On the Quasi-Elastic "Diffraction" Scattering of High-Energy Protons

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ADDENDUM to

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A B S T R A C T

The observations of Cocconi et al., on the "quasi-elastic diffraction" scattering of 10 - 25 GeV/c protons by nucleons, are explained as resulting from "isobar" excitation of the target nucleons. The experiments are shown to provide evidence on the excitation of the first three levels. Some physical arguments are presented concerning the nature of the excitation process and its energy and angular dependence.

Note added 26 April, 1961

Similar arguments, but based on a perturbation field-theoretic approach, were applied by Selove [Phys. Rev. Letters 5, 163 (1960)] in discussing the Brookhaven results [5]. We are grateful to A. Roberts and G. B. Chadwick for recalling this work to our attention.

It has also been pointed out to us by a number of our colleagues, and most persuasively by J.S. Bell, that since, when viewed in the c.m.s. the levels of the target nucleon will suffer a "red shift", $\Delta_i \longrightarrow \Delta_i/\gamma_o$, the maximum excitation energy should be $\gamma_o\Delta_o$ rather than Δ_o [see Eq. (9) and Table 3, column 2]. Correspondingly, it is not clear whether the maximum angular momentum transfer should be taken as ~ 1 [Eq. (11)] or as $\sim \gamma_o$. Eq. (10), for the momentum transfer, holds in any case. These considerations weaken the conclusions drawn at the end of section 3 insofar as the CERN experiments are concerned, in the direction of enhancing the expectations for observing the excitation of all the known nucleon isobars in this energy range. But they do not appreciably alter our conclusions with respect to the Brookhaven results.

Cocconi, Diddens, Littlethun and Wetherall [1] have observed, in the laboratory system spectrum of protons scattered by free nucleons through small angles, a peak of inelastically scattered protons of momentum ~ 1 GeV/c less than that of the elastically scattered protons. This peak appears to be characterized by a "diffraction" angular distribution, of angular spread roughly comparable to that of the elastic "shadow" scattering. The most striking feature of the phenomenon, however, is the apparent constant energy difference between the elastic and the inelastic peaks, independent of the incident proton momentum (which was varied in the experiments between 9 and 25 GeV/c) and of the scattering angle (between 20 and 60 milliradians in the lab. system).

We present, below, some arguments in favour of an interpretation of these observations as resulting from processes in which the target nucleon is left in a definite excited state, or states, the ones corresponding to the $D_{3/2}$ or $F_{5/2}$ pion - nucleon resonance, of isotopic spin $\frac{1}{2}$, observed with pions of kinetic energy ~ 600 and ~ 900 MeV [2]. The arguments are based primarily on kinematical considerations. But, in section 3, we attempt to give a physical justification of our interpretation and discuss the circumstances which would favour the excitation, in such processes, of the various known nucleon "isobars".

1. Kinematical Description

Van Hove [3] has pointed out that the observations can be described by the kinematical properties of the scattering process represented in Fig. 1. M^* represents an "excited" nucleon, of excitation energy Δ (or rest-mass $M^*=M+\Delta$). Consider an incident proton of total energy [*)] u. In this case,

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*) Unless specifically stated to the contrary, all masses are given in units of the proton mass M, all momenta in units of Mc and all energy in units of $Mc^2=0.938$ GeV. As usual, $c=\hbar=1$.

when projectile and target masses are equal (and equal to 1), we have for the usual quantities describing the c.m.s. motion

$$\gamma_o = u_o = (1-\beta_o^2)^{-\frac{1}{2}} = [(u+1)/2]^{\frac{1}{2}} \tag{1}$$

$$\beta_o \gamma_o = p_o = [(u-1)/2]^{\frac{1}{2}} \tag{2}$$

Consider, first, elastic scattering ($\Delta = 0$, $M = M^*$) through the small angle θ_o. Then, in the small angle approximation,

$$\theta = \theta_o / 2\gamma_o \tag{3}$$

$$(p-p')/p \equiv \Delta p_1/p = \tfrac{1}{4}\left(\frac{u}{u+1}\right)\theta_o^2 = \tfrac{1}{2}\, u\, \theta^2 \tag{4}$$

In Table 1 we have tabulated the values of Δp_1 observed by Cocconi et al [1] as well as the values computed from Eq. (4).

