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ABSTRACT

The time-reversal *(T)* operation with respect to a reaction or decay process reverses all momenta and spins and exchanges the initial and final states. The naive time-reversal *(T_N)* operation reverses momenta and spins *without* exchanging initial and final states. It is shown that the T_N operation is simply an *ad hoc* means of explaining the vanishing of an experimental observable, and that the observable vanishes for other clearly defined reasons of symmetries and dynamics. Thus, the T_N operation has no validity with respect to *T* symmetry. Other misconceptions concerning relationships between *T*-odd operators and (so called) *T*-odd observables are discussed.

A recent paper describes the production of polarized Z^o bosons in $e^+ e^-$ annihilation with longitudinally polarized electrons and a measured correlation in the Z^o decays into three jets [1]. The correlation, shown in Fig. 1, is that between the Z^o polarization $\boldsymbol{p}^Z$ and the normal to the decay plane, $\boldsymbol{y} = \boldsymbol{k_1} \times \boldsymbol{k_2}$, defined by the momenta of the two highest-energy jets. With $p_y = \boldsymbol{p}^Z \cdot \boldsymbol{y} = p^Z \cos\omega$, the measured observable is the decay analyzing-power component A_y [2]. That is, the decay intensity for the Z^o (spin-1) vector polarization component p_y is given by

$$I_y = I(1 + \frac{3}{2} p_y A_y) \qquad (1)$$

where I is the intensity for unpolarized Z^o decays. In the notation of [1], $p^Z = A_Z$, $p_y = A_Z \cos\omega$, and $\frac{3}{2} A_y = \beta$.

It was claimed that the corresponding Z^o spin-operator component,

$$\boldsymbol{S} \cdot \boldsymbol{k_1} \times \boldsymbol{k_2} \equiv S_y, \qquad (2)$$

is odd under T_N, "naive time reversal" [4], which reverses momenta and spins without interchanging the initial and final states. As such, not being a true time-reversal operation, a nonzero value of the corresponding (T_N -odd) experimental observable A_y would not signify any violation of T symmetry. However, an immediate inconsistency follows, because a long accepted test of T symmetry has been associated with exactly the same spin-operator component

$$\boldsymbol{S} \cdot \boldsymbol{k_e} \times \boldsymbol{k_\nu} \equiv S_y \qquad (3)$$

in β-decay of polarized nuclei [5]. Since the spin operator S_y changes sign under T and is, thus, T-odd, the argument has been that the corresponding experimental observable, here A_y, is similarly T-odd and is required by T symmetry to vanish. However, that argument, without qualification with respect to the interaction dynamics, is itself in conflict with a theorem that states that there can be no null test of T symmetry [6,7]; i.e., T symmetry alone does not require any observable to vanish [8]. So, with respect to these inconsistencies, the following three issues will be discussed:

1) the relationship between T-odd spin-operators and the corresponding observables that emerges from T symmetry alone,

2) the further conditions that are imposed on the transition amplitudes and observables in first-order electromagnetic and weak processes, leading to the introduction of the T_N operation, and

3) the resulting interpretation of the Z^o decay analyzing-power result [1].

1) For illustration, consider a reaction with the simple spin-structure $\frac{1}{2} + 0 \rightarrow \frac{1}{2} + 0$. Choosing the center of mass helicity frame , unit vectors along the coordinate axes are

$$\boldsymbol{z_i}\ (\boldsymbol{z_f}) = \boldsymbol{k_i}\ (\boldsymbol{k_f}) \qquad \boldsymbol{y} = \boldsymbol{k_i} \times \boldsymbol{k_f} \qquad \boldsymbol{x_i}\ (\boldsymbol{x_f}) = \boldsymbol{y} \times \boldsymbol{z_i}\ (\boldsymbol{z_f}) , \qquad (4)$$

where $\boldsymbol{k_i}\ (\boldsymbol{k_f})$ is the c.m. momentum of the projectile (ejectile). Then with the T transformation $\boldsymbol{k_i} \leftrightarrow -\boldsymbol{k_f}$, $\sigma \rightarrow -\sigma$, and noting that $\sigma_x \equiv \sigma \cdot \boldsymbol{x}$ etc., one has the following transformations under the T operation:

$$T: \quad \sigma_x, \sigma_y, \sigma_z \rightarrow -\sigma_x, \sigma_y, \sigma_z . \qquad (5)$$

