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Scalar-Isoscalar Meson Exchange in the Calculation of the Nucleon-Nucleon Interaction

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Abstract

We provide a unified description of (i) scalar-isoscalar exchange in the nucleon-nucleon interaction, (ii) the pion-nucleon sigma term, and (iii) the scalar form factor of the nucleon. Our analysis requires that we specify a parameter that appears in the description of a nucleon valence-quark "core". Other parameters are fixed either by our analysis of the Nambu–Jona-Lasinio model, or with reference to a recent lattice simulation of QCD in which the scalar form factor of the nucleon was calculated. We find that our model has some predictive power. Once the parameters are fixed, we find that we reproduce the values of the scalar form factor of the nucleon, as determined in the lattice simulation. We also predict the strength of the scalar-isoscalar *NN* potential for the particular OBE model considered here, where the effects of (virtual) delta excitation are treated in an explicit fashion. However, the overall strength of the force obtained in this work is sensitive to the approximations used in the calculation.

I. Introduction

In this work we will use some information gained from a recent QCD lattice simulation [1], where a study was made of the pion-nucleon sigma term and the scalar form factor of the nucleon. One of our goals is to study the scalar-isoscalar component of the nucleon-nucleon interaction, making use of an extended version of the Nambu–Jona-Lasinio model that we have developed [2-7]. In particular, we are interested in understanding the parametrization of the one-boson-exchange (OBE) model of the nucleon-nucleon interaction [7] in the case of scalar exchange.

In the OBE model, the intermediate-range attraction is obtained via the exchange of a scalar meson (sigma) of mass 550 MeV. There is no such meson in the data tables, so that the use of such a meson in the OBE model is problematic. This problem may be resolved in our extended version of the NJL model [3]. The standard bosonization procedure of the NJL model leads to a scalar meson of mass $m_\sigma^2 = 4m_q^2 + m_\pi^2$, where m_q is the constituent quark mass. In one case, we found $m_q = 260$ MeV and $m_\sigma = 540$ MeV [5]. Inclusion of a model of confinement moves the energy of the scalar meson out of the low-energy region, so that $m_\sigma \geq 900$ MeV. However, we have shown that, for spacelike meson momenta, the theory behaves as if there were a pole at $m_\sigma = 540$ MeV [3]. Since the meson momenta in the OBE model are spacelike, we see how the problem mentioned above may be resolved.

It is useful to start our discussion with a review of our calculation of the nucleon scalar-isoscalar form factor at zero momentum transfer [5]. Let us define the scalar form factor, $F_S(q^2)$, such that

$$F_S(q^2)\bar{u}(\vec{p}+\vec{q}, s')u(\vec{p}, s)\delta_{\tau\tau'} = \langle N, \vec{p}+\vec{q}, s', \tau' \,|\, \bar{q}(0)q(0) \,|\, N, \vec{p}, s, \tau \rangle \quad . \tag{1.1}$$

(Here $q(x)$ is the quark field and $u(\vec{p}, s)$ is a Dirac spinor.) In our earlier work, we calculated $F_S(0)$ in a sigma-dominance model [5]. (See Fig. 1.) In Fig. 1 we show the various ingredients of the calculation. The operator $\bar{q}(0)q(0)$ is represented by a large filled circle. The single lines represented (constituent) quarks, while the string of $q\bar{q}$ loops may be expressed in terms of the basic quark-antiquark loop integral of the NJL model, $J_S(q^2)$ [3,5].

The first diagram gave a contribution to $F_S(0)$ that we may denote as $\langle N \,|\, \bar{q}q \,|\, N \rangle_{val}$, where the subscript indicates that we have separated the form factor into a valence part and a part due to the "meson cloud" [1,5]. Thus

$$F_S(0) = \langle N \,|\, \bar{q}q \,|\, N \rangle_{val} + \langle N \,|\, \bar{q}q \,|\, N \rangle_C \quad . \tag{1.2}$$

The value obtained for $F_S(0)$ in the QCD simulation was $F_S(0) \simeq 10.0$, with 85% of this value coming from the up and down quark contribution. It was also found that $\langle N \,|\, \bar{q}q \,|\, N \rangle_{val} = 3.02$ [1]. (See Table 1.) The diagram representing the valence contribution in Ref. [1] was the same as the diagram of Fig. 1a. In Ref. [1] that diagram represented a connected amplitude, while the remaining (meson cloud) contribution was given by a disconnected diagram. It was also found that the meson cloud contribution was about twice the valence contribution. It is of interest to see that our sigma-dominance calculation has the same feature. To understand that comment, we present the result of the analysis given in Ref. [5]. There we had

$$\langle N \,|\, \bar{q}q \,|\, N \rangle = \frac{1}{1 - G_S J_S(0)} \langle N \,|\, \bar{q}q \,|\, N \rangle_{val} \tag{1.3}$$

where the factor $[1 - G_S J_S(0)]^{-1}$ serves to generate the series depicted in Figs. 1a, 1b, 1c, etc.