Now, we consider the inelastic scattering process depicted in Fig. 1. For fixed Δ ($M^* \equiv 1+\Delta$), the final c.m.s. momenta $p_o' \equiv p_o - \delta_o$ are determined entirely by conservation of energy and momentum, and are given (for $\delta_o \ll p_o$) by [*)]

$$\delta_o = \frac{\Delta(1+\Delta/2)}{2p_o} \equiv \frac{\Delta'}{2p_o} \tag{5}$$

Corresponding to this c.m.s. momentum change, we may compute the momentum shift of the line corresponding to the inelastically scattered projectile in the small angle approximation

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*) The exact formula is

$$\delta_o = \frac{\Delta'}{2p_o}\left[1 - \frac{\Delta'}{2(u+1)}\right] \tag{5'}$$

$$p - p' \equiv \Delta p_1 + \Delta p_2 \tag{6a}$$

$$\Delta p_2(\theta) = \Delta p_2(0)\left(1 - \frac{u_o}{u}\Delta p_1\right) \tag{6b}$$

$$\Delta p_2(0) = \frac{u}{u_o}\,\delta_o \cong \frac{u}{p}\;\Delta' \tag{6c}$$

We observe from Eqs. (6) that to a very good approximation, at the energies and angles covered by the experiments under consideration, constant Δp_2 implies constant target nucleon excitation Δ (Δ'). To obtain the observed values of $\Delta p_2 \simeq 1.0$, see Table 1, we require

$$\Delta \cong 0.7 \pm 0.1$$

TABLE 1

Observations of Cocconi et al. [1] on the positions of the elastic and inelastic peaks

p(GeV/c)	θ(mrad)	Δp_1(GeV/c)	Δp_2(GeV/c)	(calc.) $\Delta p_{elastic}$(GeV/c)
8.95	60	0.35 ± 0.2	1.0 ± 0.1	0.16
12.1	60	0.5 ± 0.2	1.0 ± 0.1	0.28
15.2	60	0.7 ± 0.2	1.0 ± 0.1	0.44
16.0	40	0.2 ± 0.2	1.0 ± 0.1	0.22
16.2	60	0.7 ± 0.2	0.9 ± 0.1	0.50
17.6	60	0.7 ± 0.2	1.0 ± 0.1	0.60
18.6	60	0.7 ± 0.2	1.0 ± 0.1	0.66
19.5	60	0.8 ± 0.2	1.0 ± 0.2	0.73
21.9	60	0.8 ± 0.3	~ 1	0.92
24.2	20	0.3 ± 0.1	0.8 ± 0.1	0.13
24.2	40	0.2 ± 0.2	1.0 ± 0.1	0.53

The width of the inelastic peak will be determined both by the experimental resolution, which is what should determine the width of the elastic peak, and by the natural spread in the excitation energies of the excited target *). Let the natural width of the excited nucleon state (M^*) be Γ_o. Differentiation of Eq. (6c) gives, for the momentum spread in the l.s. of the inelastic peak

$$\Gamma \cong \frac{u}{p}(1+\Delta)\Gamma_o = \frac{u}{p}\left(\frac{M^*}{M}\right)\Gamma_o \qquad (7)$$

Actually, owing to the large (non-resonant inelastic) background and the finite resolution of the experimental observations, it is not possible to quote an experimental value of Γ. While it appears that the observed Γ is somewhat larger than the instrumental resolution, its evaluation must await improved experiments.

