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BRS Cohomology of Zero Curvature Systems

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Abstract

The computation of the BRS cohomology classes associated to the Wess-Zumino consistency condition in local field theories is presented within a zero curvature formalism. This approach relies on the existence of an operator δ which decomposes the exterior space-time derivative as a BRS commutator. As explicit examples, the three dimensional Chern-Simons gauge model and the B-C string ghost system will be discussed in detail.

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1 Introduction

Nowadays it is an established fact that the search of the possible anomalies and of the countert-erms which arise at the quantum level in local field theories can be done in a purely algebraic way [1]. Following the standard general BRS procedure, a local field theory is characterized by a set of fields $\{\phi\}$ (gauge fields, ghosts, matter...), a set of anti-fields $\{\phi^*\}$ (the BRS sources needed to properly quantize systems with non-linear symmetries) and a set of transformations described by means of a nilpotent operator b whose action on the fields and on the anti-fields can be generically written as

$$\begin{aligned} b\phi &= \mathcal{Q}(\phi,\phi^*) \ , \\ b\phi^* &= \mathcal{P}(\phi,\phi^*) \ , \\ b^2 &= 0 \ , \end{aligned} \tag{1.1}$$

with $\mathcal{Q},\mathcal{P}$ local polynomials in (ϕ,ϕ^*). In addition, if

$$S = \int d^D x \, \mathcal{L}(\phi,\phi^*) \ , \tag{1.2}$$

denotes the fully quantized classical action, we have the Slavnov-Taylor (or Master Equation) identity

$$bS = 0 \ , \tag{1.3}$$

summarizing the symmetry content of the model, i.e. the invariance of S under the transformations (1.1). At the quantum level, the radiative corrections lead to an effective action

$$\Gamma = S + \hbar\Gamma^{(1)} + \hbar^2\Gamma^{(2)} + \ldots \ , \tag{1.4}$$

which obeys the broken Slavnov-Taylor identity

$$b\Gamma = \hbar^n \mathcal{A} + O(\hbar^{n+1}) \ , \qquad n \geq 1 \ , \tag{1.5}$$

where, according to the Quantum Action Principle, $\mathcal{A}$ is an integrated local polynomial in (ϕ,ϕ^*) and their derivatives whose ultraviolet dimensions are bounded by power counting requirements [3]. As it is well known, the breaking term $\mathcal{A}$, due to the nilpotency of the operator b, obeys the Wess-Zumino consistency condition, i.e.

$$b\mathcal{A} = 0 \ . \tag{1.6}$$

Supposing now that the most general soultion of (1.6) can be written as a $b-$variation of a local polynomial $\hat{\Delta}$,

$$\mathcal{A} = b\hat{\Delta} \ , \tag{1.7}$$

then it is very easy to check that the redefined quantum action

$$\bar{\Gamma} = \Gamma - \hbar^n\hat{\Delta} \ , \tag{1.8}$$

turns out to be symmetric up to the order $\hbar^{n+1}$,

$$b\bar{\Gamma} = O(\hbar^{n+1}) \ . \tag{1.9}$$

This equation tells us that if the breaking $\mathcal{A}$ is a pure $b-$variation, then it is always possible to extend the invariance of the classical action S at the quantum level by means of the introduction of appropriate local counterterms. The procedure can be iterated by induction, allowing

[3] We shall consider here only power counting renormalizable models.

ourselves to prove that the Slavnov-Taylor identity (1.3) can be maintained at all orders of perturbation theory. Otherwise, if $\mathcal{A}$ cannot be written as a $b-$variation of a local polynomial,

$$\mathcal{A}^n \neq b\hat{\Delta} \ , \tag{1.10}$$

then the symmetry cannot be restored and the theory displays an anomaly. In particular, equations (1.6), (1.10) show that the existence of an anomaly is related to a nonvanishing cohomology of the operator b in the space of integrated local polynomials. It is apparent then that the knowledge of the cohomology classes of b is of great importance in order to establish if a given model is anomalous or not.

