)$

then $S_{ma}\ (2\pi a\ t_o)\left[\frac{a}{2.3y_o} + \frac{2.3y_o}{a}\right] = p\pi a^2 \sqrt{1 + \frac{5.29y_o^2}{a^2}}$

which reduces to

$$S_{ma} = \frac{1.15\, p y_o}{t_o\sqrt{1 + 5.29y_o^2/a^2}}$$

$S = A\varepsilon^n$ in general

Then min. meridional strain $\varepsilon^n_{min} = \frac{1.15 p y_o}{A t_o\sqrt{1+5.29y_o^2/a^2}}$ at x=a

From above, $\epsilon_{max}^{n} = \dfrac{pa^2}{3.6\ At_1 y_o}$

and $\left(\dfrac{\epsilon_{min}}{\epsilon_{max}}\right)^n = \left(\dfrac{1.15\, py_o}{At_o\sqrt{1 + 5.29 y_o^2/a^2}} \cdot \dfrac{3.6\ At_1 y_o}{pa^2}\right)$

or $\dfrac{\epsilon_{min}}{\epsilon_{max}} = \left[\left(\dfrac{4.14 y_o^2 t_1}{t_o a^2\sqrt{1 + 5.29 y_o^2/a^2}}\right)\right]^{\frac{1}{n}} = \left(\dfrac{Zt_1}{t_o}\right)^{\frac{1}{n}}$

where $\dfrac{1}{n} = 5.62$ and $Z = \dfrac{4.14 y_o^2 t_1}{t_o a^2 \sqrt{1 + 5.29 y_o^2/a^2}}$

t_1/t_o from above $= (1 - 0.6\ \epsilon_{max})$

and $\dfrac{\epsilon_{min}}{\epsilon_{max}} = \left[Z(1 - 0.6\ \epsilon_{max})\right]^{\frac{1}{n}}$

To eliminate ϵ_{min}, let

$$\epsilon_{min} + \epsilon_{max} = B$$

then $B - \epsilon_{max} = \epsilon_{max}\left[Z(1 - 0.6\ \epsilon_{max})\right]^{5.62}$

Solve for ϵ_{max} by trial and error knowing Z, B and n for various values of y_o/a. Note that for values of y_o/a up to 0.1, ϵ_{min} at the edge of the membrane is essentially zero.

Maximum stress, S_{max}, can then be obtained as well as the useful ratio $pa/_{to}$. Both are tabulated below for different values of $y_{o/a}$ and plotted on the graph of Fig. 10.

TABLE V

$\frac{y_o}{a}$	Z	B = $\varepsilon_{max} + \varepsilon_{min}$	ε_{min}	ε_{max}	$\frac{t_1}{t_o}$	S_{max} = $7390\varepsilon^{.178}$ kg/cm^2	$\frac{pa}{to}$
.01	.000414	.0002	~ 0	.0002	.9999	* 394	14.2
.02	.001654	.0008	↓	.0008	.9995	1580	113.2
.05	.01028	.005		.005	.9970	2880	517
.1	.04035	.0134	.00007	.0134	.9920	3400	1215
.2	.15048	.053	.0003	.053	.9682	4375	3045
.3	.3068	.118	.0007	.1173	.9292	5050	5075
.4	.4877	.202	.0017	.2003	.8798	5555	7045
.5	.6795	.308	.012	.296	.8224	5750	8850
.6	.8745	.430	.041	.389	.7666	6250	10400
.7	1.0703	.566	.068	.498	.7012	6550	11580

NOMINAL FAILURE ⟶ ** .528 ⟶ ** 6050 -|- 12800 → 1800

* S_{max} values obtained from $S_{max} = 1.97\ (10^6)\ \varepsilon_{max}$ for the region below yield stress.

Note: $\frac{pa}{to} = S_{max}\ (3.6\ \frac{y_o}{a})\ (1 - .6\ \varepsilon_{max})$

** Typical handbook values for certain 300 series stainless steels. Actual range of values from 5 tests at failure. Generally, as might be expected, the very thin material ruptured at the higher values of $\frac{pa}{to}$. Only two thicknesses of sheet were used; 35μ and .1mm (see Table VI below).

TABLE VI

Test data on round 300 series stainless steel windows:

Clamped edges

Uniform pressure loading.

All tested to rupture.

y_o/a	Rupture at : $\frac{pa}{t_o}$ kg/cm^2	Original Material thickness, t_o
.59	17750	35 μ
.61	16900 ± 600	35 μ
.735	15000	0.1 mm
.74	15000	0.1 mm
.74	15000	0.1 mm

Bibliography

1. PPA Technical Notes - report no. A-389, Sept. 4, 1968, D.L. Collins

2. ISR - EN / 70-32

3. H.H. Stevens, "Behaviour of Circular Membranes Stretched above the Elastic Limit by Air Pressure", Experimental Stress Analysis, Vol II, No. 1, (1944)pp. 139-146.