2. Isobar Interpretation.

Both the constancy of Δp_2 (therefore of Δ) and the apparent narrow width of the inelastic peaks suggest that we may be dealing, in these observations, with the excitation of a nucleon "isobar", possibly one of the levels which give rise to the peaks in the pion-nucleon cross-section [2].

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*) Note that the process under consideration corresponds to the excitation of the target nucleon, with the projectile remaining unaltered and continuing forward in the c.m.s. Since the situation is symmetrical in the c.m.s., it should also be possible to excite the forward scattered nucleon. However, in this case, owing to the subsequent rapid decay

$$M^* \rightarrow M+n\pi ,$$

the inelastic protons will be spread over a broad range of l.s. momenta, and will be lost in the general background of inelastically scattered protons.

In Table 2 we list the properties of the observed levels, together with the expected values of Δp_2 and Γ corresponding to their excitation by nucleons of initial l.s. energy $\gtrsim$ 10 GeV.

Of the levels listed, only the second (1,3) and third (1,5) are candidates for explaining the observed quasi-elastic peak. The experiments [1] are, however, not inconsistent with an unresolved superposition of peaks from the two levels, both from the point of view of the position and the observed widths.

TABLE 2

Properties of nucleon "isobars" and their "quasi-elastic" excitation

description	isotopic spin	angular momentum	pion l.s. kinetic energy at resonance (MeV)	pion c.m.s. kinetic energy (MeV)	Δ	Γ_0 (MeV)	$\Delta p_2(0)$ (BeV/c)	Γ (MeV)
(3,3)	3/2	$3/2^+$	200	160	0.32	105	0.35	139
(1,3)	1/2	$3/2^-$	600	440	0.62	130	0.76	210
(1,5)	1/2	$5/2^+$	900	615	0.80	115	1.06	208
(3,?)	3/2	$\geqslant 5/2$	1350	835	1.04	~200	1.48	~ 410

It remains, however, to understand the apparent absence of peaks corresponding to the other resonances. In the case of the lowest (3,3) state, it cannot yet be excluded experimentally that a peak corresponding to its excitation may be "hidden" in the observed elastic peak. In fact, comparison of the observed and computed values of Δp_1, in Table 1, lends some evidence to this possibility, especially at the lower energies, since the observed values of Δp_1 are consistently greater than the computed $\Delta p_{elastic}$. This could be due to a shifting of the elastic peak by unresolved combination with the (3,3) inelastic peak, but it could also be due to experimental errors in the determination of Δp_1.

However, further weight is given to the possibility of "quasi-elastic" excitation of the (3,3) isobar, especially at low incident energies, by the observations of the Brookhaven group [4] on the spectra of protons, of initial (total) energies between ~ 2 and 2.5 GeV, scattered at the l.s. angle of $\sim 5^o$. Clear indications were found for the excitation of the (3,3) isobar, this time resolved (but, apparently, not completely) from the elastic peak. (Values of $\Delta \simeq 0.2$ were derived from the observations). The Brookhaven group also looked for indications of excitation of the higher resonances; but, although the available energies were sufficient for excitation of the next two resonances, they found no indication of peaks corresponding to their excitation.

As for the highest (3,?) isobar, there have so far been no evidences of its excitation, although it could possibly be lost in the background of inelastically scattered protons observed in the measurements of Cocconi et al. [1].

3. Physical Considerations.

Diffraction (or shadow) elastic scattering is generally described as a purely optical phenomenon accompanying the absorption, within a radius R, of a beam of particles of wavelength $\lambdabar \ll R$. The characteristic scattering angle is $\theta_0 \simeq \lambdabar/R = (kR)^{-1}$. Alternatively, the process may be characterized by the transverse transfer of momentum $(\hbar/R)$, presumably through the exchange of a meson *), as indicated schematically in Fig. 2a.

