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SPECTRAL ANALYSIS OF TRANSVERSE COHERENT OSCILLATIONS OF INTENSE COASTING BEAMS

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In order to study space charge effects on the transverse coherent motion of a coasting beam we perform computer simulations of transverse single-particle and coherent oscillations with high space charge derived from Fourier spectra of the time evolution of the electric potentials. Results with solenoidal and periodic focusing with initial K-V and waterbag distributions are compared with those obtained from the envelope equation.

1. Introduction

Heavy ion inertial confinement fusion requires beams of highest intensities. At high space charge densities single particle (s.p.) motion and coherent oscillations of the beam might lead to instabilities and in circular machines to dangerous crossings of integer or half-integer resonances. For circular machines such space charge phenomena have been studied previously [1] for simplified lattices. A comprehensive treatment in a realistic ring by Machida [2] focuses on the emittance growth. There elliptical symmetry has been assumed for the oscillations of the charge density.

It is the purpose of this work to study the limits of space charge and the influence of space charge on the coherent and incoherent frequencies of quadrupole and higher order oscillations of a coasting beam without limitation to elliptical symmetry. For that purpose we perform computer simulations with a Particle-In-Cell (PIC) code with solenoidal focusing and in simplified lattices. Coasting beams with initially spherical or elliptical cross sections are simulated by following the trajectories of N (up to 65536) macro-particles moving in a two dimensional $(x-y)$ transverse constant focusing field of betatron tunes ν_{x0}, ν_{y0} and in the self-consistent electric field obtained by solving the Poisson equation on a grid (up to 101x101) with a rectangular boundary condition.

The time varying self-consistent space charge potential is calculated for a number of (up to 1000) revolutions and is recorded on virtual horizontal and vertical pick-ups, as well as on an (unphysical) central one, see fig.1. After many betatron oscillations it is Fourier transformed in order to get the Schottky spectra $\Phi(\omega)$. From them the frequency shifts and the s.p. and coherent tunes can be read off.

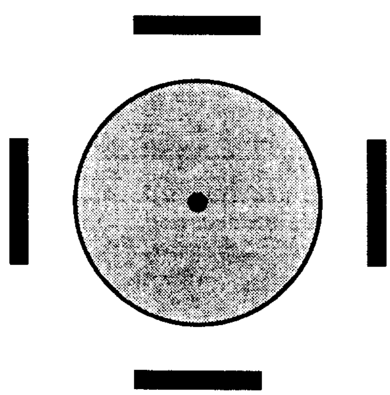

Figure 1: Arrangement of the virtual vertical, horizontal and central pickups.

With solenoidal focusing an initial K-V distribution is employed and the results are compared with those obtained from solutions of the envelope equation. With periodic lattices of type FODO, on the other hand, the space charge density is increased by cooling the macro-particles. Single particle tunes are obtained by following the phase space angle of particle j, $\Phi_i^j = \mathrm{arctg}(v_i^j/\omega_r x_i^j)$, where ω_r is the revolution frequency, and measuring the time when Φ_i^j jumps by 2π.

As results, in particular the coherent even and odd frequencies split up if the fucusing constants are different in horizontal and vertical directions and for beams with non-spherical cross sections and the two branches can cross whereas the incoherent particle tunes do not play an influential role. Also, even with very strong cooling one cannot cross systematic stop-bands of periodic lattices.

2. Solenoidal focusing

Initially the envelopes of the particles with emittances ϵ_i and envelopes a_i are in a matched state obtained from a solution of the envelope equation with a K-V distribution,

$$\nu_{i0}^2\, a_i - \frac{\epsilon_i^2}{a_i^3} - \frac{\nu_p^2 a_x a_y}{a_x + a_y} = 0, \qquad i = x, y\ . \tag{1}$$

In a K-V distribution all particles have the same transverse energy and are distributed randomly within the space (a_i) and velocity ($v_i = \epsilon_i/a_i$) boundaries. By virtue of the random initial positions and momenta all types of coherent and incoherent modes are excited automatically.