We may write Eq. (1.3) as

$$\langle N|\bar{q}q|N\rangle = \langle N|\bar{q}q|N\rangle_{val} + \left[\frac{1}{1-G_S J_S(0)} - 1\right]\langle N|\bar{q}q|N\rangle_{val} \ , \tag{1.4}$$

where the second term is the "meson cloud" contribution.

We used $G_S = 7.91\ \text{GeV}^{-2}$ and found $[1 - G_S J_S(0)]^{-1} = 3.12$ in Ref. [5]. (See Table 2.) Thus, since we had taken $\langle N|\bar{q}q|N\rangle_{val} = 3.0$, we obtained $\langle N|\bar{q}q|N\rangle = 9.36$. In Eq. (1.4) the meson cloud contribution is 6.36, which is 2.12 times the valence contribution. While it is not clear whether our meson cloud contribution is directly related to the disconnected (meson cloud) contribution of Ref. [1], the numerical values obtained are remarkably similar. Note further that we may define the pion-nucleon sigma term for $q^2 = 0$, with $\bar{q}q = \bar{u}u + \bar{d}d$, as

$$\sigma_N = \left[\frac{m_u^0 + m_d^0}{2}\right]\langle N|\bar{q}q|N\rangle \ , \tag{1.5}$$

where m_u^0 and m_d^0 are the current quark masses. In our earlier work we found that the average current quark mass was 5.50 MeV, if we were to obtain $m_\pi = 138$ MeV, when we used a Euclidean cutoff of $\Lambda_E = 1000$ MeV. (See Table 2.) Thus, we had $\sigma_N = 51.5$ MeV, which is very close to the value obtained in the QCD lattice simulation: $\sigma_N = 49.7\ (2.6)$ MeV. (Here 2.6 MeV represents a measure of the theoretical uncertainty.) Our result is also consistent with the analysis of Vogl and Weise [8] that is based upon the use of the Feynman-Hellman theorem [9,10] in the calculation of σ_N. Vogl and Weise calculate a pion-quark sigma term and then multiply by 3 to obtain the pion-nucleon sigma term. It was found that [8,11]

$$\sigma_N = 3m_\pi^2 \left[\frac{m_u}{4m_u^2 + m_\pi^2} \right] \quad . \tag{1.6}$$

Our analysis of the NJL model found $m_q = 260$ MeV for $\Lambda_E = 1.0$ GeV. Therefore, from Eq. (1.6), we have $\sigma_N = 51.3$ MeV, which is quite close to the value $\sigma_N = 51.5$ MeV obtained from our study of the scalar form factor [5]. We remark that, in the work of Vogl and Weise [8], up and down quark masses of about 360 MeV are used. Use of $m_u = 360$ MeV in Eq. (1.6) leads to $\sigma_N = 38$ MeV. That value is considered to be too small and, therefore, other features (such as diquark correlations in the nucleon) that enhance the calculated value of σ_N are studied [8].

In order to introduce the sigma "meson" into the analysis, it is useful to write

$$\frac{G_S}{1 - G_S J_S(q^2)} = - \frac{g_{\sigma qq}^2(q^2)}{q^2 - m_\sigma^2} \tag{1.7}$$

for $q^2 < 0$. Equation (1.7) serves to define the momentum-dependent meson-quark coupling parameter $g_{\sigma qq}(q^2)$. For values of $-q^2$ that are positive, and not too large, we may write [8]

$$\frac{G_S}{1 - G_S J_S(q^2)} \simeq - \frac{g_{\sigma qq}^2}{q^2 - m_\sigma^2} \quad , \tag{1.8}$$

where $g_{\sigma qq} = 2.58$ and $m_\sigma = 540$ MeV [5]. Thus,

$$\frac{1}{1 - G_S J_S(q^2)} = - \left[\frac{g_{\sigma qq}}{G_S} \right] \frac{1}{q^2 - m_\sigma^2} g_{\sigma qq} \quad . \tag{1.9}$$