The diagram corresponding to the "quasi-elastic" diffraction scattering, Fig. 2b, differs from that for elastic scattering only in that the target nucleon emerges from the "exchange" in an excited state; the momentum transfer is still $\sim \mu c$, but this time there must be a longitudinal component to permit conservation of energy and momentum in the collision.

Although it is possible that the properties of such collisions might be analyzed by means of the dispersion approach [5], we have preferred to consider them from a somewhat more classical, albeit quite qualitative point of view. We regard the nucleon as a "complex" system, with energy levels separated from the ground state by energies Δ_i. If a time varying perturbation is applied to such a system then, quite generally, those levels will be excited most strongly for which the frequency of the perturbing force is "in resonance with" the level, $\omega_i \simeq \Delta_i$.

Now, consider a peripheral collision, at an impact parameter $R \simeq \mu^{-1}$. In the c.m.s., the collision corresponds to an impulse of duration

$$\tau_0 \simeq R/\beta_0\gamma_0 \tag{8}$$

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*) We consider one-meson exchange diagrams because of the long range $R \simeq \hbar/\mu c$ and small transverse momentum transfer $\Delta p_t \simeq \mu c$ characteristic of the observed elastic scattering.

Such an impact may be characterised by a relatively flat Fourier spectrum of frequencies up to *) $\omega_0 \gtrsim 2/\tau_0$. Accordingly, those levels will be most strongly excited for which

$$\Delta_i \lesssim \Delta_0 \simeq \frac{2}{\tau_0} \simeq 2\beta_0\gamma_0\mu = 2\mu p_0 \quad . \tag{9}$$

Furthermore, the longitudinal momentum transfer in such collisions is, from Eq. (5)

$$\delta_0 \lesssim \frac{\Delta_0}{2p_0} \approx \mu \quad , \tag{10}$$

in justification of our earlier statement concerning the small momentum transfers in Fig. 2.

An additional consequence of this approach is that it suggests the possibility of a limitation, also on the change of <u>internal</u> angular momentum of the target in the excitation process **). Thus, assuming that it remains appropriate to view these processes in the c.m.s. (Fig. 2), the possible internal angular momentum transfer in the impact is

$$\Delta_j \lesssim R\delta_0^{(max)} \approx 1 \quad , \tag{11}$$

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*) For example, a "step" impulse of duration τ_0 has the Fourier spectrum $j_1(\omega\tau_0)/\omega\tau_0$, which falls to half the initial value at $\omega_0 \simeq 2.2/\tau_0$, and to zero at $\omega_0 \simeq 3.8/\tau_0$.

**) Note that it is important, for these considerations, that the internal state of the projectile nucleon be unchanged. Whether this applies also to its charge is a problem whose discussion we postpone until the next section.

Clearly, the details depend on the nature of the nucleonic system and of the impacts; but, in general, those levels will tend to be excited most strongly for which the relations (9)–(11) tend to be obeyed.

Table 3 lists values of Δ_o [Eq.(9)] also for a number of incident proton energies, and also lists the required longitudinal momentum transfers for the excitation of the first four nucleonic isobars [Eq.(5)]. It is clear from these numbers that collisions in the 1-5 GeV range favour excitation of the (3,3) level only, in agreement with the Brookhaven observations [4]; that excitation becomes favourable for the next two, (1,3) and (1,5), levels in the 10-25 GeV range [1]; our angular momentum considerations, insofar as they are important, favour the (1,3) level excitation over both the (3,3) and the (1,5), especially at the upper end of this energy range. Finally, since the maximum frequency in the impact, Δ_o, as well as the momentum transfer δ_o, increase only as $u^{\frac{1}{2}}$, it will be necessary to increase the incident energy quite considerably in order to produce conditions favourable for the excitation of still higher isobar levels.

