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Period changes of AR Lacertae between 1900 and 1989

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Abstract. The epochs of the primary and secondary minima of AR Lac between 1900 and 1989 are analysed with two non-parametric methods of searching for periodicity in a weighted time point series. An analytical model for transforming the O−C data of eclipsing binaries into the period domain is presented and is applied to AR Lac. The abrupt period changes of AR Lac reported in several previous studies are most probably a consequence of an oversimplified interpretation of the O−C data, since these changes are as likely to be continuous, and even the possibility of a cycle in the period cannot be definitely ruled out.

Key words: Stars: AR Lacertae – eclipsing binaries, Methods: analytical – statistical

1. Introduction

The following physical parameters are listed for the eclipsing RS CVn star AR Lac (HR8448, HD210334) in the catalogue of chromospherically active binaries by Strassmeier et al. (1993): G2IV/K0IV, double–line spectrum with strong CaII H&K emission from both components, variable Hα emission and $P_{\rm orb} = 1.^{\rm d}983222 \approx P_{\rm phot}$. The variability of AR Lac was first detected by Leavitt in 1907 (Pickering 1907). A continuous light curve is difficult to obtain, because the orbital period is very close to two days, and hence over two decades elapsed, before Jacchia (1929) found AR Lac to be an eclipsing binary. Since then numerous photometric studies have been made (e.g. see our Table 1) and the geometric, photometric and orbital elements of this binary system have been thoroughly examined (e.g. Chambliss 1976, Park 1984, Lee et al. 1986). Hall (1976) published the definition of the RS CVn class of binaries and classified AR Lac as one member of this class. An important detail to remember is his remark that another long period RS CVn star HK Lac (HD 209813, see Blanco & Catalano 1970) was unfortunately used as a photometric comparison of AR Lac by Wood (1946) and Kron (1947), which means that some epochs of the earlier photometric minima may not be very reliable.

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Numerous studies of the orbital period variations of AR Lac have been made (e.g. Wood 1946, Cester 1967, Chambliss 1976, Lee et al. 1986). For example, Kim (1991: his Table 2) found that the range of these period changes is between $1.^{\rm d}9831674$ and $1.^{\rm d}9832601$ (i.e. about $0.^{\rm d}00009$). The possible physical phenomena responsible for the period changes of AR Lac have been discussed by Hall & Kreiner (1980), and later by, e.g., Panchatsarem & Abhyankar (1982) and Kim (1991). The proposed phenomena include: presence of a third body in the system, effects of starspots, apsidal motion, mass transfer, mass loss, to mention a few. Most of these alternatives have been critically examined by Kim (1991), who argued that the correct model to explain the period variation of AR Lac has not yet been found. A more general discussion of the different physical processes that may explain the short– and long–term period changes of eclipsing binaries can be found, e.g., in Hall (1990). A model where the orbital period modulations of eclipsing binaries are connected to magnetic activity, was quite recently proposed by Applegate (1992), and different types of observational data seem to support several predictions of this model (e.g. Hall 1990, Hall 1991, Rodonò et al. 1995). This model has not yet been tested in the case of AR Lac. Our paper presents a method for determining the period variations of eclipsing binaries from the epochs of the primary and secondary minima, and this is applied to the currently available data for AR Lac.

2. Observations

The epochs of the primary and secondary minima of AR Lac in Table 1 were collected from numerous sources. The *earliest* observations were compiled from Hall & Kreiner (1980), and their values with zero weights have been omitted. Some modifications have been made, the most important being that when several values were available for the same minimum, the values in Table 1 are then mean epochs. The original references for the data denoted by Hall & Kreiner (1980) as "B.B.S.A.G observers" are given, but the first of these values has been omitted (i.e. HJD2440933.330), because it is not listed in the BBSAG Bulletin. Although we did not find the original references for the data referred to as "B.A.N. observers" in Hall & Kreiner (1980), these data have been included in Table 1. The more *recent* observations in Table 1 were compiled from Kim (1991),

Table 1. The epochs of the primary and secondary minima of AR Lac [HJD–2400000]: the references are [1]Dugan & Wright (1939), [2]Jacchia (1930), [3]Loreta (1930), [4]Schneller & Plaut (1932), [5]Parenago (1930), [6]Parenago (1938), [7]Rügemer (1931), [8]Zverev (1936), [9]Himpel (1936), [10]Wood (1946), [11]Gainullin (1943), [12]Ahnert (1949), [13]Svechnikov (1955), [14]Wroblewski (1956), [15]Makarov et al. (1957), [16]Karetnikov (1959), [17]Aleksandrovich (1959), [18]Karetnikov (1961), [19]Karle (1962), [20]B.A.N. observers, [21]Oburka (1964), [22]Ahnert (1965), [23]Oburka (1965), [24]Hall (1968), [25]Pohl & Kizilirmak (1966), [26]Ahnert (1966), [27]Kizilirmak & Pohl (1968) [28]Cester (1967), [29]Karle et al. (1977), [30]Pohl & Kizilirmak (1970), [31]Oburka & Silhan (1970), [32]Nha & Kang (1982), [33]Battistini et al. (1973), [34]Pickup (1972), [35]Peter (1972), [36]Kizilirmak & Pohl (1974), [37]Chambliss (1974), [38]Isles (1973), [39]Isles (1975), [40]Pokorny (1974), [41]Scarfe & Barlow (1978), [42]Srivastava (1981), [43]Kurutac et al. (1981), [44]Ertan et al. (1982), [45]Pohl et al. (1982), [46]Park (1984), [47]Caton (1983), [48]Evren et al. (1983), [49]Kim (1991), [50]Pagano (1990), [51]Nezry (1988), [52]Martignoni (1995) and [53]Panov (1987). The notations for different systems (v, f, p, pv and e) are explained in the text (Sect. 2). The epochs of the secondary minima are denoted by •