The influence of space charge is controlled by the parameter η or the tune ν_0 or the plasma frequency ω_p derived from the current I according to:

$$\eta = Q/2r_0^2\nu_0^2 = \omega_p^2/2\nu_0^2 = 1 - \nu^2/\nu_0^2, \tag{2}$$

where $Q = 4qI/Mv^3\gamma^3$ is an effective current, r_0 is the beam radius, I the current, M the mass, v the velocity, γ the relativistic factor. $\eta = 0$, thus, means no space charge and $\eta = 1$ corresponds to the limiting space charge where the incoherent frequency vanishes.

A K-V distribution of charged particles creates an inverted harmonic potential which partly compensates the harmonic solenoidal focusing potential, thus effectively reducing the s.p frequency to $\nu = (\nu_0^2 - \omega_p^2/2)^{1/2}$ for a round beam.

2.1 Incoherent (single-particle) tunes

Elliptical focusing is introduced by the frequency asymmetry parameter τ, viz $\nu_{x0} = \nu_0/\tau$, $\nu_{y0} = \nu_0\tau$. Furthermore, non-spherical beams are characterized by the spatial asymmetry parameter σ, viz $a_x = r_0\sigma$, $a_y = r_0/\sigma$. Then the incoherent s.p. frequencies of a K-V distribution become

$$\nu_x^2 = \omega_0^2/\tau^2 - \omega_p^2/(1+\sigma^2), \qquad \nu_y^2 = \omega_0^2\tau^2 - \omega_p^2\sigma^2/(1+\sigma^2)\ . \tag{3}$$

A waterbag distribution, on the other hand, creates a non-harmonic self-consistent potential so that the s.p tunes are spread over a certain interval. However, if the initial waterbag envelopes are rms matched according to the K-V initial envelopes, the average tunes are expected to follow the same systematics of eq.(3). Our results show that this is the case even for large space charges. Only for very high space charges larger than about 90% of the space charge limit there are deviations from eq.(3). These deviations are smaller if using a waterbag distribution thus indicating that the particles arrange more in a waterbag rather than in a K-V distribution.

2.2 Coherent tunes

The coherent oscillation frequencies of a round K-V beam have been calculated by Gluckstern [3] by decomposing the electric field in the Vlasov equation according to the radial (j) and azimuthal (m) node numbers. This yields the characteristic equation

$$\frac{j!\,m!}{(m+j-1)!}\sum_{\mu=0}^{j}\frac{(m+j+\mu-1)!}{\mu!(j-\mu)!(m+\mu)!}\int_0^\infty \mathrm{d}s\, i\lambda e^{i\lambda s}\cos^{m+2\mu}(s)+\delta_{j0}+\frac{2(-)^j\,m!\,j!\,\nu^2}{(m+j-1)!\,\omega_p^2}\equiv 0\,, \quad (4)$$

where $\lambda=\omega/\nu$. The integral in eq.(4) is solved analytically with the help of Mathematica in form of a polynomial. Note that $j=0, m=0$ does not exist and that the dipole mode $j=0, m=1$ is unshifted, $\omega_{\ell=1}=\nu_0$. The lowest nontrivial excitations branch down from $\omega=2\nu_0$,

$$j=1, m=0: \quad \omega_{\ell=0}^2 = 2\nu_0^2+2\nu^2 \text{ monopole (breathing)} \quad (5)$$

$$j=0, m=2: \quad \omega_{\ell=2}^2 = \nu_0^2+3\nu^2 \text{ quadrupole.} \quad (6)$$

However, also the modes $(j,m)=(4,2)$, $(1,2)$, $(2,0)$ have branches at $\omega=2\nu_0$. Its lowest one, $(2,0)$, results from

$$\omega_{\ell=3}^4 - 2\omega_{\ell=3}^2(\nu_0^2+9\nu^2) = 4\nu_0^2(\nu_0^2-17\nu^2). \quad (7)$$

The coherent monopole ($\ell=0$ or *even*) and octupole ($\ell=3$) oscillations are only observed with the center signal because the field of a round beam with conserved charge is not influenced outside the beam. These modes are shown in fig.2. The monopole and quadrupole frequencies follow well the analytical result. The frequencies of the $\ell=3$ mode, however, being very sensitive to the actual distribution [4] are less degressed than for a pure K-V distribution.

