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Field-theoretic approach for systems of composite hadrons

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Abstract

Systems containing simultaneously hadrons and their constituents are most easily described by treating composite hadron field operators on the same kinematical footing as the constituent ones. Introduction of a unitary transformation allows redescription of hadrons by elementary-particle field operators. Transformation of the microscopic Hamiltonian leads to effective Hamiltonians describing all possible processes involving hadrons and their constituents.

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1. Introduction. The quark-gluon description of the interactions among hadrons and the properties of high temperature and/or hadronic matter is one of the most central problems of contemporary nuclear physics. Such problems are characterized by processes that involve the simultaneous presence of hadrons and their constituents. The mathematical description of these processes requires approximations where a drastic reduction of the degrees of freedom is unavoidable. In this sense, one would expect simplifications by describing the hadrons participating in the processes in terms of macroscopic hadron field operators, instead of the microscopic constituent ones. The problem with such a description is the noncanonical nature of the composite hadron field operators which complicates the use of the traditional field theoretical methods. Of course, the problem can be formulated in terms of constituent field operators only, but then one would have to deal with other difficulties such as singularities in certain Green's functions of the system which reflect the presence of bound states.

In this Letter we present a generalization to hadronic physics of a field theoretical formalism developed in atomic physics for problems where atoms, electrons and nuclei are simultaneously present in the system [1]- [2]. This formalism is based on a change of representation by means of a unitary transformation in which composites are redescribed by elementary-hadron field operators. In the new representation all field operators are canonical and the traditional methods of quantum field theory can be employed.

The original atomic physics formalism, useful for problems with a fixed number of particles, is generalized for hadron states with an indefinite number of constituents. This is important for the implementation of the formalism to models where creation and annihilation of quarks and gluons play an important role and, therefore, processes such as creation and annihilation of mesons and baryons are naturally taken into account.

2. Formalism. One starts with the Fock space ($\mathcal{F}$) representation of the system. Let $A^\dagger_\alpha$ be the creation operator of a single hadron state, $|\alpha>= A^\dagger_\alpha|0>$, where $|0>$ is the vacuum state:

$$A^\dagger_\alpha = \sum_{n_q n_{\bar q} n_g}^{\infty} (n_q! n_{\bar q}! n_g!)^{-1/2} \Phi_\alpha^{\mu_1 \ldots \mu_{n_q} \nu_1 \ldots \nu_{n_{\bar q}} \sigma_1 \ldots \sigma_{n_g}} \, q^\dagger_{\mu_1} \cdots q^\dagger_{\mu_{n_q}} \bar q^\dagger_{\nu_1} \cdots \bar q^\dagger_{\nu_{n_{\bar q}}} g^\dagger_{\sigma_1} \cdots g^\dagger_{\sigma_{n_g}}. \quad (1)$$

α represents the set of all quantum numbers (spatial and internal) of the hadron and μ, ... are the quantum numbers of the constituents. For the vacuum state one has $q_\mu|0>= \bar q_\nu|0>= g_\sigma|0>= 0$. Φ is the amplitude of the Fock component with n_q quarks, $n_{\bar q}$ antiquarks and n_g gluons; it is taken orthonormalized and antisymmetric (symmetric) in the quark and antiquark (gluon) indices. An Einstein-like summation convention over repeated indices is used.

While the quark and gluon creation and annihilation operators satisfy canonical (anti)commutation relations, the hadron operators satisfy:

$$A_\alpha \cdot A^\dagger_{\alpha'} = \delta_{\alpha\alpha'} + C_{\alpha\alpha'}, \quad A_\alpha \cdot A_{\alpha'} = A^\dagger_\alpha \cdot A^\dagger_{\alpha'} = 0, \quad (2)$$

where the dot products denote graded Lie brackets (commutators or anticommutators) [3]. The presence of $C_{\alpha\alpha'}$ reflects the internal structure of the hadrons and is responsible for effects such as quark-gluon exchange in hadronic collisions. In general, it can be decomposed as follows:

$$C_{\alpha\alpha'} = C^0_{\alpha\alpha'} + C^+_{\alpha\alpha'} + C^-_{\alpha\alpha'} \;, \tag{3}$$

where the components on the right are defined implicitly by the following symmetry and vacuum annihilation properties:

$$\begin{aligned}(C^0_{\alpha\alpha'})^\dagger = C^0_{\alpha'\alpha},\ \left(C^+_{\alpha\alpha'}\right)^\dagger = C^-_{\alpha'\alpha},\ \left(C^-_{\alpha\alpha'}\right)^\dagger = C^+_{\alpha'\alpha} \\ C^0_{\alpha\alpha'}|0>= C^-_{\alpha\alpha'}|0>=0,\ C^+_{\alpha\alpha'}|0>\neq 0.\end{aligned} \tag{4}$$

The symmetry and vacuum properties follow respectively from the behavior of the graded Lie bracket under hermitian conjugation and from the fact that different Fock components have different number of particles.