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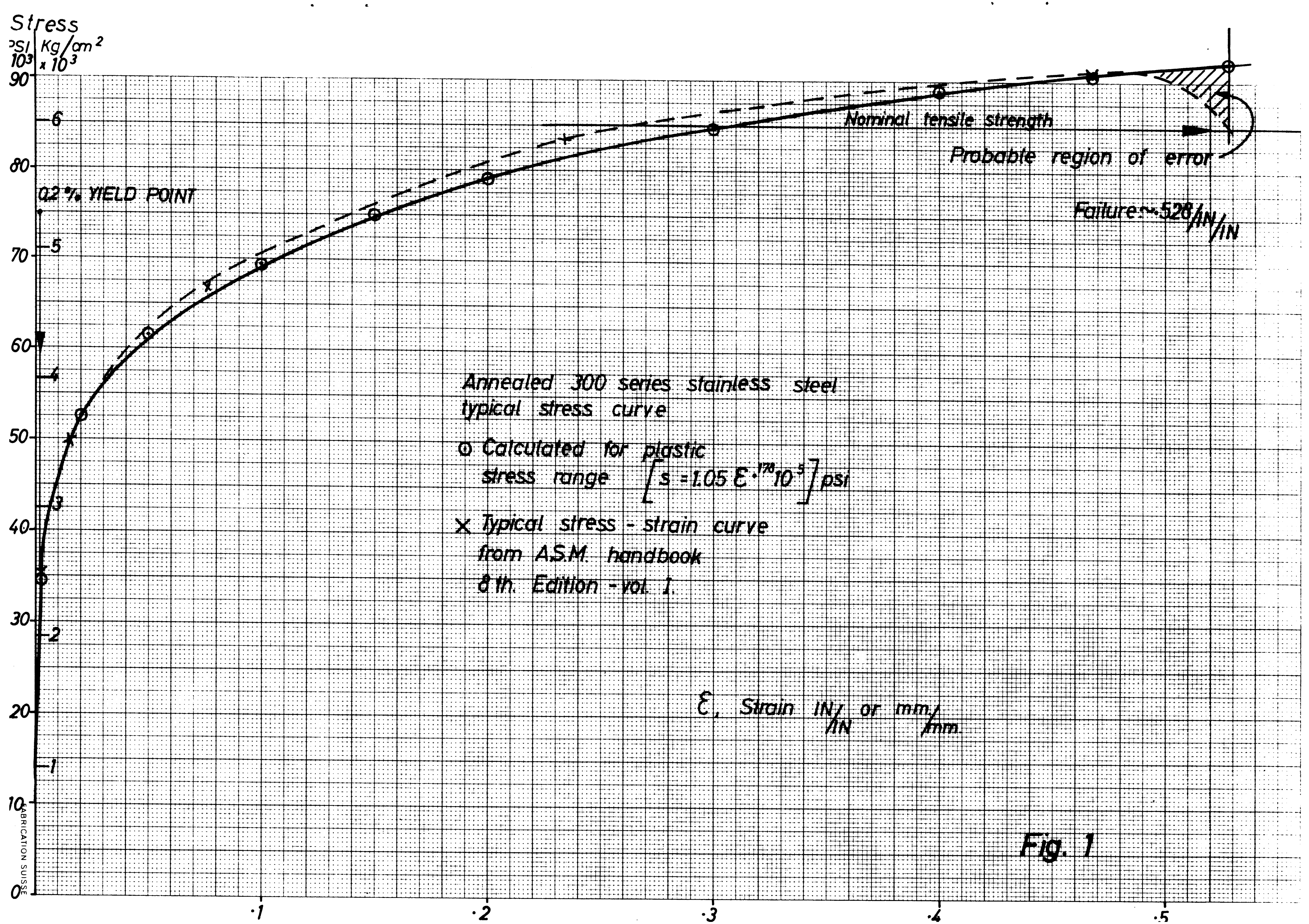


Stress
PSI 10³
Kg/cm² x 10³
90
80
70
60
50
40
30
20
10
0
6
5
4
3
2
1
0.2 % YIELD POINT
Nominal tensile strength
Probable region of error
Failure: ~.528 IN/IN
Annealed 300 series stainless steel
typical stress curve
⊙ Calculated for plastic
stress range [s = 1.05 ε^.178 10^5] psi
× Typical stress - strain curve
from A.S.M. handbook
8 th. Edition - vol. I.
ε, Strain IN/IN or mm/mm.
·1
·2
·3
·4
·5
FABRICATION SUISSE

