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INTERPRETATION OF THE TRANSVERSE MOMENTUM DISTRIBUTION OF PARTICLES IN HIGH-ENERGY HADRON COLLISIONS

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1. INTRODUCTION

A well-known and most characteristic property of high-energy hadron collisions is the fact that the average transverse momentum, $\langle p_\perp \rangle = \langle p \sin \vartheta \rangle$, of all long-lived secondaries produced in strong interactions is, at least in first approximation, constant[1]:

$$\langle p_\perp \rangle \approx 0.4 \text{ GeV/c} . \tag{1}$$

This fact has been verified on the secondaries produced in collisions with nucleons by hadrons having momenta p_0 extending from a few GeV/c to cosmic-ray momenta of 10^4–10^5 GeV/c, and consequently for secondary particles covering a similar range of momenta, p.

When this "universal law" was investigated in more detail, two other main features were discovered.

First[2], that the value of $\langle p_\perp \rangle$ is mass dependent, increasing as the mass of the secondary considered increases. Typical are the following values:

$$\begin{array}{llll} \pi \text{ mesons}, & \langle p_\perp \rangle = & 0.30 & \text{GeV/c} \\ \text{protons}, & \quad '' & 0.44 & \quad '' \\ \Sigma \text{ mesons}, & \quad '' & 0.51 & \quad '' \quad . \end{array} \tag{2}$$

Second[3], that the distribution of the transverse momenta of the secondaries is well represented by an exponential law:

$$N(p_\perp)\, dp_\perp \propto p_\perp\, e^{-p_\perp/b}\, dp_\perp , \tag{3}$$

with $b = \langle p_\perp \rangle / 2 \approx 150$–$200$ MeV/c depending on the mass of the secondary. In the high-energy case when

$$p_\perp = p\vartheta \qquad (p >> p_\perp) , \tag{4}$$

the distribution (3) gives for the secondaries of momentum $p = p_\perp/\vartheta$ the following differential cross-section:

$$\frac{d\sigma}{d\omega} \propto \frac{N(p_\perp)dp_\perp}{2\pi\vartheta d\vartheta} \propto e^{-p_\perp/b} . \qquad (5)$$

These relations have been checked, at least for the most abundant secondaries produced in inelastic collisions, over a range of transverse momentum going from ~ 10 MeV/c to ~ 1.5 GeV/c.

It is remarkable that relation (5) was then also found to apply, with a value of b similar to that found for the inelastic events, for describing the over-all behaviour at large angles of the elastic p-p differential cross-section [Orear's formula[4])]. In this case the values of $p_\perp$ for which Eq. (5) holds, go from ~ 1 GeV/c to ~ 4 GeV/c, within which range the cross-section decreases by a factor of 10^8.

Rules (1), (2), and (5) are empirical, and their justification on first principles has thus far proved difficult. In the case of secondary production in inelastic collisions, the exponential behaviour of Eq. (5) is hard to understand because of the following fact.

The study of the differential cross-section of inelastic interactions in which two bodies are produced has shown that, apart from peculiarities at the smallest angles, the cross-sections all have the following behaviour:

$$\frac{d\sigma}{d\omega} \propto e^{At} \quad (|t| < 0.5 \text{ GeV/c}^2) , \qquad (6)$$

where t is the square of the four-momentum transfer. At high energies, the production angles are small and

$$t = -(p\vartheta)^2 = -p_\perp{}^2 .$$

Expression (6) then becomes:

$$\frac{d\sigma}{d\omega} \propto e^{-Ap_\perp{}^2} , \qquad (7)$$

a Gaussian (not an exponential) in transverse momentum, with

$$\langle p_\perp \rangle = \left(\frac{\pi}{4A}\right)^{1/2} . \tag{8}$$

Expression (7) has been found[5] to hold for two-body inelastic reactions involving all sorts of particles, from the long-lived mesons and baryons to the most complex short-lived resonant states that fill the Rosenfeld tables (and our pockets!). For the most common reactions, the value of the parameter A is typically around 10 $(\mathrm{GeV}/c)^{-2}$.

Since these two-body channels are already responsible for a good fraction of the total inelastic cross-section, the question arises: how is it that, whilst at least a good percentage of the secondaries have their transverse momenta distributed according to a Gaussian [Eq. (7)] in $p_\perp$, the global distribution of $p_\perp$ when all secondaries are considering together is exponential [Eq. (5)]?