We adjoin to the Fock space $\mathcal{F}$ the "ideal hadron space" $\mathcal{H}$ with extra degrees of freedom described by "ideal hadron" operators a_α and $a^\dagger_\alpha$ satisfying canonical (anti)commutation relations:

$$a_\alpha \cdot a^\dagger_{\alpha'} = \delta_{\alpha\alpha'}, \quad a_\alpha \cdot a_{\alpha'} = a^\dagger_\alpha \cdot a^\dagger_{\alpha'} = 0. \tag{5}$$

These operators (anti)commute with the quark and gluon operators. Next, we define the graded direct product of $\mathcal{F}$ and $\mathcal{H}$: $\mathcal{F}\times\mathcal{H} \equiv \mathcal{I}$ [4]. We introduce a new vacuum $|0)$ which is the direct product of the vacua of $\mathcal{F}$ and $\mathcal{H}$, and a constraint on allowed physical states $|\psi>$, $a_\alpha|\psi>=0$, which guarantees that allowed states (those satisfying the constraint) constitute a subspace isomorphic to the original Fock space.

Physical content is then ascribed to the new degrees of freedom by carrying out a unitary transformation U which transforms the *single* quark-gluon states $|\alpha>= A^\dagger_\alpha|0>$ into *single* elementary-particle states $|\alpha) = a^\dagger_\alpha|0)$:

$$|\alpha>= U|\alpha), \qquad |\alpha) = U^{-1}|\alpha>= U^\dagger|\alpha> \;. \tag{6}$$

Fock space matrix elements of any observable O between states $|\psi>$ and $|\psi'>$ are equal to those of the transformed observable $U^{-1}OU$ between the transformed states $|\psi)$ and $|\psi')$. Defining $U^{-1}a_\alpha U = D_\alpha$, then the transformed constraint equation in the new representation is $D_\alpha|\psi) = 0$. Although states $|\psi)$ satisfying this constraint describe the same physics as the corresponding states $|\psi>$, their mathematical representation is more convenient because in $|\psi)$ all bound quark-gluon composites are described in terms of canonical operators $a^\dagger_\alpha$, and the $q^\dagger_\mu$, $\bar{q}^\dagger_\nu$ and $g^\dagger_\sigma$ describe free particles only.

By analogy with prior work [1]- [5], the expression for the unitary operator U is of the form

$$U = \exp\left(\frac{\pi}{2}F\right). \tag{7}$$

F is a normally-ordered series of products of quark, antiquark, gluon and ideal hadron operators:

$$F = B^\dagger_\alpha a_\alpha - a^\dagger_\alpha B_\alpha, \tag{8}$$

where the B_α are canonical operators:

$$B_\alpha \cdot B^\dagger_{\alpha'} = \delta_{\alpha\alpha'} \;, \quad B_\alpha \cdot B_{\alpha'} = B^\dagger_\alpha \cdot B^\dagger_{\alpha'} = 0. \tag{9}$$

B_α is constructed by an iterative procedure as a power series in the Fock amplitudes Φ by enforcing Eq. (9) order by order. The B_α operator derived in the original atomic physics formalism [5] cannot be simply transcribed to the present case. Technically, this is because the $C_{\alpha\alpha'}$ in Eq. (2) does not annihilate the vacuum, a feature that was originally the key element for transforming real states into ideal ones. Physically this is a consequence of the fact that different Fock components of an hadron state have different numbers of constituents. The crucial observation is that the different parts of $C_{\alpha\alpha'}$, Eq. (3), have to be combined in an appropriate way. It turns out that [6] B_α, up to third order in Φ (the highest order required for our purposes here), is given by:

$$B_\alpha = A_\alpha + \left(C^+_{\alpha\alpha'} + \frac{1}{2}C^0_{\alpha\alpha'}\right)A_{\alpha'} + (-1)^{NN'}A^\dagger_{\alpha'}\left[A_\alpha \cdot \left(C^-_{\alpha'\alpha''} + \frac{1}{2}C^0_{\alpha'\alpha''}\right)\right]A_{\alpha''}. \tag{10}$$

The construction of such a term encompasses the required generalization for operators as in Eq. (1).