Fig. 1

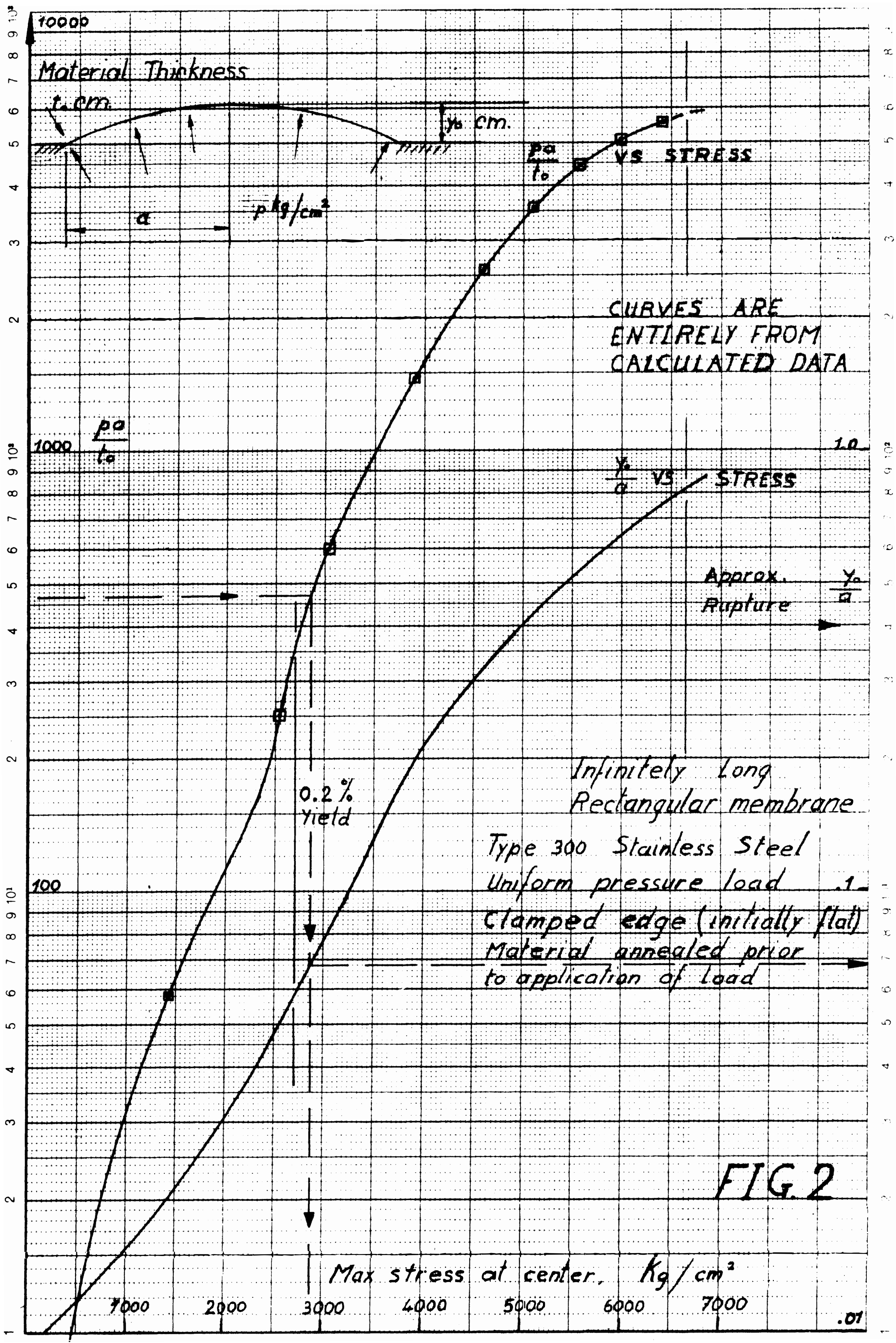


FIG. 2

210-552-4