15300.0590[1v]	29404.8470[10e]	38670.4815[22v,23v]	41274.4170[34v]	42319.5750[39v]	44550.6417[46e]	46349.3690[51v]
17005.6490[1v]	29535.7390[10e]	38672.4435[22v,23v]	41506.4430[35v]	42634.8850[29e]	44809.4452•[44e,45e]	46351.3620[51v]
18889.7290[1v]	29692.4110[11f]	38680.3820[22v]	41513.3920•[36e]	42715.2081•[42e]	44817.3817•[45e]	46397.9601•[49e]
20991.9730[1v]	32889.3360[12v]	38975.8796[24e]	41592.7219•[37e]	42716.1968[42e]	44818.3702[45e]	46726.1722[49e]
22995.0380[1v]	34646.4700[13v]	38996.7067•[24e]	41593.7124[37e]	42717.1861•[42e]	44823.3279•[44e]	46728.1583[49e]
25196.4440[1v]	34888.4290[14v]	39019.5255[25v]	41604.6224•[37e]	43084.0812•[32e]	44898.6836•[47e]	46738.0734[49e]
25555.4280[2f]	34890.4060[14v]	39027.4425[20v,26f]	41635.3860[38v]	43420.2290[32e]	44977.0223[46e]	46742.0394[49e]
25573.2780[3v]	34902.3230[14v]	39029.4260[20v,26f]	41637.3440[38v]	43427.1700•[32e]	45163.4375[48e]	46745.0181•[49e]
25712.0860[4f]	36080.3340[15v]	39259.4913[26f,27v]	41639.3505[38v]	43428.1606[32e]	45164.4375•[48e]	46752.9532•[49e]
25801.3280[5v]	36082.3220[15v]	39376.4926[28e]	41911.0170[39v]	43739.5194[43e]	45165.4189[48e]	47042.5320•[52v]
25803.3200[6v]	36084.3030[15v]	39383.4386•[28e]	41920.9480[39v]	43740.5104•[43e]	45166.4098•[48e]	47045.5290[52v]
26146.4050[7p]	36766.5250[16v]	39386.4040[26v]	41922.9270[39v]	43745.4701[43e]	45296.3065[48e]	47053.3907[50e,53e]
26489.5260[7p]	36774.4740[16v]	39691.8224[29e]	41936.8022[37e]	43747.4535[43e]	45333.9946[49e]	47054.3796•[50e]
26592.6510[7f]	36778.4460[16v]	39695.7941[29e]	41938.7874[37e]	43750.4274•[43e]	45337.9580[49e]	47055.3720[53e]
26604.5480[8v]	36784.4175[16v,17v]	39699.7590[29e]	41962.6030[40v]	43751.4182[43e]	45612.6224•[50e]	47056.3610•[53e]
26622.3950[7v]	37137.4410[18v]	39701.7410[29e]	41968.5640[40v]	43752.4108•[43e]	45674.1062•[49e]	47494.6518•[50e]
26624.3780[7f]	37569.7977[19e]	39876.2680[30e]	41972.5360[40v]	43755.3858[43e]	45680.0533•[49e]	47495.6385[50e]
26626.3610[7v]	37623.3320[20v]	40046.8186[29e]	41974.5170[40v]	44113.3584•[43e]	45683.0263[49e]	47733.6202[50e]
26842.5420[4v]	37958.4740[20v]	40443.4710[31v]	42285.8406[29e]	44114.3380[43e]	45691.9558•[49e]	
26991.2860[9pv]	38315.4520[21v]	40546.5834[32e]	42287.8274[29e,41e]	44449.4985[44e,45e]	46032.0635[49e]	
29178.7610[10e]	38321.3930[21v]	40932.3168•[32e,33e]	42288.8215•[29e,41e]	44451.4809[44e,45e]	46041.9815[49e]	
29186.6930[10e]	38323.3910[20v,21v]	41268.4610[34v]	42289.8120[29e]	44458.4232•[44e,45e]	46344.4039•[50e]	
29188.6750[10e]	38333.3450[20v]	41270.4350[34v]	42292.7872•[29e]	44549.6484•[46e]	46345.4280[51v]	

who revised the epochs of some previously published minima, as well as discarding some values by Srivastava (1981) and Nha & Kang (1982). However, the data by Ishchenko (1963) were omitted, because Hall & Kreiner (1980) gave zero weights to these data. The *new* epochs in Table 1 are from Panov (1987), Nezry (1988), Pagano (1990) and Martignoni (1995).

The data in Table 1 have been obtained with different techniques, and we use the notations by Hall & Kreiner (1980): visual estimates ("v"), a series of photographic exposures ("f"), mid-time of exposures on which the object appeared faint ("p"), a series of measurements with a visual polarizing photometer or similar device ("pv") and, finally, photoelectric measurements ("e"). Because the accuracy of the data is not the same in all systems and most of the references do not contain an error estimate, Hall & Kreiner (1980) gave the weight $w=3$ for the "e" data, while observations in all "other" systems were given the weight $w=1$. Therefore, the measurements performed in "v", "f", "p" and "pv" are also referred to as "other" systems in this paper. For reasons to be discussed later (see Table 3), our error estimates are $\sigma_e = 0.^d004$ and $\sigma_{other} = 0.^d011$.

3. Time series analysis

The standard technique to determine the ephemerides of eclipsing binaries is to make linear or quadratic least squares fits to the O−C data. This procedure has been applied for AR Lac, e.g., by Chambliss (1976) and Hall & Kreiner (1980). Examples of linear least squares fits to parts of the data can be found in, e.g., Chambliss (1976) and Kim (1991). In this paper the ephemeris of AR Lac is determined with two nonparametric methods of searching for periodicity in a weighted time point series, presented by Jetsu & Pelt (1996: hereafter Paper I), which were already applied in Jetsu et al. (1995) and Jetsu (1996). These methods analyse circular data, i.e. a random sample of single measurements representing directions in a plane or phases at different time values, folded with a fixed period. The data in Table 1 are circular when folded with any arbitrary period, and are typical of a case where the model is unknown. Thus nonparametric (i.e. model independent) methods offer an ideal approach to study these data. The following abbreviations are used throughout this paper: the WK–method (weighted version of the test by Kuiper (1960), see Sect. 3.3. in Paper I) and the WSD–method (weighted version