Before proceeding further, let us end this introduction with two pertinent remarks.

Firstly, the Gaussian distribution (7), with values for the parameter A around 10 $(\mathrm{GeV}/c)^{-2}$, also describes very well the small-angle ($p\vartheta < 0.5$ GeV/c) distribution of all the two-body reactions par excellence, i.e. the elastic hadron scatterings. In that case the Gaussian distribution is interpreted as the result of the diffraction scattering by an absorbing region of characteristic radius

$$R = 2\,A^{1/2} \approx 1.2 \times 10^{-13}\ \mathrm{cm}\ , \tag{9}$$

since the small-angle expansion of the Bessel function that describes the diffraction pattern is a Gaussian of the argument.

It is appealing to think that the same physical process, i.e. diffraction, that describes elastic scattering can also describe the inelastic two-body reactions. A particularly attractive detailed mechanism that also gives quantitative predictions about the absolute values of the cross-sections, is that based on the quark model. We shall come back to this point later.

Secondly, very different reasons have been invoked for explaining the exponential distribution (5) observed for the global ensemble of long-lived secondaries. Since a simple exponential resembles a Boltzmann factor, the most popular interpretation is the statistical one, and consequently the constant b of Eq. (3) has been identified with a temperature T (KT $\simeq$ 160 MeV), characteristic of hadronic matter[6].

This is how the problem stands at present.

The aim of this paper is to show that the exponential law (5) can be interpreted as the consequence of the superimposition of many Gaussian distributions, similar to that of Eq. (7), but with different values of the parameter A, and that this point of view fits naturally into a model where hadrons are assumed to be composed of several sub-units, that eventually could be identified with quarks.

2. THE DEPENDENCE OF A ON THE MASS

In the introduction it has been mentioned that, as a rule, the average transverse momentum of the secondaries produced in high-energy inelastic collisions is the greater, the larger the mass of the secondary considered. Recent work[7] on the differential cross-sections of two-body channels in 8 GeV/c π^+p and 10 GeV/c K^-p interactions observed in a bubble chamber has indicated that there is more meaning in that statement than previously assumed.

As an example of what has been done in the work of Ref. 7, consider the case of the reaction

$$K^- + p \rightarrow p + \underbrace{K^-\pi^+\pi^-}_{m} \quad , \qquad (10)$$

where 10 GeV K^- produce three mesons that can be thought of as arising from a single body of mass m. The value of m depends on the relative momentum of the three mesons and can range from 0.8 to ~ 3 GeV/c².

In Fig. 1, taken from the above-quoted paper, the upper histogram gives the number of cases of reaction (10) observed as a function of m, whilst in the lower part are plotted the values of the parameter A of Eq. (7) that fits the Gaussian distribution of the two-body process,