As a first step in the computation of the cohomology of b, let us translate the integrated consistency condition (1.6) at the nonintgrated level. Writing

$$\mathcal{A} = \int \omega^1_D \ , \tag{1.11}$$

ω^1_D denoting a local polynomial in the fields of ghost number 1 and form degree D, the integrated condition (1.6) is equivalent to

$$b\,\omega^1_D + d\,\omega^2_{D-1} = 0 \ , \tag{1.12}$$

$d = dx^\mu \partial_\mu$ being the exterior space-time derivative which together with the operator b satisfies the algebraic relations

$$b^2 \ = \ d^2 \ = \ b\,d + d\,b \ = 0 \ . \tag{1.13}$$

For the sake of generality, equation (1.12) is usually referred to an arbitrary value G of the ghost number, i.e.

$$b\,\omega^G_D + d\,\omega^{G+1}_{D-1} = 0 \ , \tag{1.14}$$

the values $G = 0, 1$ correspond respectively to invariant counterterms and anomalies. Acting now with the operator b on the equation (1.14) and making use of the algebraic relations (1.13), we get

$$d\,b\,\omega^{G+1}_{D-1} = 0 \, , \tag{1.15}$$

which, from the algebraic Poincare Lemma [2], i.e.

$$d\,\Delta^Q_{D-1} \ = \ 0 \quad \Rightarrow \quad \Delta^Q_{D-1} \ = \ d\,\hat{\Delta}^Q_{D-2} \ , \tag{1.16}$$

implies the new equation

$$b\,\omega^{G+1}_{D-1} + d\,\omega^{G+2}_{D-2} = 0 \ , \tag{1.17}$$

with ω^{G+2}_{D-2} local polynomial of ghost number $G+2$ and form degree $D-2$. Iteration of this procedure yields a system of equations usually called descent equations (see [1] and refs. therein):

$$\begin{aligned}
&b\,\omega^G_D \quad + d\,\omega^{G+1}_{D-1} = 0 \ , \\
&b\,\omega^{G+1}_{D-1} + d\,\omega^{G+2}_{D-2} = 0 \ , \\
&\qquad \cdots\cdots\cdots\cdots \\
&\qquad \cdots\cdots\cdots\cdots \\
&b\,\omega^{G+D-1}_1 + d\,\omega^{G+D}_0 = 0 \ , \\
&b\,\omega^{G+D}_0 = 0 \ ,
\end{aligned} \tag{1.18}$$

where the $\omega^{G+D-j}_j (\,0 \le j \le D)$ are local polynomials in the fields of ghost number $(G+D-j)$ and form degree j. The problem of solving the descent equations (1.18) is a problem of cohomology of b modulo d, the corresponding cohomology classes being given by solutions of (1.18) which are not of the type

$$\begin{aligned} \omega_m^{G+D-m} &= b\,\hat{\omega}_m^{G+D-m-1} + d\,\hat{\omega}_{m-1}^{G+D-m} , \qquad 1 \le m \le D , \\ \omega_0^{G+D} &= b\,\hat{\omega}_0^{G+D-1} , \end{aligned} \tag{1.19}$$

with $\hat{\omega}$'s local polynomials. Notice also that at the nonintegrated level one looses the property of making integration by parts. This implies that the fields and their derivatives have to be considered as independent variables.

Of course, the knowledge of the most general nontrivial solution of the descent equations (1.18) immediately yields the integrated cohomology classes of the operator b. Indeed, once the full system (1.18) has been solved, integration on space-time of the equation (1.14) will give the general solution of the consistency condition (1.6). The problem of finding the solutions of the equations (1.18) will be the main subject of this talk.

It should be remarked that the last equation of the system (1.18) is a problem of local cohomology instead of a modulo d one. Actually the latter can be handled by means of several tools as, for instance, the spectral sequences technique. We shall therefore assume that the most general solution of the last of the equations (1.18) is known.

For instance, in the case of the standard renormalizable Yang-Mills theory for which the BRS transformations of the gauge field $A = A^a T^a$ (T^a being the generators of the gauge group) and of the Faddeev-Popov ghost $c = c^a T^a$ are

$$bA = -dc - i[A, c] , \qquad bc = ic^2 , \tag{1.20}$$

the cocycle ω_0^{G+D} is given by polynomials in the ghost field c built up with invariant monomials of the type [3, 4, 5, 6, 7, 8]

$$\mathrm{Tr}\,\frac{c^{G+D}}{(G+D)!} , \qquad (G+D) \ \text{odd} . \tag{1.21}$$

2 Solution of the Descent Equations

We face now the problem of finding the solution of the full system of descent equations (1.18). To this purpose we introduce an operator δ which decomposes the exterior derivative d as a BRS commutator [9], i.e.