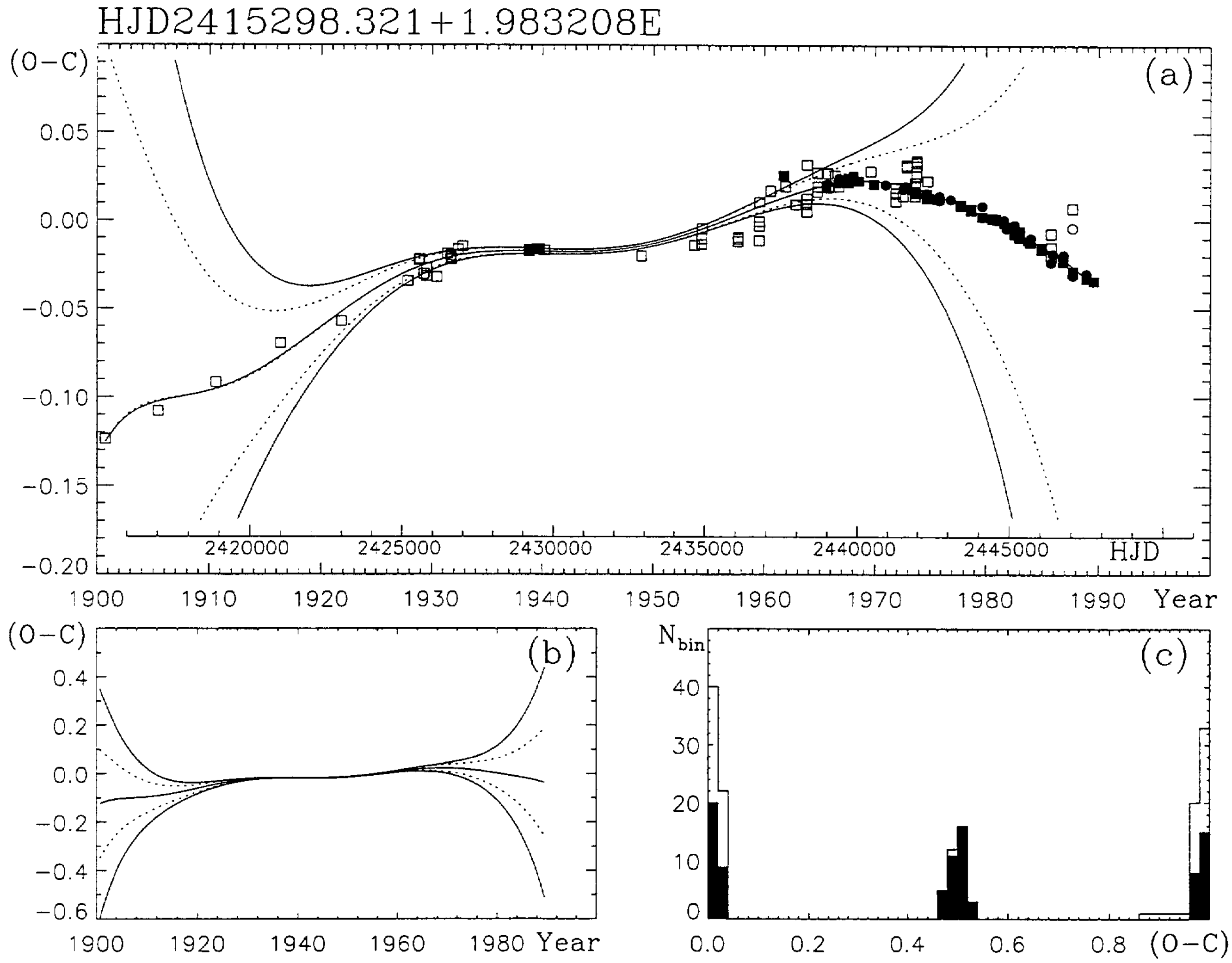

Fig. 1. a) The O−C variations of AR Lac with the ephemeris (1): primary minima ("e"≡ closed squares, "other" ≡ open squares), secondary minima ("e"≡ closed circles, "other" ≡ open circles). The $(O-C)(T, P_0)$ curves (Eq. 5: $C_i(P_0)$ from Table 2) for $K=8$ (dotted line) and $K=9$ (continuous line) and their 1σ error limits (Eq. 7). **b)** The total range of 1σ error limits of $(O-C)(T, P_0)$. **c)** The O−C distribution in bins of 0.02 for the "e" (dark) and "other" data (white)

of the test by Swanepoel & De Beer (1990), see Sects. 2.2.–2.3. in Paper I). The number of analysed time points (t_i) with errors σ_i ($0.^d004$ or $0.^d011$) is $n=156$. The ratio of the weights ($w_i = \sigma_i^{-2}$) between "e" and "other" systems is ~ 7.6, which is more than twice as large as that of Hall & Kreiner (1980). The means of all available values for any particular minimum were derived, because all values for the same minimum would be close in phase to any tested period, which might mislead the period analysis. Because both methods are suitable for the analysis of multimodal distributions, the data of the primary and secondary minima can be analysed simultaneously (see Figs. 1c and 2c). The results of these tests using trial periods between $1.^d96$ and $2.^d00$ are given in the ephemerides (1) and (2). Both periods are extremely significant, because the solutions for the critical levels of Eqs. 19 and 30 in Paper I exceed the available computing capacity, i.e. they are below 10^{-200}! No other significant periodicities are present within the above period interval, since all other periods revealed by the periodograms are spurious values resulting from the data spacing of 365^d (see Jetsu 1996: Eq. 6). The ephemerides obtained were

$$\mathrm{HJD}2415298.321(\pm.003)+1.983208(\pm.000006)E_{\mathrm{WK}} \quad (1)$$

$$\mathrm{HJD}2415298.646(\pm.005)+1.983184(\pm.000007)E_{\mathrm{WSD}}. \quad (2)$$

The error estimates for the periods (P_0) of the ephemerides (1) and (2) were determined with the bootstrap approach explained in Sect. 4 of Paper I. However, the method for determining the zero point (t_0) for these ephemerides is formulated here for the first time. The notation is the same as in Paper I. The original data are denoted by the vectors $\bar{t}$ and $\bar{w}$, and a large number (q_1) of random samples are drawn from these original data vectors, and are denoted as $\bar{t}_1^*, ..., \bar{t}_{q_1}^*$ and $\bar{w}_1^*, ..., \bar{w}_{q_1}^*$. This random selection procedure is the same as explained in Paper I, i.e. the connection that w_i is the weight of t_i is preserved in every random sample. Each random sample ($k=1,...,q_1$) yields the following estimate for the mean of the O−C values for a constant P_0 and a fixed zero point (t')