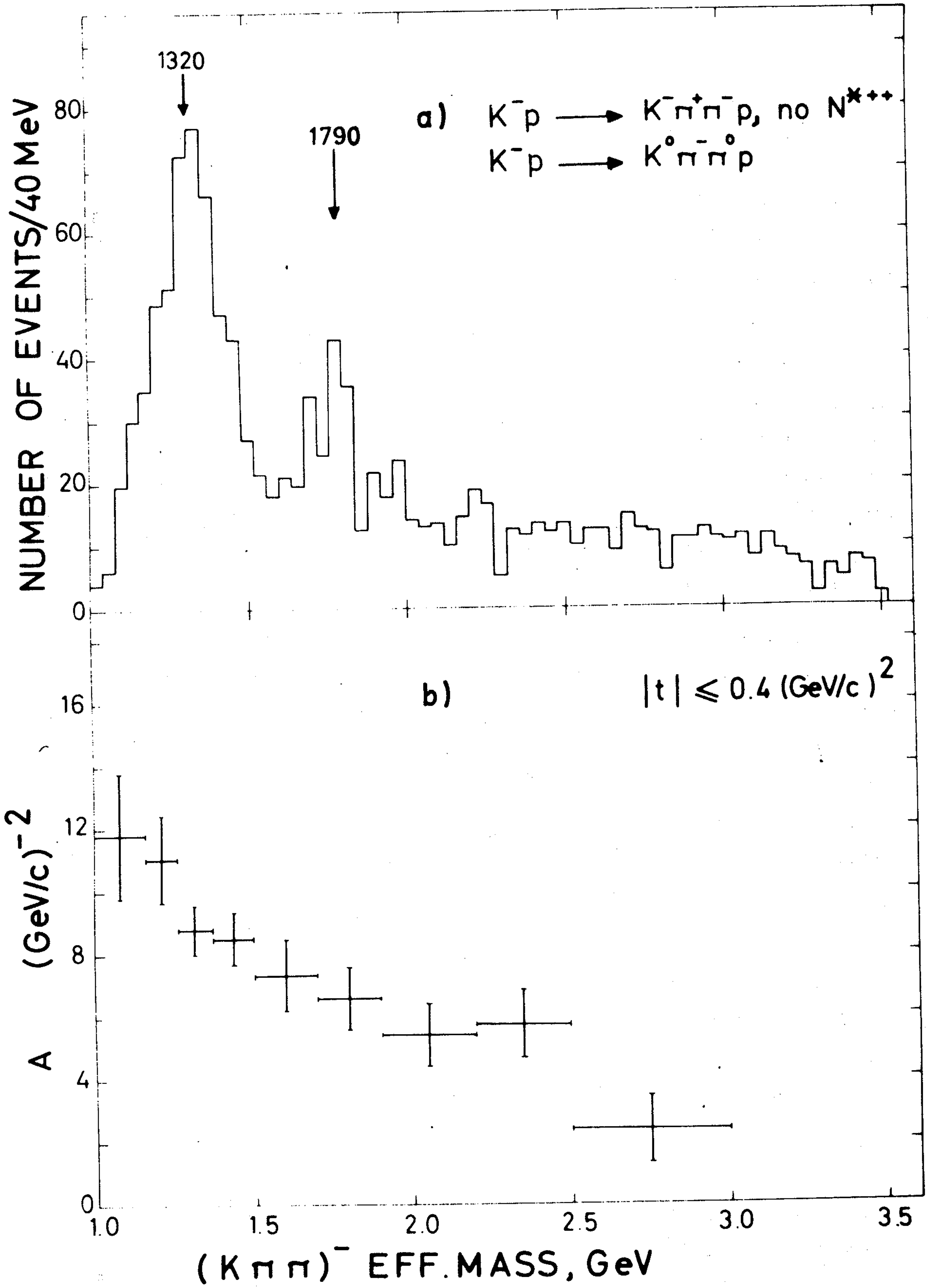

Fig. 1

10 GeV/c K^-p

[Aachen-Berlin-London(I.C.)-Vienna Collaboration] again as a function of m. What is remarkable in the data plotted in Fig. 1 is that at all values of m the Gaussian distribution gives a good fit, that the value of A does not show any great fluctuation when m corresponds to the excitation of known resonances, and finally that the value of A steadily decreases as m increases. Figure 1 is typical, and the same behaviour is observed whenever inelastic interactions are plotted in a similar manner, independent of whether the primary particle is a pion or a kaon. It can be verified that the situation is not different for incoming nucleons.

These facts justify the conclusion that in general inelastic interactions, when analysed in terms of two-body reactions, present the property that the differential cross-sections are Gaussians in $p_\perp$, with coefficients A that decrease as the mass of one of the two bodies increases. Since mass can be considered as a rough measure of the complexity of the structure of a particle, it can be said that the more complex is one of the particles produced the wider is the Gaussian describing its production cross-section.

This behaviour can be justified in a rather intuitive way if we assume that hadrons are composed of several sub-units, which one might want to call quarks, but which in the present context could still be different from the particles postulated by Gell-Mann and by Zweig. We also assume that when two hadrons collide, the interactions between these units can be evaluated, at high energies, utilizing the Glauber multiple scattering formalism[8]. Such a model has been already used successfully by Chou and Yang[9] in analysing p-p elastic scattering, and has recently been extended to interpret some inelastic processes by Dean[10].

The latter author has shown that processes involving double charge or hypercharge exchange, which are forbidden in the simple additive quark model, are allowed when double quark scattering is also taken into consideration. The reactions considered belonged to the family:

$$\pi p \rightarrow K\Sigma,\ Kp \rightarrow \pi\Sigma,\ Kp \rightarrow K\Xi,\ \bar{p}p \rightarrow \bar{\Sigma}\Sigma,\ \bar{p}p \rightarrow \bar{\Xi}\Xi\ ,$$

and the estimated cross-sections and the angular distributions fit reasonably well the existing experimental data.