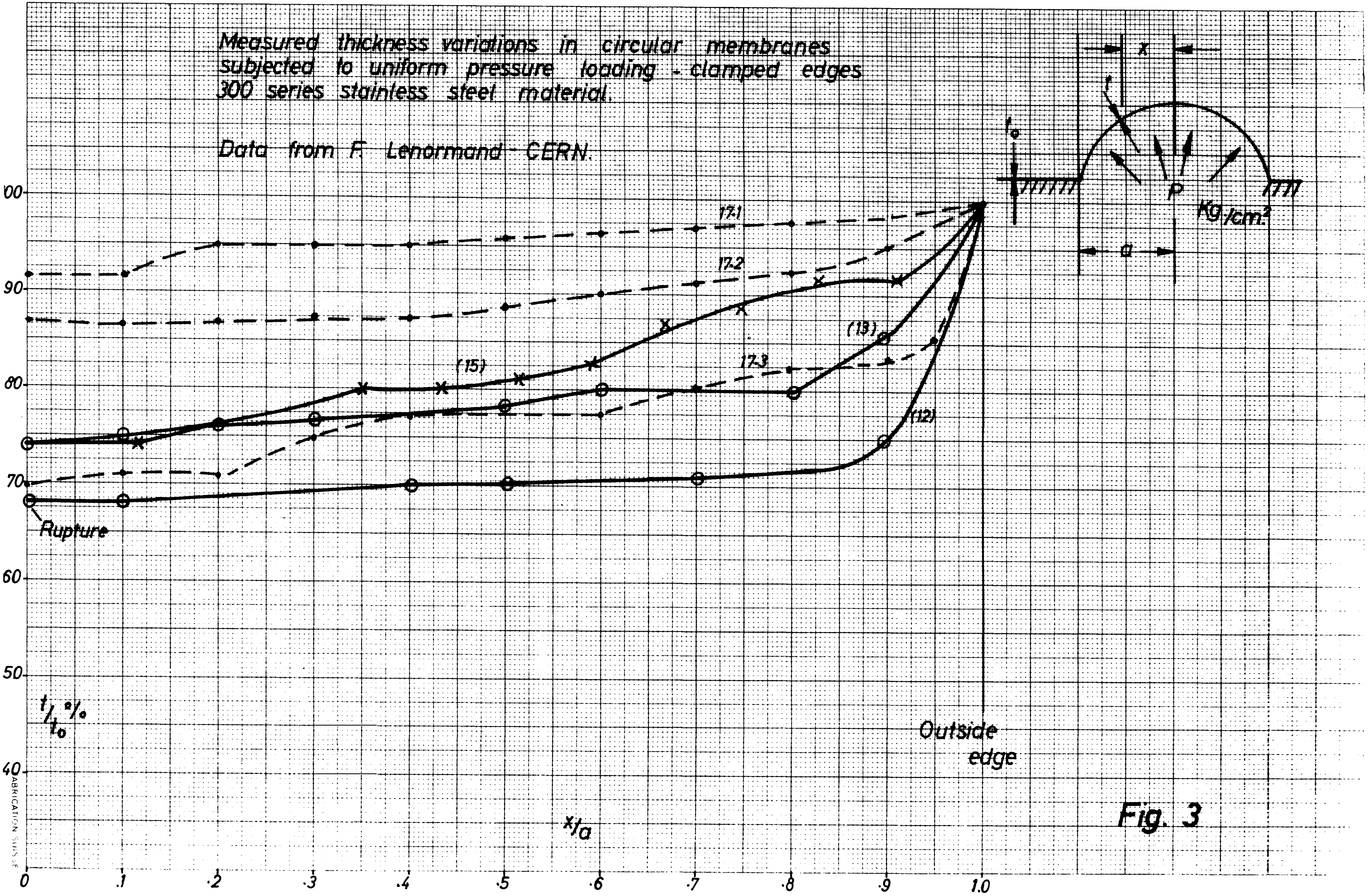
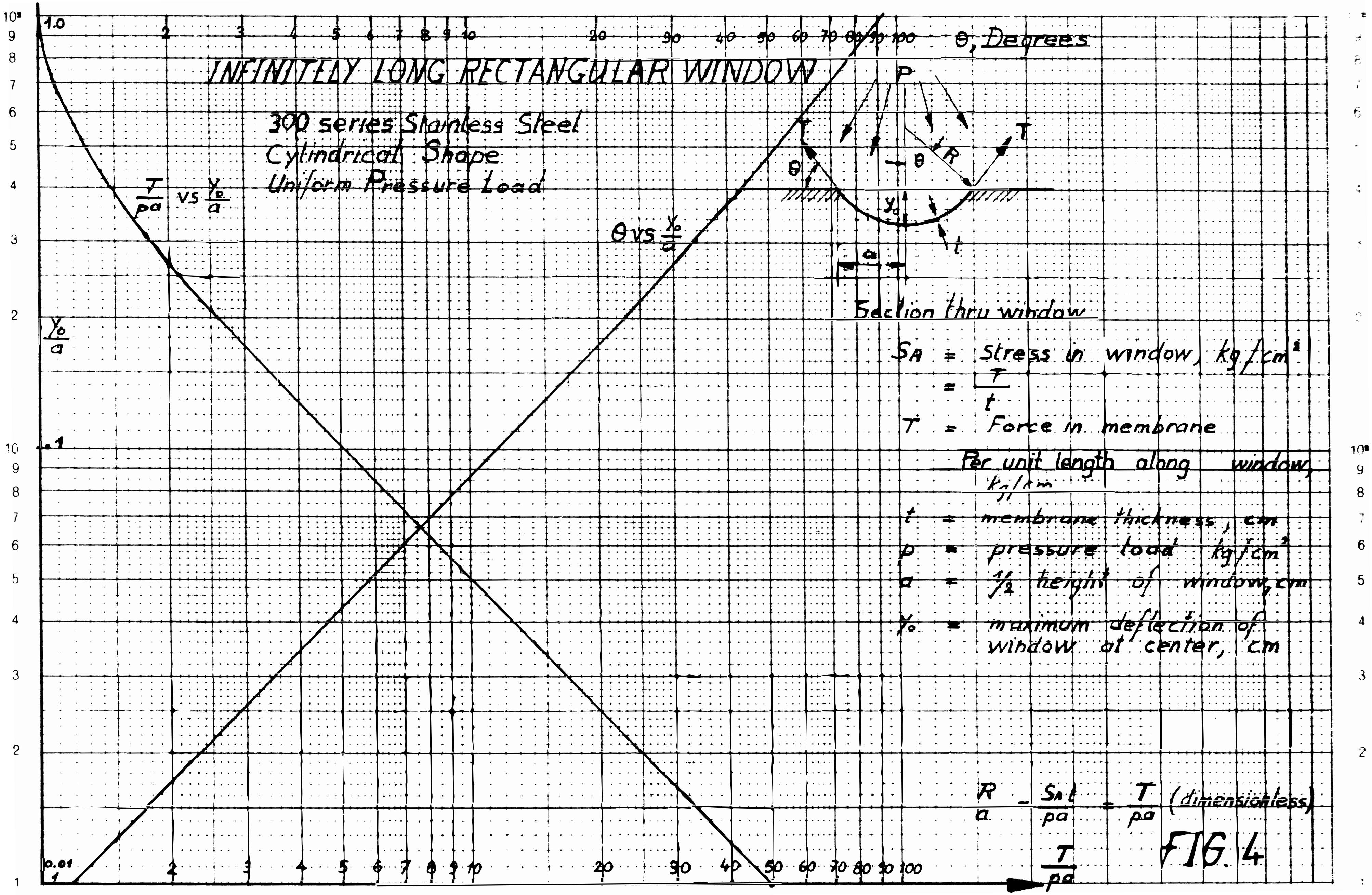