Here, then, by definition, σ_x, is a T-odd operator. However, it has been shown [7] that the conditions on the observables that follow from (5) are, for example,

$$A_x = -P^t{}_x , \quad A_y = P^t{}_y , \quad A_z = P^t{}_z . \tag{6}$$

That is, the analyzing-power components A_j are equal to the (±) polarizing-power components $P^t{}_j$ in the inverse reaction. Thus, the even/odd character of the operators in (5) translates into the even/odd character of *pairs* of observables in (6), and it is interesting to note that the rather standard nuclear physics test of $A_y = P^t{}_y$ is a T-even test of T symmetry.

The same arguments apply in the case of vector meson strong decay into three particles, e.g.,

$$\omega \rightarrow \pi^+(k_1) + \pi^-(k_2) + \pi^0(k_3). \tag{7}$$

However, with the ω rest (helicity) frame, analagous to (4), now defined by

$$z = k_1 , \qquad y = k_1 \times k_2 , \qquad x = y \times z , \tag{8}$$

(5) becomes (for spin-1)

$$T: \quad S_x, S_y, S_z \rightarrow S_x, -S_y, S_z , \tag{9}$$

and now S_y is the T-odd operator, corresponding to which the decay analyzing-power $A_y = -P^t{}_y$, the inverse-decay polarizing power, which is an experimentally inaccessible observable [9]. So, in neither the strong-interaction reaction or decay process does the T-odd operator have a corresponding T-odd observable that vanishes from T symmetry. In fact, in each case the expectation value $<\sigma_x>$ or $<S_y>$ is simply the initial-state polarization p_x or p_y , respectively, whereas the observable measured is the corresponding analyzing power [7].

2) When, in combination with T symmetry, the dynamical restrictions of first-order electromagnetic or weak interactions are imposed on the amplitudes of the transition matrix, M, between initial and final helicity states, limited null-tests of T symmetry become available [7]. Specifically, from S matrix unitarity,

$$SS^\dagger = (1 + iM)(1 - iM^\dagger) = 1 + i(M - M^\dagger) + MM^\dagger = 1, \tag{10}$$

so, to first order, M is Hermitian. Then in a process $a(\alpha) + b(\beta) \rightarrow c(\gamma) + d(\delta)$, where $\alpha, \beta, \gamma, \delta$ are the particle helicities, from T symmetry and hermiticity (H) [10, 11],

$$\text{T:} \quad (k_a/k_c)\, M_{\alpha\beta,\gamma\delta} = (-1)^{\alpha-\beta-\gamma+\delta}\, M_{\gamma\delta,\alpha\beta} , \tag{11a}$$

$$\text{H:} \quad M^*_{\alpha\beta,\gamma\delta} = (-1)^{\alpha-\beta-\gamma+\delta}\, M_{\gamma\delta,\alpha\beta} . \tag{11b}$$

In general, $M_{\alpha\beta,\gamma\delta}$ and $M_{\gamma\delta,\alpha\beta}$ are elements of the separate matrices that correspond to the processes $ab \rightarrow cd$ and $cd \rightarrow ab$, respectively, and only for elastic scattering are they elements of the same M matrix. The common phase-factor in (11) comes from the interchange of initial and final states [12]. Thus, from the *combination* of T and H, the transition amplitudes $M_{\alpha\beta,\gamma\delta}$ are real, whereas neither T nor H , *separately*, imposes any restriction on them. Since all observables are sums of bilinear combinations of these amplitudes, any observable that is given by the imaginary part of such a sum then vanishes from T and H.

With all particles in the reaction having spin-$\frac{1}{2}$, for example, the analyzing powers for polarized (beam) particles *a* are given by [7]

$$A_{jo} = Tr\, M\, \sigma_{jo}\, M^{\dagger} / Tr\, M\, M^{\dagger}, \qquad j = x, y, z, \tag{12}$$

where $\sigma_{jo} \equiv \sigma_j \otimes \sigma_o$ and $\sigma_o = 1$. Since the amplitudes $M_{\alpha\beta,\gamma\delta}$ and σ_{xo} , σ_{zo} are all real and σ_{yo} is imaginary, only A_{yo} is given by the imaginary part of such a sum and, thus, vanishes from *T* and *H*. The same argument applies with respect to the target analyzing-power A_{oy}.