The factor $(- g_{\sigma qq}/G_S)$ in Eq. (1.9) may be understood as arising from the bosonization relation

$$\sigma(x) = -\frac{G_S}{g_{\sigma qq}}\bar{q}(x)q(x) \qquad (1.10)$$

that appears in the simplest bosonization scheme used for the NJL model. (In vacuum, Eq. (1.9) may be identified with the Goldberger-Treiman relation $f_\pi = m_q/g_\pi$, where for exact chiral symmetry, $g_\pi \equiv g_{\sigma qq} = g_{\pi qq}$.) Equation (1.8) and Eq. (1.3) may be used to generate the diagrams of the sigma-dominance model, such as that shown in Fig. 1d.

While the comments made above are suggestive, we still have to describe how these ideas may be used to discuss sigma exchange in the nucleon-nucleon interaction. That discussion will be taken up in the next section.

II. Sigma Exchange in the OBE Model of the Nucleon-Nucleon Interaction

In earlier work we have discussed a relation between the OBE model of the nucleon-nucleon interaction and the NJL model [12-14]. To describe that relation, it is useful to define a parameter, λ_σ, that governs the momentum dependence of the valence scalar form factor of the nucleon:

$$F_S^{val}(q^2) = F_S^{val}(0)\left[\frac{\lambda_\sigma^2}{\lambda_\sigma^2 - q^2}\right] , \tag{2.1}$$

with $F_S^{val}(0) = \langle N|\bar{q}q|N\rangle_{val}$. We put $F_S^{val}(0) = 3.02$ in accordance with the analysis of Ref. [1]. (See Table 1.)

In the OBE model there are vertex cutoffs that are dependent upon a parameter Λ_σ. Thus the OBE force in the scalar channel is [7]

$$V_\sigma^{OBE}(q^2) = g_{\sigma NN}^2\left[\frac{\Lambda_\sigma^2 - m_\sigma^2}{\Lambda_\sigma^2 - q^2}\right]^2 \frac{1}{q^2 - m_\sigma^2} . \tag{2.2}$$

(We suppress reference to Dirac and isospin matrices for simplicity.) In Table B.1 of Ref. [7], we find $\Lambda_\sigma = 1.5$ GeV, $g_{\sigma NN}^2/4\pi = 6.32$, and $m_\sigma = 550$ MeV. (These parameters are given for the case where the effects of the virtual excitation of the delta are treated explicitly.) It is useful to define

$$\frac{\left(G_{\sigma NN}^{OBE}\right)^2}{4\pi} = \frac{g_{\sigma NN}^2}{4\pi}\left[\frac{\Lambda_\sigma^2 - m_\sigma^2}{\Lambda_\sigma^2}\right]^2 . \tag{2.3}$$

With the values given above, we find $G_{\sigma NN}^{OBE} = 7.71$.

The corresponding amplitude in the NJL model is given in terms of the quark-quark T matrix, $t_{qq}(q^2)$, such that

$$V_\sigma^{NJL}(q^2) = t_{qq}(q^2)\left[F_S^{val}(0)\right]^2 \left[\frac{\lambda_\sigma^2}{\lambda_\sigma^2 - q^2}\right]^2 \tag{2.4}$$

$$= \frac{g_{\sigma qq}^2}{q^2 - m_\sigma^2}\left[F_S^{val}(0)\right]^2 \left[\frac{\lambda_\sigma^2}{\lambda_\sigma^2 - q^2}\right]^2 . \tag{2.5}$$

(See Fig. 2b.) For consistency we should have $G_{\sigma NN}^{NJL} = G_{\sigma NN}^{OBE}$, with

$$G_{\sigma NN}^{NJL} = g_{\sigma qq} F_S^{val}(0) . \tag{2.6}$$

Evaluation of the right-hand side of Eq. (2.6) with $g_{\sigma qq} = 2.58$ and $F_S^{val}(0) = 3.02$ yields $G_{\sigma NN}^{NJL} = 7.79$, so that $G_{\sigma NN}^{OBE} / G_{\sigma NN}^{NJL} = 0.99$. Thus, we see that the strength of the force is predicted accurately for this choice of parameters.