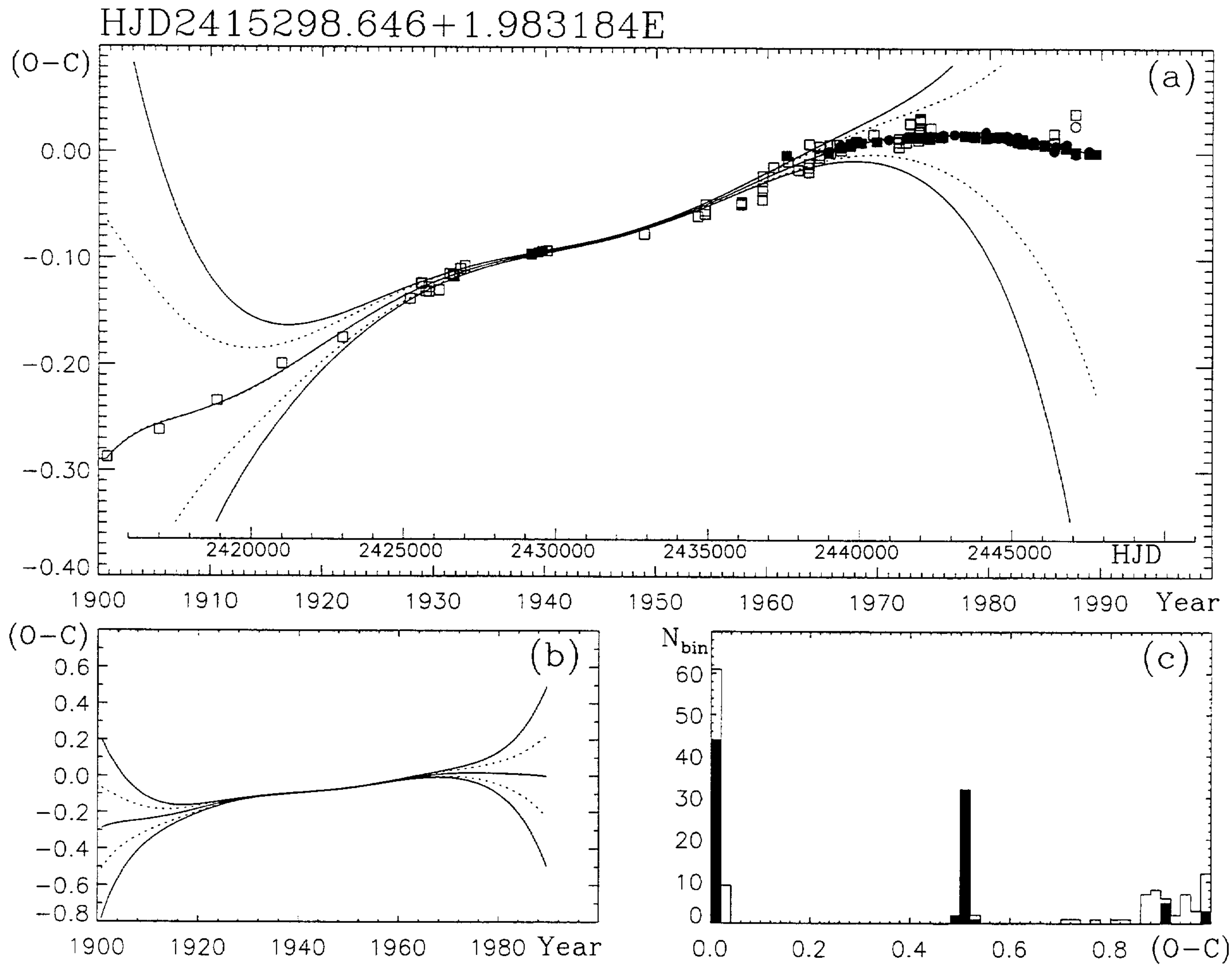

Fig. 2. The same as Fig. 1, using the ephemeris (2)

$$\phi_{\rm k} = \left[\sum_{i=1}^{n} w_i^* \left(\text{FRAC}\left[(t_i^* - t')P_0^{-1}\right] - a\right)\right]\left[\sum_{i=1}^{n} w_i^*\right]^{-1}, \quad (3)$$

where the notation FRAC means that the integer part of $(t_i^* - t')P_0^{-1}$ is removed, while $a = 0.0$ and 0.5 for the primary and secondary minima, respectively. The final result for the zero point of the ephemeris is

$$t_0 = t' + P_0(\langle\phi_{\rm k}\rangle \pm \sigma_{\phi_{\rm k}}), \quad (4)$$

where $\langle\phi_{\rm k}\rangle$ and $\sigma_{\phi_{\rm k}}$ are the average and the standard deviation of the q_1 estimates of $\phi_{\rm k}$. The O−C variations for the ephemerides (1) and (2) are shown in Figs. 1a and 2a.

4. The O-C and P variations

Note that 0.5 has been subtracted from the O−C values of the secondary minima in Figs. 1a and 2a to shift them to the level of the primary minima. The standard technique in studying the period variations of eclipsing binaries is to fit linear or higher order polynomials within limited time intervals to the O−C values, and the previous studies of AR Lac contain examples of this approach (e.g. Chambliss 1976, Kim 1991). For example, visual inspection might suggest that the variations in Fig. 1a could be adequately presented by five or six linear fits, which would more or less resemble the approach chosen by Kim (1991: his Fig. 1). On the other hand, perhaps four or five linear fits might suffice for rough modelling of Fig. 2a. The interpretations would then be that the period has been decreasing from the beginning of 1960 (Fig. 1a), or increasing from the beginning of the century to 1980 (Fig. 2a). One might even claim that Fig. 1a may indicate periodicity in the O−C variations. But the main problem with these interpretations is that abrupt and discontinuous period changes seem to occur at the epochs where the lines fitted to the O−C data intersect. Furthermore, there is plenty of freedom in choosing the intervals for fitting. Unfortunately, the results also depend on the chosen P_0 to derive the O−C values.

In reality, the function for modelling the O−C data is unknown. Furthermore, the shape of this function depends on P_0.

The weighted fits shown in Figs. 1ab and 2ab are

$$(\mathrm{O-C})(T, P_0) = \sum_{i=0}^{K} C_i(P_0) T^i, \quad (5)$$

where $K = 8$ or 9. The time scale is $T = (t - t'')/36525$, where $t'' =$HJD2431516.8396 is the mid point of the time interval of the data. The values $C_0(P_0)$, ..., $C_K(P_0)$ for the curves in Figs. 1ab are given in Table 2. A simple connection exists between the models (Eq. 5) for the data in Figs. 1ab and 2ab, because the transformation between them is

$$(\mathrm{O-C})(T, P_1) - (\mathrm{O-C})(T, P_2) = \frac{P_2 - P_1}{P_1 P_2} T + \frac{P_1 T_2 - P_2 T_1}{P_1 P_2}, \quad (6)$$

where T_1 and P_1, and T_2 and P_2 are the zero points and periods of the ephemerides (1) and (2), respectively. The model gives different values only for $C_0(P_0)$ and $C_1(P_0)$, while values of $C_2(P_0), ..., C_K(P_0)$ are the same for $P_0 = P_1$ and P_2. The coefficients on the right side of Eq. 6 determine the differences in $C_0(P_0)$ and $C_1(P_0)$. Although the O$-$C variations in Figs. 1a and 2a do not appear similar, Eq. 6 shows that the model does not strongly depend on the choice of P_0.