Ironically, by (6), A_{oy} is a *T*-even observable, and it appears that the T_N operation was introduced in order to explain the vanishing analyzing power A_{oy} in the elastic scattering of electrons from polarized protons [13]. That is, in (5) σ_y becomes a T_N-odd operator so A_{oy} becomes a T_N-odd observable. Specifically, with $M_{\alpha\beta,\gamma\delta} \equiv M_{if}$, [13] defines the "T-odd effect" as any observable that is proportional to the difference of probabilities

$$\Delta M^2 \equiv |M_{if}(k_i, k_f, p_i, p_f)|^2 - |M_{if}(-k_i, -k_f, -p_i, -p_f)|^2, \tag{13}$$

where the momenta and spin polarizations of the initial and final states have been reversed in the second term, *but the states have not been interchanged.* The result of this quite *ad hoc* operation, somewhere later termed the T_N operation, is immediately apparent. As will be shown, ΔM^2 is directly proportional to the analyzing power, so A_{oy} vanishes when ΔM^2 does, but *without any restriction that* M_{if} *be real,* which is required by the *T* and *H* condition on the amplitudes.

Reference 13 uses the transversity frame, with the quantization *(z)* axis taken along the normal (my *y)* to the reaction plane, so if one takes I_+ (I_-) to be the cross-section with the proton spin along $+z$ $(-z)$, the analyzing power (my A_{oy}) is

$$A_{oz} = \frac{I_+ - I_-}{I_+ + I_-} = \sum \frac{|M_{if}(+)|^2 - |M_{if}(-)|^2}{|M_{if}(+)|^2 + |M_{if}(-)|^2}, \tag{14}$$

where the summation is taken over the spin projections of the other particles [3]. The *k* arguments in (13) are redundant since $(-k_i, -k_f) \to (k_i, k_f)$ and $(-p_x, -p_y, -p_z) \to (p_x, p_y, -p_z)$ in a rotation of the second term by π around the transverse *z*-axis. Thus, the net result is the difference in probabilities for opposite transverse proton spin-states, which is just the numerator in (14). So, (13) simply defines A_{oz} to vanish when the two terms are equal. However, as will be shown, they are *not* equal in the first-order, one-photon exchange, calculation of A_{oy}.

Expressing the required 4×4 *M* matrix in terms of the matrices $\sigma_{jk} \equiv \sigma_j \otimes \sigma_k$ as

$$M = \sum_{j,k} a_{jk}\, \sigma_{jk} \qquad j, k = o, x, y, z\ , \tag{15}$$

it has been shown [3] for *ep* elastic scattering with $m_e/E_e << 1$ that *M* is reduced to three terms by parity conservation and *T* symmetry, and in the helicity frame is

$$M = a_{oo} + a_{oy}\, \sigma_{oy} + a_{zz}\, \sigma_{zz}\, . \tag{16}$$

Then

$$IA_{oy} = \frac{1}{4}\, Tr\, M\, \sigma_{oy}\, M^{\dagger} = 2\, Re\, a_{oo} a_{oy}^{*}\, , \tag{17}$$

which is not identically zero, but which does vanish from T and H since a_{oo} is real and a_{oy} is imaginary; the latter appears as a term in M of the form ia_{oy}. Transforming to the transversity (t) frame by a rotation of $\frac{\pi}{2}$ around the x axis,

$$x \to {}^t x, \quad y \to -{}^t z, \quad z \to {}^t y, \tag{18}$$

(16) becomes

$${}^t M = {}^t a_{oo} - {}^t a_{oz}\, \sigma_{oz} + {}^t a_{yy}\, \sigma_{yy}, \tag{19}$$

and

$$I^t A_{oz} = \frac{1}{4}\, Tr\, {}^t M\, \sigma_{oz}\, {}^t M^{\dagger} = -2\, Re\, {}^t a_{oo} {}^t a_{oz}^{*}. \tag{20}$$