Crucial to this model is the fact that the double scattering, which allows the more complex quark arrangements, also gives rise to a Gaussian distribution of the differential cross-sections as a function of $p_\perp$ which is about twice as wide as that found for the simpler processes produced by single scattering.

This is a general property of the diffraction by complex bodies that the Glauber formalism has made particularly easy to understand. The higher the order of scattering in a complex structure, the wider is the angular distribution of the product.

We believe that basically the same phenomenon is responsible for the observations reported in Ref. 7 because, in general, the larger the mass of a composite particle, the greater is its complexity and hence the order of the "quark" scattering that can produce it.

This statement can be synthetized in the following expression of the differential cross-section for a two-body process in which a particle of mass m is produced;

$$\left(\frac{d\sigma}{d\omega}\right)_m \propto \frac{1}{m} \exp\left(-A_0 \frac{m_0}{m} p_\perp^2\right) . \tag{11}$$

In this equation, $A_0 \simeq 10\ (\mathrm{GeV}/c)^{-2}$ is the value of A in Eq. (7) which characterizes the simplest two-body channels; in the composite model, these channels are interpreted as produced by a single scattering of a sub-unit, just as elastic scattering. m_0 is the value of the mass that corresponds to the simplest structures. The factor $1/m$ comes from the normalization. This is the expression that we shall use in the next paragraph to evaluate the over-all angular distribution of the secondaries; it also fits reasonably well the trend observed in the experiments of Ref. 7.

3. THE GLOBAL $p_\perp$ DISTRIBUTION

The evaluation of the over-all transverse momentum distribution of the secondaries of one kind, e.g. K mesons, emerging from the two-body channels produced in the collision of two hadrons is then reduced to the evaluation of a weighted sum over all channels, and is given by the following expression:

$$\frac{d\sigma}{d\omega} \propto \int_{m_{min}}^{m_{max}} C(m)\,\frac{1}{m}\,\exp\left(-A_0\,\frac{m_0}{m}\,p_\perp^{\,2}\right) dm\ , \qquad (12)$$

where C(m) is proportional to the probability that in the two-body channel the mass m is formed. The value of m_{min} is around the mass of the particle considered whilst m_{max} depends on the energy available in the centre of mass of the collision. To simplify the arithmetic, we shall consider the asymptotic case, where $m_{max} \to \infty$.

Looking at the behaviour of the histogram plotted in the upper part of Fig. 1, and using some obvious physical intuition, one can predict that the value of C(m) will vanish when $m \to 0$, have a maximum somewhere around $m = m_0$, and then decrease as m increases. It is also plausible that its behaviour should be roughly independent of the primary energy and of the nature of the hadrons involved, because the grouping of the sub-units produced in the interaction in all possible combinations of two, three, etc., is a probabilistic problem, to some extent independent of the number of units available, provided this number is sufficiently high.

For these reasons we believe that the following expression has a good chance of representing the main characteristics of the phenomenon

$$C(m) \propto m\, e^{-m/m_0}\ . \qquad (13)$$

With this choice and with the approximation $m_{min} = 0$, Eq. (12) gives:

$$\frac{d\sigma}{d\omega} \propto \int_0^\infty \exp\left(-\frac{m}{m_0} - \frac{m_0 A_0}{m}\,p_\perp^{\,2}\right) dm = \sqrt{4A_0\, m_0^2\, p_\perp^{\,2}}\; K_1\left(\sqrt{4A_0\, p_\perp^{\,2}}\right), \qquad (14)$$

where $K_1(x)$ is the modified Bessel function[11]. For the values of the argument in which we are interested, the following approximate expression can be used:

$$K(x) \simeq \frac{1}{x} e^{-x} ,$$

and from Eq. (14) one obtains

$$\frac{d\sigma}{d\omega} \propto \exp\left(- p_\perp \sqrt{4A_0} \right) = \exp\left(- \frac{p_\perp}{b} \right) , \qquad (15)$$

an equation equal to Eq. (5).