$$d = -[b, \delta] . \tag{2.1}$$

Although not necessary, we shall also suppose for simplicity that

$$[d, \delta] = 0 . \tag{2.2}$$

It is easily proven now that, once the decomposition (2.1) has been found, repeated applications of the operator δ on the polynomial ω_0^{G+D} which solves the last of the equations (1.18) will give an explicit nontrivial solution for the higher cocycles ω_j^{G+D-j}

$$\omega_j^{G+D-j} = \frac{\delta^j}{j!}\,\omega_0^{G+D} , \qquad j = 1, \ldots, D . \tag{2.3}$$

This very simple and elegant formula displays the usefulness of the introduction of the operator δ whose existence turns out to be quite general. In fact the decomposition (2.1) is actually present in a large class of field theory models such as

1. Yang-Mills type theories [9, 10]

2. Gravity [11]

3. Topological models (BF models, Chern-Simons, Witten's type models, ..) [12, 13, 14]

4. String theory [15, 16]

5. $\mathcal{W}_3$ - gravity [17]

6. $N = 1$ four dimensional supersymmetric Yang-Mills theories in superspace [18].

3 The Zero-Curvature Condition

Recently we have found [19] an interesting connection between the existence of the operator δ and the possibility of encoding the full set of BRS transformations into a unique equation which takes the form of a generalized zero curvature condition

$$\widetilde{\mathcal{F}} = \widetilde{d}\widetilde{\mathcal{A}} - i\widetilde{\mathcal{A}}^2 = 0 \ . \tag{3.1}$$

the operator $\widetilde{d}$ and the generalized gauge connection $\widetilde{\mathcal{A}}$ being respectively the transformations under δ of the BRS operator b and of the Faddeev-Popov ghost, i.e.

$$\widetilde{d} = e^{\delta}\, b\, e^{-\delta} \ , \qquad \widetilde{\mathcal{A}} = e^{\delta}\, c \ . \tag{3.2}$$

In particular, in the case in which eq.(2.2) holds, we have

$$\widetilde{d} = b + d \ . \tag{3.3}$$

Notice that, as a consequence of eq.(3.1), the operator $\widetilde{d}$ turns out to be nilpotent

$$\widetilde{d}^2 = 0 \ . \tag{3.4}$$

The zero curvature condition (3.1) has a very simple interpretation and its origin is deeply related to the existence of the operator δ entering the decomposition (2.1). Indeed, acting with the operator e^{δ} on the equation (1.20) expressing the BRS transformation of the Faddeev-Popov ghost we obtain

$$e^{\delta} b e^{-\delta} e^{\delta} c = i\, e^{\delta}\, c^2 \qquad \Rightarrow \qquad \widetilde{d}\widetilde{\mathcal{A}} = i\widetilde{\mathcal{A}}^2. \tag{3.5}$$

This is not surprising since, as it is well known, the ghost field c identifies the so called Maurer-Cartan form of the gauge group G and its BRS transformation is nothing but the corresponding Maurer-Cartan equation [20] which is in fact a zero curvature condition.

Turning now to the cohomology of the operator $\widetilde{d}$, it is apparent to see that the cohomology classes of $\widetilde{d}$ are obtained by $\delta-$transforming the corresponding cohomology classes of the BRS operator b. In other words, it is very easy to check that the generalized cocycles

$$\mathrm{Tr}\, \frac{\widetilde{\mathcal{A}}^{2n+1}}{(2n+1)!} = e^{\delta}\, \mathrm{Tr}\, \frac{c^{2n+1}}{(2n+1)!} \ , \tag{3.6}$$

identify cohomology classes of $\widetilde{d}$.

Introducing now the generalized cocycle

$$\widetilde{\omega}^{G+D} = \sum_{j=0}^{D} \omega_j^{G+D-j} \ . \tag{3.7}$$

the full system of descent equations (1.18) can be cast in the compact form

$$\tilde{d}\,\tilde{\omega}^{G+D} = 0 \ , \tag{3.8}$$

from which one sees that $\tilde{\omega}^{G+D}$ belongs to the cohomology of $\tilde{d}$. It follows then that $\tilde{\omega}^{G+D}$ is simply given by polynomials of the type (3.6), i.e.

$$\tilde{\omega}^{G+D} = e^{\delta}\omega_0^{G+D} \ . \tag{3.9}$$

This equation shows that the solution of the descent equations (1.18) are related to the cohomology of the operator $\tilde{d}$ entering the zero curvature equation (3.1). Moreover, the cohomology of the operator $\tilde{d}$ is given by $\delta-$transforming the cohomology of the BRS operator b. It is clear thus that the existence of the operator δ as well as the zero curvature condition (3.1) give a complete and very elegant algebraic set up in order to deal with the solutions of the descent equations. Let us conclude by remarking that, although referred to a nonabelian type theory, the zero curvature condition can be generalized to gauge theories whose ghost content is different from the usual Yang-Mills Faddeev-Popov fields. An example of this will be provided later on by the so called $B - C$ string ghost system.