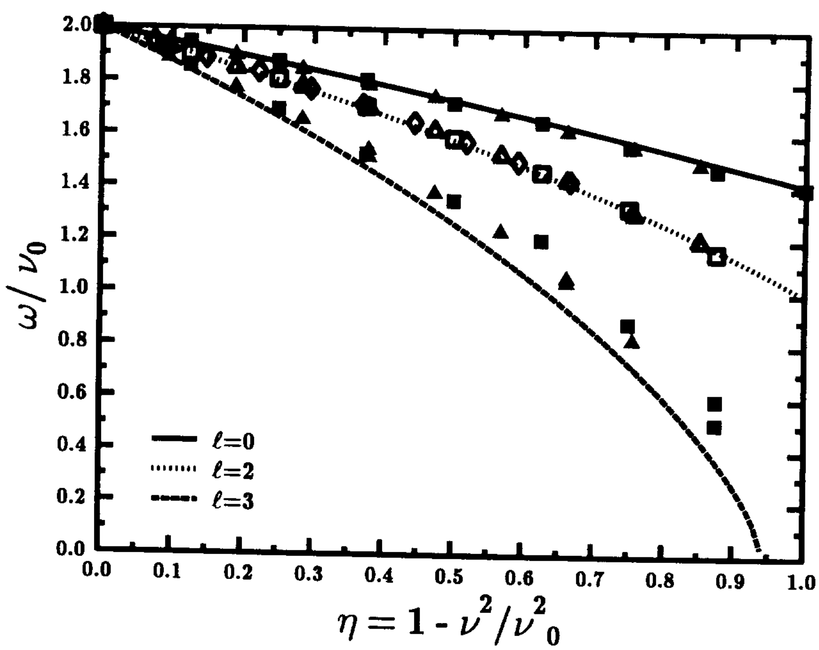

Figure 2: Coherent frequencies branching from $\omega=2\nu_0$. The lines are theoretical results from eq.(4).

3. Periodic lattices

For non-solenoidal focusing as is the case in periodic lattices, the waterbag distribution is more realistic. The initial emittances, envelopes and velocity boundaries in the horizontal and vertical directions are calculated from the envelope equation and we carry out an rms matching. The initial conditions of the particles are then obtained by randomly choosing the positions and velocities within these rms boundaries. This always yields a well matched beam to start with.

We study two lattice cells with quadrupole magnets and free sections shown in fig.3. Cell (a) has an envelope stop-band and cell (b) which is similar to cell (a) but has a s.p. stop-band instead. Cell (b) is similar to cell (a) but has shorter drift sections between the FD-magnets and longer drift sections between the two cells (a). A realistic lattice consists of four to eight such cells and tunes multiply accordingly. The beam properties are: charge $q = 1$, mass $A = 208$, energy $E =$ 48 MeV/A, velocity $\beta = 0.3092$, and emittances $\epsilon_x = \epsilon_y = 6.25$ (π) mm mrad. Both cells have a stiffness of $B\rho = 2.1 \times 10^8$ G cm. Cell (a) has length 2.8 m, focusing gradient $B' = 3000$ G/cm and defocusing gradient $B' = -3135$ G/cm and cell (b) has length 2.7 m, $B' = 3109$ G/cm and $B' = -3036$ G/cm. The current I, beta functions $\beta_{x,y}$, s.p. phase advances $\sigma_{x,y}$, and coherent even and odd phase advances $\phi_{0,2}$ in these cells are listed in the following table.