Why should the model of Eq. 5 be better than some more arbitrary model? What is the reason for choosing $K = 8$ or 9 in Figs. 1ab and 2ab? The answer to the first question is that whatever the form of the unknown function suitable for modelling the O$-$C data, it can be expanded in a Taylor series, unless it or its derivatives are discontinuous. The coefficients of Eq. 5 can be interpreted as coefficients of the Taylor expansion of this unknown function. In conclusion, this is all the information that can be extracted from the O$-$C data with an unknown model. The O$-$C variations of AR Lac may indeed be discontinuous, but the observations in "e" system certainly do not suggest this (Figs. 1a and 2a: closed symbols). It rather seems that the accuracy of the data in "other" systems is relatively low. While the continuity of the O$-$C curve of AR Lac can not be proved beyond doubt, we will continue by assuming that this is the case. The 1σ error limits for $(\mathrm{O-C})(T, P_0)$ in this model (Eq. 5) are

$$\sigma_{\mathrm{O-C}}(T, P_0) = \sqrt{\sum_{i=0}^{K} (T^i \sigma_{C_i})^2}, \quad (7)$$

where σ_{C_i} are the 1σ errors of the free parameters $C_i(P_0)$. Because only a Taylor expansion of the unknown model is available, these error estimates diverge strongly at both ends of the time interval of the data, as shown in Figs. 1b and 2b.

The question of choosing the order (K) in Eq. 5 can be settled by setting all weights to unity and modelling Eq. 5 to different orders. The mean residuals given in Table 3 stop decreasing at $K = 9$ for both the "e" and the "other" data. These mean residuals with weights of unity should satisfy an approximation $\langle\sigma_e\rangle \approx \langle\epsilon_e\rangle = 0.^d004$ and $\langle\sigma_{\mathrm{other}}\rangle \approx \langle\epsilon_{\mathrm{other}}\rangle = 0.^d011$ (see, e.g., Press et al. 1986) and thus they were chosen as error estimates for the data in Table 1. Inspection of Table 3 also reveals that it is unnecessary to model the data in higher orders (i.e. $K > 9$), because the mean residuals will not decrease. The weighted modelling for $K = 9$ with the above error estimates yields $\chi^2 = 287$ for the data of Fig. 1a. The probability for this or a smaller value of χ^2 is unity, i.e. the fit is not significant. However, this result is due to outliers. For example, the three residuals for the data by Karle (1962) and Martignoni (1995) contribute 31% to this χ^2 of 156 residuals, while the mean "e" residual is already $\langle\epsilon_e\rangle = 0.^d0031$. But the elimination of outliers is a questionable procedure, especially for an unknown model. Therefore we conclude that modelling the data in Fig. 1a with $K = 9$ is sufficient, although the fit is not significant due to outliers in the "other" systems. However, we do note that an alternative approach to decide the limiting order K in models similar to our Eq. 5 has been proposed, e.g., by Kalimeris et al. (1995). They concluded that the order is sufficient, if "no oscillatory term can be traced in the residuals" of the O$-$C modelling. Another approach might be to formulate criteria based on the "expected noise" level (e.g. correlations between residuals or χ^2) but this is quite difficult, if the residuals are unevenly spaced in time and/or the errors of the data are unknown.

Table 2. The coefficients $C_i(P_0)$ of Eq. 5 in Figs. 1ab (i.e. $P_0 = P_1$)

$K = 9$	$C_0 = -0.0174 \pm 0.0013$	$C_4 = -23.4 \pm 3.0$	$C_8 = -362 \pm 74$
	$C_1 = 0.036 \pm 0.012$	$C_5 = -41 \pm 12$	$C_9 = -77 \pm 220$
	$C_2 = 0.91 \pm 0.11$	$C_6 = 158 \pm 27$	
	$C_3 = 4.15 \pm 0.68$	$C_7 = 126 \pm 88$	
$K = 8$	$C_0 = -0.0175 \pm 0.0012$	$C_4 = -24.3 \pm 2.3$	$C_8 = -383 \pm 55$
	$C_1 = 0.038 \pm 0.010$	$C_5 = -37.7 \pm 5.2$	
	$C_2 = 0.94 \pm 0.10$	$C_6 = 166 \pm 20$	
	$C_3 = 4.01 \pm 0.45$	$C_7 = 96 \pm 17$	

Although trivial, we note that the χ^2 values for the data in Figs. 1 and 2 are equal for $K = 8$ and 9 (see Eq. 6), and the residuals are also the same (i.e. Table 3). The results for the model with $K = 8$ are also outlined in all our figures, because they show how strongly the error estimates depend on the order of the model (Eq. 5), although the solutions for $(\mathrm{O-C})(T, P_0)$ and $P(T, P_0)$ (see Eq. 8 below) remain nearly unchanged.

Table 3. The mean residuals $\langle\epsilon_e\rangle$ and $\langle\epsilon_{\mathrm{other}}\rangle$ [d] for the nonweighted fits to the data of Fig. 1a in orders K of Eq. 5

K	$\langle\epsilon_e\rangle$	$\langle\epsilon_{\mathrm{other}}\rangle$	K	$\langle\epsilon_e\rangle$	$\langle\epsilon_{\mathrm{other}}\rangle$	K	$\langle\epsilon_e\rangle$	$\langle\epsilon_{\mathrm{other}}\rangle$
4	0.006	0.018	8	0.005	0.012	12	0.004	0.011
5	0.007	0.017	9	0.004	0.011	13	0.004	0.011
6	0.005	0.014	10	0.004	0.011	14	0.004	0.011
7	0.006	0.012	11	0.004	0.011	15	0.004	0.011

The nonparametric bootstrap for regression coefficients formulated by Efron & Tibshirani (1986) was chosen to estimate the errors σ_{C_i} of the free parameters $C_i(P_0)$, because outliers are present and the error estimates for the data are based on the residuals of the model. An example of this method can be found in Jetsu (1993) and the same notation is adopted here. If the model g of Eq. 5 is denoted as $g = (T, P_0, \bar{a})$, the vector of the free parameters is $\bar{a} = [C_0(P_0), C_1(P_0), ..., C_K(P_0)]$. The random samples $y^*(T_i)$ for a fixed P_0 are derived as explained in Jetsu (1993: Eq. 5). The only difference is that the correspondence that w_i is the weight of the residual ϵ_i is conserved when selecting the random samples $\bar{\epsilon}^*$ and the modelling is