Although the physical interpretation of the B_α operator is not very transparent, the fact it satisfies Eq. (9) leads to the interpretation of U as a rotation of $\pi/2$ in the space spanned by the operators a_α and B_α. An important consequence of this is that the effective Hamiltonians in the new representation are free of the post-prior discrepancy (i.e. lack of symmetry under exchange of initial and final states) [7]. This discrepancy usually plagues composite-particles formalisms and might have catastrophic consequences for scattering amplitudes. Another important feature is that D_α does not contain a_α or $a^\dagger_\alpha$ and has at least one annihilation operator on the right [6]. Therefore, the state in which the constituents are confined in single hadrons has the simple form:

$$|\psi) = \sum_{n=0}^{\infty}\sum_{\alpha_1\ldots\alpha_n} \Psi_n(\alpha_1,\ldots,\alpha_n)\, a^\dagger_{\alpha_1}\cdots a^\dagger_{\alpha_n}|0). \tag{11}$$

Note that $|\alpha_1\ldots\alpha_n) = a^\dagger_{\alpha_1}\cdots a^\dagger_{\alpha_n}|0)$ *is not* in general simply the transform of $|\alpha_1\ldots\alpha_n>= A^\dagger_{\alpha_1}\cdots A^\dagger_{\alpha_n}|0)$ since $a^\dagger_\alpha$ differs from $U^{-1}A^\dagger_\alpha U$ by terms representing complicated effects of quarks and gluons.

In a variety of applications using field theoretical many-body techniques one will be interested in the Hamiltonian in the new representation. Application of the transformation on the microscopic Hamiltonian leads to an effective Hamiltonian of the general form:

$$U^{-1}HU = H_{\bar{q}qg} + H_a + H_{a\bar{q}qg}. \tag{12}$$

The subscripts identify the type of operators upon which the Hamiltonians depend on. One distinctive feature of $H_{\bar{q}qg}$ is that it cannot form the single hadron bound-states which are mapped into ideal hadrons; it describes scattering processes only. This is similar to the "quasi-particle" formalism of Weinberg [8], in which bound-states are redescribed by quasi-particles and their effects are subtracted from the original microscopic interaction such that the convergence of the Born series is improved. H_a describes baryon-baryon, meson-meson, and baryon-meson interactions and $H_{a\bar{q}qg}$ describes processes such as hadron-quark scatterings and hadron breakup and quark-gluon recombination into hadrons.

3. Effective Baryon Hamiltonian. With the purpose of demonstrating and exemplifying in a transparent way the characteristics of the transformed Hamiltonian, we discuss baryons in a simple quark model. We consider a general class of models in which baryons are three-quark bound states, the quarks interact by two-body forces, and neglect antiquarks and gluons. Most of the calculations of baryon-baryon interactions using quark models [9]-[11], have been performed with such models. The microscopic quark Hamiltonian can be written as:

$$H = T(\mu) q^\dagger_\mu q_\mu + \frac{1}{2} V_{qq}(\mu\nu;\sigma\rho)\, q^\dagger_\mu q^\dagger_\nu q_\rho q_\sigma, \tag{13}$$

where T is the kinetic energy and V_{qq} is the quark-quark interaction. The baryon creation operator is given by Eq. (1), with $n_q = 3$ and $n_{\bar{q}} = n_g = 0$. For this case the $C^{\pm} = 0$ and:

$$C^0_{\alpha\alpha'} = -3\Phi^{*\mu_1\mu_2\mu_3}_\alpha \Phi^{\mu_1\mu_2\mu'_3}_{\alpha'} q^\dagger_{\mu'_3} q_{\mu_3} + \frac{3}{2}\Phi^{*\mu_1\mu_2\mu_3}_\alpha \Phi^{\mu_1\mu'_2\mu'_3}_{\alpha'} q^\dagger_{\mu'_3} q^\dagger_{\mu'_2} q_{\mu_2} q_{\mu_3}. \tag{14}$$

In free-space, a single baryon is an eigenstate of H:

$$H(\mu\nu;\sigma\rho)\Phi^{\sigma\rho\lambda}_\alpha = 3\left[\delta_{[\mu]\sigma}\delta_{\nu\rho}T([\mu]) + V_{qq}(\mu\nu;\sigma\rho)\right]\Phi^{\sigma\rho\lambda}_\alpha = E_{[\alpha]}\Phi^{\mu\nu\lambda}_{[\alpha]}, \tag{15}$$

where we are using the convention that there is no sum over repeated indices inside square brackets, E_α is the total energy of the baryon.