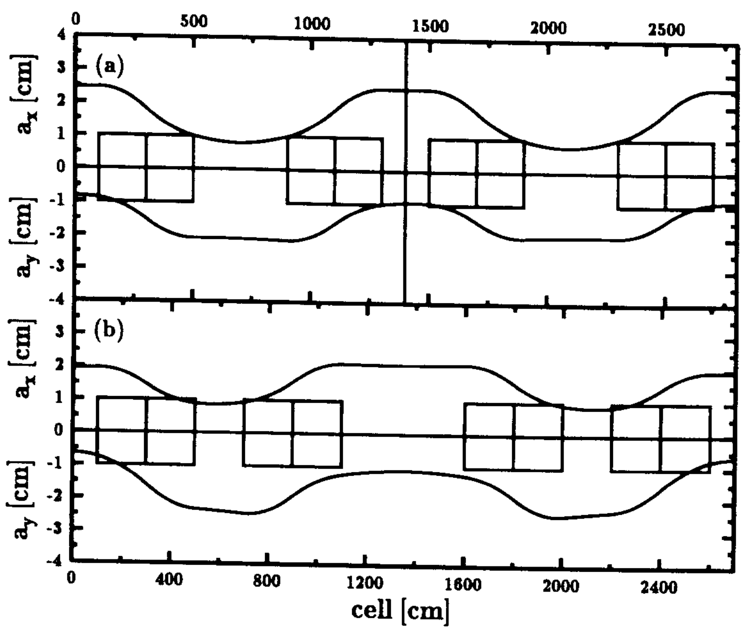

Figure 3: Lattice cells with horizontal (upper part) and vertical (lower) envelopes. (a): double cell with an envelope stop-band and (b): single cell similar to (a) but with a single particle stop-band.

Table: Currents I, matched betatron functions $\beta_{x,y}$, s.p. tunes $\sigma_{x,y}$ and coherent even and odd tunes $\phi_{0,2}$ of lattices (a) and (b) as obtained from the envelope equation.

Cell (a): I [kA]	$\beta_x[\frac{\text{cm}}{\text{rad}}]$	$\beta_y[\frac{\text{cm}}{\text{rad}}]$	σ_x	σ_y	ϕ_0	ϕ_2
0	2393.5	281.9	118.0°	118.0°	236.0°	236.0°
111.4	2423.8	357.7	102.4°	102.6°	231.6°	218.9°
243.1	2614.9	453.6	86.3°	86.9°	196.7°	184.3°

Cell (b): I [kA]	$\beta_x[\frac{\text{cm}}{\text{rad}}]$	$\beta_y[\frac{\text{cm}}{\text{rad}}]$	σ_x	σ_y	ϕ_0	ϕ_2
0	1506.1	173.6	228.0°	228.0°	456.0°	456.0°
115.9	1453.4	140.9	198.8°	199.2°	415.9°	426.6°
228.8	1536.1	82.4	177.5°	186.4°	381.4°	422.0°

An increase in space charge density is simulated through cooling by reducing the kinetic energy of the relative motion of the particles by 2% per cell. The total time spectrum is recorded at the virtual pick-ups for 125 transits through the cells and is then Fourier transformed. As can be seen from fig.4, after passage through about 50 cells in both cases the horizontal and vertical s.p.

tunes become equal after about 40 cells, thus indicating that the quarter-integer stop-band (90° per cell, tune 0.25 in fig.4) and half-integer resonance (180° per super-cell, tune 0.5 in fig.4), respectively, have been reached and that they cannot be crossed. The emittances, envelopes and average velocities, too, level off, however not at the same values in horizontal and vertical directions.