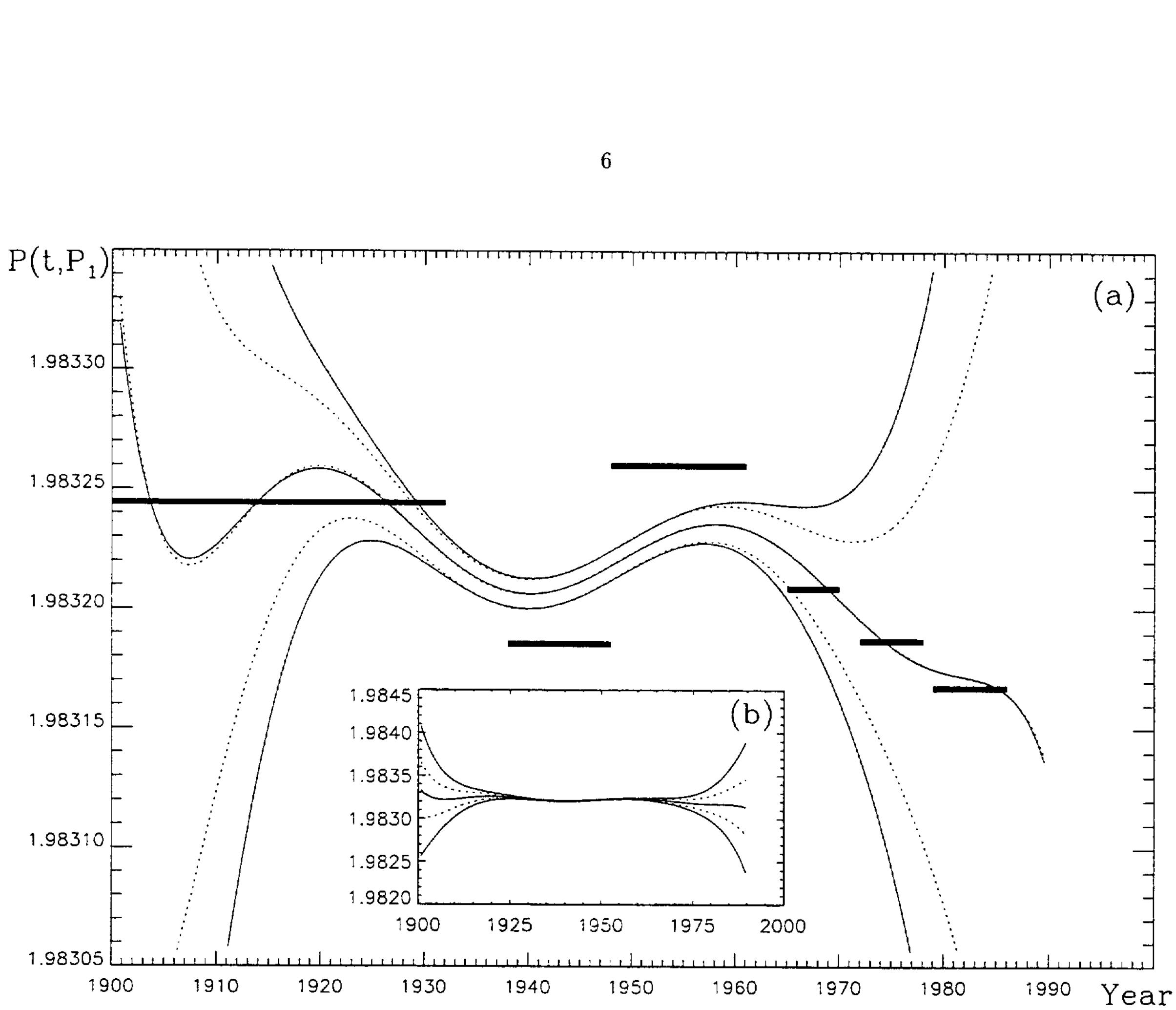

Fig. 3. a) The period variations $P(T,P_1)$ and their 1σ error limits (Eqs. 8 and 9) with $K=8$ (dotted line) and $K=9$ (continuous line). The results by Kim (1991: his Table 2) are outlined with thick horizontal lines. **b)** The total range of 1σ error limits of $P(T,P_1)$

performed for the weighted samples $\bar{y}^*$. The estimates of σ_{C_i} obtained with this bootstrap procedure for the models in Figs. 1ab are given in Table 2.

Because the variations of $(O-C)(T,P_0)$ for $P_0=P_1$ and P_2 are not very large over 90 years, the ratio $P(T,P_0)/P_0$ remains close to unity, and the relation

$$\Delta(O-C)(T,P_0) = \frac{\Delta T}{P_0} - \frac{\Delta T}{P(T,P_0)},$$

is satisfied during any short time interval ΔT. This means that the period variation for the model (Eq. 5) is

$$P(T,P_0) = \left[\frac{1}{P_0} - \sum_{i=1}^{K} iC_i(P_0)T^{i-1}\right]^{-1} \quad (8)$$

while the error estimate of $P(T,P_0)$ is

$$\sigma_{P(T,P_0)} = [P(t,P_0)]^2 \sqrt{\left[\sum_{i=1}^{K} (iT^{i-1}\sigma_{C_i})^2\right] + \left(\frac{\sigma_{P_0}}{P_0^2}\right)^2}. \quad (9)$$

The transformation of Eq. 6 shows that the difference between the coefficients $C_1(P_1)$ and $C_1(P_2)$ for the model of Eq. 5 is $(P_2-P_1)(P_1P_2)^{-1}$. Because all other coefficients $C_2(P_0)$, ..., $C_K(P_0)$ are equal for both periods, it is quite easy to show that

$$P(T,P_1) = P(T,P_2). \quad (10)$$

In conclusion, the results for the period variations are the same for the $O-C$ data of Figs. 1 and 2. The $P(T,P_1)$ variation of AR Lac is shown in Fig. 3.

Finally, we note that higher order polynomials have been previously used in modelling the $O-C$ variations of eclipsing binaries. For example, Wood & Forbes (1963) derived ephemerides based on third order polynomials. In particular, the method formulated by Kalimeris et al. (1994, 1995) utilizes higher order polynomials, but the period variations are not solved directly from the free parameters, and no error estimates are presented. Although Kalimeris et al. (1994) correctly emphasized the uncertainties connected with the modelling of the data in parts, they sometimes found necessary to perform this procedure with spline interpolation between different parts of the data. If "no oscillatory term" could be traced in the residuals, they concluded that the degree of the polynomial model was sufficient.