An estimate of the coefficient b, taking $A_0 = 10\ (\mathrm{GeV}/c)^{-2}$, gives

$$b = \frac{1}{\sqrt{4A_0}} = 160\ \mathrm{MeV}/c, \qquad (16)$$

in good agreement with the experimental results*)

Although the derivation of Eq. (15) cannot be justified in detail (mostly because the phenomena that are supposed to be responsible for it are still known only in a rather qualitative fashion), it makes clear that when many Gaussian distributions of different width contribute with different weight to the over-all distribution of the secondaries in high-energy hadron interactions, the consequence is that the over-all transverse momentum distribution of these secondaries is exponential.

The way of deriving the exponential distribution, as described above, shows that the constant $b \approx 160$ MeV/c is not related to any characteristic

*) This result is valid only for particles directly produced in one of the two-body channels. Actually, in the over-all $p_\perp$ distribution one should include the contributions coming from the products of the disintegrations of heavier particles. A consequence of this, as well as of the fact that the lower limit of the integral in Eq. (14) is not zero, but rather the mass of the particle considered, is the fact that the value of $\langle p_\perp \rangle$ depends on the mass, as discussed in the Introduction [Eq. (2)].

temperature of hadronic matter, as was concluded when Eq. (5) was justified with statistical arguments; rather it is correlated directly to A_0, i.e. as shown by Eq. (9), to the characteristic dimensions (~ 1 fermi) of the region within which the sub-units are confined.

We believe that this conclusion is of some importance, because it permits one to think in terms of a unified, though still grossly undetailed picture of all strong interaction phenomena.

Coming back to Eq. (14), the integration up to $m_{max} = \infty$ is valid only when the centre-of-mass energy of the collision is very high. Otherwise, m_{max} is finite and is roughly proportional to the c.m. energy. Although this fact modifies only slightly the final result for the values of $p_\perp$ with which we are concerned($p_\perp < 1.5$ GeV/c), it indicates that the higher the c.m. energy, the greater is the percentage of secondaries with large transverse momentum. Thus, according to Eq. (14), it should be expected that the average transverse momentum of the secondaries increases slowly as the energy of the primary hadron increases.

4. THE ELASTIC HADRON SCATTERING

From the previous discussion it is clear that when hadrons are visualized as a collection of sub-units, there is a parallelism between our interpretation of the formation of complex hadronic structures in inelastic collisions and the large-angle scattering in elastic collisions between two hadrons. Both phenomena are the consequence of the multiple scattering of these sub-units.

In both cases, when only one scattering is predominant (small mass ≡ small scattering angles), the angular distribution is Gaussian, with $A = A_0 \approx 10$ $(GeV/c)^{-2}$. In both cases, when the effect of multiple scatterings begins to be felt (large masses ≡ large scattering angles), the over-all angular distributions depart from Gaussians and tend toward exponentials; actually, about the same exponentials, because Orear's law is practically equal to that expressed by Eq. (5).

One is led to think that (if one disregards the fact that in elastic events the various orders of scattering, in the Glauber formalism, can interfere and give rise to dips and bumps) the general behaviour of the elastic scattering differential cross-section at large angles can be obtained by integrating an equation similar to Eq. (14).

This can be easily justified, because in the case of elastic scattering there exists an equation very similar to Eq. (11). If n is now the order of the scattering, one can write:

$$\left(\frac{d\sigma}{d\omega}\right)_n \propto \frac{1}{n} \exp\left(-\frac{A_0}{n} p_\perp^2\right) . \qquad (17)$$

As far as C(n) is concerned, one can find arguments for choosing the following expression, similar to Eq. (13):

$$C(n) \propto n\, e^{-n/n_0} \qquad (n > 1) . \qquad (18)$$

The combination of Eqs. (17) and (18) gives:

$$\frac{d\sigma}{d\omega} \propto \int_0^\infty \exp\left(-\frac{n}{n_0} - \frac{A_0}{n} p_\perp^2\right) dn \propto \exp\left(-p_\perp \sqrt{\frac{4A_0}{n_0}}\right) . \qquad (19)$$

If one assumes that $n_0 \approx 1$, this is Orear's law with the right numerical value in the exponent. In fact:

$$b = \frac{1}{\sqrt{4A_0}} = 160 \text{ MeV/c} . \qquad (20)$$

If this treatment of the elastic scattering of hadrons turns out to be fundamentally correct, then it is clear that all features of elastic scatterings are determined by the diffraction of the sub-units contained in the hadrons. It is then natural to identify the characteristic dip of the differential cross-sections that follows the forward peak with the first diffraction minimum, and use the same explanation for the symmetric peak often observed in the backward direction, and for the corresponding dip, the backward scattering being a particular example of an inelastic channel.