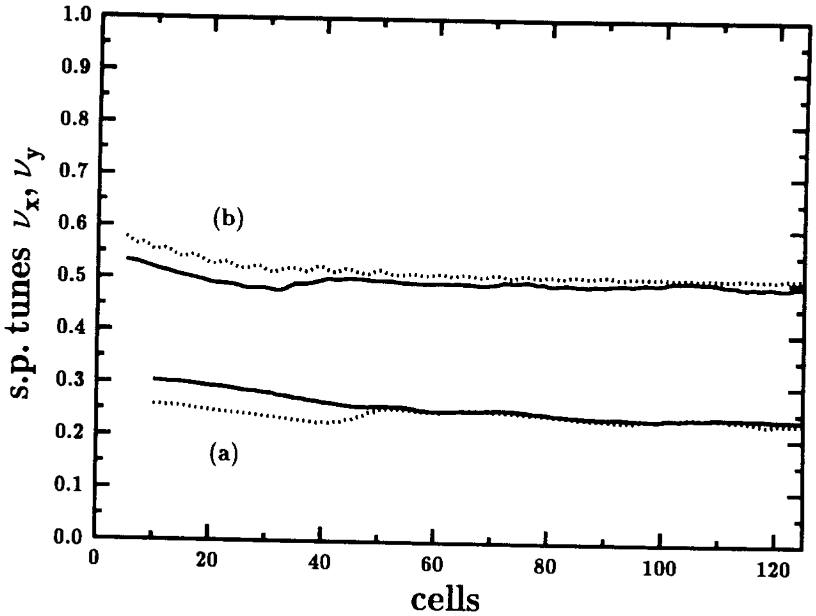

Figure 4: Time evolution of the horizontal (full lines) and vertical (dotted lines) single particle tunes as fractions of the zero-intensity phase advances per single cell in cells (a) and (b) of fig.3.

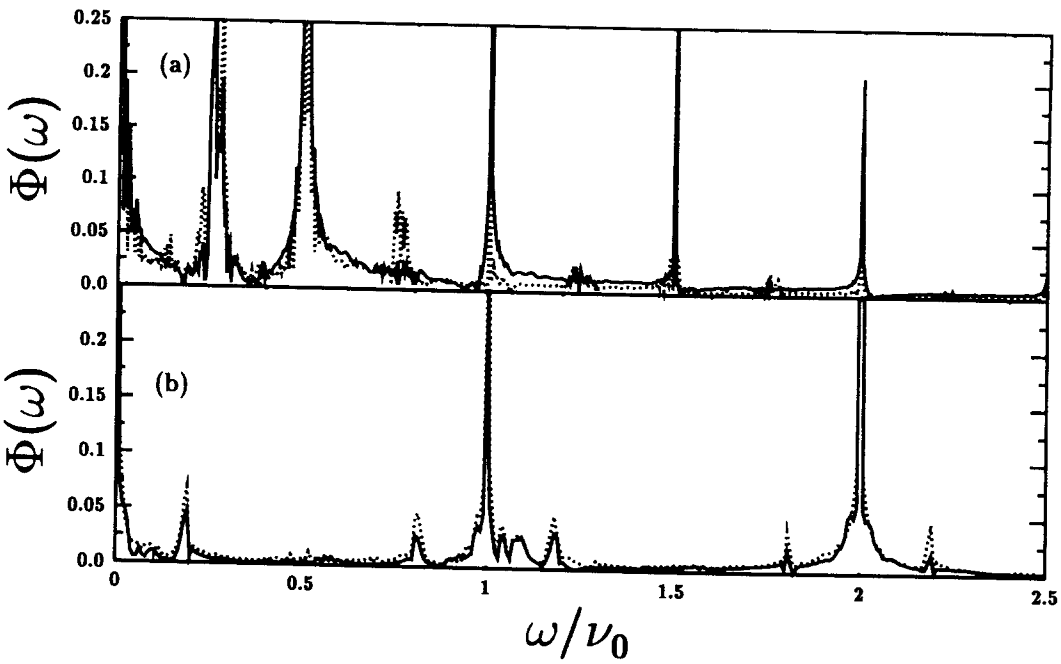

Figure 5: Horizontal (full lines) and vertical (dotted lines) frequency spectra after 125 transits through cells (a) and (b) of fig.3.

In fig.5 are shown the frequency spectra in these two cells. In case (a) there is a pronounced coherent response at one half of the revolution frequency (and a multiple thereof) corresponding to twice the s.p. stop-band frequency. Here a typical envelope resonance [5] has been reached. For case (b), on the other hand, the envelope equation predicts vertical and horizontal coherent phase advances of about $380^o(\omega/\nu_0 = 1.055)$ and $420^o(\omega/\nu_0 = 1.17)$ according to the table which also can be found in fig.5(b).

4. Summary

We found that the s.p tunes obtained in the simulations with solenoidal focusing agree well with those obtained from the envelope equation (with a K-V distribution) sufficiently well far away from the space charge limit. Close to the space charge limit, however, a non-standard redistribution of the particles occurs which calls for further investigation. The lowest coherent transverse oscillation frequencies also agree with those obtained from the characteristic equation derived with a K-V distribution. With periodic focusing we could not cross stop-bands by (even strong) cooling.

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