5. Discussion and conclusions

Hall & Kreiner (1980) reviewed several physical phenomena that might explain the period variations of AR Lac: third body, magnetically driven anisotropic mass ejection, effects of starspots, apsidal motion, etc. More recently, Kim (1991) suggested a new alternative, a beat phenomenon, where several periodicities due to different physical mechanisms interact to produce the observed aperiodic and seemingly irregular $O-C$ variations. He also summarized the reasons for rejecting most of the previously proposed models for explaining the period changes. Our paper has concentrated mainly on developing a method for extracting information about the period changes of AR Lac, as well as of other eclipsing binaries, based on analysing the epochs of the primary and secondary minima. However, we do note that it is possible to explain the period variations of AR Lac without abrupt changes. In fact, our Fig. 3a does not even rule out the possibility of a cyclic variation in $P(T, P_0)$. The period changes of AR Lac may well be continuous and Fig. 3a might indicate the presence of *quasiperiodic* trends with a period of about 30 years. Unfortunately, the error estimates for the period variations are as yet too large to verify this possibility (corresponding, e.g., to the presence of a third body), but a few decades of new photoelectric determinations of the epochs of the primary and secondary minima will certainly determine whether a cycle in $P(T, P_0)$ exists. In general, it might be more fruitful to transform the O−C data to the period domain and then consider the alternative physical processes, because direct interpretations of the O−C data with any P_0 have their disadvantages, as explained below.

The time series analysis of the epochs of the primary and secondary minima of AR Lac with the nonparametric WK– and WSD–methods gave two different periods for studying the O−C variations. This contradiction is apparent for three reasons. Firstly, although both methods are nonparametric (i.e. model independent), they are not equally sensitive to different types of distributions, as discussed in greater detail in Jetsu et al. (1995). Secondly, the period of AR Lac is variable, and the available data cannot be used to decide whether these variations are stationary. If they are nonstationary, i.e. both the mean and the standard deviation of the period are not constant over a longer time interval, then a unique period for determining the O−C values does not exist or, equivalently, any time series analysis method will fail. Thirdly, although the different periods (P_0) detected with the WK– and WSD–methods yield different O−C values, the temporal variations of the period of AR Lac with the model of Eq. 5 are the same for both values of P_0.

All O−C data of AR Lac was modelled (Eq. 5) with $K+1 =$ 9 or 10 free parameters in Figs. 1 and 2. However, the error estimates of $(O-C)(T, P_0)$ and $P(T, P_0)$ diverge strongly at both ends of the time interval of data. But this is unavoidable, if only a Taylor series of an unknown model is available, and the aim is to analyse the whole time series simultaneously. Naturally, the centre (t") of the Taylor expansion can be shifted, for example, closer to the recent data, but the cost will be that the error estimates diverge even more strongly for the earlier data. In any case, the results of the modelling of Eq. 5 are independent of the chosen t". Another approach would be to determine these expansions for parts of the data, but this would most probably only yield discontinuous period curves, which have been amply discussed in the earlier literature of AR Lac, as well as that of other eclipsing binaries. Furthermore, the selection of these parts of the data for modelling would be subjective. For example, Kim (1991) modelled the period variations of AR Lac with six linear fits for separate parts of the data and a similar approach can also be found in Chambliss (1976: Table 2). The six linear fits in Kim (1991) required 12 free parameters, while the epochs selected to subdivide the data represented 5 additional free parameters, giving total of 17 free parameters. Moreover, the subdivision was made by eye and would most probably have been different, for example, if the value of P_0 in our Fig. 2 had been used to derive the O−C values. That model results in discontinuous period variations, interpreted as abrupt period changes (see Fig. 3: thick horizontal lines). The results by Kim (1991) are in quantitative agreement with our $P(T, P_0)$ curve, except between 1938 and 1961. This is mainly due to omitting the data by Ishchenko (1963) from our Table 1, while these data were analysed by Kim (1991). As already mentioned earlier, Hall & Kreiner (1980) gave zero weights to these data. Had we included the data by Ishchenko (1963), then the other data with zero weights in Hall & Kreiner (1980) should have also been included, but this would have only introduced more outliers to the modelling. Note that this problem of outliers was discussed in Sect. 4., in connection with the data of Karle (1962) and Martignoni (1995).

There are at least two approaches that could reduce the uncertainties in the determination of the period variation. If the model remains unknown, *new data* will eventually reveal more details of the period variation of AR Lac, and there are certainly eclipsing binaries where the model of Eq. 5 will not require so many orders (K) and/or the quality of the data is better. Hall & Kreiner (1980) note that AR Lac has one of the most "baffling" O−C curves among eclipsing binaries, which in terms of our model means a high value of K in Eq. 5. For example, modelling the O−C variations of RT And or SV Cam would most probably succeed with $K < 9$ in Eq. 5 and give more accurate estimates of the period changes (see Hall & Kreiner 1980: Figs. 1 and 5). Another possibility is that a *known* model with less free parameters than in ours is developed, that is, some parameters can be fixed on physical or other grounds. The partial derivatives of this hypothetical model with respect to the free parameters should not have a similar time dependence as in our model (Eq. 5), e.g. trigonometric functions might be utilized. In any case, the method outlined in this paper ought to motivate observers to obtain new epochs of the minima of eclipsing binaries.

The most important conclusions of this study are:

1. The period variations $P(T, P_0)$ of AR Lac can be modelled (Eq. 5) with a continuous curve, and the results are independent of the period P_0 used to derive the O−C values.
2. A decrease or increase of O−C values should not be interpreted as a decrease or increase of the period, respectively. These changes of O−C depend on the chosen P_0, while the real period changes depend on the derivative of the O−C curve, and are independent of P_0. For example, the O−C values will always decrease when $P(T, P_0) < P_0$, regardless of whether $P(T, P_0)$ itself is decreasing or increasing.
3. Different types of regularities, as for example periodic variations of the O−C values, are meaningless, because the period is not constant. These apparent regularities depend solely on the chosen P_0, which can not be uniquely determined. Moreover, the period changes may be nonstationary, i.e. the long–term mean and variance may not be constant. Even periodic changes of the orbital period do not necessarily show as periodic changes in the O−C data. For example, let us assume that T_1 is the time interval for the period to increase from the minimum ($P_{\min}$) to the maximum ($P_{\max}$), and the corresponding time interval for the decrease back to $P_{\min}$ is T_2. If the period changes are not symmetric, e.g. $T_1 \neq T_2$, then the O−C data do not necessarily show this cycle (i.e. $T_1 + T_2$), although the O−C values were derived with the mean period $(P_{\max} + P_{\min})/2$.