This way of interpreting the general features of elastic hadron scatterings is in line with the ideas proposed by Wu and Yang[12] in order to explain the results on p-p large-angle scattering. However, it now appears that the large angle scatterings are not due to single events where great amounts of four momentum are transferred but rather to a succession of small momentum transfers, as in multiple scattering phenomena.

5. THE ELECTRON PROTON SCATTERING

According to what has been found in the previous paragraphs, it seems plausible that the large angle scattering of hadrons colliding against hadrons is produced by the multiple scattering of the hadron sub-units. It is then reasonable to think that, if Nature has found a way of obtaining large momentum transfers when hadrons are concerned, it could follow the same path also for other kinds of particles.

This argument, together with the recent evidence that electro-magnetic interactions at high energy seem to be mediated via the virtual production of vector mesons (e.g. the ρ meson), leads us to seek whether there is any evidence for multiple scattering of sub-units in the case of electron-proton elastic scattering. Actually, there is.

Following the point of view of Wu and Yang[12], we shall equate the four-momentum transfer, q, in e-p scattering, with the transverse momentum, $p_\perp$, and compare our differential cross-sections with the square of the proton form factor deduced from e-p elastic scattering experiments.

First we note that, for small values of $p_\perp$

$$G_M^2 \propto \exp\,(-A_1 p_\perp^2) \qquad (p_\perp < 0.7 \text{ GeV/c}) \tag{21}$$

a Gaussian in $p_\perp$, as in the case of p-p scattering (Eq. 7). Here, however,

$$A_1 = \tfrac{1}{2}\,A_0 = 5\ (\text{GeV/c})^{-2}\ . \tag{22}$$

This fact has already been interpreted by Wu and Yang[12] and by Van Hove[13] as due to the presence of sub-units in the proton within a radius of 0.8 fermi [Eq. (9)].

At larger values of $p_\perp$, the square of the form factor becomes proportional to an exponential[14], and precisely

$$G_M^2 \propto e^{-p_\perp/b_1} \qquad (0.7 < p_\perp < 2.5 \text{ GeV/c}) \qquad (23)$$

where

$$b_1 = \frac{1}{4.6} = 0.22 \text{ GeV/c} . \qquad (24)$$

Now, this numerical value is very close to what is expected from the straight application of Eq. (20), found valid for the p-p scattering, in fact

$$\sqrt{4A_1} = 4.5 \text{ GeV/c}^{-1} .$$

Relation (23) is thus the equivalent, for e-p elastic scattering, of the Orear's law.

This could be a numerical coincidence, but also could mean that the electromagnetic form factor of the proton should be interpreted in terms of the composite nature of its hadrons not only for the smallest values of $p_\perp$, but also for values of $p_\perp$ as large as 2.5 GeV/c, a very interesting possibility.

It must be pointed out that, at still larger $p_\perp$ the constant b_1 in Eq. (23) changes and seems to assume the value[14]

$$b_1 \approx 1/2 = 0.5 \text{ GeV/c} \qquad (p_\perp > 3.0 \text{ GeV/c}).$$

We have no simple explanation for this.

6. CONCLUSION

The similarity between processes that lead to elastic and to inelastic hadron interactions has several consequences, besides those already mentioned, that can in some cases be experimentally verified.

One is that the dependence on angles of two-body inelastic channels leading to well-defined particles should be similar to that of the elastic channel. It can thus be expected that, at intermediate angles, structures will be found, and that at the largest angles the cross-sections will decrease in proportion to those of the elastic channels. The last point has been verified already, at 8 GeV, for the production of nucleon isobars in p-p interactions[15].

As an over-all conclusion, we must admit that our arguments lack rigour, and that the agreement with the known facts is sometimes <u>a priori</u> forced into them. However, it is inspiring to see that, simply by assuming that hadrons are composed of sub-units confined within a sphere of about 1 fermi radius, it is possible to justify the main characteristics of the angular distribution of the particles produced in strong interactions. When one considers that with the same model, and by identifying the sub-units with quarks, besides satisfying the symmetry of many of the known particles, one can also predict correctly their mass differences, their total cross-sections[16] and the mass spectrum[17], one is led to think that the long list of successes cannot be due to chance.

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