The evaluation of the transformed Hamiltonian follows the iterative technique of early applications of the formalism [1], [12]. The term that involves only quark operators is given by:

$$\begin{aligned} H_q = H &- \frac{1}{6}\left[\Delta(\nu_1\nu_2\nu_3,\mu\nu\lambda)\, H(\mu\nu;\sigma\rho)\, q^\dagger_{\nu_1} q^\dagger_{\nu_2} q^\dagger_{\nu_3} q_\rho q_\sigma q_\lambda + \text{h.c.}\right] \\ &- \frac{1}{6}\Delta(\nu_1\nu_2\nu_3,\mu\nu\lambda)\, H(\mu\nu;\sigma\rho)\,\Delta(\sigma\rho\lambda,\mu_1\mu_2\mu_3)\, q^\dagger_{\nu_1} q^\dagger_{\nu_2} q^\dagger_{\nu_3} q_{\mu_1} q_{\mu_2} q_{\mu_3} \end{aligned} \tag{16}$$

where $\Delta(\mu\nu\lambda,\mu'\nu'\lambda') = \Phi^{\mu\nu\lambda}_\alpha \Phi^{*\mu'\nu'\lambda'}_\alpha$. It is not difficult to show that if the Φ_α's satisfy Eq. (15), then H_q has no baryon bound states. This guarantees that H_q describes only true scattering processes, the binding of the baryon is described by H_a.

The baryon Hamiltonian, H_a, is given by:

$$H_a = \Phi^{*\mu\nu\lambda}_\alpha H(\mu\nu;\sigma\rho)\Phi^{\sigma\rho\lambda}_\beta a^\dagger_\alpha a_\beta + \frac{1}{2}V_{aa}(\alpha\beta;\gamma\delta)\, a^\dagger_\alpha a^\dagger_\beta a_\delta a_\gamma\ , \tag{17}$$

where V_{aa} is an effective two-baryon interaction given by:

$$\begin{aligned} V_{aa}(\alpha\beta;\delta\gamma) = 9\ V_{qq}(\mu\nu;\sigma\rho)&\Big[\Phi^{*\mu\mu_2\mu_3}_\alpha \Phi^{*\nu\nu_2\nu_3}_\beta \left(\Phi^{\rho\nu_2\nu_3}_\gamma \Phi^{\sigma\mu_2\mu_3}_\delta - \Phi^{\sigma\nu_2\nu_3}_\gamma \Phi^{\rho\mu_2\mu_3}_\delta - 4\Phi^{\rho\nu_2\mu_3}_\gamma \Phi^{\sigma\mu_2\nu_3}_\delta\right) \\ &+ 2\Phi^{*\mu\nu\mu_3}_\alpha \Phi^{*\nu_1\nu_2\nu_3}_\beta \Phi^{\rho\nu_2\nu_3}_\gamma \Phi^{\nu_1\sigma\mu_3}_\delta - 2\Phi^{*\mu\mu_2\mu_3}_\alpha \Phi^{*\nu_1\nu_2\nu}_\beta \Phi^{\nu_1\nu_2\mu_3}_\gamma \Phi^{\sigma\mu_2\rho}_\delta \Big\} \\ &- 3H(\mu\nu;\sigma\rho)\Big(\Phi^{*\mu\nu\mu_3}_\alpha \Phi^{*\nu_1\nu_2\nu_3}_\beta \Phi^{\nu_1\nu_2\mu_3}_\gamma \Phi^{\sigma\rho\nu_3}_\delta + \Phi^{*\mu\mu_2\nu}_\alpha \Phi^{*\nu_1\nu_2\nu_3}_\beta \Phi^{\nu_1\nu_2\rho}_\gamma \Phi^{\sigma\mu_2\nu_3}_\delta \\ &+ \Phi^{*\mu\mu_2\mu_3}_\alpha \Phi^{*\nu_1\nu_2\nu}_\beta \Phi^{\nu_1\nu_2\mu_3}_\gamma \Phi^{\sigma\mu_2\rho}_\delta\ \Big). \end{aligned} \tag{18}$$

Note that this effective baryon Hamiltonian is of general validity in that the Φ's are not restricted to baryon ground states, neither are they restricted to be eigenstates of the microscopic quark Hamiltonian, Eq. (15). Also, it is not difficult to show that it is symmetrical under exchange of initial and final states and, therefore, free of the post-prior discrepancy.