4. Linear, or even higher order, fits to *parts* of the data are oversimplifications which give misleading results for the period variation. Moreover, because the whole data set is not analysed simultaneously, discontinuous period changes will result.

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References

Ahnert P. 1949, Astr. Nachr. 277, 187
Ahnert P. 1965, Mitt. Veränderl. Sterne, Sonneberg 2, 170
Ahnert P. 1966, Mitt. Veränderl. Sterne, Sonneberg 4, 137
Aleksandrovich Y. R. 1959, Astron. Circ. U.S.S.R. 205, 19
Applegate J.H. 1992, ApJ 385, 621
Battistini P., Bonifazi A., Guarnieri A. 1973, IBVS 817
Blanco C., Catalano S. 1970, A&A 4, 482
Caton D.B. 1983, IBVS 2303
Cester B. 1967, Mem. Soc. Astron. Ital. 38, 707
Chambliss C.R. 1974, IBVS 883
Chambliss C.R. 1976, PASP 88, 762
Dugan R.S., Wright F.W. 1939, Princeton Univ. Obs. Contr. No. 19
Efron B., Tibshirani R. 1986, Statistical Science 1, 54
Ertan A.Y., Tumer O., Tunca Z., et al. 1982, Ap&SS 87, 255
Evren S., Ibanoglu C., Tumer O., Tunca Z., Ertan A.Y. 1983, Ap&SS 95, 401
Gainullin A.Sh. 1943, Bull. Engelhardt Obs. 22, 3
Hall D.S. 1968, IBVS 281
Hall D.S. 1976, in Multiple Periodic Variable Stars, ed. W.S. Fitch, I.A.U. Coll. 29, Reidel, Dordrecht, p. 287
Hall D.S. 1990, in Active Close Binaries, ed. C. Ibanoglu, NATO ASI series C–Vol. 319, Kluwer, Dordrecht, p. 95
Hall D.S. 1991, ApJ 380, L85
Hall D.S., Kreiner J.M. 1980, Acta Astron. 30, 387
Himpel K. 1936, Astr. Nachr. 261, 233
Ishchenko I.M. 1963, Trudy Tashkent Astr. Obs. ser. II, 9, 28
Isles J.E. 1973, J. Br. Astron. Ass. 83, 452
Isles J.E. 1975, J. Br. Astron. Ass. 85, 443
Jacchia L. 1929, Gaz. Astron. 16, No. 184
Jacchia L. 1930, Astr. Nachr. 237, 249
Jetsu L. 1993, A&A 276, 345
Jetsu L. 1996, A&A, in press
Jetsu L., Pelt J. 1996, A&AS, in press (Paper I)
Jetsu L., Pohjolainen S., Pelt J., Tuominen I. 1995, A&A, submitted
Kalimeris A., Rovithis–Livaniou H., Rovithis P. 1994, A&A 282, 775
Kalimeris A., Mitrou C.K., Doyle J.G., Antonopoulou E., Rovithis–Livaniou H. 1995, A&A 293, 371
Karetnikov V.G. 1959, Astron. Circ. U.S.S.R. 207, 16
Karetnikov V.G. 1961, Perem. Zvezdy 13, 420
Karle J.H. 1962, PASP 74, 244
Karle J.H., Vaucher Ch., Gaston B., Sherman E. 1977, Acta Astron. 27, 93
Kim C.-H. 1991, AJ 102, 1784
Kizilirmak A., Pohl E. 1968, Astr. Nachr. 291, 111
Kizilirmak A., Pohl E. 1974, IBVS 937
Kron G.E. 1947, PASP 59, 261
Kuiper N.H. 1960, Proc. Koningkl. Nederl. Akad. Van Wettenschappen, Series A, 63, 38
Kurutac M., Ibanoglu C., Tunca Z., et al. 1981, Ap&SS 77, 325
Lee E.-H., Chen K.-Y., Nha I.-S. 1986, AJ 91, 1438
Loreta E. 1930, Gaz. Astron. 17, 7
Makarov V., Mandel O., Panaioti A. 1957, Astron. Circ. U.S.S.R. 187, 16
Martignoni M. 1995, BBSAG Bull. 109
Nezry E. 1988, BBSAG Bull. 89
Nha I.-S., Kang Y.-W. 1982, PASP 94, 496
Oburka O. 1964, Bull. Astr. Inst. Czech. 15, 250
Oburka O. 1965, Bull. Astr. Inst. Czech. 16, 212
Oburka O., Silhan J. 1970, Contr. Obs. and Planetarium Brno, No 9, 27 pp.
Pagano I. 1990, Degree Thesis, Univ. of Catania
Panchatsarem T., Abhyankar K.D. 1982, in Binary and Multiple Stars as Tracers of Stellar Evolution, eds. Z. Kopal and J. Rahe, Reidel, Dordrecht, p. 47
Panov K.P. 1987, private communication
Parenago P. 1930, Astr. Nachr. 238, 209
Parenago P. 1938, Publ. Sternberg 12, 35
Park H.S. 1984, J. Astron. Space Sci. 1, 67
Peter H. 1972, BBSAG Bull. 4
Pickering E.C. 1907, Harvard Coll. Obs. Circ. 130, 4
Pickup D.A. 1972, BBSAG Bull. 2
Pohl E., Kizilirmak A. 1966, Astr. Nachr. 289, 191
Pohl E., Kizilirmak A. 1970, IBVS 456
Pohl E., Evren S., Tumer O., Sezer C. 1982, IBVS 2189
Pokorny Z. 1974, Contr. Obs. and Planetarium Brno, No 17, 15 pp.
Press W.H., Flannery B.P., Teukolsky S.A., Vetterling W.T. 1986, Numerical Recipes, Cambridge University Press, New York
Rodonò M., Lanza A. F., Catalano S. 1995, A&A 301, 75
Rügemer H. 1931, Astr. Nachr. 245, 39
Scarfe C.D., Barlow D.J. 1978, IBVS 1379
Schneller H., Plaut L. 1932, Astr. Nachr. 245, 387
Srivastava R.K. 1981, Ap&SS 78, 123
Strassmeier K.G., Hall D.S., Fekel F.C., Scheck M. 1993, A&AS 100, 173
Svechnikov M.A. 1955, Perem. Zvezdy 10, 262
Swanepoel J.W.H, De Beer C.F. 1990, ApJ 350, 754
Wood D.B., Forbes J.E. 1963, AJ 68, 257
Wood F.B. 1946, Princeton Univ. Obs. Contr. No. 21
Wroblewski A. 1956, Acta Astron. 6, 146
Zverev M. 1936, Publ. Sternberg 8, 43

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