Measured thickness variations in circular membranes
subjected to uniform pressure loading - clamped edges
300 series stainless steel material.
Data from F. Lenormand - CERN.
17.1
17.2
17.3
(15)
(13)
(12)
Rupture
Outside edge
t
t_o
x
a
P
Kg/cm²
t/t_o %
x/a
100
90
80
70
60
50
40
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0

Fig. 3



INFINITELY LONG RECTANGULAR WINDOW
300 series Stainless Steel
Cylindrical Shape
Uniform Pressure Load
θ, Degrees
T/pa vs Y₀/a
θ vs Y₀/a
Y₀/a
Section thru window
P
T
R
θ
t
a
S_A = Stress in window, kg/cm²
= T/t
T = Force in membrane
Per unit length along window, kg/cm
t = membrane thickness, cm
p = pressure load kg/cm²
a = ½ height of window, cm
Y₀ = maximum deflection of window at center, cm
R/a = S_At/pa = T/pa (dimensionless)
T/pa

FIG. 4

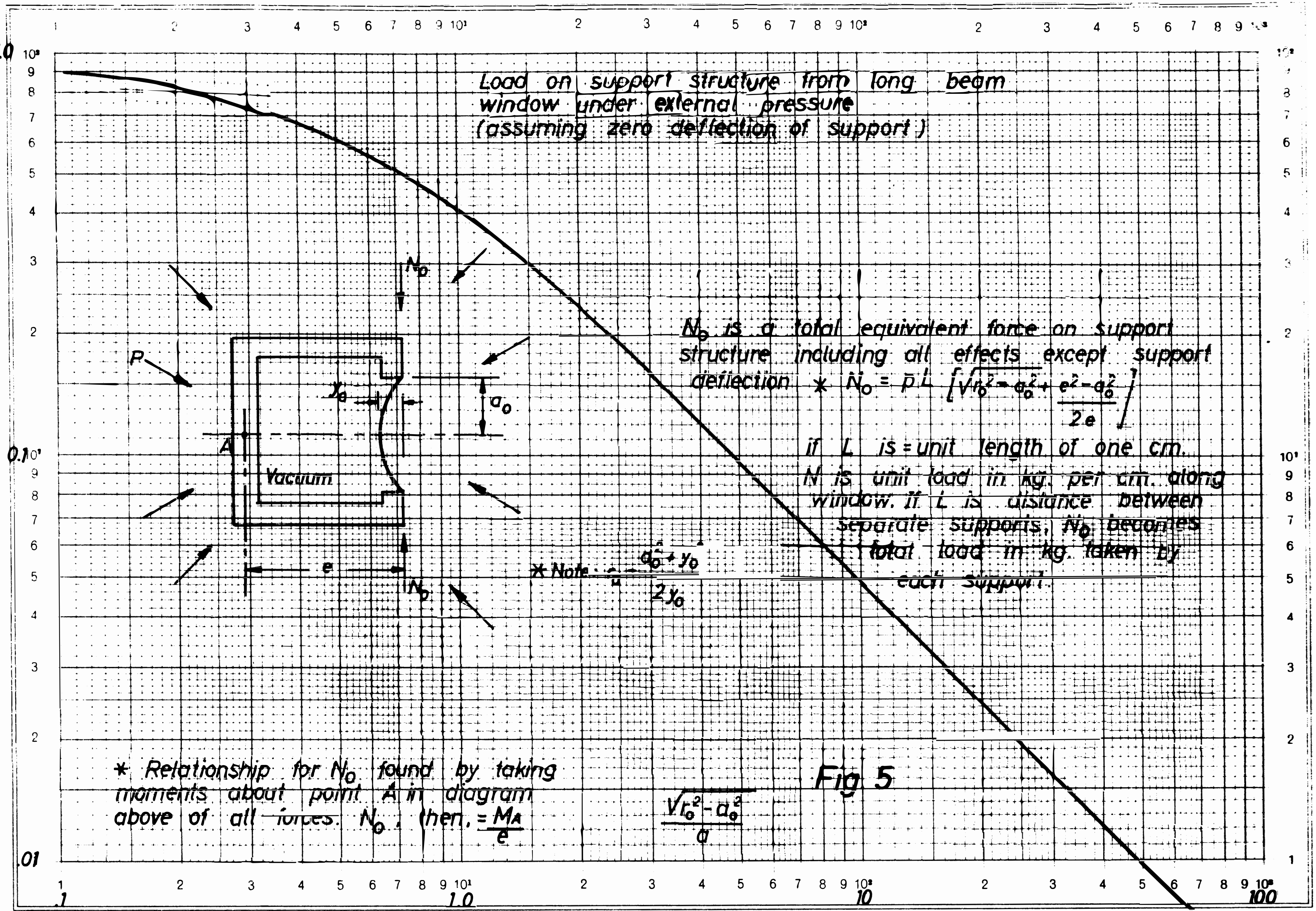

Load on support structure from long beam window under external pressure (assuming zero deflection of support)
N_o is a total equivalent force on support structure including all effects except support deflection