As in our previous work, we determine λ_σ by equating $V_\sigma^{NJL}(q^2)$ and $V_\sigma^{OBE}(q^2)$. We find that $\lambda_\sigma = 1.1$ GeV yields an excellent fit. For example, we may put

$$V_\sigma^{NJL}(q^2) = V_\sigma^{NJL}(0)\, h_\sigma^{NJL}(q^2) , \tag{2.7}$$

and

$$V_\sigma^{OBE}(q^2) = V_\sigma^{OBE}(0)\, h_\sigma^{OBE}(q^2) , \tag{2.8}$$

where $h_\sigma^{NJL}(0) = h_\sigma^{OBE}(0) = 1$. We compare the values of $h_\sigma^{OBE}(q^2)$ and $h_\sigma^{NJL}(q^2)$ in Fig. 3 for the case $\lambda_\sigma = 1.1$ GeV. It is seen that the fit is excellent for the chosen value of λ_σ.

III. The Scalar Form Factor of the Nucleon

We now ask whether our formalism has any further predictive power. To that end, we may now calculate our values for the scalar form factor of the nucleon. We see that in our sigma-dominance model

$$F_S(q^2) = \frac{1}{1 - G_S J_S(q^2)} F_S^{val}(0) \left[\frac{\lambda_\sigma^2}{\lambda_\sigma^2 - q^2} \right] \quad , \tag{3.1}$$

where $F_S(0) = 3.02$, as determined in Ref. [1]. (See Table 1.)

We now define

$$f_S(q^2) = \frac{F_S(q^2)}{F_S(0)} \tag{3.2}$$

$$= \frac{[1 - G_S J_S(0)]}{[1 - G_S J_S(q^2)]} \left[\frac{\lambda_\sigma^2}{\lambda_\sigma^2 - q^2} \right] \quad . \tag{3.3}$$

In Fig. 4, the solid line represents $f_S(q^2)$, while the "data points" are the result of the QCD lattice simulation of the scalar form factor [1]. Again, we find a good fit for our choice of $\lambda_\sigma = 1.1$ GeV.

IV. Discussion

We have seen that we have obtained an excellent fit to the strength of the OBE scalar-isoscalar force and to the q^2-dependence of that force. We also fit the values of the scalar form factor of the nucleon that were obtained in a QCD lattice simulation [1]. However, some cautionary remarks are in order. Our results are sensitive to the value obtained for $[1 - G_S J_S(0)]$. Various modifications may be considered. For example, if we include a model of confinement, $J_S(0)$ is replaced by $\hat{J}_S(0)$ which is about 10 percent smaller than $J_S(0)$ [12,13]. However, we may also include coupling to the two-pion continuum, as discussed in our earlier work [2,3]. If both effects are included, $[1 - G_S J_S(0)]$ is replaced by $\{1 - G_S[\hat{J}_S(0) + \hat{K}_S(0)]\}$, where $\hat{K}_S(q^2)$ describes the coupling of the $q\bar{q}$ states to the two-pion continuum. We remark that $\hat{K}_S(0)$ is positive and about 10 percent of the value of $J_S(0)$. Therefore, our results are only slightly modified in a first approximation, if we include such effects. These corrections suggest that the rather precise agreement found for $G_{\sigma NN}^{NJL}$ and $G_{\sigma NN}^{OBE}$ may be somewhat fortuitous. However, small modifications of the value of $[1 - G_S J_S(0)]$ will not change the general success of our analysis. We have also seen that the value of the enhancement factors, $[1 - G_S J_S(0)]^{-1} = 3.12$, found in Ref. [5] and used here, provides a satisfactory value for σ_N. The associated value of the constituent quark mass, $m_q = 260$ MeV, also yields a good phenomenological description of the nucleon scalar form factor and the OBE potential in the scalar-isoscalar channel.

In a future work we hope to extend our model to study the SU(3)-flavor version of the NJL model so that we can make a more detailed comparison to the results of Ref. [1].

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Table 1. Results of lattice simulation and of the SU(2) NJL model. (We identify $F_S^{val}(0)$ with $g_{S,con}$ of Ref. [1].)