The baryon breakup Hamiltonian is given by:

$$H_{a\to qqq} = \frac{3}{4}\Phi^{\mu_1\mu_2\rho}_\alpha \Phi^{\nu_1\nu_2\sigma}_\beta V(\mu\nu\sigma\rho)\, q^\dagger_\mu q^\dagger_\nu q^\dagger_{\mu_1} q^\dagger_{\mu_2} q^\dagger_{\nu_2} q^\dagger_{\nu_1} a_\beta a_\alpha, \tag{19}$$

and $H_{qqq\to a} = H^\dagger_{a\to qqq}$ is the recombination Hamiltonian. In models where quarks are confined, these terms contribute to a free-space baryon-baryon process in intermediate states only, because free quarks should not be produced as asymptotic states in a free-space hadron-hadron collision. The exact way this will happen depends on the particular confining mechanism of the underlying microscopic quark model. The breakup and recombination Hamiltonians can give rise to effects similar to quark delocalization [13]. Also, in a high temperature and/or density environment, where hadrons and quarks coexist, the breakup and recombination processes can play an important role.

The higher order terms of B_α give rise to many-baryon (>2-baryon) forces and also orthogonality corrections. The orthogonality corrections correspond to the "renormalization" of the relative wave function by the square-root of the normalization kernel in a resonating group calculation [9], [10]. Among other effects, these weaken the "intra-exchange" interactions, i.e., interactions where the microscopic quark-quark interaction occurs within a single hadron [1], [5]. For Φ's that are eigenstates of the microscopic Hamiltonian, the orthogonality corrections cancel, to lowest order, the term proportional to $H(\mu\nu;\sigma\rho)$ in Eq. (18).

4. An Example. We have derived an effective nucleon-nucleon potential, consistent with lowest order orthogonality, from a microscopic quark-quark interaction of the form:

$$V_{qq} = \frac{1}{4}\lambda^1_a\lambda^2_a\left(v_c + \sigma^1_i\sigma^2_j v^{ij}_\sigma\right), \tag{20}$$

where λ_a are the color $SU(3)$ matrices, σ_j are spin $SU(2)$ matrices, and v_c and v_σ are arbitrary functions of the momentum transfer **q**. v_c can describe quark confinement and other spin-independent quark-quark interactions, whereas v^{ij}_σ describes spin-spin and tensor forces. For simplicity we are not considering spin-orbit interactions. We use s-wave nonrelativistic nucleon wave-functions, and perform the sum over quark color-spin-flavor indices in closed form using the method of Ref. [14] to obtain a spin-isospin effective NN potential $U(\mathbf{p},\mathbf{p}')$:

$$U = u_c + \tau^1_N\cdot\tau^2_N u_\tau + \sigma^1_{Ni}\sigma^2_{Nj}\left(u^{ij}_\sigma + \tau^1_N\cdot\tau^2_N u^{ij}_{\tau\sigma}\right), \tag{21}$$

σ_N, τ_N refer to *nucleon* spin and isospin $SU(2)$ matrices and $u_c, u_\tau, \cdots$ are functions of the center-of-mass initial and final momenta $\mathbf{p}, \mathbf{p}'$ given by:

$$u_c = \frac{1}{4}\left[I_1(v_c + 1/9v_\sigma) + I_2(v_c + v_\sigma) + I_3(v_c - 1/3v_\sigma)\right], \tag{22}$$

$$u_\tau = \frac{1}{36}\left[I_1(v_c + 1/9v_\sigma) + I_2(v_c + v_\sigma) + I_3(v_c + 1/3v_\sigma)\right], \tag{23}$$

$$u^{ij}_\sigma = \frac{1}{36}\left[I_1(\delta^{ij}v_c + v^{ij}_\sigma) + I_2(\delta^{ij}v_c - \delta^{ij}v_\sigma + 2v^{ij}_\sigma) + I_3(\delta^{ij}v_c + v^{ij}_\sigma)\right], \tag{24}$$

$$u^{ij}_{\tau\sigma} = \frac{1}{324}\left[I_1(\delta^{ij}25v_c + v^{ij}_\sigma) + 25I_2(\delta^{ij}v_c - \delta^{ij}v_\sigma + 2v^{ij}_\sigma) + 5I_3(\delta^{ij}5v_c - v^{ij}_\sigma)\right], \tag{25}$$