* $N_o = \bar{p} L \left[\sqrt{r_o^2 - a_o^2} + \frac{e^2 - a_o^2}{2e} \right]$
if L is = unit length of one cm. N is unit load in kg. per cm. along window. If L is distance between separate supports, N_o becomes total load in kg. taken by each support.
* Note $\frac{a_o^2 + y_o}{2 y_o}$
* Relationship for N_o found by taking moments about point A in diagram above of all forces. N_o then $= \frac{M_A}{e}$
$\frac{\sqrt{r_o^2 - a_o^2}}{a}$
$\frac{y_o}{a_o}$
P
A
Vacuum
y_o
a_o
e
N_o
1.0
0.1
.01
.1
1.0
10
100

Fig 5

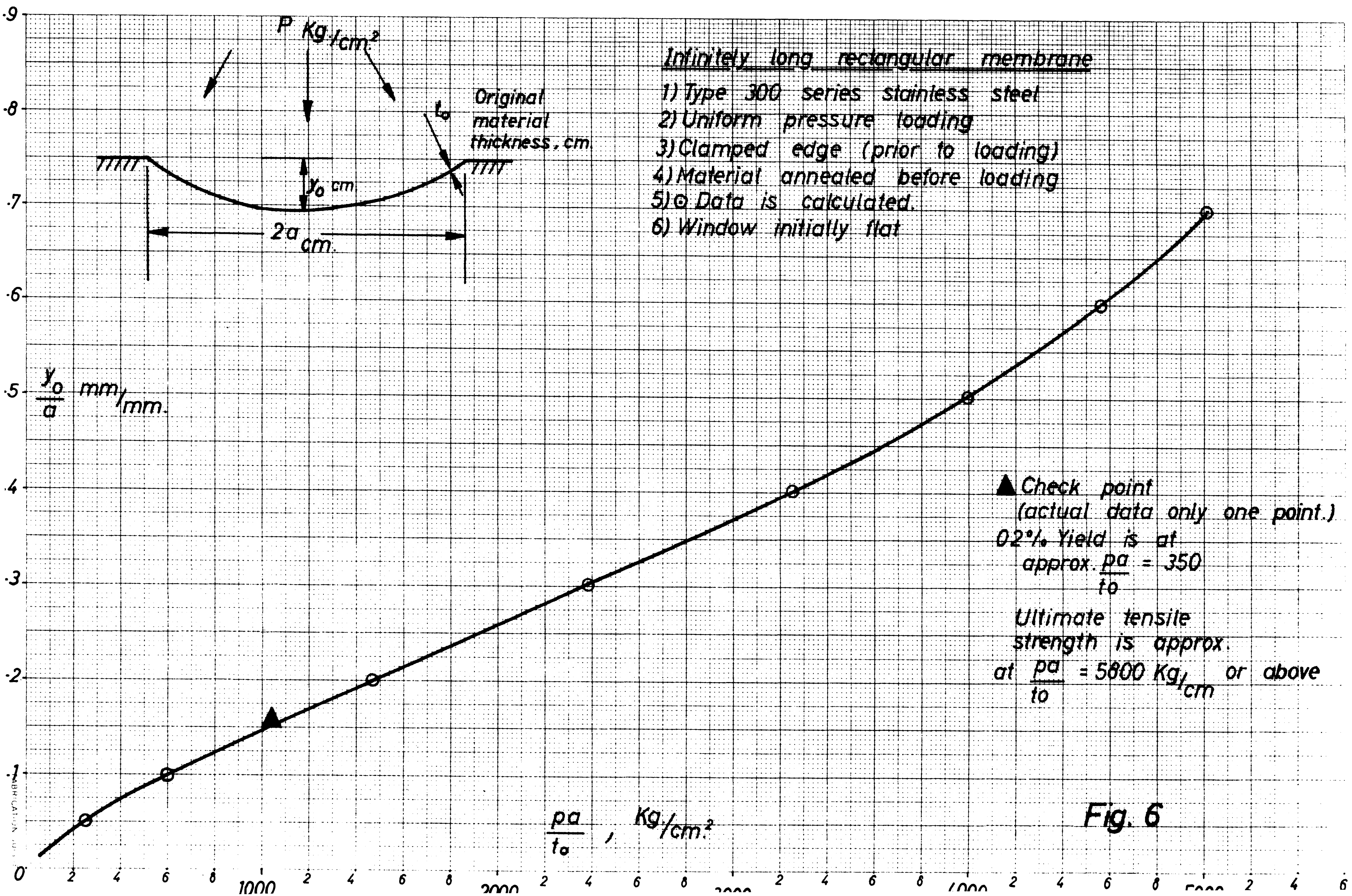

P Kg/cm²
Original material thickness, cm
t_o
y_o cm
2a cm.
Infinitely long rectangular membrane
1) Type 300 series stainless steel
2) Uniform pressure loading
3) Clamped edge (prior to loading)
4) Material annealed before loading
5) ⊙ Data is calculated.
6) Window initially flat
$\frac{y_o}{a}$ mm/mm.
▲ Check point
(actual data only one point.)
0.2% Yield is at
approx. $\frac{pa}{to}$ = 350
Ultimate tensile
strength is approx.
at $\frac{pa}{to}$ = 5800 Kg/cm or above
$\frac{pa}{t_o}$, Kg/cm²
·9
·8
·7
·6
·5
·4
·3
·2
·1
0
2
4
6
8
1000
2000
4000
5000