4 Example I : D = 3 Chern-Simons Gauge Theories

For a better understanding of the previous construction let us discuss in details the case of the three dimensional Chern-Simons theory, corresponding to $G = 0$ and $D = 3$.

The relevant fields here are the one form gauge field and the zero form Faddeev-Popov ghost

$$\begin{aligned} A &= T^a A^a_\mu dx^\mu \ , \\ c &= T^a c^a \ , \end{aligned} \tag{4.1}$$

and the corresponding antifields, respectively a two form γ (associated to the nonlinear transformation of A) and a three form τ (associated to c)

$$\begin{aligned} \gamma &= \frac{1}{2} T^a \gamma^a_{\mu\nu} dx^\mu \wedge dx^\nu \ , \\ \tau &= \frac{1}{3!} T^a \tau^a_{\mu\nu\rho} dx^\mu \wedge dx^\nu \wedge dx^\rho \ . \end{aligned} \tag{4.2}$$

The invariant quantized action can be written as

$$S = \int \mathrm{Tr}\,(AF + i\frac{A^3}{3} + \gamma Dc + i\tau c^2), \tag{4.3}$$

F being the two-form gauge field strength $F = dA - iA^2$ and Dc the covariant derivative

$$Dc = dc - i[A, c] \ . \tag{4.4}$$

The action (4.3) is invariant under the following set of transformations:

$$\begin{aligned} bc &= ic^2 \ , \\ bA &= -dc + i\,[c, A] \ , \\ b\gamma &= -F + i\,[c, \gamma] \ , \\ b\tau &= -d\gamma + i\,[c, \tau] + i\,[A, \gamma] \ , \\ b^2 &= 0 \ . \end{aligned} \tag{4.5}$$

Introducing now the operator δ [13] defined as

$$
\begin{aligned}
\delta c &= A \ , \\
\delta A &= 2\gamma \ , \\
\delta\gamma &= 3\tau \ , \\
\delta\tau &= 0 \ ,
\end{aligned}
\tag{4.6}
$$

one easily checks that equations (2.1) and (2.2) are verified, i.e. δ decomposes the exterior derivative d as a BRS commutator. For the generalized connection of eq. (3.2) we get

$$\widetilde{\mathcal{A}} = e^{\delta} c = c + A + \gamma + \tau \ . \tag{4.7}$$

Remark that the generalized connection $\widetilde{\mathcal{A}}$ collects all the relevant fields, meaning that the external sources γ and τ are naturally included in the zero curvature formalism.
The zero curvature condition

$$\widetilde{d}\widetilde{\mathcal{A}} = i\widetilde{\mathcal{A}}^2 \tag{4.8}$$

reads now

$$(b + d)(c + A + \gamma + \tau) = i(c + A + \gamma + \tau)^2 \ , \tag{4.9}$$

which is easily seen to reproduce the transformations (4.5).

As explained before, in order to find a solution of the descent equations

$$
\begin{aligned}
& b\,\omega^j_{3-j} + d\,\omega^{j+1}_{2-j} = 0 \ , \qquad 0 \le j \le 2 \ , \\
& b\omega^3_0 = 0 \ ,
\end{aligned}
\tag{4.10}
$$

it is sufficient to expand the generalized cocycle of total degree three

$$\widetilde{\omega}^3 = \frac{1}{3!}\mathrm{Tr}\,\widetilde{\mathcal{A}}^3 \ . \tag{4.11}$$

After an easy computation we get

$$\frac{1}{3!}\mathrm{Tr}\,\widetilde{\mathcal{A}}^3 = \omega^0_3 + \omega^1_2 + \omega^2_1 + \omega^3_0 \ , \tag{4.12}$$

with

$$
\begin{aligned}
\omega^3_0 &= \frac{1}{3!}\mathrm{Tr}\,c^3 \ , \\
\omega^2_1 &= \frac{1}{2}\mathrm{Tr}\,c^2 A \ , \\
\omega^1_2 &= \frac{1}{2}\mathrm{Tr}\,\left(c^2\gamma + cA^2\right) \ , \\
\omega^0_3 &= \frac{1}{2}\mathrm{Tr}\,\left(c^2\tau + cA\gamma + c\gamma A + \frac{A^3}{3}\right) \ .
\end{aligned}
\tag{4.13}
$$