Displayed in its matrix form,

$${}^t M = \begin{pmatrix} M_1 & 0 & 0 & -M_3 \\ 0 & M_2 & M_3 & 0 \\ 0 & M_3 & M_1 & 0 \\ -M_3 & 0 & 0 & M_2 \end{pmatrix}, \tag{21}$$

with

$$M_1 = {}^t a_{oo} - {}^t a_{oz}, \quad M_2 = {}^t a_{oo} + {}^t a_{oz}, \quad M_3 = {}^t a_{yy}, \tag{22}$$

the numerator in (14) does not vanish identically. Expressing (14) as

$$I^t A_{oz} = \frac{1}{4} \sum \left(|{}^t M_{if}(+)|^2 - |{}^t M_{if}(-)|^2 \right), \tag{23}$$

and with (21) and (22),

$$I^t A_{oz} = 2(|M_1|^2 - |M_2|^2) = -2\, Re\, {}^t a_{oo} {}^t a_{oz}^{*}, \tag{24}$$

in agreement with (17) via (20).

Since it is clear that the second term in (13) does not correspond to a T transformation of the first term, there is no reason to imply that the consequences of (13) have anything to do with T symmetry. In fact, reference 13 remarks that A_{oz} vanishes in the (first-order) one-photon exchange approximation, explicitly including T symmetry, which are the T and H conditions; and their calculations were made in order to show quantitatively that $A_{oz} \neq 0$ when the (second-order) two-photon exchange contribution was included. As is shown in that calculation, this result is due entirely to the non-Hermitian nature of the amplitudes, so to term it a "T-odd effect" contribution to a T-even observable was, minimally, confusing.

Finally, even though the analyzing power A_{oy} vanishes, in general, from T and H as described above, in the case of ep elastic scattering, where $M_{\alpha\beta,\gamma\delta}$ and $M_{\gamma\delta,\alpha\beta}$ are elements of the same matrix, it is not even necessary to invoke T symmetry since the vanishing of A_{oy} is already assured by the combination of H and hadronic current conservation [14]. A detailed examination of this assertion will be provided elsewhere [15].

Later, the same "T-odd effect" argument, along with the T_N operation, was used in connection with a calculation of the decay analyzing power A_y, corresponding to (2), in the decay of vector meson states into three gluons [16]. In a decay process, however, the effect of the T_N operation is exactly the same as that of T, since the operator S_y in (2) changes sign under either operation. This results from the fact that $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$ are both final-state vectors (or initial-state vectors in the inverse process), whereas in (4) $\boldsymbol{k_i}$ $(\boldsymbol{k_f})$ is an initial (final)-state vector. This difference is responsible for the opposite T transformations of σ_y and S_y in (5) and (9), respectively. There are recent examples, of course, in calculations of $e^+e^- \rightarrow q\ \bar{q}\ g$ processes, in which S_y has been correctly identified as a T-odd operator [17]. Thus, it is wrong to imply that the T_{N}- operation has any justification, and its use should be discontinued even though the concept, where used, does not invalidate any of the actual calculations of observables in various decay processes [1,4,14,18].

3) The T_N concept can, however, alter the basic interpretation of results. For example, reference 1 states that $S_y(2)$ is CP-even, but that a non-zero value of the corresponding T_N-odd observable A_y would not signal CPT violation. Now, since A_y is a genuinely T-odd observable, the opposite conclusion follows, i.e., it would signal CPT violation. Of course, the non-zero value would have to exceed that contributed by the various final-state interaction processes before any T symmetry violation could be claimed. This is, then, a limited null-test of T, in that the ultimate precision attainable in such a test is limited by that available in the calculation of these non-zero contributions, and not by the experimental precision itself. As an example, at the value of (D coefficient) $A_y = (0.5 \pm 1.4) \times 10^{-3}$ achieved in neutron β–decay [19], the upper limit of these contributions, of order 10^{-5} [20], has not yet been approached.

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Figure Caption

Fig. 1. p_y is the component of the Z^o polarization normal to the decay plane.