	Reference [1]	This work
$\left[\frac{m_u^0 + m_d^0}{2}\right]$	5.84 (13)	5.50 MeV
$\langle p \mid \bar{u}u \mid p \rangle$	4.55 (16)	--
$\langle p \mid \bar{d}d \mid p \rangle$	3.92 (16)	--
$\langle N \mid \bar{s}s \mid N \rangle$	1.53 (7)	0
$\langle N \mid \bar{u}u + \bar{d}d \mid N \rangle$	8.47 (24)	9.42
$F_S(0) = \langle N \mid \bar{u}u + \bar{d}d + \bar{s}s \mid N \rangle$	10.00 (25)	9.42
$\Delta\sigma_N = \sigma_N(2m_\pi^2) - \sigma_N(0)$ $\sigma_N(0)$	6.62 (59) MeV 49.7 (2.6) MeV	~ 6.6 MeV 51.8 MeV
$F_S^{val}(0)$	3.02 ± 0.09	3.02 (taken from Ref. [1])
$y = \frac{2\langle N \mid \bar{s}s \mid N \rangle}{\langle N \mid \bar{u}u + \bar{d}d \mid N \rangle}$	0.36 (3)	0

Table 2. The parameters of the NJL model (Λ_E, m_q^0, G_S) used in Ref. [5] and the quantities calculated in Ref. [5] are given.

Λ_E	1.0 GeV
m_q^0	5.5 MeV
G_S	7.91 GeV^{-2}
m_q	260 MeV
m_σ	538 MeV
$1 - G_S J_S(0)$	0.321
$[1 - G_S J_S(0)]^{-1}$	3.12
$g_{\sigma qq}$	2.58
$g_{\pi qq}$	2.68
f_π	93 MeV
m_π	138 MeV
$-2m_q^0\langle 0 \mid \bar{u}u \mid 0 \rangle$	1.76×10^8 MeV^4
$\langle 0 \mid \bar{u}u \mid 0 \rangle^{1/3}$	-252 MeV
σ_N	51.5 MeV

Figure Captions

Fig. 1 Calculation of the scalar form factor of the nucleon. The operator $\bar{q}(0)q(0)$ is denoted by the large filled circle. The single lines represent quarks or antiquarks.

(a) The valence contribution is shown.

(b)-(c) A series of quark-antiquark loop diagrams are shown.

(d) A sigma dominance model representing the diagrams shown in a, b, c, etc. Here the small open circle represents $g_{\sigma qq}$.

Fig. 2 (a) The OBE amplitude due to sigma exchange between nucleons. The large open circles denote the vertex cutoffs of the OBE model [7].

(b) The representation of sigma exchange in the NJL model based upon the use of the valence form factor seen in Fig. 1a. The small open circles represent $g_{\sigma qq}$.

(c) The nucleon-nucleon interaction is related to a quark-quark T matrix. A sigma-dominance model of the T matrix is shown in b).

Fig. 3 The values of $h_\sigma^{OBE}(q^2)$ [dotted line] and $h_\sigma^{NJL}(q^2)$ [solid line] are shown. Here $\Lambda_\sigma = 1.5$ GeV and $\lambda_\sigma = 1.10$ GeV. For the OBE amplitude $m_\sigma = 0.550$ GeV. (See Eqs. (2.7) and (2.8).)

Fig. 4 The values calculated for $f_S(q^2) = F_S(q^2)/F_S(0)$ are shown. (See Eqs. (3.1) and (3.3).) Here $\lambda_\sigma = 1.10$ GeV. The circles with error bars are results of the QCD lattice simulation of Ref. [1].