TABLE 3

Maximum excitation energies and required transverse momentum transfers for isobar excitation

u	$\Delta_o=2\mu p_o$	$\delta_o/\mu(3,3)$	$\delta_o/\mu(1,3)$	$\delta_o/\mu(1,5)$	$\delta_o/\mu(3,?)$
2	0.21	1.66	3.34	4.37	5.55
5	0.42	0.86	1.80	2.44	3.27
10	0.63	0.58	1.24	1.71	2.33
15	0.78	0.47	1.00	1.39	1.91
20	0.91	0.40	0.87	1.21	1.67
25	1.03	0.36	0.77	1.07	1.49
50	1.47	0.25	0.55	0.76	1.06

4. Summary, Conclusions and Discussion.

We have shown that the observations of Cocconi et al. [1] on the position of the elastic and "quasi-elastic" peaks in p-N scattering, are kinematically consistent with a diffraction scattering mechanism, in which the target nucleon can be excited into a low-lying "isobar" level as well as left in its ground state. Some physical considerations have been presented which, although they cannot be considered as a "derivation" of the effect, indicate the plausibility of the excitation mechanism assumed to explain the observations and give rise to certain kinematical restrictions on the levels it is possible to excite as a function of the projectile energy.

Our description has certain features in common with the "diffraction dissociation" mechanism, discussed by a number of authors [6),7)] and suggested by Van Hove [3] as the source of the quasi-elastic scattering process. However, at least as most commonly interpreted, the diffraction dissociation process cannot lead to a change in any of the internal quantum numbers of the system - charge, spin, isotopic spin, strangeness, etc.; if this were true, none of the isobar levels could be excited. On the contrary, our mechanism permits changes in the quantum numbers, although it does suggest certain restrictions on the possible changes.

In particular, our model gives rise to a number of further consequences, some of which can be tested experimentally :

1) Since the isotopic spin of the excited nucleon is fixed, depending on the level excited, the charge of the scattered nucleon is also determined. Thus, in p-p collisions leading to the quasi-elastic peak of Cocconi et al., the scattered nucleon must always be a proton, since the levels which could be responsible for this peak both have $t=\frac{1}{2}$. For the excitation of the (3,3) level, on the other hand, the final state must have the definite ratio of 1:3 between protons and neutrons, since the total isotopic spin of the system is fixed $(t=1,\ t_3=1)$.

2) For p-n collisions, change-exchange excitation becomes quite possible, even for levels with $t=\frac{1}{2}$. However, it is not possible, to make an *a priori* prediction of the proton to neutron ratio in the final state *), since this depends on the relative magnitude and phase of the amplitudes corresponding to the two values of the total isotopic spin (0 and 1) present in the initial state. For excitation of the (3,3) level, however, the charge prediction again becomes unique, and is 1:1 for the final proton-to-neutron ratio.

3) Of course, the pion and nucleon charge and momentum distributions from the decay of the excited target should also be predictable. Their observation, however, will require visual techniques, such as bubble chambers or emulsions

4) For the case of incident π- or K-mesons, instead of protons, the same type of quasi-elastic scattering processes should be observed, with Δp_1, Δp_2, and Γ given by Eqs. (4), (6) and (7), respectively, to a good approximation; for these quantities depend mainly on the mass and excitation of the target nucleon, and only very weakly on the mass of the incident particle, at least in the energy range of interest to us. The observation of such quasi-elastic scattering for mesons would provide an excellent test of this model, as compared to other models, such as one which describes the effect to the diffraction scattering of the nucleon by a pion in the field of the target [8]; or one in which the effect is due to the exchange of a single π^o [1].

We are grateful to W.M. Layson, S. Okubo and to other members of the CERN theoretical group, for stimulating discussions and searching criticisms.

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*) It should be noted that the same type of argument might also be applied to the diffraction elastic scattering, since it is also possible that the amplitudes for this type of scattering differ in the two isotopic spin states.

R E F E R E N C E S

1) G. Cocconi, A.N. Diddens, E. Lillethun and A.M. Wetherall, Phys.Rev. Letters 6, 231 (1961).