From

$$-i\mathrm{Tr}\,\left(c^2\tau + cA\gamma + c\gamma A\right) = -\mathrm{Tr}\,AF + b\,\mathrm{Tr}\,(c\tau + A\gamma) + d\,\mathrm{Tr}\,c\gamma \ , \tag{4.14}$$

the three-form ω^0_3 can be rewritten as

$$\omega^0_3 = \frac{-i}{2}\mathrm{Tr}\,(AF + i\frac{A^3}{3}) + \frac{i}{2}b\,\mathrm{Tr}\,(c\tau + A\gamma) + \frac{i}{2}d\,\mathrm{Tr}\,c\gamma \ , \tag{4.15}$$

yielding thus the invariant action

$$S = i\int\omega^0_3 = \frac{1}{2}\int\mathrm{Tr}\,(AF + i\frac{A^3}{3}) - \frac{1}{2}b\int\mathrm{Tr}\,(c\tau + A\gamma) , \tag{4.16}$$

which is easily recognized as the action of the fully quantized Chern-Simons gauge theory (4.3).

5 Example II: the Bosonic String

Let us discuss now, as the second example of the zero curvature construction, the so called $B-C$ model whose action is given by

$$S_{B-C} = \int dzd\bar{z}\, B\,\bar{\partial}C \ . \tag{5.1}$$

The fields $B = B_{z\,z}$ and $C = C^z$ are anticommuting and carry respectively ghost number -1 and $+1$. The action (5.1) is recognized to be the ghost part of the quantized bosonic string action. It is usually accompanied by its complex conjugate. However, the inclusion of the latter in the present framework does not require any additional difficulty.

As it is well known, the action (5.1) is left invariant by the following nonlinear BRS transformations

$$\begin{aligned} sC &= C\,\partial C \ , \\ sB &= -(\partial B)C - 2B\,\partial C \ . \end{aligned} \tag{5.2}$$

Transformations (5.2) being nonlinear, one needs to introduce two external invariant sources $\mu = \mu^z{}_{\bar{z}}$ and $L = L_{z\,z\,\bar{z}}$ of ghost number respectively 0 and -2

$$S_{ext} = \int dzd\bar{z}\,(\mu\, s\,B + L\,s\,C) \ . \tag{5.3}$$

The complete action

$$S = S_{B-C} + S_{ext} \tag{5.4}$$

obeys thus the classical Slavnov-Taylor identity

$$\int dzd\bar{z}\left(\frac{\delta S}{\delta B}\frac{\delta S}{\delta\mu} + \frac{\delta S}{\delta L}\frac{\delta S}{\delta C}\right) = 0 = \frac{1}{2}\,b\,S \ , \tag{5.5}$$

b denoting the nilpotent linearized operator

$$b = \int dzd\bar{z}\left(\frac{\delta S}{\delta B}\frac{\delta}{\delta\mu} + \frac{\delta S}{\delta\mu}\frac{\delta}{\delta B} + \frac{\delta S}{\delta L}\frac{\delta}{\delta C} + \frac{\delta S}{\delta C}\frac{\delta}{\delta L}\right) \ . \tag{5.6}$$

The operator b acts on the fields and on the external sources in the following way

$$\begin{aligned} bC &= sC = C\,\partial C \ , \\ b\mu &= \bar{\partial}C + (\partial\,\mu)\,C - \mu\,(\partial C) \ , \end{aligned} \tag{5.7}$$

and

$$\begin{aligned} bB &= sB = -(\partial B)C - 2B\,\partial C \ , \\ bL &= \bar{\partial}\,B - (2\,B)\,\partial\mu - \mu\partial\,B + (\partial\,L)\,C + 2L\,\partial\,C \ . \end{aligned} \tag{5.8}$$