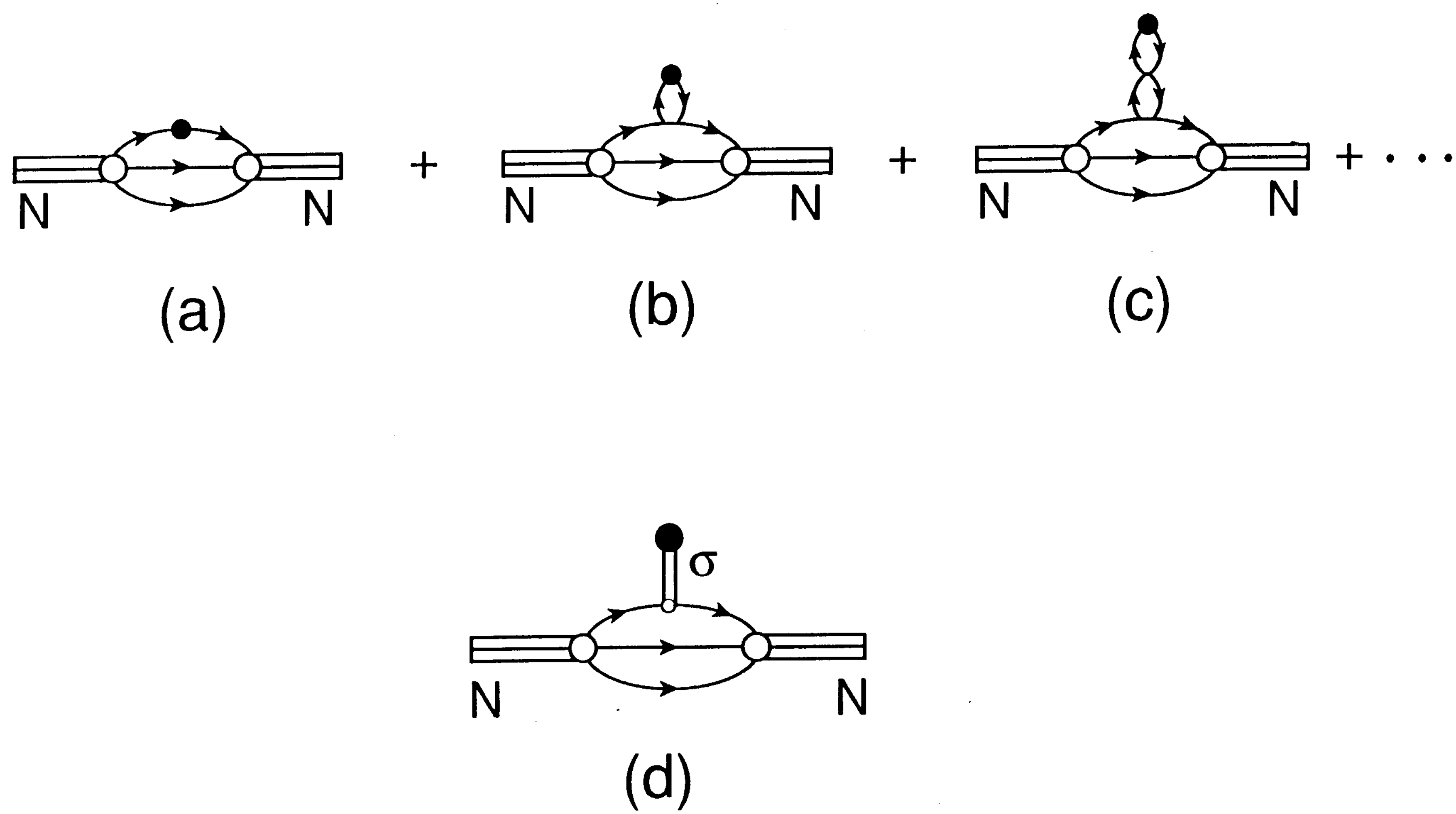

FIG. 1

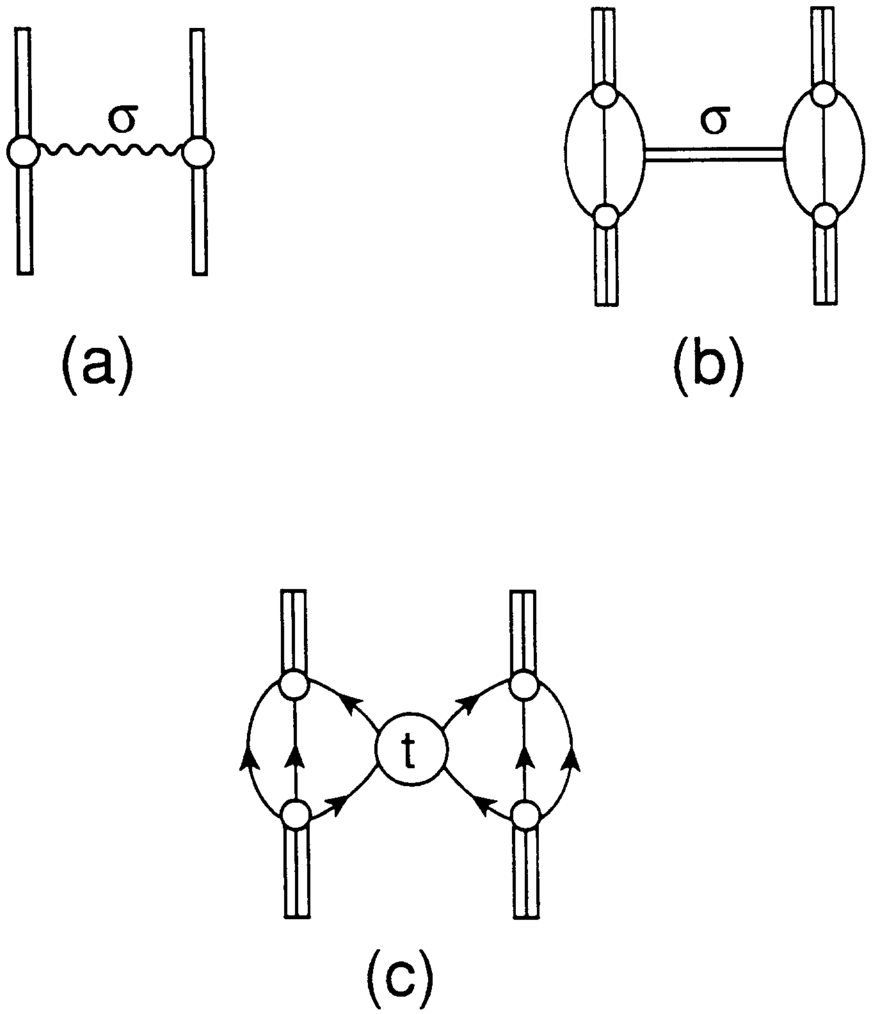

FIG. 2

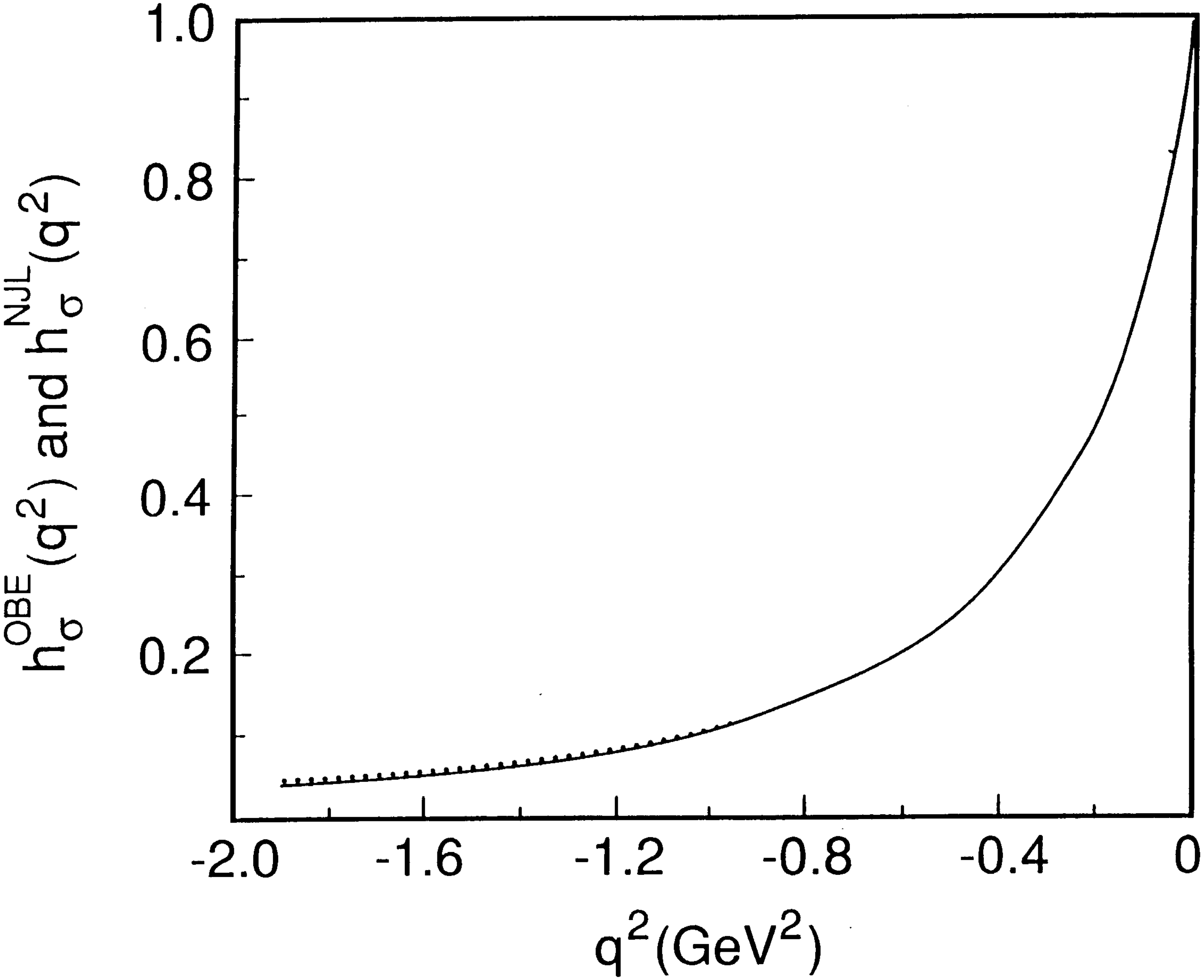
1.0
0.8
0.6
0.4
0.2
$h_\sigma^{OBE}(q^2)$ and $h_\sigma^{NJL}(q^2)$
-2.0
-1.6
-1.2
-0.8
-0.4
0
$q^2(GeV^2)$