Fig. 6

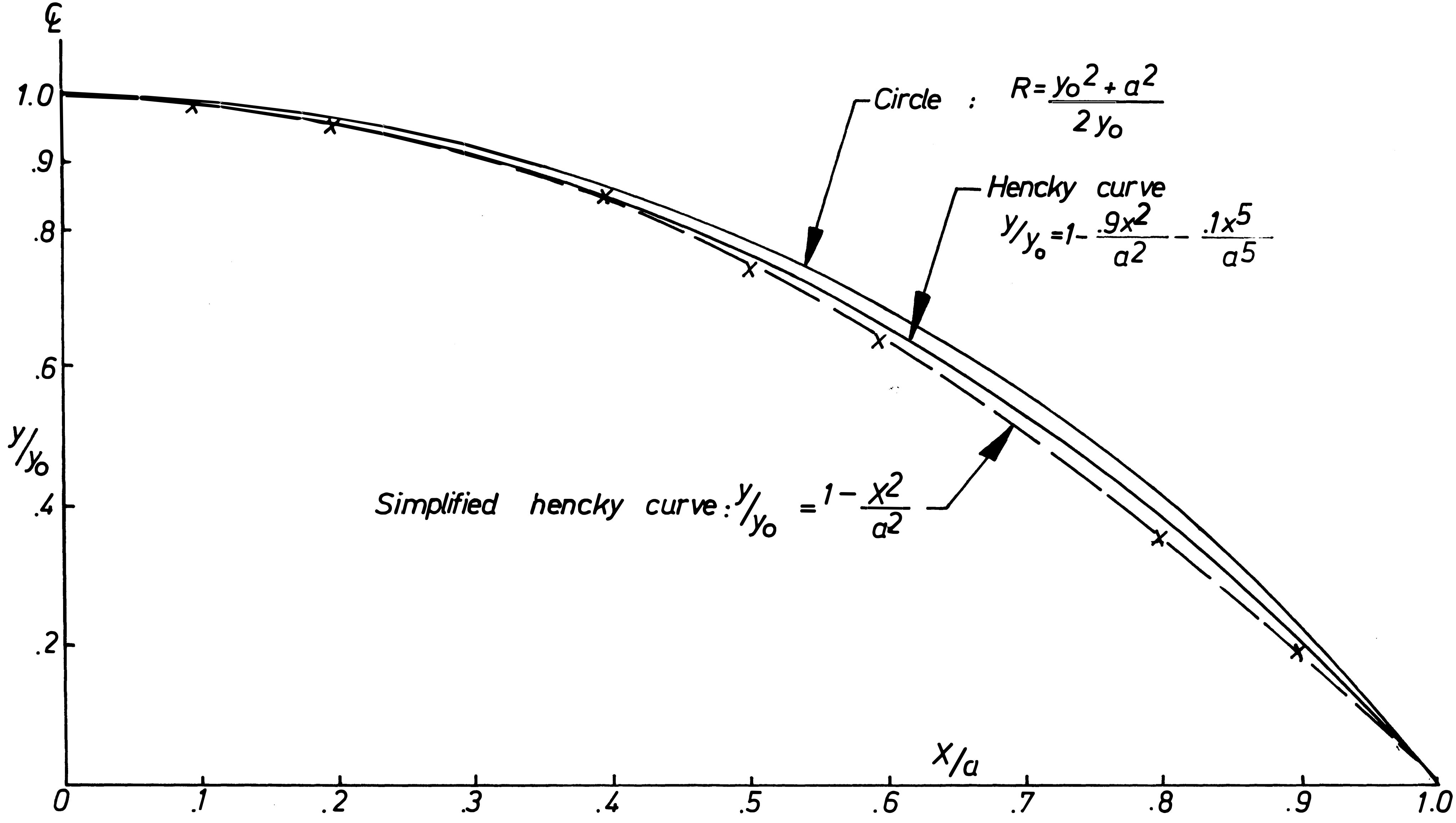
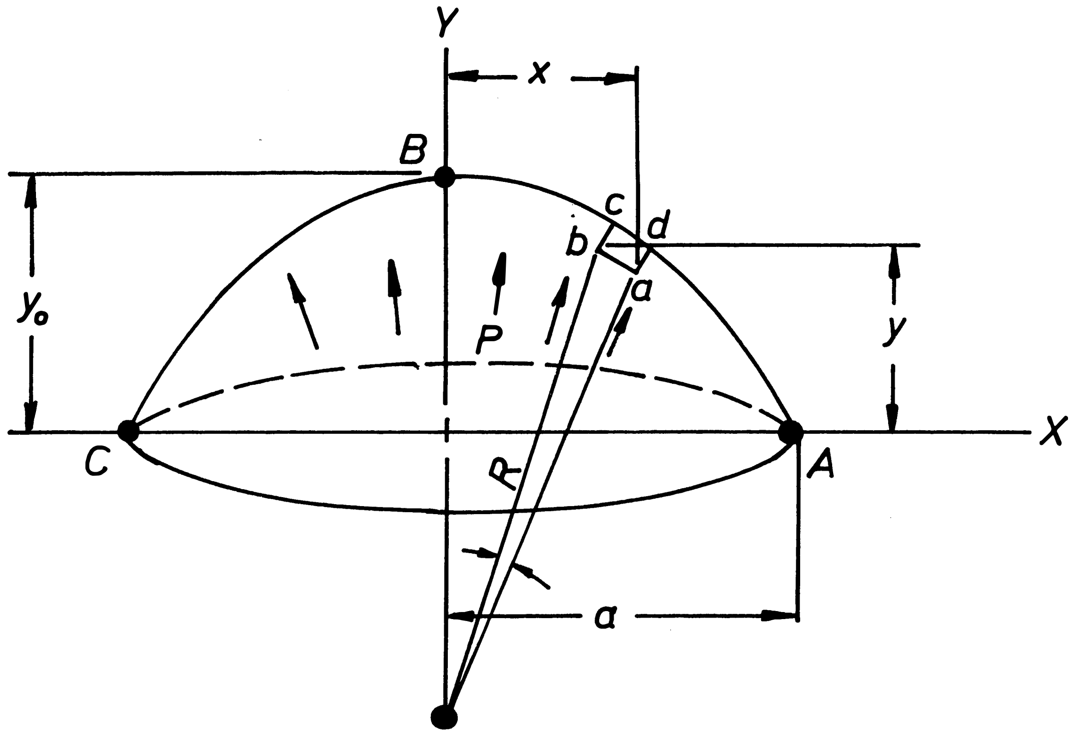
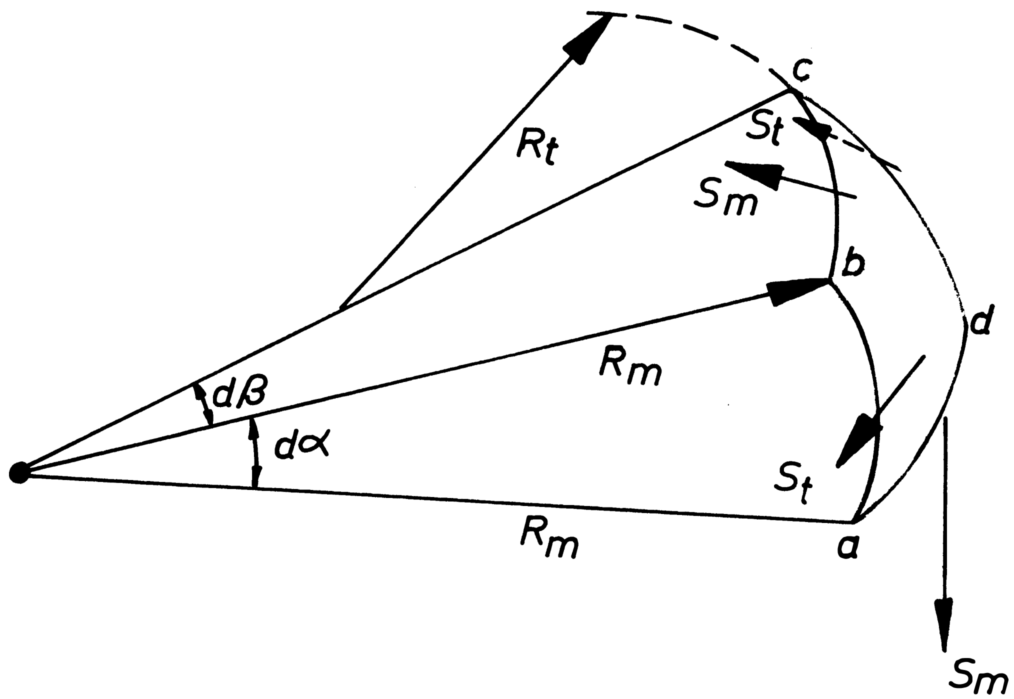