where $I_i(v)$, $i = 1, \cdots 4$ are 12-dimensional integrals which, in general, must be integrated numerically, and $v_\sigma = v_\sigma^{ii}$. However, when the quark wave functions are approximated by gaussians, one can integrate the majority of the integrals analytically and obtain:

$$I_i(v) = e^{-5/12a^2(\mathbf{p}^2+\mathbf{p}'^2)+1/2a^2\mathbf{p}\cdot\mathbf{p}'} \int d\mathbf{q}\, v(\mathbf{q})\, f_i(\mathbf{q};\mathbf{p},\mathbf{p}'), \tag{26}$$

where

$$f_1 = e^{-a^2\mathbf{q}^2-a^2\mathbf{q}\cdot(\mathbf{p}-\mathbf{p}')}, \tag{27}$$

$$f_2 = e^{-3/4a^2\mathbf{q}^2+1/2a^2\mathbf{q}\cdot(\mathbf{p}+\mathbf{p}')}, \tag{28}$$

$$f_3 = e^{-11/16a^2\mathbf{q}^2}\left[e^{1/4a^2\mathbf{q}\cdot(3\mathbf{p}-\mathbf{p}')} + e^{1/4a^2\mathbf{q}\cdot(3\mathbf{p}'-\mathbf{p})}\right], \tag{29}$$

where a is the nucleon r.m.s. radius. These last integrals must in general be done numerically. To make contact with previous approaches, we mention that the integrals I_i correspond to four diagrams of Eqs. (9-12) of Ref. [11] (the other four are obtained by antisymmetrization of initial or final hadron states), and are related by a Fourier transform to Eqs. (3.18d-3.18g) of Ref. [10]. Note that since all terms in Eqs. (21) involve quark exchange between the nucleons, U describes only the short-range part of the NN interaction. When considering in Eq. (20) only that part proportional to $\sigma^1 \cdot \sigma^2$, one arrives at the recent quark Born diagram result [11] for the Born-order *on-shell* T-matrix. When solving the Lippman-Schwinger equation with the full non-local expression for U, one obtains [12] results numerically similar to the resonating group ones [9,10].

5. Conclusions. We have generalized a field theoretic formalism for composite particles originally developed in atomic physics to treat composite hadron interactions in quark models. We have derived a unitary operator which transforms a general single composite hadron state in Fock space into an elementary-hadron state. When the unitary operator is applied to the microscopic Hamiltonian one derives effective Hamiltonians describing all possible processes involving hadrons and their constituents. As an example we explained the formalism using a simple quark model where baryons are composites of three constituent quarks which interact by two-body forces.

We have shown that one can formally construct the unitary transformation for a generic Fock space decomposition of a single hadron state. In practice, however, one will in general need truncation of the Fock space. The cloudy bag model [15] is a typical example of such a truncation, the nucleon state is a superposition of a three-quarks state and a three-quarks plus one pion state. In general, when using a relativistic quantum field model one will have to face renormalization and truncation of the Fock space will then introduce problems with divergencies related to the vacuum and violation of Lorentz boost invariance. One expects that such problems can be solved in a light-cone formulation [16] of the model. In such a formulation, the vacuum seems to be "simple" and hadron states are described by a finite Fock space basis in a Tamm-Dancoff approximation. In this sense, given the renormalized Hamiltonian and the hadron states, one can construct the unitary transformation and obtain the effective Hamiltonians describing effective hadron-hadron interactions. The effects of divergencies are then transferred to the matrix elements of the effective interactions.

Recently techniques similar to the one presented here, originally developed in the context of nuclear structure, have been used in hadronic physics [17]. Their emphasis and aim, however, are quite different from ours; whereas in references [17] the transformation is defined on many-composite states, here it is defined on single hadron states. Another important difference is that the present formalism allows applications to relativistic quantum field models in which hadrons are bound states of an indefinite number of constituents. In this sense, it would be interesting to apply the present approach in connection with the one of Ref. [18], where creation and annihilation operators of composites are defined in the context of the reduction formulas of the LSZ formalism. The treatment of composites in terms of ideal operators will facilitate, in view of their canonical nature, the use of Feynman rules for calculating Green's functions and S-matrix elements involving bound-states.

The formalism developed here offers great opportunities for several interesting applications where field theoretical methods are required. As in the case of atomic physics studies of medium effects [19] and electromagnetic plasmas [20], the formalism finds natural applicability in studies of the properties of hot and/or dense hadronic matter.

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