FIG. 3

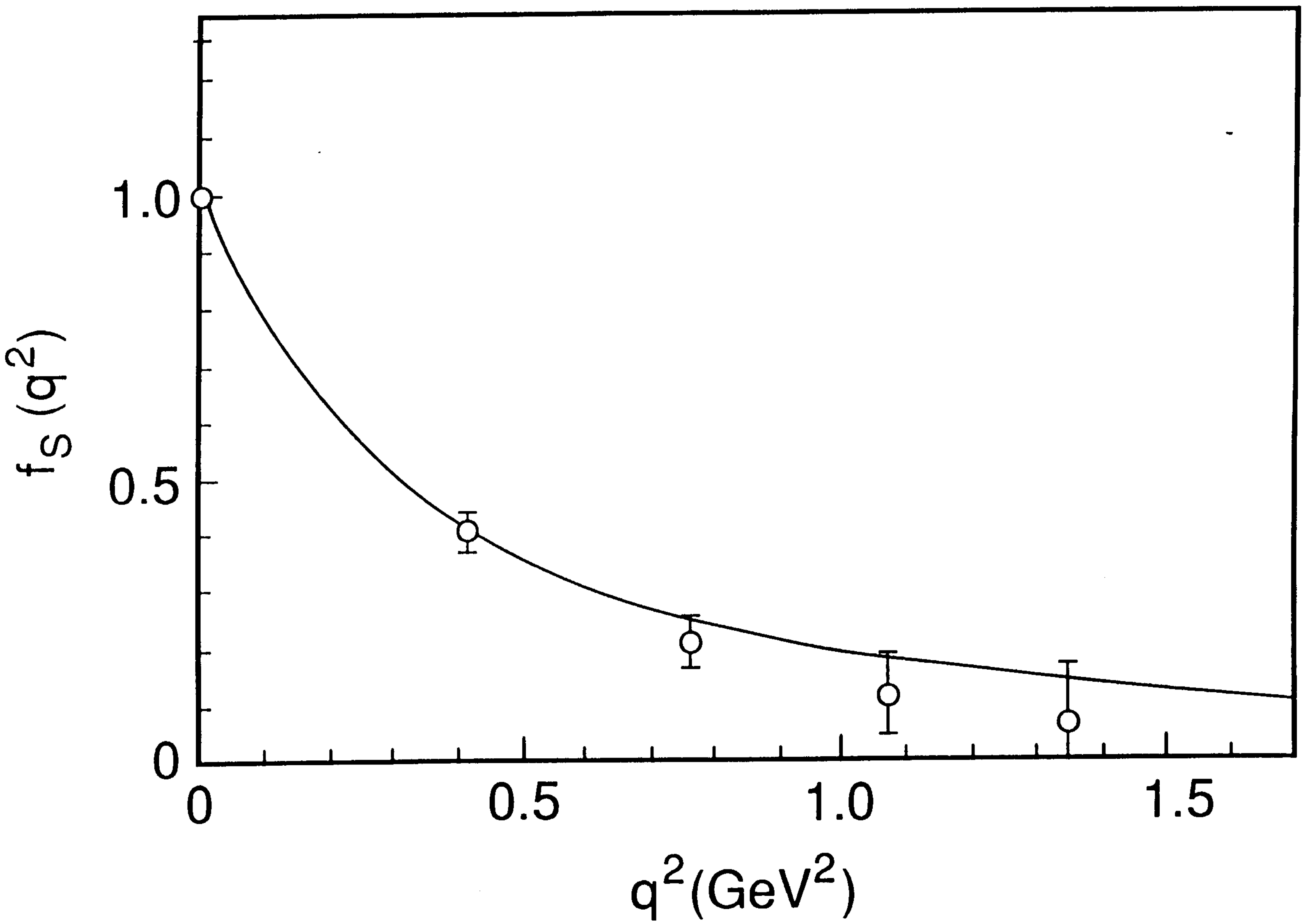

FIG.4