Introducing now the two functional operators [15]

$$\mathcal{W} = \int dzd\bar{z}\,\frac{\delta}{\delta C} \ , \qquad \overline{\mathcal{W}} = \int dzd\bar{z}\left(\mu\,\frac{\delta}{\delta C} + L\,\frac{\delta}{\delta B}\right) \ , \tag{5.9}$$

one easily proves that

$$\delta = dz\,\mathcal{W} + d\bar{z}\,\overline{\mathcal{W}} \tag{5.10}$$

obeys to

$$d = -[b,\delta] \ , \qquad [d,\delta] = 0 \ , \tag{5.11}$$

d being the exterior derivative $d = dz\partial + d\bar{z}\bar{\partial}$. We have thus realized the decomposition (2.1). In order to derive the transformations (5.7), (5.8) from a zero curvature condition, we proceed as before and we define the generalized field

$$\tilde{C}^z = e^{\delta}\, C^z = C^z + dz + d\bar{z}\,\mu_{\bar{z}}^{z} \ , \tag{5.12}$$

so that introducing the holomorphic generalized vector field $\tilde{C} = \tilde{C}^z\,\partial_z$, it is easily checked that equations (5.7) can be cast in the form of a zero curvature condition

$$\tilde{d}\,\tilde{C} = \frac{1}{2}\left[\tilde{C},\tilde{C}\right] = \mathcal{L}_{\tilde{C}}\,\tilde{C} \ , \tag{5.13}$$

where, as usual, $\tilde{d}$ is the operator

$$\tilde{d} = e^{\delta}\, b\, e^{-\delta} = b + d \, , \tag{5.14}$$

and $\mathcal{L}_{\tilde{C}}$ denotes the Lie derivative with respect to the vector field[4] $\tilde{C}$.

Concerning now the second set of transformations (5.8), we define a second generalized field $\widetilde{\mathcal{B}}_{z\,z}$ as

$$\widetilde{\mathcal{B}}_{z\,z} = e^{\delta}\mathcal{B}_{z\,z} = \mathcal{B}_{z\,z} + d\bar{z}\,L_{z\,z\,\bar{z}} \ . \tag{5.15}$$

To expression (5.15) one can naturally associate the generalized holomorphic quadratic differential

$$\widetilde{\mathcal{B}} = \widetilde{\mathcal{B}}_{z\,z}dz \otimes dz \ . \tag{5.16}$$

Therefore, transformations (5.8) can be rewritten as

$$\tilde{d}\,\widetilde{\mathcal{B}} - \mathcal{L}_{\tilde{C}}\,\widetilde{\mathcal{B}} = 0 \ . \tag{5.17}$$

Let us consider now the problem of identifying the anomalies which affect the Slavnov-Taylor identity (5.5) at the quantum level. We look then at the solution of the descent equations

$$\begin{aligned} b\,\omega_2^1 + d\,\omega_1^2 &= 0 \ , \\ b\,\omega_1^2 + d\,\omega_0^3 &= 0 \ , \\ b\,\omega_0^3 &= 0 \ . \end{aligned} \tag{5.18}$$

As it has been proven in refs. [15, 21], the cohomology of the BRS operator in the sector of the zero-forms with ghost number three contains, in the present case, a unique element given by

$$\omega_0^3 = C\,\partial C\,\partial^2 C \ . \tag{5.19}$$

From the zero curvature condition (5.13), it follows then that the generalized cocycle of total degree three

$$\tilde{\omega}^3 = \tilde{C}\,\partial\tilde{C}\,\partial^2\tilde{C} \ . \tag{5.20}$$

belongs to the cohomology of $\tilde{d}$. The expansion of $\tilde{\omega}^3$ will give thus a solution of the ladder (5.18), *i.e.*

$$\tilde{\omega}^3 = \omega_0^3 + \omega_1^2 + \omega_2^1 \ . \tag{5.21}$$

with ω_1^2, ω_2^1 given respectively by

$$\begin{aligned} \omega_1^2 &= (C\,\partial C\partial^2\mu - C\,\partial^2 C\,\partial\mu + \mu\,\partial C\partial^2 C)\,d\bar{z} + (\partial C)\,(\partial^2 C)\,dz \ , \\ \omega_2^1 &= (-\partial\, C\partial^2\mu + \partial\,\mu\partial^2 C)\,dz\wedge d\bar{z} \ . \end{aligned} \tag{5.22}$$

In particular,

$$\int \omega_2^1 = 2\int dz d\bar{z}\; C\partial^3\mu \tag{5.23}$$

is recognized to be the well known two-dimensional diffeomorphism anomaly characterizing the central charge of the energy-momentum current algebra.

[4] Of course, the bracket $[\tilde{C},\tilde{C}]$ in eq.(5.14) refers now to the Lie bracket of vector fields.

Conclusion

We have shown that the zero curvature formulation can be obtained as a consequence of the existence of the operator δ realizing the decomposition (2.1). This formalism enables us to encode into a unique equation all the relevant informations concerning the BRS cohomology classes.

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