Fig. 7

Shape of loaded membrane

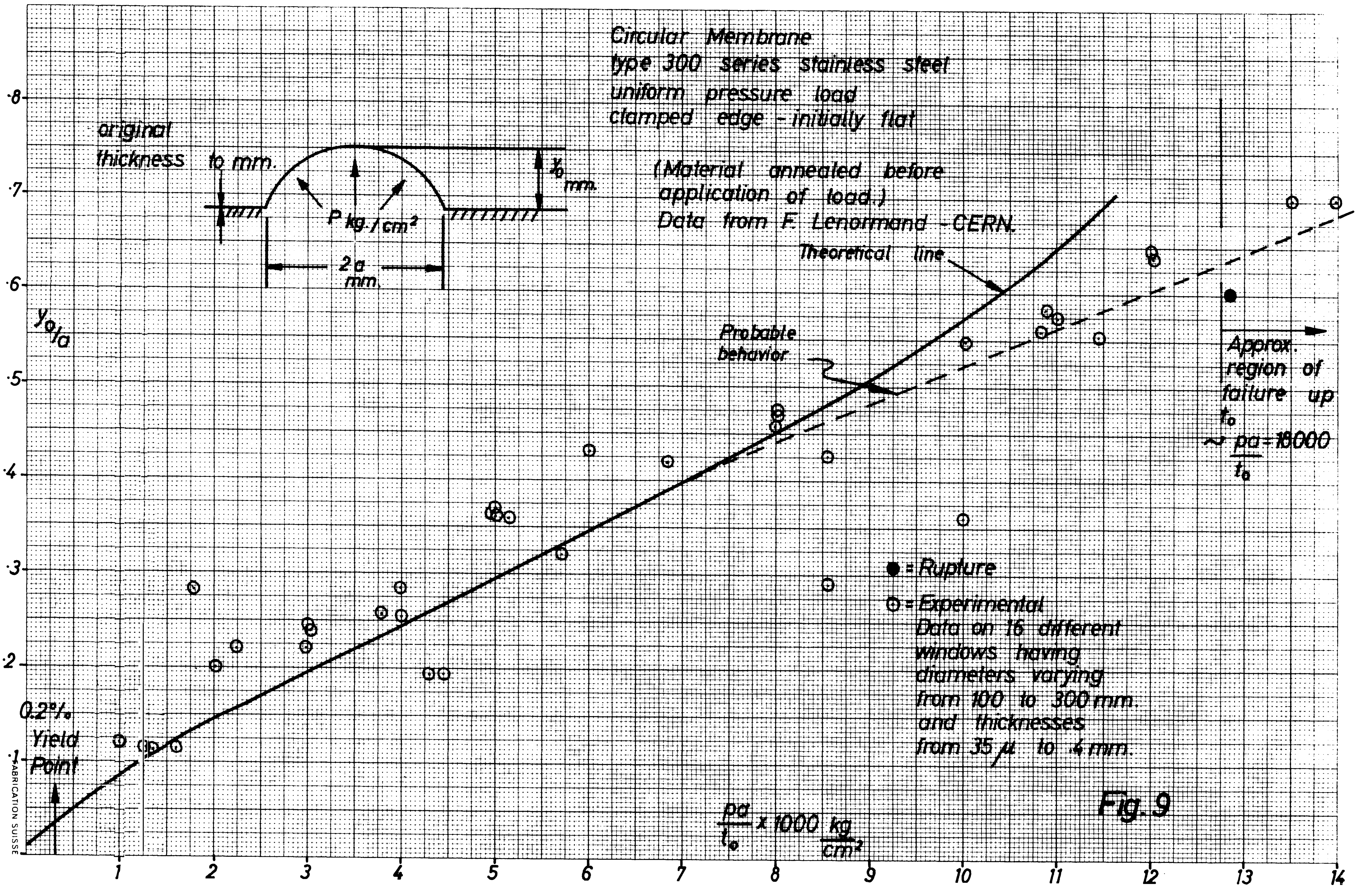




Element abcd of loaded surface

Fig. 8

Circular membrane
Uniform pressure load
Clamped edge



Circular Membrane
type 300 series stainless steel
uniform pressure load
clamped edge - initially flat
(Material annealed before
application of load.)
Data from F. Lenormand - CERN.
original
thickness to mm.
y_o mm.
P kg./cm²
2a
mm.
Theoretical line
Probable
behavior
Approx.
region of
failure up
~ pa/to = 18000
● = Rupture
⊙ = Experimental
Data on 16 different
windows having
diameters varying
from 100 to 300 mm.
and thicknesses
from 35 µ to 4 mm.
y_o/a
0.2%
Yield
Point
pa/to x 1000 kg/cm²
·8
·7
·6
·5
·4
·3
·2
·1
1
2
3
4
5
6
7
8
9
10
11
12
13
14
FABRICATION SUISSE

Fig. 9

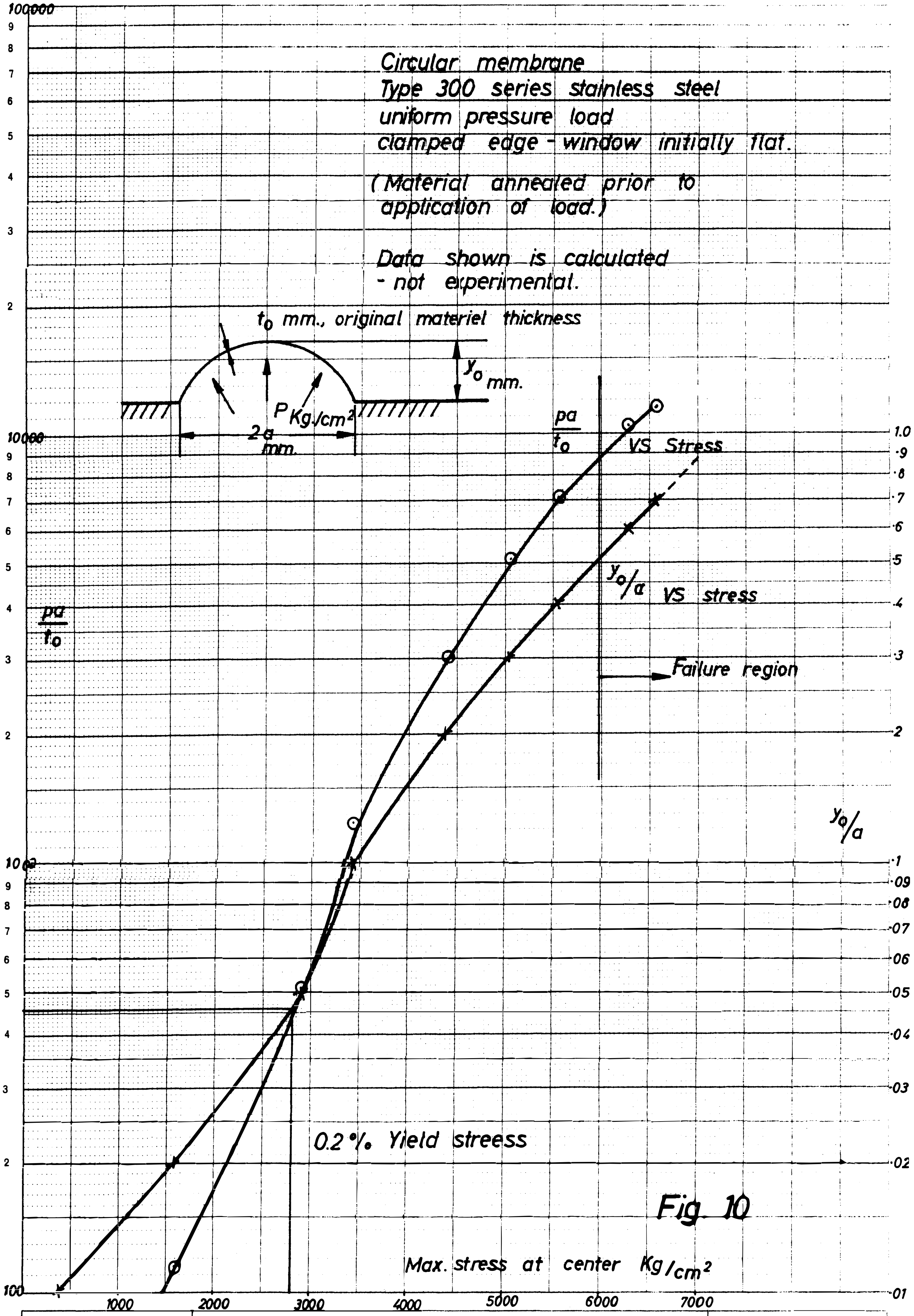


Fig. 10

Gez.
Gepr.
Ges.