2) See, for example, P. Falk-Vairant and G. Valladas, Proceedings 1960 Conference on High Energy Physics, Univ. of Rochester, p. 38.

3) L. Van Hove, private communication; see also ref. [1].

4) Chadwick, Collins, Swartz, Roberts, De Benedetti, Hien and Duke, Phys.Rev. Letters 4, 611 (1960).

5) E.g., S.D. Drell, "Peripheral Contributions of High-Energy Interaction Processes", presented at the Conference on Strong Interactions (Berkeley, Calif., Dec. 27-29, 1960), Rev. Modern Phys. (in press).

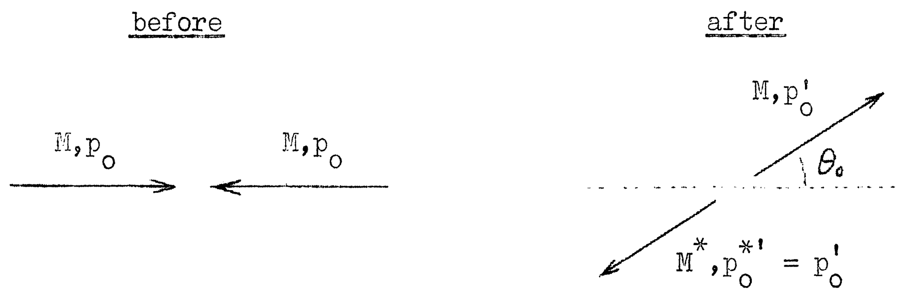
6) E.L. Feinberg and J.I. Pomeranchuk, Suppl. Nuovo Cimento 3, 652 (1956).

7) M.L. Good and W.D. Walker, Phys.Rev. 120, 1857 (1960).

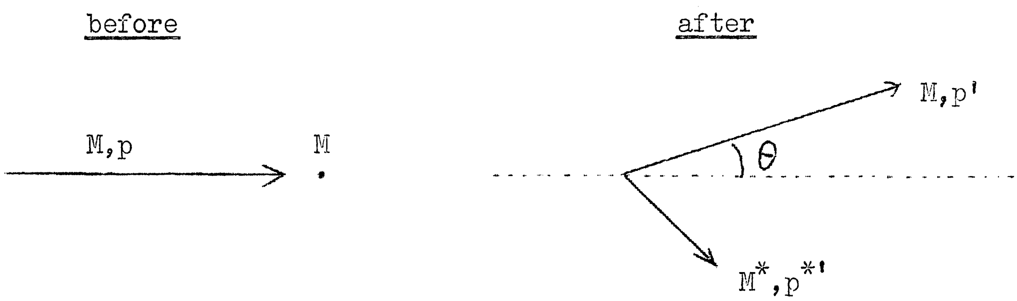
8) D. Amati, private communication.

FIGURE 1

(a) Kinematics in the centre of mass system (c.m.s.)



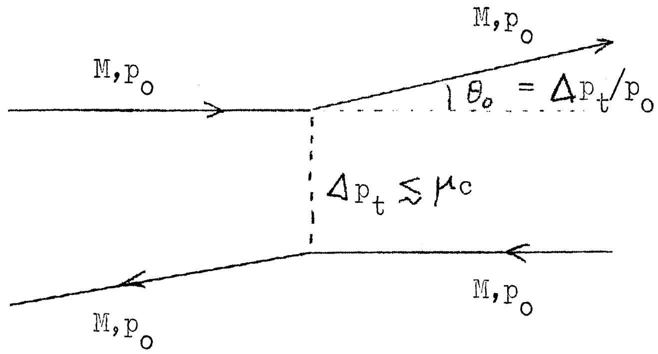
(b) Kinematics in the laborators system (l.s.)



Note that the subscript "o" stands for c.m.s.

FIGURE 2

(a) Diagram corresponding to diffraction elastic scattering.



(b) Diagram corresponding to "quasi-elastic" diffraction scattering.